THE UNIVERSITY of EDINBURGH

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# Optimisation Model for Strategic Planning - scheduling and advertising in the TV sector

## Scheduling Optimization Model for Channel A

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# Executive Summary

This report provides an in-depth analysis of a strategic scheduling approach aimed at maximising viewership when advertisements are to be sold and thus maximising Channel's A revenue. It examines how the intelligent placement of advertisements and the careful scheduling of movies can significantly boost audience engagement. By considering viewer demographics and their peak viewing times,the programming schedule is optimise to attract a larger audiences and maximise market share. Competitors schedules and demographic data is incorporated to ensure that advertising is specific and targeted. The report proposes an optimised advertising strategy, that can effectively complement our programming to generate higher viewership. Overall, this comprehensive approach is designed to ensures that Channel A remains competitive and in tune with the preferences of audiences.

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## List of Abbreviations

**SPOT** Scheduling Programs Optimally for Television

**MIP** Mixed integer programming

**MVP** Minimum Viable Product

**GPP** Gross Profit Percentage

**HPC** High Performance Computing

# 1 Introduction

Competing against three other channels in a market of 1 million viewers, Channel A aims to enhance its viewership and maximise revenue through intelligent scheduling and strategic advertising. To achieve this, the report proposes a Mixed-Integer Programming (MIP) model that optimises Channel A's scheduling and advertising strategy. By aligning ad slot usage with viewership data, the model enables Channel A to increase its audience while maximising the value of its ad slots, directly linked to revenue generation. Furthermore, a sensitivity analysis is conducted to explore the relationship between the assigned budget for licensing fees and advertisement of the scheduled movies, and the return on viewership achieved. The model and implementation in this report is an MVP, and several recommendations are provided to extend the given approach throughout the report.

Recent studies reveal a significant advantage for broadcasting channels with data-driven scheduling strategies, where movie scheduling decisions rely on detailed analyses of audience preferences, competitor behavior, and content performance metrics. There has been a significant shift in channels leveraging demographic insights, viewership baselines, and genre-based preferences to predict viewer behavior when scheduling [FilmGrail, 2024]. To this end demographic preference and viewing habits are incorporated when scheduling movies, and competitor's schedules are considered when choosing ad slots, to maximise viewer conversions.

# 2 Literature Review

The optimisation of television scheduling and advertisement is extensively explored in operational research. This section reviews key contributions to the field with a focus on mathematical modelling techniques, demographic targeting, competitor analysis, and advertising strategies.

Mathematical modelling techniques such as MIP and heuristic algorithms are widely applied to solve complex scheduling problems. These models prove suitable as they balance multiple factors, including audience demographics, licensing costs, and advertising revenue potential, to provide an optimal schedule.

Reddy for example introduced the SPOT model, which uses a mixed-integer network flow methodology to optimise prime-time schedules by considering ratings, revenues, and licensing costs and explicitly considered advertising revenues alongside program costs to maximise network profitability [Reddy et al., 1998]. This model demonstrated superior flexibility by incorporating managerial inputs and constraints like lead-in effects and time slot restrictions. It also emphasises the effect of competitor's schedules on an advertisement.

Furthermore, Danaher explored how conversion rates vary based on genre compatibility between advertised content and competitor programming [Danaher and Mawhinney, 2001]. For example, advertising a horror movie during a similar genre on a competitor channel yields higher conversion rates than advertising it during a romance-comedy slot.

Overall, these findings highlight the significance of genre-based targeting in optimising ad placements and reveal gaps in existing models. To address these gaps, the model integrates competitor's schedules to optimise ad placements and maximise audience conversion rates.

# 3 Problem Statement and Exploration

This report addresses the challenges of maximising Channel A's viewership through a mathematically optimised, data-driven scheduling approach. The problem requires selecting movies that appeal to targeted demographics while considering competitor's schedules, licensing and advertising costs, and ad placement strategy. The problem is cast as a MIP problem and solved using the Xpress optimisation package in Python. The objective of this approach is to maximise viewership in time slots designated for selling ads.

