

Lecture 3:

1.2. Theory: The Malthus-Solow Economic Growth Model

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Original course by Melissa Dell (Harvard Econ 1342), revised by Brad DeLong, research assistance by Anish Biligiri

Lecture Outline

Read Beforehand: J. Bradford DeLong: *Lecture Notes: Malthusian Economies* <<https://tinyurl.com/dl-2020-01-18g>>

Read Beforehand: Greg Clark: *A Farewell to Alms*, selections <<https://delong.typepad.com/files/clark-alms-selections.pdf>>

Slides: <<https://github.com;braddelong/public-files/blob/master/econ-135-lecture-3.pptx>>

1. **Review:** Long-run shape of global economic growth

2. **Lecture:** Solow-Malthus model basics

3. **Review:** Solow model essentials

4. **Lecture:** Applying the Solow-Malthus model

- <https://datahub.berkeley.edu/user/delong@econ.berkeley.edu/notebooks/LS2019/2019-10-14-Ancient_Economies.ipynb>
- <https://nbviewer.jupyter.org/github;braddelong/LS2019/blob/master/2019-10-14-Ancient_Economies.ipynb>

5. **Assignment:** What is economics? paper <<https://bcourses.berkeley.edu/courses/1487685/assignments/8065184>>

6. **Big Ideas:** Principal takeaways from this class

7. **MOAR** references:

- C.I. Jones: The Facts of Economic Growth <<https://web.stanford.edu/~chadj/facts.pdf>>...
- Gregory Clark (2005): The Condition of the Working Class in England, 1209-2003 <<https://delong.typepad.com/files/clark-condition.pdf>>...
- Ian Morris (2010): Why the West Rules—For Now, chapter 3: Taking the Measure of the Past <<https://delong.typepad.com/files/morris-rules-3.pdf>>...
- Patricia Crone: Pre-Industrial Societies, selections <<https://delong.typepad.com/files/crone-pre-selections.pdf>>...

Changeup Review: One Table: Average Global Numbers

Date	ideas Level H	Total Real World Income Y (billions)	Average Real Income per Capita y (per year)	Total Human Population L (millions)
-68000	1.0	\$0	\$1,200	0.1
-8000	5.0	\$3	\$1,200	2.5
-6000	6.3	\$6	\$900	7
-3000	9.2	\$14	\$900	15
-1000	16.8	\$45	\$900	50
0	30.9	\$153	\$900	170
800	41.1	\$270	\$900	300
1500	53.0	\$450	\$900	500
1770	79.4	\$825	\$1,100	750
1870	123.5	\$1,690	\$1,300	1300
2020	2720.5	\$90,000	\$11,842	7600

You've Seen This Before...

Approximately what was the growth rate of the human useful-ideas stock between the year 0 and 1500?

- A. About 0.036%/year
- B. About 3.6%/year
- C. About 2.06%/year
- D. About 0.206%/year
- E. None of the above are close

One Table: Average Global Numbers

Date	ideas Level H	Total Real World Income Y (billions)	Average Real Income per Capita y (per year)	Total Human Population L (millions)	Rate of Population and Labor Force Growth n	Rate of Efficiency-of-Labor Growth g	Rate of Ideas-Stock Growth h
-68000	1.0	\$0	\$1,200	0.1			
-8000	5.0	\$3	\$1,200	2.5	0.005%	0.000%	0.003%
-6000	6.3	\$6	\$900	7	0.051%	-0.014%	0.011%
-3000	9.2	\$14	\$900	15	0.025%	0.000%	0.013%
-1000	16.8	\$45	\$900	50	0.060%	0.000%	0.030%
0	30.9	\$153	\$900	170	0.122%	0.000%	0.061%
800	41.1	\$270	\$900	300	0.071%	0.000%	0.035%
1500	53.0	\$450	\$900	500	0.073%	0.000%	0.036%
1770	79.4	\$825	\$1,100	750	0.150%	0.074%	0.149%
1870	123.5	\$1,690	\$1,300	1300	0.550%	0.167%	0.442%
2020	2720.5	\$90,000	\$11,842	7600			

Approximately what has been the growth rate of the human useful-ideas stock between the year 1870 and today?

- A. About 0.036%/year
- B. About 3.6%/year
- C. **About 2.06%/year**
- D. About 0.206%/year
- E. None of the above are close

Why Is This Interesting?

Date	ideas Level H	Total Real World Income Y (billions)	Average Real Income per Capita y (per year)	Total Human Population L (millions)	Rate of Population and Labor Force Growth n	Rate of Efficiency-of-Labor Growth g	Rate of Ideas-Stock Growth h
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2020	2720.5	\$90,000	\$11,842	7600			

2.06/.0036 is about 60, no?

- What else do you find interesting about this table to the left?

One Table: Average Global Numbers

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1770	79.4	\$825	\$1,100	750	0.150%	0.074%	0.149%
1870	123.5	\$1,690	\$1,300	1300	0.550%	0.167%	0.442%
2020	2720.5	\$90,000	\$11,842	7600	1.177%	1.473%	2.061%

Questions

1. What assumption that Aristotle made—perhaps (probably?) without thinking about it, because it seemed most obvious to him—struck you as the most wrong or repugnant or weird?
2. What role do the “statues of Daedalus, [and] the tripods [robotic catering vessels] of Hephaestus” play in Aristotle’s argument about how the economy of his age is, must be, and should be structured?
3. What role does Aristotle’s claim that “Of the art of acquisition [*ktētike*] then there is one kind which by nature is a part of the management of a household.... There is another... commonly and rightly called an art of wealth-getting [*chrēmatistikē*]... [with] the notion that riches and property have no limit...” play in his argument?
4. Did you find any advice—even indirect and oblique advice—from Aristotle in this passage about what the economic policy of a city-state should be? If so, what was it? If not, why do you think he fails to offer advice here (he offers lots of advice as to the organization and policies of city-states later on in the book)?
5. What does Aristotle say are the four tasks of the Greek man in managing his household? Why these four?
6. What are these four in rank order of importance



