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How to teach special relativity

I have for long thought that if I had the opportunity to teach this subject, I would emphasize the continuity with earlier ideas. Usually it is the discontinuity which is stressed, the radical break with more primitive notions of space and time. Often the result is to destroy completely the confidence of the student in perfectly sound and useful concepts already acquired[1].

If you doubt this, then you might try the experiment of confronting your students with the following situation[2]. Three small spaceships, A, B, and C, drift freely in a region of space remote from other matter, without rotation and without relative motion, with B and C equidistant from A (Fig. 1).

On reception of a signal from A the motors of B and C are ignited and they accelerate gently[3] (Fig. 2).

Let ships B and C be identical, and have identical acceleration programmes. Then (as reckoned by an observer in A) they will have at every moment the same velocity, and so remain displaced one from the other by a fixed distance. Suppose that a fragile thread is tied initially between projections from B and C (Fig. 3). If it is just long enough to span the required distance initially, then as the rockets speed up, it will become too short, because of its need to Fitzgerald contract, and must finally break. It must break when, at a sufficiently high velocity, the artificial prevention of the natural contraction imposes intolerable stress.

Fig. 1.

Is it really so? This old problem came up for discussion once in the CERN canteen. A distinguished experimental physicist refused to accept that the thread would break, and regarded my assertion, that indeed it would, as a personal misinterpretation of special relativity. We decided to appeal to the CERN Theory Division for arbitration, and made a (not very systematic) canvas of opinion in it. There emerged a clear consensus that the thread would **not** break!

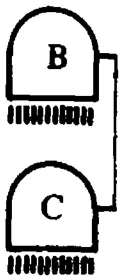
Of course many people who give this wrong answer at first get the right answer on further reflection. Usually they feel obliged to work out how things look to observers B or C. They find that B, for example, sees C drifting further and further behind, so that a given piece of thread can no longer span the distance. It is only after working this out, and perhaps only with a residual feeling of unease, that such people finally accept a conclusion which is perfectly trivial in terms of A's account of things, including the Fitzgerald contraction. It is my impression that those with a more classical education, knowing something of the reasoning of Larmor, Lorentz, and Poincaré, as well as that of Einstein, have stronger and sounder instincts. I will try to sketch here a simplified version of the Larmor–Lorentz–Poincaré approach that some students might find helpful.

Some familiarity with Maxwell's equations is assumed, so that the calculation of the field of a moving point charge can be followed, or at least the result accepted without mystification. For a charge Ze moving with constant velocity V along the z axis the nonvanishing field components are:

$$\left.\begin{aligned} E_z &= Zez'(x^2+y^2+z'^2)^{-3/2} \\ E_x &= Zex(x^2+y^2+z'^2)^{-3/2}(1-V^2/c^2)^{-1/2} \\ E_y &= Zey(x^2+y^2+z'^2)^{-3/2}(1-V^2/c^2)^{-1/2} \\ B_x &= -(V/c)E_y \\ B_y &= +(V/c)E_x \end{aligned}\right\} \tag{1}$$

Fig. 2.

Fig. 3.