Specifically, the model is implemented to maximise viewership when ads are sold, which in turn maximises revenue. The trade off between selling ad slots and advertising our own movies at no cost is carefully handled. The model will account for competitors' schedules when selecting ad slots to advertise movies to target audiences most likely to watch the advertised movie based on genre similarity. The genres of each movie are considered when advertising, so that only movies similar to those being shown on the competitor's channel are advertised in the same time slot

Three demographics are considered in our approach: children, adults, and retirees. It is important to consider demographic preference when selecting movies and their viewing habits. There is a total viewer population of $1,000,000$ which Channel A and the three competitor channels are competing to capture.

This problem involves four key data sets: a database of potential movies, Channel A's schedule, the schedules of three competitors, and each competitor's advertising conversion rates by genre. Below is a summary of the exploratory data analysis conducted and how these datasets were managed.

### 3.1 Movie database

This dataset contains $5,920$ movies with the following relevant columns; `genres`, `runtime with ads`, `scaled popularity` for each demographic, and `license fee`. When running the model, several heuristics were used to reduce the size of the database due to computing power constraints.

To reduce the dataset, the movies were filtered with IMDb ratings between 5.5 and 7.5, released after 2000, and with licensing fees exceeding $1,000,000$. This selection focused on moderate-quality, high-cost movies, representing the worst-case budget scenarios. Excluding unprofitable movies was avoided as long as they met these conditions, ensuring a comprehensive yet challenging dataset for the model.

This heuristic was developed based on the Exploratory Data Analysis done in **DataAnalysis_MovieDataBase.ipynb** file on the project's GitHub repository [Group9-MMCS-Edinburgh, 2024]. The goal was to create a reduced dataset that emphasizes high-budget constraints to test the model under tough conditions. By focusing on worst-case scenarios, the recommendations would remain valid even in less challenging situations. Segmenting based on popularity ensured optimal audience targeting.

A series of criteria were applied to the given movie dataset of 5,920, reducing the dataset to 136 movies that meet the set criteria. Of these, 39 movies were chosen based on their popularity value for implementing the model.

### 3.2 Channel A's schedule

This dataset is structured as 5 minute time slots for a 12 week period, with relevant columns `Date-Time`, and a `baseline view count` for each demographic. As the focus of this report is to schedule over a one week period, the time slots beyond seven day were dropped. To reduce the size of the problem, the 5 minute time slots were aggregated into 30 minute time slots, averaging across the values of the columns when resampling. The column `Date-Time` then showed the starting time of each half hour interval. Note this column was named after the data set was available.

### 3.3 Competitor's schedules

Three competitor's schedules were provided structured similarly, over 12 weeks in 5 minute time slots. They provided information on when a movie is shown or an ad slot is available to buy. To reduce problem size these datasets were preprocessed to drop all non-advert 5 minute time slots. The relevant columns from this dataset are `Date-Time`, a `baseline view count` for each demographic, and `ad slot price`. The column `Date-Time` was similarly named after the data set was made available.

### 3.4 Genre conversion rates for ad slots

This dataset is provided for each channel and has columns for each possible genre. It provides a conversion rate for each ad slot available on that channel based on the movie that is being shown. If the movie has the genre the conversion rate is non-zero, and is zero otherwise.

## 4 General Model

### 4.1 Assumptions

There are several assumptions that were made in designing the model, to reduce complexity of the initial MVP. In future, some of these assumptions could be further explored to improve the models accuracy and robustness. Given more time and computing resources stochastic elements and further constraints could be introduced to the model. The list of assumptions is laid out below:

- All ad slots can be sold at any time and the identity of the buyer is not a significant factor.
- There is no cost to selling ad slots, *i.e.*, losing viewership if competitors advertise on Channel A.
- It is optimal to show each movie only once.
- The viewers gained from advertising a movie will watch the movie for its entire duration.
- Advertising a movie multiple times does not result in a diminishing number of viewers converted.

- A specific proportion of ad viewers are converted, *i.e.*, converting viewers does not depend on some probabilistic element.
- The model cannot produce a scheduling and advertising strategy which can return a viewership greater than the total population, *i.e.*, The total population distributed across the four channels is not tracked.
- The only revenue source for the channel is through the sale of advertisement slots.

### 4.2 Index Sets

$M$, the set of candidate movies.

$T$, the set of 30 minute time slots of our channel.

$G$, the set of unique movie genres.

$C$, the set of competitor channels.

$A_c$, the set of ad slots available to buy on competitor channel $c$, $c \in C$.