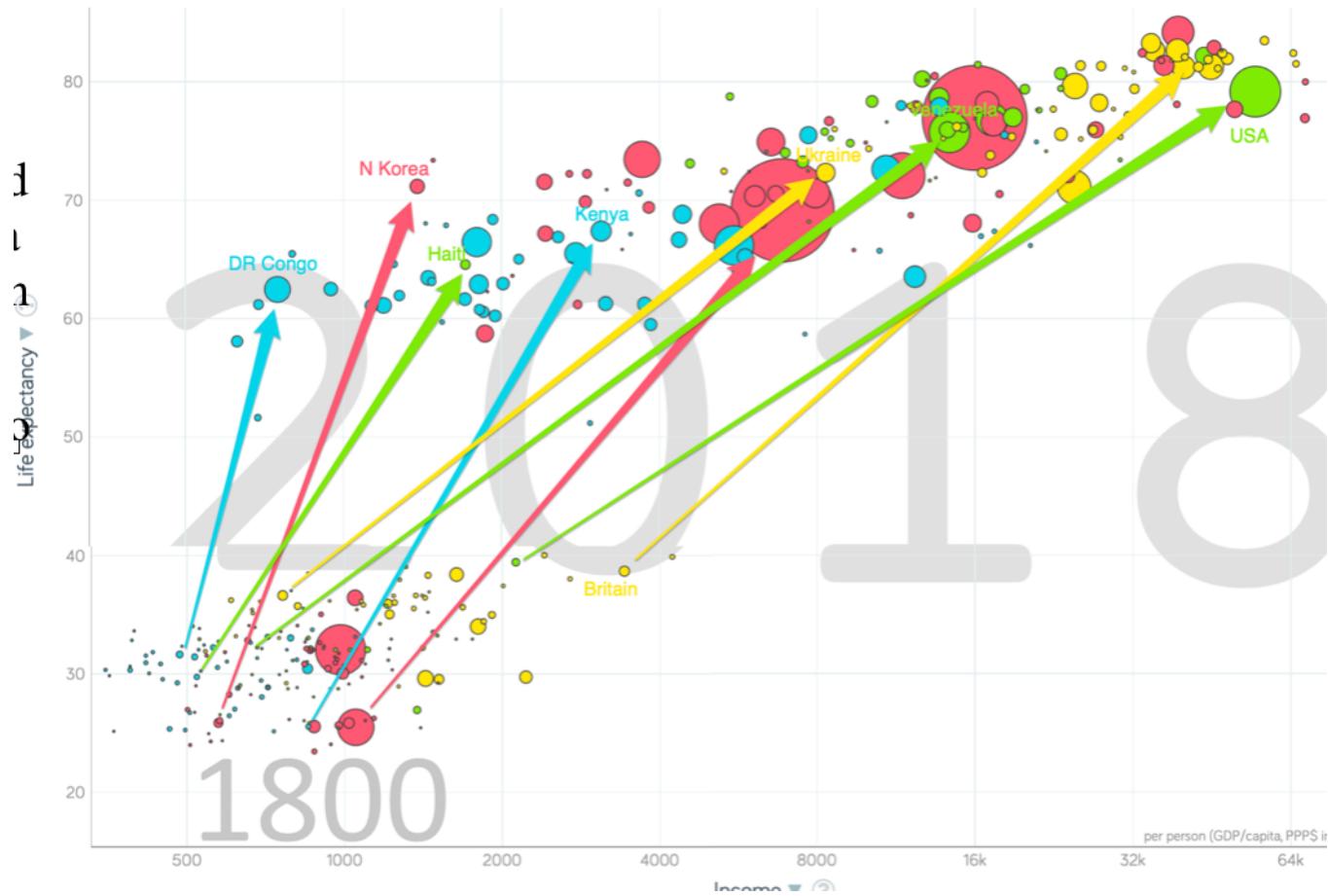
Classical Athenian 4-drachma silver coin

The “owl”, with the head of the goddess Athene on the front and her familiar bird of wisdom on the back, of the type that Aristotle (or his slaves) would have carried on their person. Weighs 3/5 of an ounce. The Athenian navy paid its oarsmen one drachma a day.

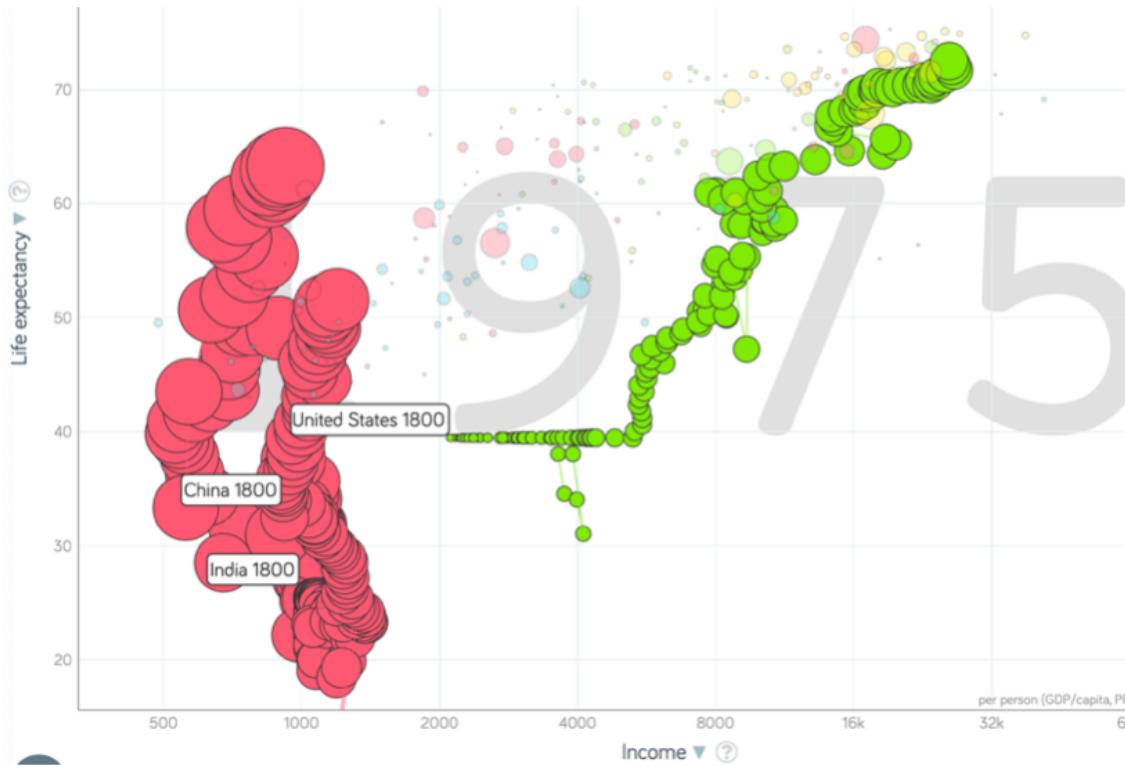
One Figure: A Great Divergence

From 1800 to 2018:

- The dots start with a 3-1 spread in incomes and a 10-year spread in life expectancy.
- All the arrows go up.
- Some arrows—mostly those already to the right—go right fast.
- Other arrows go right slowly.



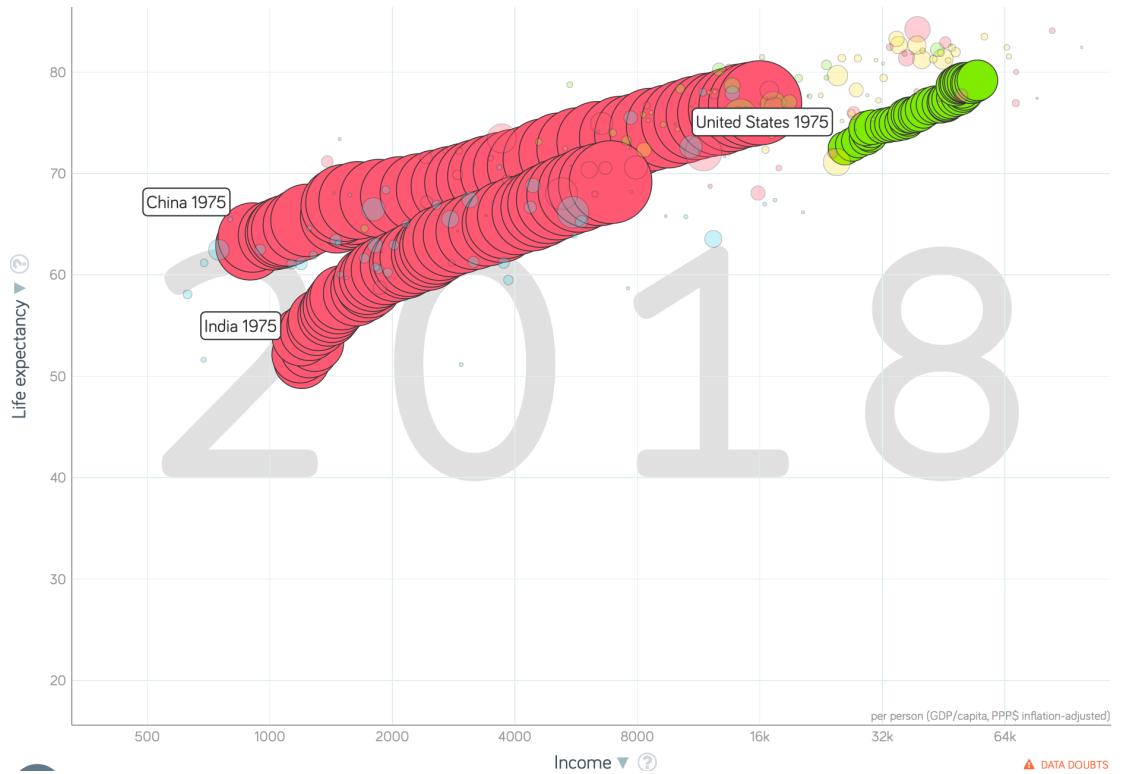
China and India and America, 1800-1975



From 1800 to 1975:

- Measured living standards and productivity levels improve fourteen-fold in the United States...
- ...& less than 30% in China & India...
- ...in spite of economic, transport, and cultural globalization...
- This is crazy!
- A “great divergence”
 - Not only were China & India relatively poor in 1800, they fell further behind thereafter

China and India and America, 1975–2018



From 1975-2018:

- Measured living standards and productivity levels...
- ... $54.9/25.9 = 2.12$ in America...
- ... $16.0/0.9 = 17.8$ in China...
- ... $6.9/1.2 = 5.8$ in India...

