Tasks chapter3: Linear regression

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For the "location" assigned to your dyad, use the SYNOP observations and forecast data from ECMWF with a forecast horizon of 36 hours contained in location_obs_ECMWF_2009-2022.rds. The file contains several forecast parameters for which acronyms, description and units are given in ECMWF_selected_parameters.pdf. The observations are rr6hObs, the precipitation sum over the previous 6 hours (mm), t2mObs, the temperature 2 m above ground (Celsius), and tdObs, the dew point temperature (Celsius). The R-lab session at the end of chapter 3 in the ISL book will show you all necessary R commands.

1. Use the lm() function for a simple linear regression with location$t2mObs as the response and location$t2m as the predictor and assign it to the variable lm.fit. R has a formula notation with the "$\sim$" sign: lm(t2mObs $\sim$ t2m, data = ibk). The data = ibk saves you from having to type ibk$t2mObs.

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2. What are the coefficients (intercept and slope) of the linear regression?
   Use print(coef(lm.fit))

$$\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x \tag{1}$$

Table 1: Coefficients of linear regression

intercept ($\hat{\beta}_0$)	**slope** ($\hat{\beta}_1$)
-1.669422	1.037230

3. Use summary(lm.fit) to print the results.

Table 2: Residuals

min	**1Q**	**median**	**3Q**	**max**
-9.9010	-0.9523	0.2410	1.2517	7.3054

The standard error of an estimator reflects how it varies under repeated sampling.

Table 3: Coefficients

	coefficient	**std.-error**	**t-values**	**p-value**
intercept	-1.669422	0.051533	-32.4	<2e-16
t2m_celsius	1.037230	0.003078	337.0	<2e-16

Residual standard error: 1.907 on 5043 degrees of freedom (52 observations deleted due to missingness).

Multiple R-squared: 0.9575, Adjusted R-squared: 0.9575

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4. Is there a relationship between the predictor and the response and how strong is it?

   R-squared = 0.9575, so the relationship between predictors and response is very strong positive!!

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5. How much of the variance is explained?

   If the R-squared values is approaching to 1, it means that our predictors explain much of our data variability (much of our variance).

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6. What does the value of the F-statistic tell you about the possibility of rejecting the null-hypothesis that the slope $\hat{\beta}_1$ is (close to) zero?

   F-statistic: 1.136e+05 on 1 and 5043 DF, p-value: < 2.2e-16
   The F-statistic quantifies the difference in error between the model, assuming your null hypothesis (i.e. $\hat{\beta}_1 = 0$) and your fitted model. It simply a formal way of determining if a coefficient belongs to the model. If the p-value recieved from the F-testing statistics is small then we reject the null (because the probability of occurrence is so small), and say $\hat{\beta}_1 = 0$ here. So assuming $\hat{\beta}_1 = 0$ and rejecting the null hypothesis here must have to something that we expect the p-value small.

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7. How large is the 95% confidence interval for the intercept and slope of the linear regression? By how many percent does the slope change from the 2.5% level to the 97.5% level of the confidence interval? By how many Kelvin the intercept? Confidence levels are contained in confint(lm.fit, level = 0.95).

Table 4: Confidence intervalls

	2.5%	**97.5%**	**change**
intercept ($\hat{\beta}_0$)	-1.770450 °C	-1.568394 °C	0.202056 K
t2m_celsius ($\hat{\beta}_1$)	1.031196	1.043264	101.2%

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8. Make a scatterplot of (location$t2m, location$t2mObs) with small symbols (option of cex=0.1) and add a red line showing the linear regression (abline(lm.fit, col = "red")). Add the pivot point (= mean of (x), mean of (y)) using points() with 'col = "orange".