### 4.3 Decision Variables

Binary decision variables:

$$y_m = \begin{cases} 1, & \text{if movie } m \text{ is to be shown, } m \in M. \\ 0, & \text{otherwise.} \end{cases}$$

$$x_{mt} = \begin{cases} 1, & \text{if we show movie } m \text{ in time slot } t,\ m \in M,\ t \in T. \\ 0, & \text{otherwise.} \end{cases}$$

$$w_{mt} = \begin{cases} 1, & \text{if we advertise movie } m \text{ in time slot } t \text{ on our own channel, } m \in M,\ t \in T. \\ 0, & \text{otherwise.} \end{cases}$$

$$v_{mt} = \begin{cases} 1, & \text{if we sell the ad slot in time slot } t \text{ when movie } m \text{ is shown, } m \in M,\ t \in T. \\ 0, & \text{otherwise.} \end{cases}$$

$$z_{cmk_c} = \begin{cases} 1, & \text{if we buy add slot } k_c\text{, on channel } c\text{, to advertise movie } m,\ m \in M,\ k_c \in A_c,\ c \in C. \\ 0, & \text{otherwise.} \end{cases}$$

Continuous decision variables:

$u_{mt}$, the total viewership gained from showing and advertising movie $m$ in time slot $t$, $m \in M$, $t \in T$.

$q_{mt}$, total viewership of movie $m$ for time slot $t$ if the ad slot is to be sold, $m \in M$, $t \in T$.

$s_m$, the start time of movie $m$ in minutes measured from 7am on the first day of the week, $m \in M$.

$e_m$, the end time of movie $m$ in minutes measured from 7am on the first day of the week, $m \in M$.

### 4.4 Parameters

$r_m$, runtime of movie $m$ in minutes, including ad breaks, $m \in M$.

$l$, time slot length, 30 minutes in this problem.

$W$, number of minutes in a week (10880).

$B$, total advertising and movie licensing budget ($\$$). This value is explored using sensitivity analysis.

$\alpha$, the date time of the first day of the week at 7am.

$\alpha_t$, the date time of the start of time slot $t$, $t \in T$.

$\beta_{ck_c}$, the date time when ad slot $k_c$ is scheduled on channel $c$, $k_c \in A_c$, $c \in C$.

$P$, the total population of viewers across our channel and the competitor channels; $P = 1,000,000$ in this problem.

$p_{md}$, the scaled popularity of movie $m$ for demographic $d$, $m \in M$, $d \in D$.

$b_{dt}$, the baseline view count of time slot $t$ for demographic $d$, $d \in D$, $t \in T$.

$\tilde{b}_{cdk_c}$, the baseline view count of ad slot $k_c$, for channel $c$, for demographic $d$, $d \in D$, $k_c \in A_c$, $c \in C$.

$\kappa_{mt}$, the conversion rate for advertising movie $m$ on time slot $t$, $m \in M$, $t \in T$. This parameter is a provisional calculation and requires further research to find a more precise method of calculation. It is calculated when the model is run using this formula:

$$\kappa_{mt} = \frac{\sum_{d \in D} p_{md} b_{dt}}{\sum_{d \in D} b_{dt}}$$

$\tilde{\kappa}_{cgk_c}$, the conversion rate for advertising on ad slot $k_c$ on channel $c$ if movie has genre $g$, $c \in C$, $k_c \in A_c$, $g \in G$.

$\delta_{mg}$, if movie $m$ has genre $g$, $\forall m \in M$, $\forall g \in G$. This is calculate before running the model. More specifically:

$$\delta_{mg} = \begin{cases} 1, & \text{if movie } m \text{ has genre } g,\ m \in M,\ g \in G. \\ 0, & \text{otherwise.} \end{cases}$$

$f_i$, licensing fee for movie $i$, $i \in M$.

$a_{k_c}$, the cost of buying ad slot $k_c$ on competitor channel $c$, $c \in C$, $k_c \in A_c$.

The following parameters $r_m$, $\beta_{ck_c}$ ,$\alpha_t$, $\beta_{d_t}$, $r_m$, $\tilde{b}_{cdk_c}$, $b_{dt}$, $p_{md}$, $a_{k_c}$, $f_i$, $\tilde{\kappa}_{cgk_c}$, were derived from the dataset provided.