A Second Table: “The West”

Date	ideas Level H	Total Real Income Y (billions)	Average Real Income per Capita y (per year)	Total “West” Population L (millions)	Rate of Population and Labor Force Growth n	Rate of Efficiency-of-Labor Growth g
-68000	1.0	\$0.01	\$1,200	0.005		
-8000	4.5	\$0.12	\$1,200	0.1	0.005%	0.000%
-6000	4.7	\$0.18	\$900	0.2	0.035%	-0.014%
-3000	7.5	\$0.45	\$900	0.5	0.031%	0.000%
-1000	15.0	\$1.80	\$900	2	0.069%	0.000%
0	23.7	\$4.50	\$900	5	0.092%	0.000%
800	30.0	\$7.20	\$900	8	0.059%	0.000%
1500	58.9	\$25.00	\$1,000	25	0.163%	0.015%
1770	142.9	\$105.00	\$1,400	75	0.407%	0.125%
1870	436.5	\$490.00	\$2,800	175	0.847%	0.693%
2020	16666.7	\$40,000.00	\$50,000	800	1.013%	1.922% ₁₄

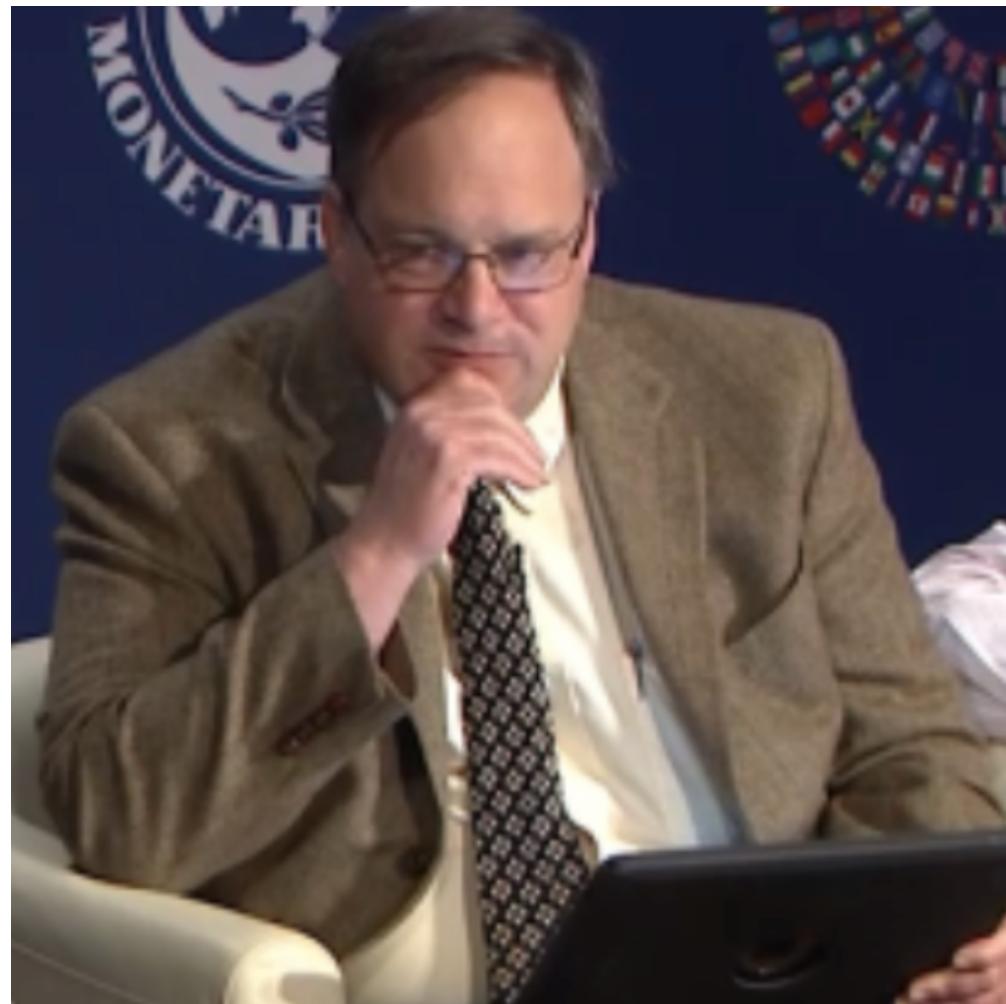
Four Major Features

The history of economic growth has four major features:

1. Poverty, in the pre-industrial ages, with population growth on average, but with average population growth (and ideas innovation!) very slow
 - $n = 0.07\%/\text{yr}$; $h = 0.035\%/\text{yr}$
2. Growing prosperity, in the Industrial Revolution and the Modern Economic Growth ages
 - In the MEG era: $h = 2.06\%/\text{yr}$
3. The great divergence since 1800
 - Globalization
 - American twentieth-century economic ascendancy
4. Pre-industrial efflorescences and declines

Catch Our Breath...

- Ask a couple of questions?
- Make a couple of comments?
- Any more readings to recommend?



Solow-Malthus Model Basics

How do we make sense of the fact that people were ingenious and inventive back before 1500, and yet standards of living did not increase?

- Although population did increase—slowly $n=0.07\%/\text{yr} \times 0.2\%/\text{generation}$
- Other parts of the model
- Balanced-growth equilibrium
- Convergence to equilibrium
- Lecture notes: <<https://nbviewer.jupyter.org/github/braddelong/long-form-drafts/blob/master/solow-model-5-pre-industrial.ipynb>>
 - datahub: <<http://datahub.berkeley.edu/user-redirect/interact?account=braddelong&repo=long-form-drafts&branch=master&path=solow-model-5-pre-industrial.ipynb>>

Solow-Malthus Model Basics: Efficiency of Labor

How do we make sense of the fact that people were ingenious and inventive back before 1500, and yet standards of living did not increase?

- We first need to make efficiency of labor a function of available natural resources per worker.
- We do this by setting the rate of efficiency of labor growth $g = h - n/\gamma$: equal to the rate h at which the stock of useful ideas grows and the rate of growth of the labor force growth n divided by a resource-importance parameter γ .
- Thus $g = 0$ if and only if: $n = n^{*\text{mal}} = h\gamma$.
- What could make this happen?