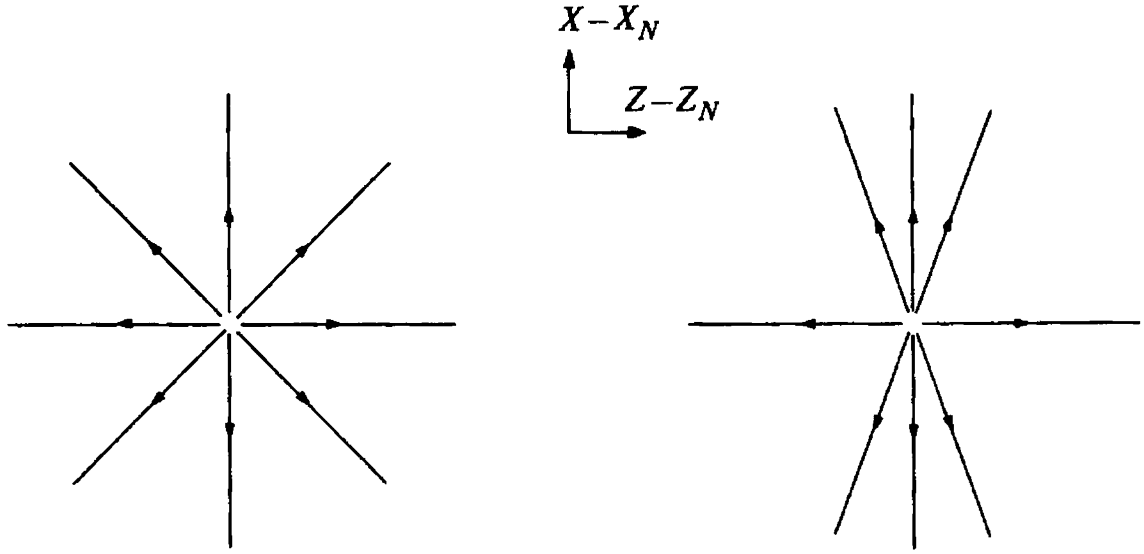
where

$$z' = (z - z_N(t))(1 - V^2/c^2)^{-1/2} \tag{2}$$

and $z_N(t)$ is the position of the charge at time t. For a charge at rest, $V = 0$, this is just the familiar Coulomb field, spherically symmetrical about the source. But when the source moves very quickly, so that V^2/c^2 is not very small, the field is no longer spherically symmetrical. The magnetic field is transverse to the direction of motion and, roughly speaking, the system of lines of electric field is flattened in the direction of motion (Fig. 4).

In so far as microscopic electrical forces are important in the structure of matter, this systematic distortion of the field of fast particles will alter the internal equilibrium of fast moving material. It is to be expected therefore that a body set in rapid motion will change shape. Such a change of shape, the Fitzgerald contraction, was in fact postulated on empirical grounds by G. F. Fitzgerald in 1889 to explain the results of certain optical experiments.

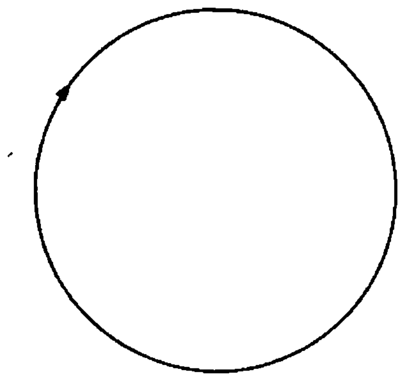
Fig. 4.



electric field of source at rest

and of source moving in z direction

Fig. 5.



The simplest piece of matter that we can discuss in this connection is a single atom. In the classical model of such an atom a number of electrons orbit around a nucleus. For simplicity take only one electron, and ignore the effect, on the relatively masssive nucleus, of the field of the electron. The dynamical problem is then that of the motion of the electron in the field of the nucleus. Let us start with the nucleus at rest and the electron, for simplicity, describing a circular orbit (Fig. 5).

What happens to this orbit when the nucleus is set in motion?[4]

If the acceleration of the nucleus is quite gentle, its field differs only slightly from (1). Moreover, the accurate expression is known[5].

In this field we have to solve the equation of motion for the electrons

$$\frac{d\mathbf{p}}{dt} = -e(\mathbf{E} + c^{-1}\dot{\mathbf{r}}_e \times \mathbf{B}) \tag{3}$$

where $\mathbf{r}_e$ is the electron position and the fields in (3) are evaluated at that position. At low velocity, momentum and velocity are related by

$$\dot{\mathbf{r}}_e = \mathbf{p}/m \tag{4}$$

But this familiar formula proves inadequate for high velocities. It would imply that by acting for long enough with a given electric field an electron could be taken to arbitrarily high velocity. But experimentally it is found that the velocity of light is a limiting value. The experimental facts are fitted by a modified formula proposed by Lorentz

$$\dot{\mathbf{r}}_e = \mathbf{p}/\sqrt{m^2 + \mathbf{p}^2 c^{-2}} \tag{5}$$

This is what we take together with (3).