### 4.5 Constraints

Only one movie can be scheduled for each half an hour time slot:

$$\sum_{m \in M} x_{mt} = 1,\ \forall t \in T \tag{1}$$

A movie is only scheduled if it is to be shown:

$$x_{mt} \leq y_m,\ \forall m \in M, t \in T \tag{2}$$

Movies must be scheduled for their whole run time :

$$l \sum_{t \in T} x_{mt} = y_m r_m, \forall m \in M \tag{3}$$

The start time and the end time of a movie must have difference equal to the length of movie. Constraint (4) also ensures that movies are not scheduled over the broadcasting gap between 12am and 7am and is linked with Constraints (5) and (6) to force that time slots for which the movie is scheduled are consecutive:

$$e_m - s_m = y_m r_m, \forall m \in M \tag{4}$$

These constraints ensure that the movie is scheduled for only time slots which are consecutive. Along with Constraint (4), this forces that the time slots which the movie is scheduled must fall within a gap that is the length of the movie. The use of parameter $W$ ensures that $s_m$ is not set to zero when the movie is not scheduled at time slot $t$:

$$e_m \geq x_{mt}(\alpha_t + l - \alpha),\ \forall m \in M,\ \forall t \in T \tag{5}$$

$$s_m \leq x_{mt}(\alpha_t - \alpha) + W(1 - x_{mt}),\ \forall m \in M,\ \forall t \in T \tag{6}$$

Only one ad can be bought per available ad slot on competitor channels:

$$\sum_{m \in M} z_{cmk_c} \leq 1,\ \forall k_c \in A_c,\ \forall c \in C \tag{7}$$

Constraint (8) ensures that for each time slot on our channel only one ad slot is shown. Either the ad slot is sold or our own movie is advertised. This constraint along with Constraint (16) controls whether or not advertisements are sold rather than advertising our own movie:

$$\sum_{m \in M} w_{mt} + \sum_{m \in M} v_{mt} = 1,\ \forall t \in T \tag{8}$$

Movies are only advertised on our channel or on competitor channels if they are to be shown:

$$z_{cmk_c} \le y_m,\ \forall m \in M,\ \forall k_c \in A_c,\ \forall c \in C \tag{9}$$

$$w_{mt} \le y_m,\ \forall m \in M,\ \forall t \in T \tag{10}$$

Ads are only sold when a movie is scheduled, if the movie is scheduled:

$$v_{mt} \le x_{mt},\ \forall m \in M, t \in T \tag{11}$$

Ads are bought to advertise a movie only before the first time slot the movie is scheduled to be shown:

$$z_{cmk_c}(\beta_{ck_c} + l - \alpha) \le s_m,\ \forall m \in M,\ \forall k_c \in A_c,\ \forall c \in C \tag{12}$$

$$w_{mt}(\alpha_t + l - \alpha) \le s_m,\ \forall m \in M,\ \forall t \in T \tag{13}$$

Constraint (14) is used to count the total viewership gained from showing and advertising a movie in each time slot it is scheduled. This constraint uses assumption that any viewers converted by advertising watch the movie for each time slot the movie is scheduled. The first sum in this constraint gets the base viewership gained from scheduling the movie in time slot $t$, which is calculated as the sum of the popularity times the baseline viewership for each demographic.

The second sum calculates the total viewership gained from advertising the movie on competitor channels. This is calculated as the sum of the popularity times the baseline viewership times a conversion rate for each genre of the movie that is being shown on the competitor channel at the time of the ad slot. Each term in the sum is also multiplied by a genre factor $\delta_{mg}$ which ensures viewers are only converted when movie $m$ and the movie showing on the competitor channel share genres.

The third sum counts the viewership gained from advertising movie $m$ on our own channel similar to the first sum, with an added conversion rate:

$$\begin{aligned} u_{mt} \le\ & P \sum_{d \in D} p_{md} b_{dt} x_{mt} \\ & + P \sum_{k_c \in A_c} \sum_{g \in G} \sum_{d \in D} p_{md} \tilde{b}_{cdk_c} \tilde{\kappa}_{cmgk_c} \delta_{mg} z_{cmk_c} \\ & + P \sum_{t \in T} \sum_{d \in D} p_{md} b_{dt} \kappa_{mt} w_{mt}, \\ & \forall m \in M,\ \forall t \in T,\ \forall c \in C \end{aligned} \tag{14}$$