Solow-Malthus Model Basics: Population and Labor-Force Growth

Back before 1500—and even later—people are anxious to have children:

- In the hope that some of them will survive to themselves reproduce.
- Plus a world without reliable and effective family planning mechanisms.
- Let's say:
 - $1/\Phi$ is the fraction of production devoted to necessities
 - y^{sub} is the “subsistence” standard of necessities consumption at which population growth averages zero:
 - Depends on sociology—marriage ages, &c....
- Then, back before the demographic transition:
 - $n = \beta(y/(\Phi y^{\text{sub}}) - 1)$

Malthusian Equilibrium: Living Standards

Population and the labor force are growing just fast enough to soak up the benefits of new useful ideas:

- $h\gamma = n^{*\text{mal}} = \beta(y/(\Phi y^{\text{sub}}) - 1)$ In a world without reliable and effective family planning mechanisms.
- This gives us: $y^{*\text{mal}} = \Phi y^{\text{sub}}(1 + h\gamma/\beta)$.
 - Productivity levels and standards of living will be high enough that necessities consumption allows for enough population growth to soak up the (slow) rate of ideas generation.
 - Productivity levels and standards of living will be above subsistence—but not that far above subsistence

Malthusian Equilibrium: Population and Labor Force Growth

How big will the population and labor force then be?:

- It needs to be at the level that preserves the Malthusian equilibrium:
 - Generates $y^{*mal} = \Phi y^{sub}(1 + h\gamma/\beta)$
 - $y^{*mal} = (s/(n+g+\delta))^{\theta} E$
 - And we know that in Malthusian equilibrium, $g=0$ and $n = h\gamma$
 - So: $y^{*mal} = (s/(h\gamma+\delta))^{\theta} E$
 - $E = HL^{-(1/\gamma)}$

Malthusian Equilibrium: Population and Labor Force Growth II

The master equations are then:

- $y^{*mal} = (s/(h\gamma + \delta))^{\theta} HL^{-(1/\gamma)}$
- $L^{*mal} = \{(H/y^{sub})(s/\delta)^{\theta}(1/\Phi)\}^{\gamma} [1+h\gamma/\delta]^{-\gamma\theta} [1+h\gamma/\beta]^{-\gamma}$
- How do we understand this?:
 - Two nuisance terms:
 - Population will be lower if population response is smaller, the resource scarcity drag is lower, or ideas generation is faster in order to generate the wedge in living standards above subsistence needed to get enough population growth to soak up the benefits of new ideas
 - Population will be lower if ideas generation is faster or the resource scarcity drag is lower because those will lower the economy's steady-state capital intensity, and make it less productive

Malthusian Equilibrium: Population and Labor Force Growth III

The master equations are then:

- $y^{*\text{mal}} = (s/(h\gamma+\delta))^{\theta} HL^{-(1/\gamma)}$
- $L^{*\text{mal}} = \{(H/y^{\text{sub}})(s/\delta)^{\theta}(1/\Phi)\}^{\gamma} [1+h\gamma/\delta]^{-\gamma\theta} [1+h\gamma/\beta]^{-\gamma}$
- Three significant terms:
 - The ratio (H/y^{sub}) of ideas to subsistence necessities consumption
 - The ratio $(s/\delta)^{\theta}$ of savings and investment to depreciation raised to the parameter θ
 - $(1/\Phi)$: the greater the taste for “luxuries”, the lower the population
 - And living in cities is perhaps the most important of the luxuries
- All these are then magnified by the parameter γ in their effect on population

Understanding Malthusian Equilibrium

$$n^{*mal} = \gamma h$$

$$y^{*mal} = \phi y^{sub} \left(1 + \frac{n^{*mal}}{\beta} \right) = \phi y^{sub} \left(1 + \frac{\gamma h}{\beta} \right)$$

$$L_t^{*mal} = \left[\left(\frac{H_t}{y^{sub}} \right) \left(\frac{s}{\delta} \right)^\theta \left(\frac{1}{\phi} \right) \left[\frac{1}{(1+\gamma h/\delta)^\theta} \frac{1}{(1+\gamma h/\beta)} \right] \right]^\gamma$$

Interpretation

- Start with the rate h of new economically-useful ideas
- Add on the resource-scarcity parameter γ
- From those derive the Malthusian population growth rate $n^{*mal} = \gamma h$
- Then derive the Malthusian standard of living y^{*mal}
- Then derive the Malthusian population and labor force L_t^{*mal}

Dynamics?

Both population adjustment and the adjustment of the capital stock happen at about the same rate:

- What if $L_t < L_t^{*mal}$ and so $y > y^{*mal}$?
 - Population growth n is high...
 - So E declines...
 - And thus y falls—how fast depends on β/γ ...
 - Complications to dynamics as κ falls and then rises...
- <<https://nbviewer.jupyter.org/github/braddelong/LS2019/blob/master/2019-09-06-210a-ancient-intro.ipynb>>
- <<https://github.com/braddelong/LS2019/blob/master/2019-09-06-210a-ancient-intro.ipynb>>
- <https://github.com/braddelong/LS2019/blob/master/2019-08-17-Ancient_Economies.ipynb>
- <https://github.com/braddelong/long-form-drafts/blob/master/malthusian_convergence.ipynb>

Changeup Review: Solow Model Essentials

Lecture Notes: <<https://www.bradford-delong.com/2020/01/lecture-notes-the-solow-growth-model-the-history-of-economic-growth-econ-135.html>>

Let's assume three things about the relationship between an economy's resources and the total output it produces and income it generates:

1. A proportional increase in the economy's capital intensity κ , measured by the capital stock divided by total production $\kappa = K/Y$, will carry with it the same (smaller) proportional increase in income and production Y no matter how rich and productive the economy is. A 1% increase in capital intensity will always increase income and production by the same proportional amount θ .
2. If two economies have the same capital intensity, defined as the same capital-output ratio κ , and have the same level of technology- and organization-driven efficiency-of-labor E , then the ratio of their levels of income and output will be equal to the ratio of their labor forces L .
3. If two economies have the same capital intensity, defined as the same capital-output ratio κ , and have the same labor forces L , then the ratio of their levels of income and output will be equal to the ratio of their technology- and organization-driven efficiencies-of-labor E

Solow Model Basics: Notes

$$(2.1.2) Y = \kappa^\theta E L ; (2.1.3) y = \kappa^\theta E ; (2.1.1) \kappa = \frac{K}{Y}$$

The code in the nbViewer documents is static. But you should also look at:

- <[http://datahub.berkeley.edu/user-redirect/interact?
account=braddelong&repo=long-form-
drafts&branch=master&path=solow-model-2-basics.ipynb](http://datahub.berkeley.edu/user-redirect/interact?account=braddelong&repo=long-form-drafts&branch=master&path=solow-model-2-basics.ipynb)>
- <[http://datahub.berkeley.edu/user-redirect/interact?
account=braddelong&repo=long-form-
drafts&branch=master&path=solow-model-3-growing.ipynb](http://datahub.berkeley.edu/user-redirect/interact?account=braddelong&repo=long-form-drafts&branch=master&path=solow-model-3-growing.ipynb)>
- <[http://datahub.berkeley.edu/user-redirect/interact?
account=braddelong&repo=long-form-
drafts&branch=master&path=solow-model-4-using.ipynb](http://datahub.berkeley.edu/user-redirect/interact?account=braddelong&repo=long-form-drafts&branch=master&path=solow-model-4-using.ipynb)>

The Rest of the Model: Growth Rates

$$\frac{dE}{dt} = gE$$

$$\frac{dL}{dt} = g_L L = nL$$

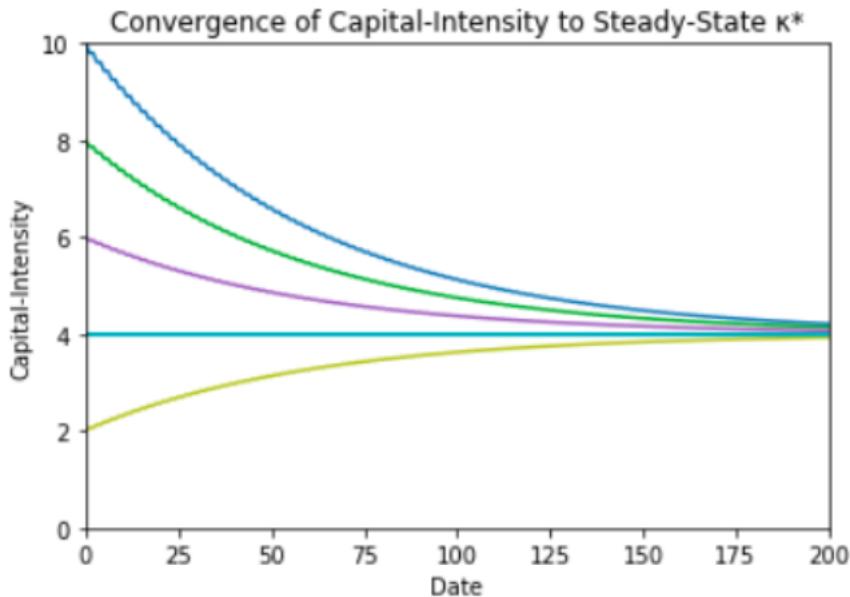
$$\frac{dK}{dt} = sY - \delta K = \left(\frac{s}{\kappa} - \delta \right) K$$

