

Instructions:

- Show the steps of your work.
- Simplify your answers.
- Use clear and correct mathematical notation.
- Graphing tool is not allowed to use while taking this test.
- Turn off your phone.

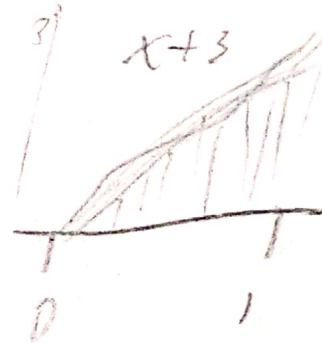
(1) Consider $f(x) = x + 3$ on the interval $[0, 1]$ (3 pts).

- (a) Find the right Riemann sum with n rectangles. Simplify the expression as much as possible.
 (b) Approximate the area by taking $n = 50$ and use the expression from (a).
 (c) Take a limit of the sum as $n \rightarrow \infty$ to calculate the exact area under the curve.

$$\Delta x = \frac{1}{n} \quad h = \left(\frac{1}{n} + 3 \right)$$

$$b \cdot h = A \quad \left(\frac{1}{n} + 3 \right)$$

$$X = \frac{1}{n} \left(\frac{1}{n} + 3 \right) = \text{Area}$$



$$\sum_{i=1}^n \left(\frac{1}{n^2} + 3 \right)$$

$$\frac{1}{n^2} \sum_{i=1}^n i + 3 \sum_{i=1}^n 1$$

$$\frac{1}{n^2} \cdot \frac{n(n+1)}{2} + \frac{3}{n} \cdot n$$

$$\frac{n^2}{2n^2} + \frac{n}{2n^2} + 3$$

$$\frac{1}{2} + \frac{1}{2n} + 3$$

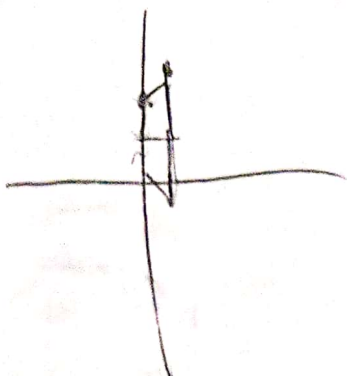
$$\frac{7}{2} + \frac{1}{100}$$

$$(b) \frac{351}{100}$$

$$(a) \frac{7}{2} + \frac{1}{2n}$$

$$(c) \lim_{n \rightarrow \infty} \left(\frac{7}{2} + \frac{1}{2n} \right) = \frac{7}{2} + 0 = \frac{7}{2}$$

- (2) Graph the function $f(x) = x + 3$. Then use **geometry** to find the area of the region between $f(x)$ and the x -axis, for $0 \leq x \leq 1$. Compare with the result from (1)c (1 pts).



$$b = \frac{1-0}{1} \quad h = 4$$

0, 1

$$6 = A$$

$$h \times \frac{a+b}{2}$$

$$A = 1 \times \frac{3+4}{2} = \frac{7}{2}$$

- (3) Simplify the following expressions by using the Fundamental Theorem of Calculus part 1 (4 pts).

(a) $\frac{d}{dx} \int_1^x \ln t \, dt$

$$\ln(t) \Big|_1^x = \ln(x) - \ln(1)$$

$$(\ln x)x - x + 1$$

(b) $\frac{d}{dx} \int_1^{\sin x} \ln t \, dt$

$$\ln(t) \Big|_1^{\sin x} = \ln(\sin x) - \ln(1)$$

$$\ln(\sin(x)) (\sin(x)) - \sin(x)$$

$$\ln(\sin x) \sin x - \sin x \cdot \left[\frac{\ln(t)}{t} \right]_{t=1}$$

- (4) Evaluate $\int_0^{\pi} \frac{\pi}{4} \sin x \, dx$ with an exact value (2 pts).

$$\int_0^{\pi} \frac{\pi}{4} \cos x \, dx$$

$$\frac{\pi}{4} \neq \frac{\pi}{4}$$

$$\frac{2\pi}{4}$$

$$-\frac{\pi}{4} \cos(\pi) + \frac{\pi}{4} \cos(0)$$

$$\frac{\pi}{2}$$