The cost of movies licensed and ad slots bought is less than or equal to the total budget allocated:

$$\sum_{m \in M} y_m f_m + \sum_{m \in M} \sum_{c \in C} \sum_{k_c \in A_c} z_{cmk_c} \le B \tag{15}$$

Constraint (16) ensures viewership gained showing and advertising a movie in a time slot is only counted in the objective function if an ad is sold in that time slot. This ensures that if the amount of viewers gained from advertising a movie which is to be shown later is greater than the amount of viewers in the current time slot, we advertise that movie rather than sell the advertisement:

$$u_{mt} \le P v_{mt},\ m \in M,\ t \in T \tag{16}$$

## 4.6 Objective Function

In the objective function, viewership for time slots in which ad slots are sold is maximised. The viewership for each time slot is tracked, which allows us to calculate the total revenue after the model is finished solving.

$$\max \left\{ \sum_{m \in M} \sum_{t \in T} u_{mt} \right\} \tag{17}$$

# 5 Implementation

Given the complexity of the problem, solving the model directly runs into runtime issues due to time frame and computing resource limitations. Instead, to illustrate the effectiveness of this modeling strategy, the model is run over a three day block for a reduced dataset of movies. This movie set was selected using the methodology laid out in Section 3.1. The budget constraint is also not used in this initial run of the model, allowing us to get an idea of what an optimal scheduling might cost over a three day period.

**This implementation demonstrates that the model does in fact schedule movies and advertises effectively, and extension to seven days is a matter of computing resources**.

Despite these limitations, this approach allows us to conduct a sensitivity analysis on the best value to set for the budget. Furthermore, it provides a useful illustration of our method in smaller time frames and emphasises the fact that the model can easily be extended to a week long interval, given the appropriate resources.

# 6 Results

After running the model for a three day period (5/10/2024-7/10/24) without a budget constraint, the total viewership gained in time slots designated to be sold is $16,502,036$. Following [TV Advertising Agency, 2024], we propose selling ad slots on a Cost Per Thousand (CPT) basis. After rounding the viewership in each time slot designated to be sold down to the nearest thousand, we get $16,502,000$. The total cost of this scheduling and advertising strategy is $\$29,143,421$. The gap between best bound and objective value for this solution was 82.76%. These values enables the calculation of Gross Profit Percentage (GPP), based on a range of selling prices. Candidate selling prices and their corresponding GPP are shown in Table 1. A further analysis of Channel A's operating cost and the current market would be required to select an optimal selling price.

Table 1: Table CPT Selling prices and their corresponding GPP

| **CPT Selling Price** | 1800 | 1820 | 1840 | 1860 | 1880 | 1900 | 1920 | 1940 | 1960 | 1980 | 2000 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **GPP** | 1.755 | 2.835 | 3.891 | 4.924 | 5.936 | 6.926 | 7.895 | 8.845 | 9.775 | 10.686 | 11.579 |

The model proposes that 45% of available ad slots on our channel are sold while 55% are used to advertise movies that will be shown at a later point. This shows that the model does not naively sell all available ad slots, but instead carefully selects which slots should be used to advertise to maximise viewership for those which will be sold. Unfortunately, further analysis of these figures highlights some limitations of the model. For instance, on 7/10/2024, "The Amazing Spider-Man" is scheduled to be shown from 21:30 until 00:00. It is advertised on our channel a total of 15 times and is not advertised on competitor channels. The model suggests that all $1,000,000$ potential viewers in the population tune in to watch the movie in each time slot the movie is scheduled. This value reflects a poor choice of conversion rate for advertising on our own channel, as well as the limiting assumption that there is no diminishing return on viewers converted when advertising the same movie multiple times on the same channel.

Despite there being 667 available slots for purchase on competitor channels and no budget constraint used in this case, the model only selects 20 competitor slots to use for advertising. This highlights the models careful handling of ad slot purchase even before considering cost. Potential slots are only considered if the movie which is to be advertised has enough genres similar to the movie being shown when the ad slot is available. This way, only viewers interested in movies similar to those advertised are targeted.

# 7 Sensitivity Analysis

The sensitivity analysis was conducted to explore how varying budget constraints influence the model's outcomes, including scheduling decisions, advertising strategies, and viewership. The approach taken began with an initial unconstrained run of the model as described in sections 5 and 6, observing costs without budget limitations. This provided valuable insights into the resource requirements and helped define realistic budget intervals for further analysis.