Variables change over time:

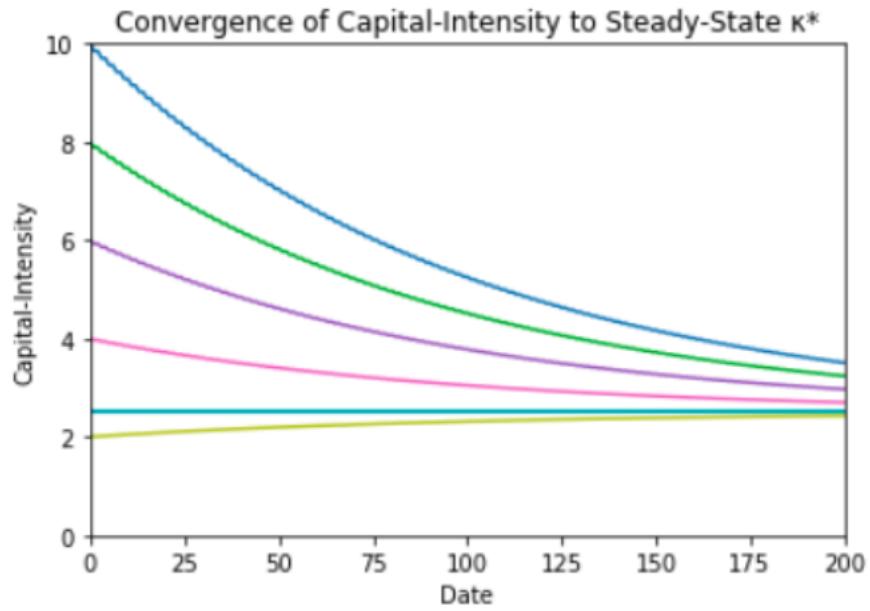
- growth of labor g_L : proportional at a constant n (for now)
- growth of labor efficiency g_E : proportional at a constant g (for now)
- rate of change of capital: savings minus depreciation
 - growth of capital $g_K = s/\kappa - \delta$
- What do these mean?

Now let's look at the rate of change of capital-intensity κ as a function of the level of capital-intensity κ , for constant n, g, s, δ , and θ ...

Solving the Model



```
k_max = 10
κ = k_max
for i in range(5):
    cg = κ_convergence_graph(κ_0=κ, s = 0.20, n = 0.01,
                             g = 0.015, δ = 0.025, θ = 1/2, T = 200)
    cg.draw()
    κ = κ-2
```



```
k_max = 10
κ = k_max
for i in range(5):
    cg = κ_convergence_graph(κ_0=κ, s = 0.15, n = 0.02,
                             g = 0.015, δ = 0.025, θ = 2, T = 200)
    cg.draw()
    κ = κ-2
```

Balanced-Growth Equilibrium: Steady-State Capital-Intensity κ^*

$$(1.16) \quad \kappa^* = \frac{s}{n+g+\delta}$$

This κ^* we define as the *steady-state balanced-growth equilibrium* value of capital-intensity in the Solow growth model. If the capital-intensity $\kappa = \kappa^*$, then it is constant, and the economy is in balanced growth, with Y and K growing at the rate $n+g$, E and y growing at the rate g , and L growing at the rate n .

Along the Balanced-Growth Path

Everything except κ —which is constant—grows at a constant proportional rate: either n , or g , or $n+g$;

- Labor force L grows at n
- Income per worker y and the efficiency of labor E grow at g
- Total income Y and the capital stock K grow at $n+g$

$$E_t^* = e^{gt} E_0$$

$$L_t^* = e^{nt} L_0$$

$$Y_t^* = (\kappa^*)^\theta E_t L_t = (\kappa^*)^\theta e^{gt} E_0 e^{nt} L_0 = (s/(n + g + \delta))^\theta e^{gt} E_0 e^{nt} L_0$$

$$K_t^* = \kappa^* Y_t^* = (s/(n + g + \delta))^{(1+\theta)} e^{gt} E_0 e^{nt} L_0$$

$$y_t^* = (\kappa^*)^\theta E_t = (\kappa^*)^\theta e^{gt} E_0 = (s/(n + g + \delta))^\theta e^{gt} E_0$$

Convergence to Steady-State Capital-Intensity

$$(1.18) \frac{d\kappa}{dt} = -\frac{n+g+\delta}{1+\theta}(\kappa - \kappa^*)$$

If we have knowledge of the initial level of an economy's capital-intensity— $\kappa = \kappa_0$ at some initial moment we index as zero—and if n, g, s, δ , and θ are constant, it immediately follows that at every time $t > 0$:

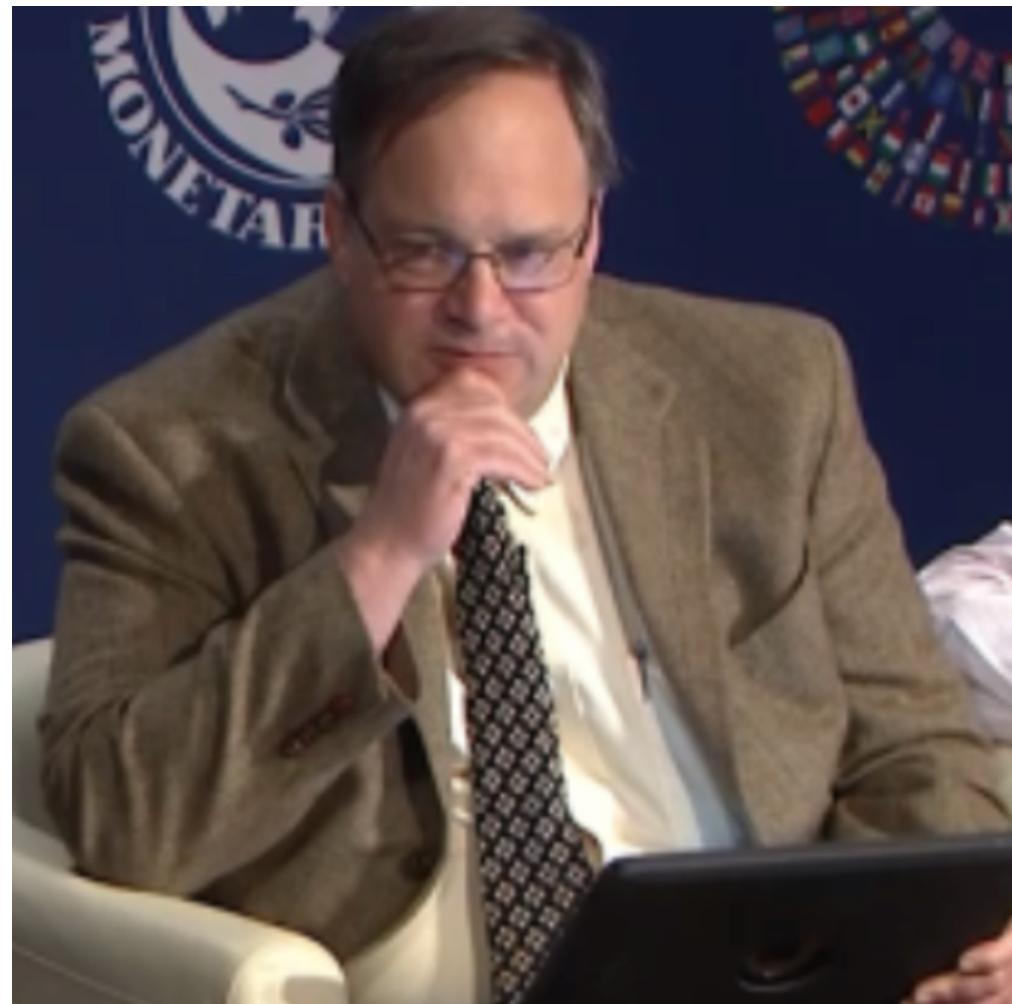
$$(1.21) \kappa_t = \kappa^* + e^{-[(n+g+\delta)/(1+\theta)]t}(\kappa_0 - \kappa^*)$$

(1.18) holds always, for that moment's values of n, g, δ, θ , and s , whatever they may be.

