Number Theory I

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Mathematics

NUMB3RS

We all use math every day; to predict weather, to tell time, to handle money. Math is more than formulas or equations; it's logic, it's rationality, it's using your mind to solve the biggest mysteries we know.

What is number theory?

Number theory is the study of the set of positive whole numbers: $1, 2, 3, \ldots$ which are often called the set of natural numbers. We will especially want to study the relationships between different sorts of numbers.

Source: https://www.math.brown.edu

- odd $1, 3, 5, 7, 9, 11, \ldots$
- even $2, 4, 6, 8, 10, \ldots$
- square $1, 4, 9, 16, 25, 36, \ldots$
- cube $1, 8, 27, 64, 125, \ldots$
- prime $2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, \ldots$
- 1 (modulo 4) $1, 5, 9, 13, 17, 21, 25, \ldots$
- triangular $1, 3, 6, 10, 15, 21, \ldots$
- perfect $6, 28, 496, \ldots$
- Fibonacci $1, 1, 2, 3, 5, 8, 13, 21, \ldots$

Divisibility

Divisor

In mathematics a divisor of an integer **n**, also called a factor of **n**, is an integer that can be multiplied by some other integer to produce **n**. An integer **n** is divisible by another integer **m** if **m** is a factor of **n**, so that dividing **n** by **m** leaves no remainder.

Definition

A **prime** number (or a prime) is a natural number greater than **1** that has no positive divisors other than **1** and itself. A natural number greater than **1** that is not a prime number is called a **composite** number.

Fundamental theorem of arithmetic

It states that every integer greater than **1** either is prime itself or is the product of prime numbers, and that this product is unique, up to the order of the factors.

Which is fastest algorithm to find prime numbers?

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- Sieve of Eratosthenes

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- Sieve of Eratosthenes
- Sieve of Atkin
- Linear Prime Sieve

GCD & LCM

Greatest common divisor

In mathematics, the greatest common divisor (*gcd*) of two or more integers, when at least one of them **is not zero**, is the largest positive integer that divides the numbers without a remainder.

Coprime numbers

Two numbers are called relatively prime, or coprime, if their greatest common divisor equals **1**. For example, 9 and 28 are relatively prime.

Least common multiple

The lowest common multiple of two integers **a** and **b**, Is the smallest positive integer that is divisible by both **a** and **b**. Since division of integers by zero is undefined, this definition has meaning **only if a and b are both different from zero**.

Calculation

Calculation

Prime factorizations

Greatest common divisors can in principle be computed by determining the prime factorizations of the two numbers and comparing factors; **gcd(18, 84)**, $18 = 2 \times 3^2$ and $84 = 2^2 \times 3 \times 7$, then $gcd(18, 84) = 6$

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Binary method

Also known as Stein's algorithm. It uses simpler arithmetic operations than the conventional Euclidean algorithm; it replaces division with arithmetic shifts, comparisons, and subtraction.

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Euclid's algorithm

The Euclidean algorithm is based on the principle that the greatest common divisor of two numbers does not change if the larger number is replaced by its difference with the smaller number.

Implementation

Recursive version

```
int gcd(int a, int b) {return (b==0)?a:gcd(b,a%b);}
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How to swap two numbers without using temp variables nor arithmetic operations?

Iterative version

```
int gcd(int a, int b) {
    while(b) {
        a %= b;
        b ^= a;
        a ^= b;
        b ^= a;
    }
    return a;
}
```

Modular Arithmetic

Big Number modulus int

Q & A

References

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