For this analysis, a range of candidate budgets were selected which can be seen in Table 2. Through this approach, the analysis aimed to highlight key aspects such as the impact of reduced budgets on viewership, the adaptability of advertising strategies, and the overall cost-effectiveness of different schedules.

Table 2: Sensitivity Analysis Results

| Budget | Cost | Average Viewership | Cost/Viewership Ratio |
|---|---|---|---|
| No Limit | 29,143,421.70 | 73,669.80 | 395.52 |
| 30,100,000 | 29,774,392.85 | 104,078.17 | 286.11 |
| **30,050,000** | 29,415,486.02 | 151,729.48 | **193.86** |
| 30,000,000 | 28,990,269.37 | 105,957.46 | 273.54 |
| 29,100,000 | 28,393,074.05 | 103,340.50 | 274.74 |
| 29,050,000 | 28,352,577.53 | 106,511.57 | 266.16 |
| 29,000,000 | 28,731,757.56 | 137,430.05 | 209.03 |
| 28,500,000 | 27,561,679.41 | 140,366.91 | 196.31 |

The Table 2 above shows that a budget of 30,050,000 offers the best cost-to-viewership ratio of 193.86, showing optimal efficiency. Higher ratios for other budgets, including the "No Limit" budget which is the worst, showing a 395.52 ratio, indicating less efficient use of resources.

Figure 1 explores the relationship between the GPP and different budget scenarios. The graph shows almost the same behavior for all the budgets, with an almost linear relationship between gross profit and selling price, accompanied by a small parabolic curvature. The main difference lies in the attainable gross profit for each budget. Interestingly, the **highest gross profit is achieved with a budget of 30,050,000**.

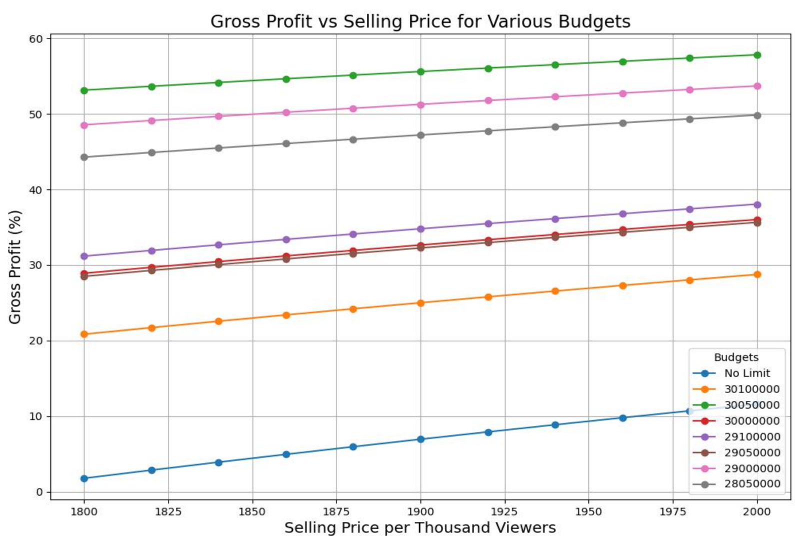


Figure 1: Gross Profit vs Selling Price for All the Budgets

We can also observe that the smallest gross profit growth with price occurs when there is no budget limit. This is coherent with our intuition; when there is no constraint on the budget, the system can become unoptimized, leading to less efficient allocation of resources and diminishing returns on gross profit growth as the price increases.

Let's finally consider the ads. Figure 2 below summarizes the distribution of ads placed on the own channel, ads sold, and ads bought from competitors across different budgets. It shows how the advertising strategy varies with budget changes, highlighting the trends for each budget level. Note that not all of our ad slots are utilised in every case as the model was run with an allowance for gap between objective value and best bound, due to computing constraints.