One can programme a computer to integrate these equations. Let the computer print out as a function of time the displacement

$$\mathbf{r}_e(t) - \mathbf{r}_N(t)$$

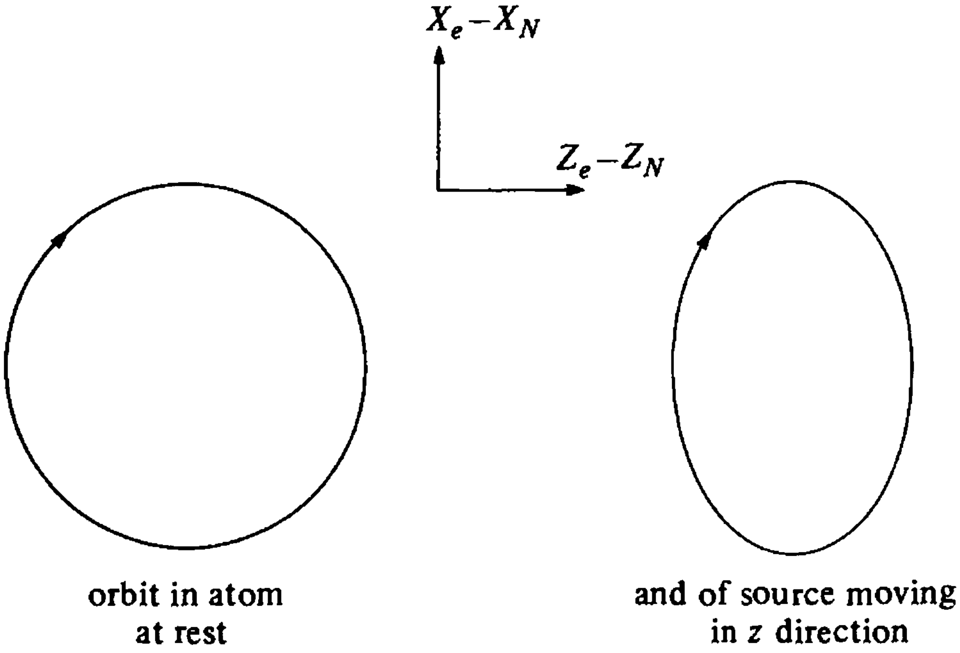
of the electron from the nucleus. Suppose the nucleus to move along the z axis, and the electron to orbit in the xz plane. Then if the acceleration of the nucleus is sufficiently gradual[6], the initially circular orbit deforms slowly into an ellipse, as in Fig. 6.

That is to say that the orbit retains its original extension in the direction transverse to the motion of the system as a whole, but contracts in the direction along that motion. The contraction is to a fraction

$$\sqrt{1 - V^2/c^2} \tag{6}$$

of the original – the Fitzgerald contraction – where V is the velocity of

Fig. 6.



the nucleus during the orbit in question. Moreover, this is performed in a period exceeding the original period by a factor

$$1/\sqrt{1 - V^2/c^2} \tag{7}$$

– the time-dilation of J. Larmor (1900).

If the period of the system at rest is T, then the total number of revolutions during a journey of time t with proton velocity $V(t)$ is

$$T^{-1} \int_0^t \mathrm{d}\tau \sqrt{1 - c^{-2} V(\tau)^2} \tag{8}$$

– which is less than that for a similar system at rest, even if the moving system is both initially and finally also at rest and initially and finally in the same position. This straightforward result of computation is the origin of the 'paradox' of the travelling twin (Le Voyageur de Langevin, en français).