(1.21) holds only while n, g, δ, θ , and s are constant. If any of them change, you then have to recalibrate and recompute, with a new initial value of κ equal to its value when the model's parameters jumped, and a new and different value of κ^* .

Catch Our Breath...

- Ask a couple of questions?
- Make a couple of comments?
- Any more readings to recommend?



Solow-Malthus Model Application

How do we make sense of the fact that people were ingenious and inventive back before 1500, and yet standards of living did not increase?

- Yet we also had “efflorescences”
- Yet efflorescences are then followed by declines—not by leveling-up elsewhere
 - The Iron Age dark age
 - The fall of the Roman Empire (and of the Han dynasty)
 - Babylon:
 - Baghdad
 - The Maya
 - The Black Death, and the conquistadores and their diseases...

Babylon, “The Gate of the Gods”: Owls and Satyrs and Wild Beasts and Dragons

Isaiah 13:

- “The burden of Babylon, which Isaiah the son of Amoz did see....
- “Babylon, the glory of kingdoms, the beauty of the Chaldees' excellency, shall be as when God overthrew Sodom and Gomorrah. It shall never be inhabited, neither shall it be dwelt in from generation to generation: neither shall the Arabian pitch tent there; neither shall the shepherds make their fold there. But wild beasts of the desert shall lie there; and their houses shall be full of doleful creatures; and owls shall dwell there, and satyrs shall dance there.
- “And the wild beasts of the islands shall cry in their desolate houses, and dragons in their pleasant palaces: and her time is near to come, and her days shall not be prolonged...”



Understanding the Solow-Mathus Equilibrium: Prosperity

Malthusian equilibrium income level

$$y^{*mal} = \phi y^{sub} \left(1 + \frac{n^{*mal}}{\beta} \right) = \phi y^{sub} \left(1 + \frac{\gamma h}{\beta} \right)$$

True zpg subsistence

Sensitivity of productivity to population

Rate of useful ideas creation

Taste for luxuries

Responsiveness of population growth to prosperity

```
graph TD; A[Malthusian equilibrium income level] --> B[y*^mal = phi * y^sub * (1 + n*^mal / beta)]; C[True zpg subsistence] --> D[phi * y^sub]; E[Sensitivity of productivity to population] --> D; F[Rate of useful ideas creation] --> G[(1 + gamma h / beta)]; H[Taste for luxuries] --> I[n*^mal / beta]; J[Responsiveness of population growth to prosperity] --> K[gamma h / beta]
```

Notes:

-

Understanding the Solow-Malthus Equilibrium: Population and Labor Force

$$L_t^{*mal} = \left[\left(\frac{H_t}{y^{sub}} \right) \left(\frac{s}{\delta} \right)^\theta \left(\frac{1}{\phi} \right) \left[\frac{1}{(1+\gamma h/\delta)^\theta} \frac{1}{(1+\gamma h/\beta)} \right] \right]^\gamma$$

The Malthusian equilibrium population

The ratio of knowledge to subsistence income

The salience of capital in determining productivity

The ratio of savings to depreciation

Nuisance terms

The inverse of the taste for luxury

The extent to which population depresses productivity

Notes:

-

How Does This System React to Shocks?

$$n^{*mal} = \gamma h$$

$$y^{*mal} = \phi y^{sub} \left(1 + \frac{n^{*mal}}{\beta} \right) = \phi y^{sub} \left(1 + \frac{\gamma h}{\beta} \right)$$

$$L_t^{*mal} = \left[\left(\frac{H_t}{y^{sub}} \right) \left(\frac{s}{\delta} \right)^\theta \left(\frac{1}{\phi} \right) \left[\frac{1}{(1+\gamma h/\delta)^\theta} \frac{1}{(1+\gamma h/\beta)} \right] \right]^\gamma$$

Let's think of some:

- a sudden major plague...
- the rise of a civilization that carries with it norms of property and law and commerce, and thus a rise in s
- the rise of an empire that raises s via the imperial peace and that also creates a rise in the taste for luxuries ϕ (and possibly reduces biological subsistence y^{sub})
- a shift in the rate of ideas growth h ...
- a shift in sociology that alters subsistence y^{sub} ...

“class malthusian”

Files:

- <https://nbviewer.jupyter.org/github;braddelong/LS2019/blob/master/2019-10-14-Ancient_Economies.ipynb>
- <http://datahub.berkeley.edu/user-redirect/interact?account=braddelong&repo=LS2019&branch=master&path=2019-10-14-Ancient_Economies.ipynb>

```
In [5]: # THE COMING OF AN IMPERIAL PEACE IN YEAR 250
# φ = 1.25, s = 0.25; Δφ = +0.25, Δs = +0.10

num_rows, num_cols = 3, 2
fig, axes = plt.subplots(num_rows, num_cols, figsize=(12, 12))
for i in range(num_rows):
    for j in range(num_cols):
        for scenario in {'base', 'with shock'}:
            seq = m.gen_seq(T1, var = figcontents[i,j][0], log = figcontents[i,j][1])
            lb = f'{scenario}'
            if scenario == 'with shock':
                m.φ = 1.25
                m.s = 0.25
            seq = np.append(seq, m.gen_seq(T2, var = figcontents[i,j][0], log = figcontents[i,j][1]))
            axes[i,j].plot(seq, 'o-', lw=2, alpha=0.5, label=lb)
            axes[i,j].set(title=figcontents[i,j][1])

axes[(0,0)].legend(loc='upper center', bbox_to_anchor=(1.1,1.3))
plt.suptitle('Malthusian Model: Simulation Run with Coming of Imperial F')
plt.show()
```

```
class malthusian:
    """
    Implements the Malthusian Model with:
    1. population growth
    n = β*(y/(φ ysub)-1)

    2. growth of efficiency-of-labor
    g = h-n/y
    """
    def __init__(self,
                 L = 1,                      # initial labor force
                 E = 1/3,                     # initial efficiency of labor
                 K = 3.0,                     # initial capital stock

                 β = 0.025,                  # determinants of n (population growth):
                                         # responsiveness of population growth to increased prosperity.
                 φ = 1,                      # luxuries parameter
                 ysub = 1,                   # subsistence level

                 h = 0,                      # determinants of g (efficiency-of-labor growth):
                                         # rate at which useful ideas are generated
                                         # effect-of-resource scarcity parameter
                 Y = 2.0,                    

                 s = 0.15,                   # savings-investment rate
                 α = 0.5,                    # orientation-of-growth-toward-capital parameter
                 δ = 0.05,                   # depreciation rate on capital parameter
                 ):
        pass
```

What happens to parameters with...