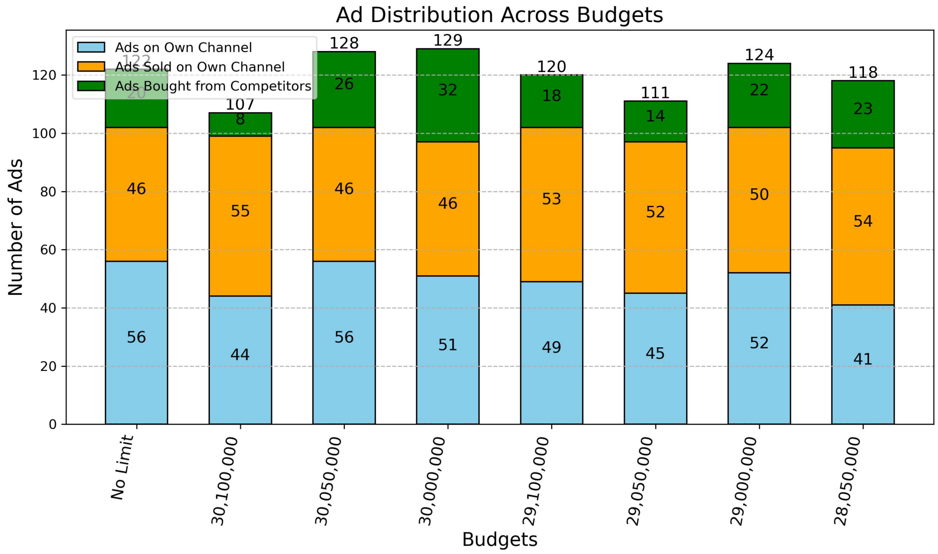


Figure 2: Ad Distribution Across Budgets

# 8 Discussion Challenges and Extension

## 8.1 Extensions and challenges

Given the time complexity of the problem, solving the model directly ran into runtime issues due to time frame and computing resource limitations. Our MVP approach aimed to produce a schedule over a three day block using the reduced movie dataset discussed in Section 3.1.

However, our implementation demonstrates the model's ability to balance demographic preferences, advertising costs, and competitor actions effectively. Despite computational constraints, the framework is implemented in a highly generalizable way, capable of handling a seven-day scheduling horizon and accommodating a much larger set of movies. It is designed to scale seamlessly with sufficient resources. Despite them being out of scope for an MVP approach, here we give an extensive explanation of limitations, and what could be done if there were no resource limitations.

Firstly, the model should be extended to a seven day period and more movies should be introduced back into the dataset. This would allow a more comprehensive exploration of which movies are beneficial to schedule based on our demographic's viewing habits.

Secondly, the model we agreed on for this final report counted viewers deterministically, as noted in our assumptions section, assuming all potential viewers from advertising would be captured. This choice was made to reduce computational cost and problem complexity while ensuring reasonable results. However, a probabilistic approach was attempted, where actual viewers were sampled based on conversion rate probabilities for a given time slot. The total conversion probability $p$ was computed as a scalar product between the conversion rate vector at an advertisement time slot and the conversion of the movie to be shown, accounting for the fact that there must be a match between the movie to advertise and the movie shown during the advertisement, with the standard deviation of the normal distribution set to $\max(0, (1-p) \cdot \mu)$, where $\mu$ was the expected viewers. This ensured higher precision when $p \to 1$, while maintaining variability for lower probabilities. If sampled viewers exceeded $\mu$, values were adjusted to lie uniformly within the upper quartile of $\mu$, ensuring realistic bounds. While this approach succeeded in capturing variability, it was computationally expensive, preventing extensive testing within the project's timeline. Interested readers can refer to the file **probabilities_integrated_1.py** on the project's GitHub repository [Group9-MMCS-Edinburgh, 2024] for implementation details and potential future extensions.

Finally, to extend the model beyond seven days adaptively, as suggested by the problem statement, two approaches were considered. The first, which we could call a "naive" approach but still reasonable, would be to run the model on the newly released schedule by other channels. This would be acceptable within the framework of the project, as inherently, there is no real game-theory-like competition. In fact, our channel has a significant advantage, as it has access to the schedules from other channels at the beginning of the week and can adapt its schedule accordingly to maximise profit, while the other channels are unaware of what we plan. Thus, the game is skewed in our favor, and the other channels cannot be considered rational or optimal players. This is why this naive approach could make sense in this specific context.