These results suggests that it may be useful to describe the moving system in terms of new variables which incorporate the Fitzgerald and Larmor effects:

$$\left.\begin{aligned} z' &= (z - z_N(t))/\sqrt{1 - c^{-2}V(t)^2} \\ x' &= x \quad y' = y \\ t' &= \int_0^t \mathrm{d}\tau \sqrt{1 - c^{-2}V(t)^2} - c^{-2}V(t)Z' \end{aligned}\right\} \tag{9}$$

The motivation for the last term in the definition of t' is not obvious, but emerges from more detailed examination of the orbit. Including this term, the orbit

$$z'_e(t') \quad , \quad x'_e(t') \tag{10}$$

is not merely circular, with period T, but is swept out with constant angular velocity. That is, *the description of the orbit of the moving atom in terms of the primed variables is identical with the description of the orbit of the stationary atom in terms of the original variables.*

As regards the electromagnetic field we have already profited from the use of the variable z' in writing (1). Going further in this direction, one can introduce

$$\left.\begin{aligned} E'_x &= (E_x - c^{-1}VB_y)/\sqrt{1-c^{-2}V^2} \\ E'_y &= (E_y + c^{-1}VB_x)/\sqrt{1-c^{-2}V^2} \\ E'_z &= E_z \\ B'_x &= (B_x + c^{-1}VE_y)/\sqrt{1-c^{-2}V^2} \\ B'_y &= (B_y - c^{-1}VE_x)/\sqrt{1-c^{-2}V^2} \\ B'_z &= B_z \end{aligned}\right\} \tag{11}$$

Then it is easy to check that *the expression of the field of the uniformly moving charge in terms of the primed variables is identical with the expression of the field of the stationary charge in terms of the original variables.*

We have been speaking of a *gently* accelerated atom. So the velocity V always remains essentially constant during many revolutions of the electron. During any such interval, one can arrange that

$$\int_0^t \mathrm{d}\tau \sqrt{1-c^{-2}V(\tau)^2} = t\sqrt{1-c^{-2}V^2} \tag{12}$$

$$z_N(t) = Vt \tag{13}$$

by a suitable choice of the origin of z and t. Then (9) can be rewritten

$$\left.\begin{aligned} z' &= (z - Vt)/\sqrt{1-V^2/c^2} \\ x' &= x \\ y' &= y \\ t' &= (t - Vx/c^2)/\sqrt{1-V^2/c^2} \end{aligned}\right\} \tag{14}$$

This is then the standard form of what is called a *Lorentz transformation.* That the use of such variables enables the moving atom to be described by the functions appropriate to the stationary atom is an illustration of the following exact mathematical fact. When Maxwell's equations

$$\frac{1}{c}\frac{\partial E_x}{\partial t}=\frac{\partial B_z}{\partial y}-\frac{\partial B_y}{\partial z}, \text{ etc.} \tag{15}$$

and the Lorentz equations

$$\left.\begin{aligned}\frac{\mathrm{d}\mathbf{p}}{\mathrm{d}t}&=-e(\mathbf{E}+c^{-1}\dot{\mathbf{r}}_e\times\mathbf{B})\\ \frac{\mathrm{d}\mathbf{r}_e}{\mathrm{d}t}&=\mathbf{p}/\sqrt{m^2+c^{-2}\mathbf{p}^2}\end{aligned}\right\} \tag{16}$$

are expressed in terms of the new variables (11) and (14) *they have exactly the same form as before*

$$\left.\begin{aligned}\frac{1}{c}\frac{\partial E'_x}{\partial t'}&=\frac{\partial B'_z}{\partial y'}-\frac{\partial B'_y}{\partial z'}, \text{ etc.}\\ \frac{\mathrm{d}\mathbf{p}'}{\mathrm{d}t'}&=-e(\mathbf{E}'+c^{-1}\dot{\mathbf{r}}'_e\times\mathbf{B}')\\ \frac{\mathrm{d}\mathbf{r}'_e}{\mathrm{d}t'}&=\mathbf{p}'/\sqrt{m^2+c^{-2}\mathbf{p}'^2}\end{aligned}\right\} \tag{17}$$