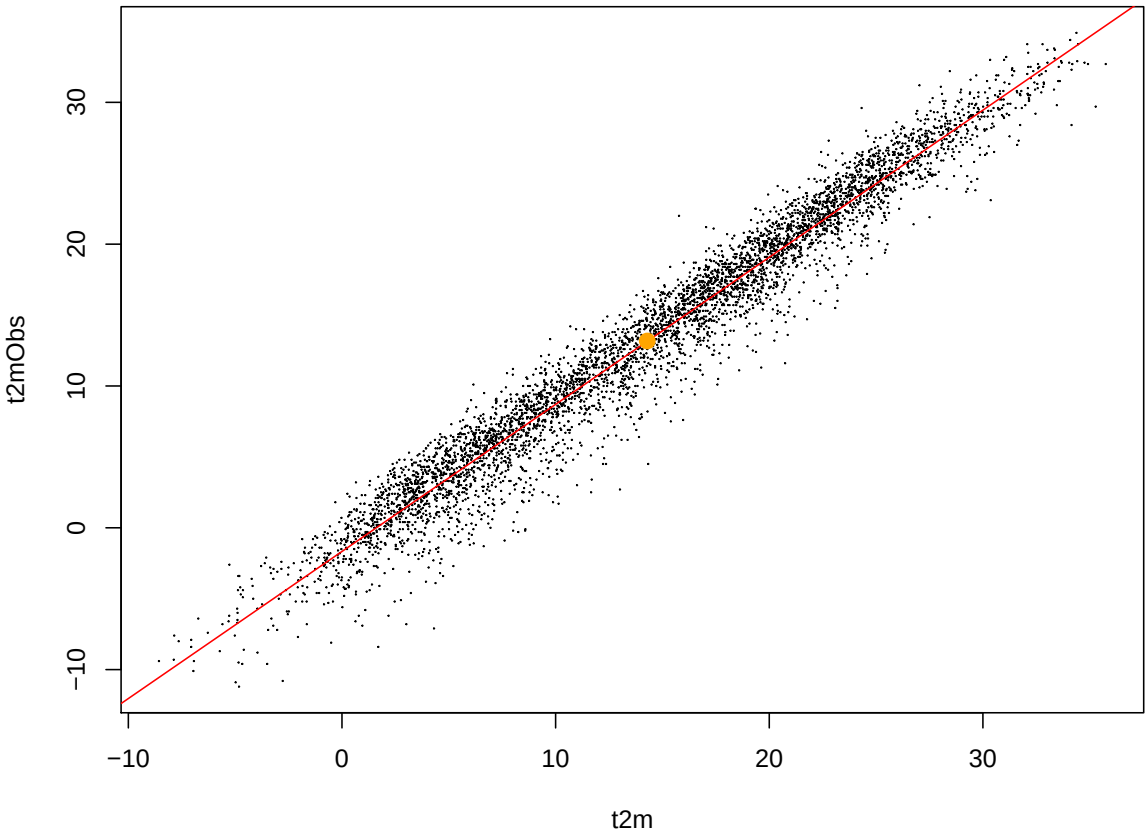

Figure 1: scatterplot with linear regression (red line)

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9. Which data point has the highest leverage? What does that mean? How much higher is it than the average leverage (cf. p99 of ISL book)? Use hatvalues(lm.fit) for the h-statistic and which.max() to get the index of the observation with the highest h-statistic (== maximum hatvalue). What temperature was that?

   - **max:** 0.00156 (data point 1129) ⇒ data point with high leverage has a large potential to affect the overall regression analysis.
   - **mean:** 0.000396
   - **difference:** 0.00116
   - **temperature at max:** -8.4°C (Obs.) -7.05°C (Pred.)
     There is no change in the predicted values, although the observations change considerably at this point.

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10. Is there any outlier? (what is the difference between outlier and high leverage point?). Use a plot of studentized residuals (almost all values should be between +-3) produced with plot(predict(lm.fit), rstudent(lm.fit))