The second approach would be to adjust the prices of our advertisement slots and decide which ad slots to disable based on the potential viewers we might lose if we sell the slot, compared to the number of viewers converted from our channel by a specific channel in previous runs (which can be considered a measure of how competitive that channel is). The price would vary depending on the channel, and ad slots would be disabled if the potential loss in viewers exceeds a certain threshold. This threshold could be updated iteratively based on the global ratio between the viewers converted from our channel previously and the maximum number of viewers that could have been converted from our channel. This would quantify the general market competitiveness, with the threshold being adjusted over time by analyzing data from previous weeks. The key aspect here is that both the number of viewers and the conversion rates would be updated dynamically, allowing the model to "learn" from past weeks performance and adapt its strategy for future decisions simply by deciding to disable or not a slot and by deciding the prices of slots for each viewership and channel. This learning process would enable the model to continuously improve its decision-making in response to evolving competitive dynamics.

## 8.2 Assumptions Revisited and Final Consultancy Recommendations

### 8.2.1 Assumptions revisited

Revisiting the assumptions laid out above, here are some recommendations as to how they could be addressed:

- The model should consider whether ad slots are being sold to competitors and consider an associated cost in the objective function for viewers who are lost to competitor advertisements.

- When a movie is advertised there should be a dampening effect on the number of viewers converted as the number of advertisements used increases, especially for repeated advertisements on the same channel. This would reflect the diminishing return of viewers converted and the same viewers being targeted repeatedly.

- As our intention is to sell ad slots for a price based a on CPT basis, it would be more optimal to cast $u$ as an integer decision variable. Then the left hand side of Constraint (14) would be written as $1000u$ and the right hand side of Constraint (16) as $\frac{P}{1000} v_{mt}$. The objective function would look similar but would now maximise every $1,000$ unit of viewers. This approach would better handle edge cases than our current implementation, *i.e.*, ensure that adverts on competitor channels are only bought when they increase viewership by at least another $1,000$ viewers. This is not implemented, as casting $u$ as an integer variable greatly increases the problem complexity.

### 8.2.2 Consultancy Recommendations

After conducting this study, we now have 10 consultancy recommendations and comments:

1. **Budget Allocation:** We recommend allocating £60M–£70M for a 7-day period, scaling from the successful £30.05M used in the 3-day model. This ensures an optimal cost-to-viewership ratio below 200. Further analysis would be required to confirm this figure once the model has been extended to a 7-day.

2. **Computational Resources:** We recommend investing in cloud or HPC solutions to handle extended 7-day scheduling and larger movie datasets, leveraging parallel computing and advanced optimization solvers.

3. **Adaptive Adjustments:** We recommend starting with weekly strategy updates based on competitor data and gradually transitioning to probabilistic modeling using conversion rates and stochastic scenarios.

4. **Targeted Advertising:** We recommend focusing on high-conversion competitor ad slots for similar genres and prime-time self-advertising, ensuring alignment with demographic preferences.

5. **Incremental Improvements:** We recommend implementing dynamic ad pricing, genre-specific targeting, and repeated ad dampening effects to optimize conversions while efficiently utilizing resources.

6. **Sensitivity Testing:** We recommend performing sensitivity tests with 10–15% budget adjustments to refine resource allocation strategies while maintaining a cost-to-viewership ratio below 200.

7. **Optimized Ad Slot Management:** We recommend selling high-value slots during peak times, buying competitor slots with high conversion rates, and prioritizing multi-genre movies to maximise audience impact.

8. **Extensions for Future Improvements:** We recommend introducing a 7-day adaptive model refining stochastic viewer sampling, and adjusting pricing thresholds dynamically.

9. **Operational Efficiency:** We recommend setting minimum revenue thresholds to prevent bankruptcy and adjusting operational resources in line with demographic trends and market demands.

10. **Long-Term Strategies:** We recommend analyzing audience demographics to expand for example children's programming, exploring shorter movies or classics, and using stochastic simulations to refine strategies over a 12-week horizon.

# 9 Conclusions

This report proposes an approach that can produce an effective and data-driven scheduling and advertising strategy for Channel A, which maximises viewership in time slots which the channel sells advertisements. The model chooses carefully whether to use ad slots for advertising Channel A's movies or to sell these ad slots. Furthermore, the model intelligently selects which competitor ad slots should be bought, considering the competitor's schedule and the movie to be advertised. That is, ad slots are chosen based on movie similarity to the movie being shown, ensuring advertisements are specific and targeted. Current results produced by the model are for three-day blocks, but extending to seven days is simply a matter of making more computing resources available.

The full implementation and code for this project can be accessed on our GitHub repository: MMCS Project Repository [Group9-MMCS-Edinburgh, 2024]

## References

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