(where the last equation can be taken as defining $\mathbf{p}'$). The equations are said to be *Lorentz invariant.* From any solution of the original equations, involving certain mathematical functions (e.g., the Coulomb field and the circular orbit in the stationary atom), one can construct a new solution by putting primes on all the variables and then eliminating these primes by means of (11) and (14) (giving, e.g., the flattened field and ellipitcal orbit of the moving atom). Moreover, by a trivial extension this reasoning applies not only to a single electron interacting with a single electromagnetic field, but to any number of charged particles, each interacting with the fields of all others. This allows an extension to very complicated systems of some of the results described above for the simple atom. Given any state of the complicated system, there is a corresponding 'primed' state which is in overall motion with respect to the original, shows the Fitzgerald contraction, and the Larmor dilation. Suppose, for example, in the original state all particles are permanently inside a region bounded by

$$z=\pm L/2$$

then the corresponding primed state has boundaries

$$z'=\pm L/2$$

or from (14)

$$z = Vt \pm 1/2L\sqrt{L - V^2/c^2}$$

i.e., they move with the velocity V and are closer together by the Fitzgerald factor.

Suppose next that in the original state something happens (e.g., an electron passes) at a place $x = x_1, y = y_1, z = z_1$ at time t_1, and again at the same place at time t_2. Then the corresponding events in the primed state occur at

$$x' = x_1 \quad y' = y_1 \quad z' = z_1 \quad t' = t_1, t_2$$

or (solving (14)) at

$$x = x_1 \quad y = y_1$$

$$z = \frac{z_1 + Vt_1}{\sqrt{1 - V^2/c^2}}, \frac{z_1 + Vt_2}{\sqrt{1 - V^2/c^2}}$$

$$t = \frac{t_1 + Vz_1/c^2}{\sqrt{1 - V^2/c^2}}, \frac{t_2 + Vz_1/c^2}{\sqrt{1 - V^2/c^2}}$$

The place of occurrence moves with velocity V, and the time interval between the two events increases by the Larmor factor.

Can we conclude then that an arbitrary system, set in motion, will show precisely the Fitzgerald and Larmor effects? Not quite. There are two provisos to be made.

The first is this: the Maxwell–Lorentz theory provides a very inadequate model of actual matter, in particular solid matter. It is not possible in a classical model to reproduce the empirical stability of such matter. Moreover, things are made worse when radiation reaction is included. Moving charges in general radiate energy and momentum, and because of this there are extra small terms in the equation of motion. Even in the simple hydrogen atom the electron then spirals in towards the proton instead of remaining in a stable orbit. These problems were among those which led to the replacement of classical by quantum theory. Moreover, even in the quantum theory electromagnetic interactions turn out to be not the only ones. For example, atomic nuclei are apparently held together by quite different 'strong' interactions. We do not need to get involved in these details if we assume with Lorentz that the *complete theory* is Lorentz invariant, in that the equations are unchanged by the change of variables (14), supplemented by some generalization of (13) to cover all the quantities in the theory. Then for any state there is again a corresponding primed state, showing the Fitzgerald and Larmor effects.

The second proviso is this. Lorentz invariance alone shows that for any state of a system at rest there is a corresponding 'primed' state of that system in motion. But it does not tell us that if the system is set anyhow in motion, it will actually go into the 'prime' of the original state, rather than into the 'prime' of some *other* state of the system at rest. In fact, it will generally do the latter. A system set brutally in motion may be bruised, or broken, or heated, or burned. For the simple classical atom similar things could have happened if the nucleus, instead of being moved smoothly, had been *jerked*. The electron could be left behind completely. Moreover, a given acceleration is or is not sufficiently gentle depending on the orbit in question. An electron in a small, high frequency, tightly bound orbit, can follow closely a nucleus that an electron in a more remote orbit – or in another atom – would not follow at all. Thus we can only assume the Fitzgerald contraction, etc., for a coherent dynamical system whose configuration is determined essentially by internal forces and only little perturbed by gentle external forces accelerating the system as a whole. Let us do so.