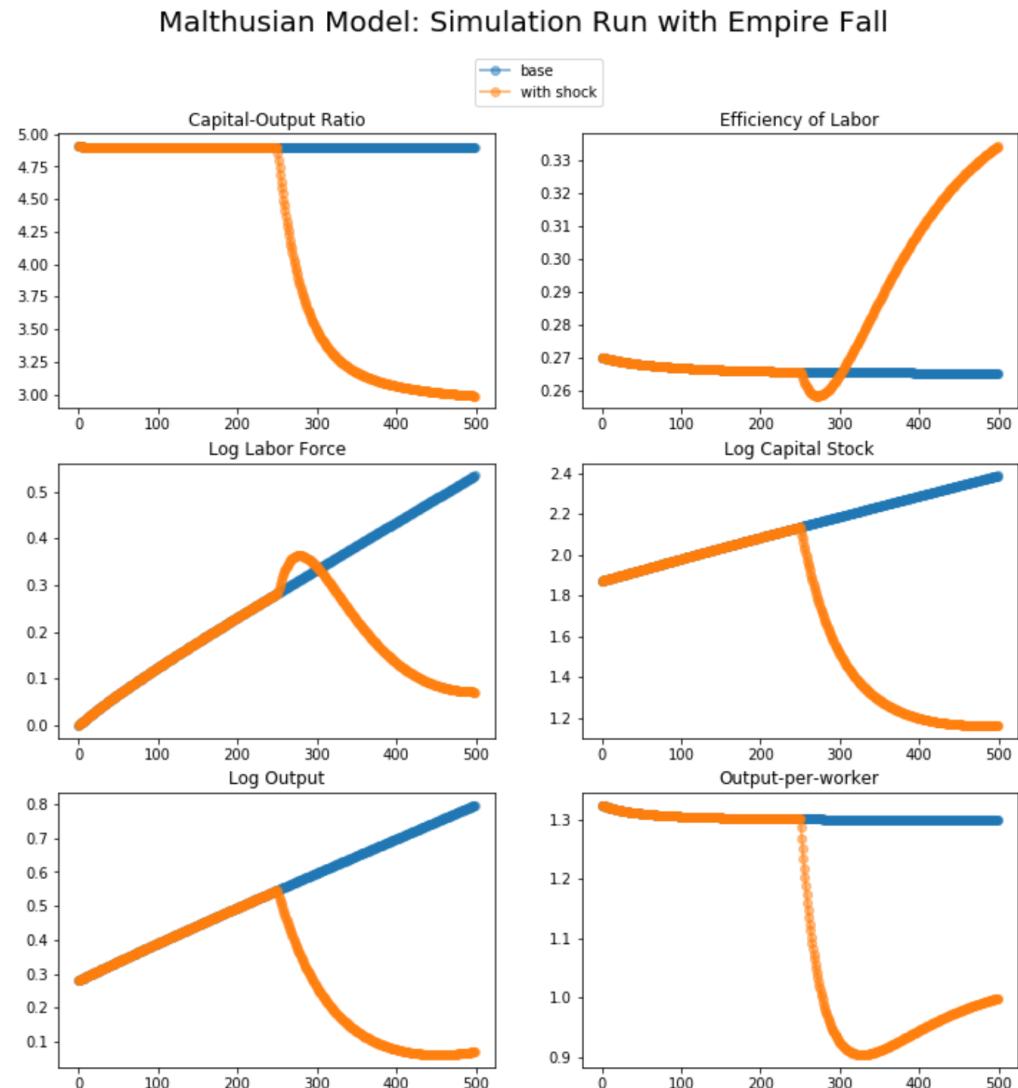
- ...a sudden major plague: $L \downarrow$ (and $K \downarrow$ too)
- ...the rise of a civilization that carries with it norms of property and law and commerce: $s \uparrow$
- ...the rise of an empire that brings both civilization in the form of an imperial peace, and that also creates a taste for luxuries: $s \uparrow, \phi \uparrow$, (and possibly $y_{sub} \downarrow$ as well)
- ...a shift in the rate of ideas growth: $h \uparrow$
- ...a shift in sociology that alters subsistence: $y_{sub} \uparrow$ or \downarrow

Steady-State and Along the Transition Path

The fall of an empire:

- <https://nbviewer.jupyter.org/github/braddejong/LS2019/blob/master/2019-10-14-Ancient_Economies.ipynb>

- A decline in inequality, taste for luxuries, and taste for urban living:
 $\Delta\varphi = -0.25$
- A decline in law-and-order that produces a sharp fall in the savings rate: $\Delta s = -0.10$



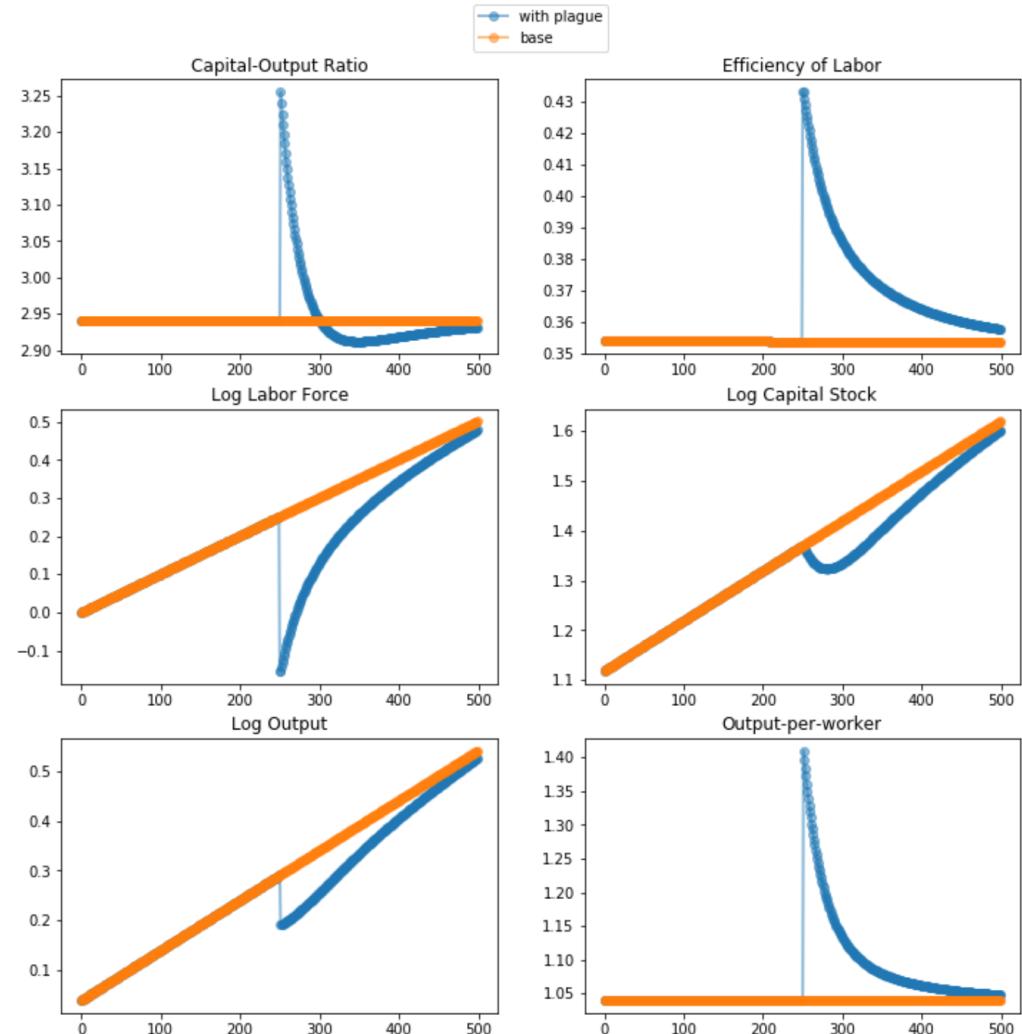
Steady-State and Along the Transition Path II

A civilization-wide great plague:

- <https://nbviewer.jupyter.org/github/braddelong/LS2019/blob/master/2019-10-14-Ancient_Economies.ipynb>