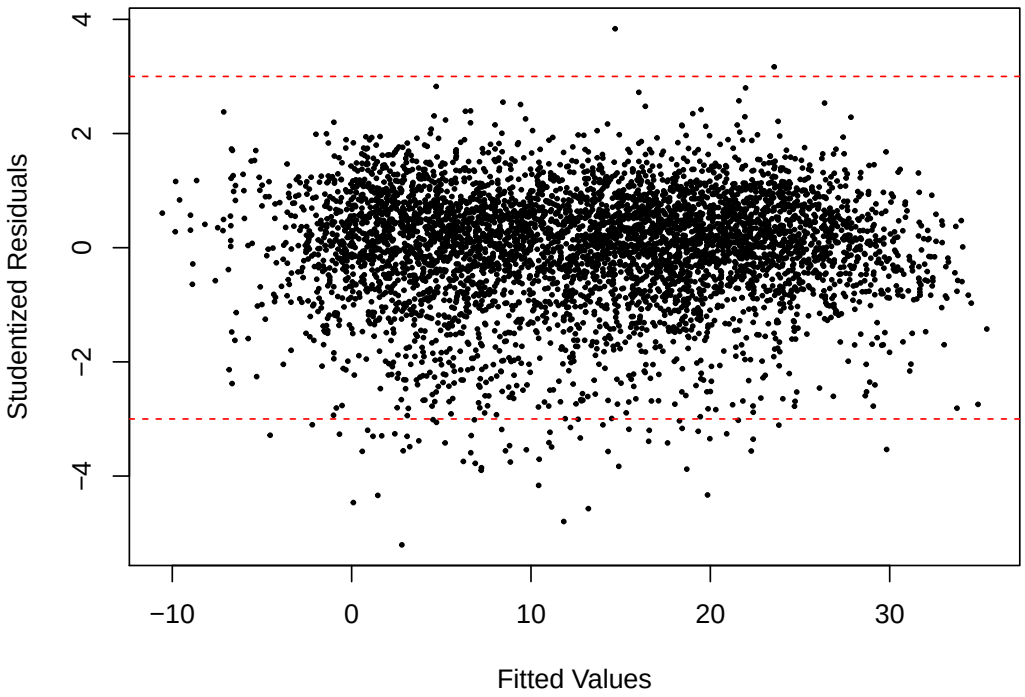

Figure 2: Residuals for identifying outliers

- Is there any outlier?
  Many negative, only two over +3
- What is the difference between outlier and high leverage point?
  A high leverage point is a data point that has a relatively extreme value for one or more predictor variables. High leverage points can have a large influence on the position of the regression line, but they may not necessarily be outliers. In some cases, high leverage points may be legitimate observations that have a unique combination of predictor variable values.

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11. Produce a 4-panel diagnostic plot of the linear regression - as shown in class. Tell R to produce a 2x2 graphics with par(mfrow = c(2,2)) and then simply do plot(lm.fit). (A remark: if you later want only 1 figure per plot reset with par(mfrow = c(1,1)).) These plots can be used to spot whether basic assumptions of least-squares regression are violated. Assumptions are that the errors (residuals) have a Gaussian distribution, that they are centered on the regression line and that their variance does not change as a function of x ("homoscedasticity". (Note that in the plots some (problematic) data points are labeled. Since we are using date (YYYY-MM-DD hh:mm:ss) as index that looks messy.)