Then, for example, in the rocket problem of the introduction, the material of the rockets, and of the thread, will Lorentz contract. A sufficiently strong thread would pull the rockets together and impose Fitzgerald contraction on the combined system. But if the rockets are too massive to be appreciably accelerated by the fragile thread, the latter has to break when the velocity becomes sufficiently great.

So far we have discussed moving *objects*, but not yet moving *subjects*. The question of moving observers is not entirely academic. Quite apart from people in rockets, it seems reasonable to regard the earth itself, orbiting the sun, as moving – at least for much of the year[7]. The important point to be made about moving observers is this, given Lorentz invariance: *the primed variables, introduced above simply for mathematical convenience, are precisely those which would naturally be adopted by an observer moving with constant velocity who imagines herself to be at rest.* Moreover, such an observer will find that the laws of physics in these terms are precisely those that she learned when at rest (if she was taught correctly).

Such an observer will naturally take for the origin of space coordinates a point at rest with respect to herself. This accounts for the Vt term in the relation

$$z' = (z - Vt)/\sqrt{1 - V^2/c^2}$$

The factor $\sqrt{1 - V^2/c^2}$ is accounted for by the Fitzgerald contraction of her metre sticks. But will she not *see* that her metre sticks are contracted when laid out in the z direction – and even decontract when turned in the x direction? No, because the retina of her eye will also be contracted, so that just the same cells receive the image of the metre stick as if both stick and observer were at rest. In the same way she will not notice that her clocks have slowed down, because she will herself be thinking more slowly. Moreover, imagining herself to be at rest, she will not know that light overtakes her, or comes to meet her, with different relative velocities $c \pm v$. This will mislead her in synchronizing clocks at different places, so that she is led to think that

$$t' = \frac{t - Vz/c^2}{\sqrt{1 - V^2/c^2}}$$

is the real time, for with this choice light again *seems* to go with velocity c in all directions. This can be checked directly, and is also a consequence of the prime Maxwell equations. In measuring electric field she will use a test charge at rest with respect to her equipment, and so measure actually a combination of **E** and **B**. Defining both **E** and **B** by requiring what looks like the familiar effects on moving charged particles, she will be led rather to **E**′ and **B**′. Then she will be able to verify that all the laws of physics are as she remembers, at the same time confirming her own good sense in the definitions and procedures that she has adopted. If something does not come out right, she will find that her apparatus is in error (perhaps damaged during acceleration) and repair it.

Our moving observer O′, imagining herself to be at rest, will imagine that it is the stationary observer O who moves. And it is as easy to express his variables in terms of hers as vice versa

$$\left.\begin{aligned} x' &= x \quad y' = y \\ z' &= \frac{z - Vt}{\sqrt{1 - V^2/c^2}} \\ t' &= \frac{t' - Vz/c^2}{\sqrt{1 - V^2/c^2}} \end{aligned}\right\} \Leftrightarrow \left\{\begin{aligned} x &= x' \quad y = y' \\ z &= \frac{z' + Vt'}{\sqrt{1 - V^2/c^2}} \\ t &= \frac{t' + Vz'/c^2}{\sqrt{1 - V^2/c^2}} \end{aligned}\right.$$

Only the sign of V changes. She will say that *his* metre sticks have contracted, that *his* clocks run slow, and that *he* has not synchronized properly clocks at different places. She will attribute his use of wrong variables to these Fitzgerald–Larmor–Lorentz–Poincaré effects in *his* equipment. Her view will be logically consistent and in perfect accord with the observable facts. He will have no way of persuading her that she is wrong.

This completes the introduction to what has come to be called 'the special theory of relativity'. It arose from experimental failure to detect any change, in the apparent laws of physics in terrestrial laboratories, with the slowly changing orbital velocity of the earth. Of particular importance was the Michelson–Morley experiment, which attempted to find some difference in the apparent velocity of light in different directions.