- A third of the population is carried off: $\Delta L = -0.33$

Malthusian Model: Simulation Run with Plague in 250



Assignment: What Is Economics? Paper

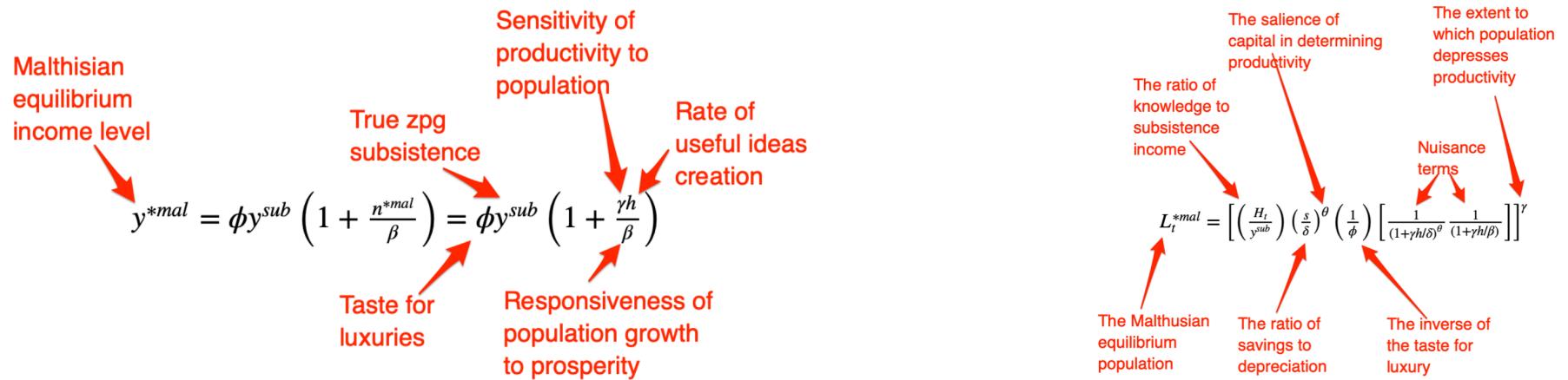
200-300 words; due Feb 1; <<https://bcourses.berkeley.edu/courses/1487685/assignments/8065184>>

- UCLA professor Stephen Bainbridge believes that Partha Dasgupta's *Economics: A Very Short Introduction* is a bad book. He wrote, in his Amazon review:
 - 1.0 out of 5 stars: Very disappointing, September 25, 2007: By Stephen M. Bainbridge: "If you're looking for a VSI to Econ 101 and 102, skip this book. The treatment of microeconomic basics consists of exactly 14 pages. Macroeconomic theory gets a whopping 4 pages. The rest consists mainly of a political tract on wealth and poverty. It's the first VSI whose title amounts to a misrepresentation..."
- Partha Dasgupta, of course disagrees. Which do you tend to agree with? (You can say that you are in the middle, but setting out and defending an "in the middle" position is actually very hard here.) Explain why and to what extent you come down on Dasgupta's or on Bainbridge's side of this dispute.
- Justify your opinions by setting out what you think economics is, or ought to be.

Big Ideas

Takeaways from this lecture:

- People were ingenious and inventive back before 1500, yet standards of living did not increase
- Populations, however, did
 - Slowly
- We explain this via:
 - Why is \mathbf{h} so low?
 - Natural resource scarcity: more heads means smaller farms which offset the productive benefit of better ideas: efficiency of labor growth $\mathbf{g} = \mathbf{h} - \mathbf{n}/\gamma$, where \mathbf{h} is ideas growth, \mathbf{n} is population and labor force growth, and γ is the resource scarcity-pressure parameter.
 - Population pressure: before the **demographic transition**, higher standards of living mean faster population growth: $\mathbf{n} = \beta(y/(\Phi y^{\text{sub}}) - 1)$, where Φ is taste for luxuries (inequality! urbanization!), y^{sub} is the income level at which we have zpg on average, and β is the population-responsiveness parameter
- Malthusian equilibrium, with efflorescences and declines:

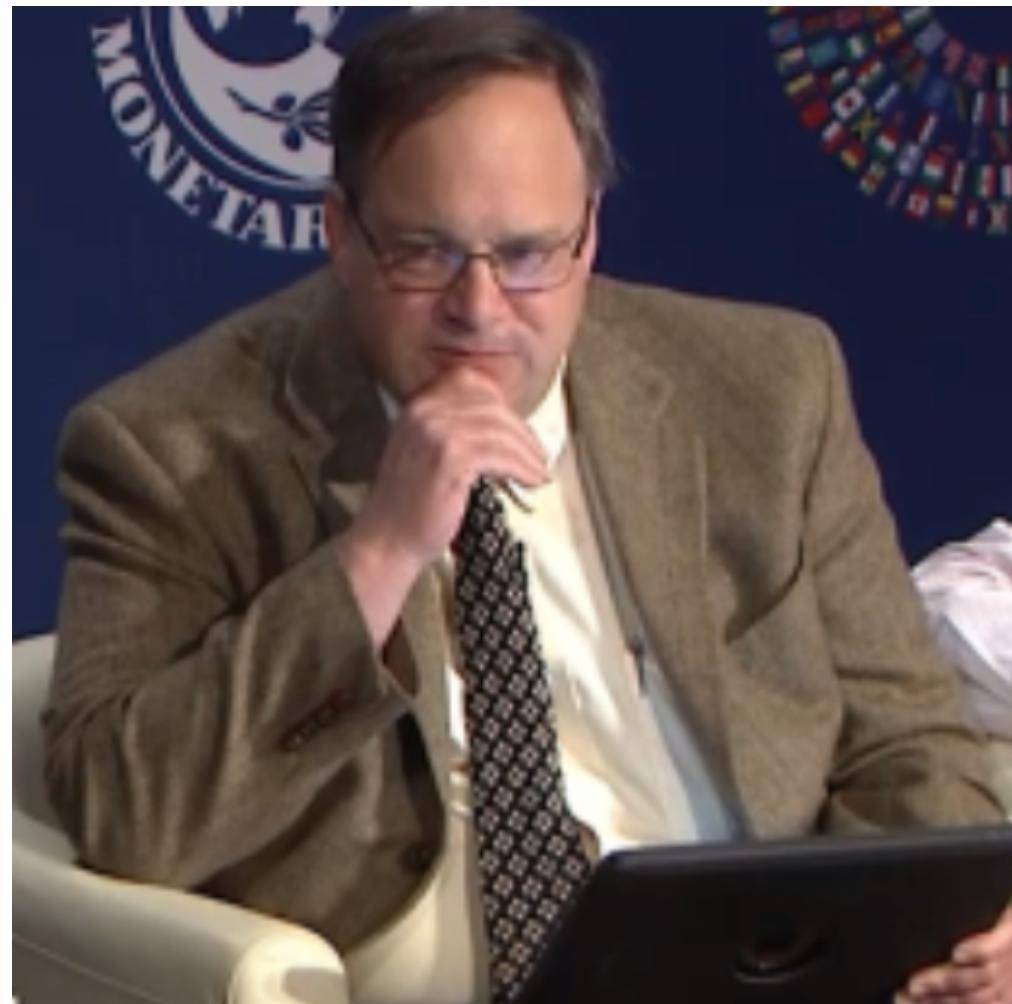


Additional Readings

- Gregory Clark (2005): The Condition of the Working Class in England, 1209-2003 <<https://delong.typepad.com/files/clark-condition.pdf>>...
- Ian Morris (2010): Why the West Rules—For Now, chapter 3: Taking the Measure of the Past <<https://delong.typepad.com/files/morris-rules-3.pdf>>...
- Patricia Crone: Pre-Industrial Societies, selections <<https://delong.typepad.com/files/crone-pre-selections.pdf>>...
- C.I. Jones: The Facts of Economic Growth <<https://web.stanford.edu/~chadj/facts.pdf>>...

Catch Our Breath...

- Ask a couple of questions?
- Make a couple of comments?
- Any more readings to recommend?



Notes