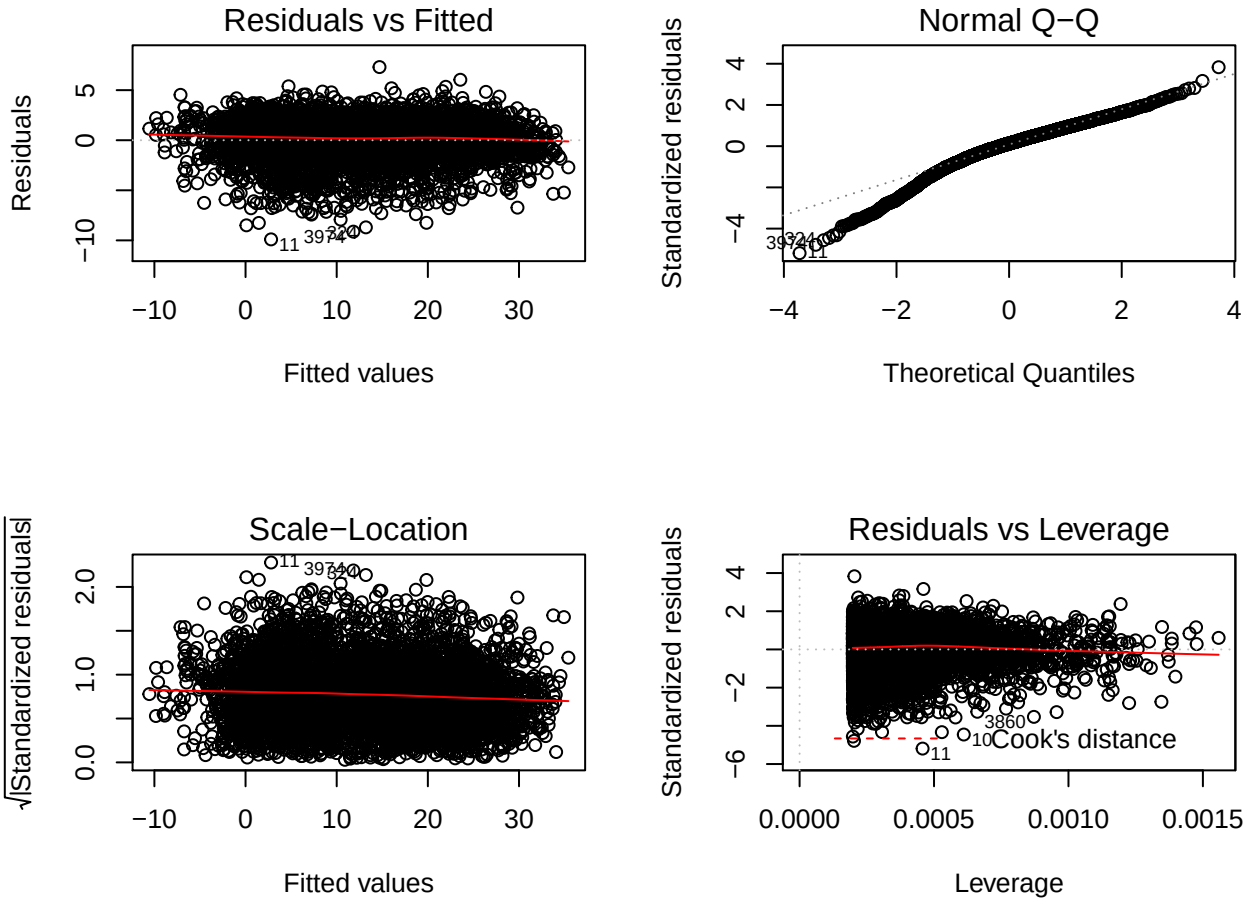
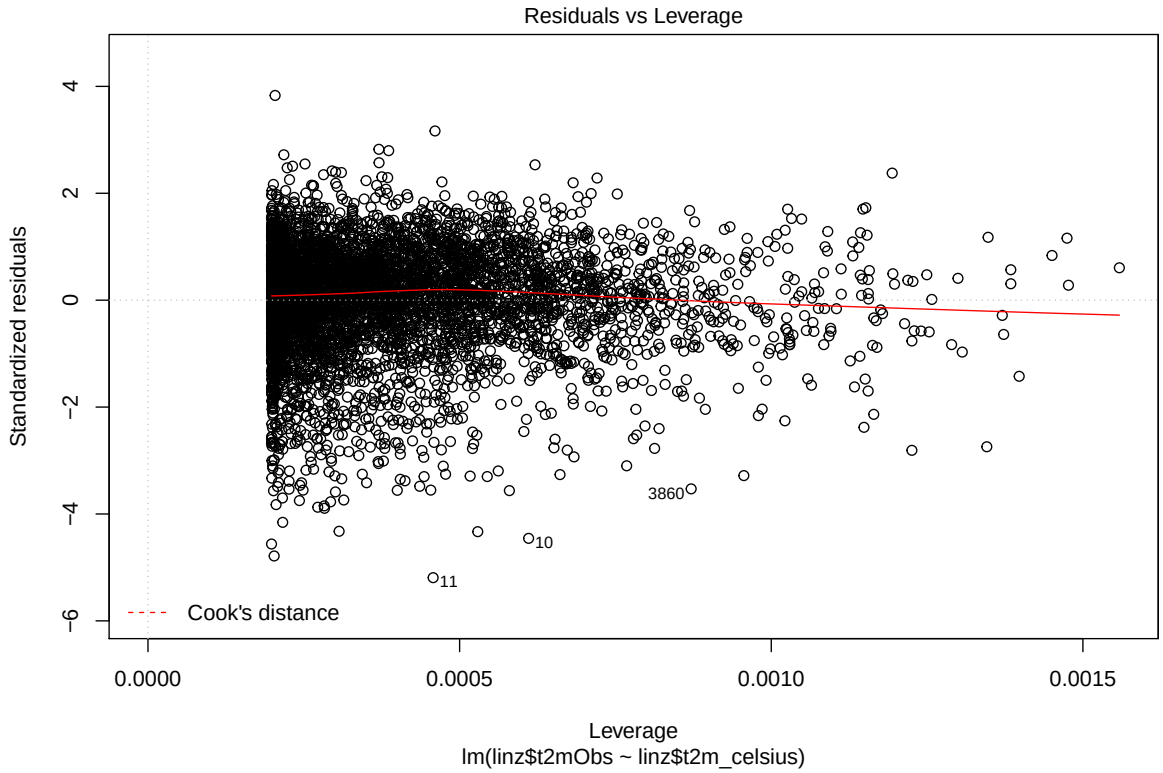


Figure 3: diagnostics of linear regression



No point falls out of cooks distance. This means there aren't any overly influential points in our dataset. So no basic assumptions are violated.