We have followed here very much the approach of H. A. Lorentz. Assuming physical laws in terms of certain variables (t, x, y, z), an investigation is made of how things look to observers who, with their equipment, in terms of these variables, move. It is found that if physical laws are Lorentz invariant, such moving observers will be unable to detect their motion. As a result it is not possible experimentally to determine which, if either, of two uniformly moving systems, is really at rest, and which moving. All this for *uniform* motion: accelerated observers are not considered in the 'special' theory.

The approach of Einstein differs from that of Lorentz in two major ways. There is a difference of philosophy, and a difference of style.

The difference of philosophy is this. Since it is experimentally impossible to say which of two uniformly moving systems is *really* at rest, Einstein declares the notions 'really resting' and 'really moving' as meaningless. For him only the *relative* motion of two or more uniformly moving objects is real. Lorentz, on the other hand, preferred the view that there is indeed a state of *real* rest, defined by the 'aether', even though the laws of physics conspire to prevent us identifying it experimentally. The facts of physics do not oblige us to accept one philosophy rather than the other. And we need not accept Lorentz's philosophy to accept a Lorentzian pedagogy. Its special merit is to drive home the lesson that the laws of physics in any *one* reference frame account for all physical phenomena, including the observations of moving observers. And it is often simpler to work in a single frame, rather than to hurry after each moving object in turn.

The difference of style is that instead of inferring the experience of moving observers from known and conjectured laws of physics, Einstein starts from the *hypothesis* that the laws will look the same to all observers in uniform motion. This permits a very concise and elegant formulation of the theory, as often happens when one big assumption can be made to cover several less big ones. There is no intention here to make any reservation whatever about the power and precision of Einstein's approach. But in my opinion there is also something to be said for taking students along the road made by Fitzgerald, Larmor, Lorentz and Poincaré[8]. The longer road sometimes gives more familiarity with the country.

In connection with this paper I warmly acknowledge the counsels of

M. Bell, F. Farley, S. Kolbig, H. Wind, A. Zichichi and H. Øveras. I thank especially H. D. Deas for discussion of these ideas at an early stage.

Notes and references

1 Notes are to be ignored in a first reading.

2 E. Dewan & M. Beran, *Am. J. Phys.* **27**, 517, 1959. A. A. Evett & R. K. Wangsness, *Am. J. Phys.* **28**, 566, 1960. E. M. Dewan, *Am. J. Phys.* **31**, 383, 1963. A. A. Evett, *Am. J. Phys.* **40**, 1170, 1972.

3 Violent acceleration could break the thread just because of its own inertia while velocities are still small. This is not the effect of interest here. With gentle acceleration the breakage occurs when a certain *velocity* is reached, a function of the degree to which the thread permits stretching beyond its natural length.

4 This method of acceleration, applying somehow a force to the nucleus without any direct effect on the electron, is not very realistic. However, as explained later, it follows from Lorentz invariance and stability considerations that any sufficiently smooth acceleration process will produce the same Fitzgerald contraction and Larmor dilation. The student is invited to attach a meaning to this statement also in the more general cases of non-circular orbits and when the acceleration is not in the plane of the orbit.

5 For a source of charge Ze the fields are[9], in c.g.s. units,

$$\mathbf{E} = \frac{Ze}{s^3}\left\{\left(\mathbf{r} - r\frac{[\mathbf{v}]}{c}\right)\left(1 - \frac{[\mathbf{v}]^2}{c^2}\right) + \left(\left(\mathbf{r} - r\frac{[\mathbf{v}]}{c}\right) \times \frac{[\mathbf{A}]}{c^2}\right)\right\} \tag{5.1}$$

$$\mathbf{B} = \mathbf{r} \times \mathbf{E}/r$$

where

$$\mathbf{r} = \mathbf{r}_e - [\mathbf{r}_N]$$

$$s = r - \mathbf{r}\cdot[\mathbf{v}]/c.$$

These are the fields at position $\mathbf{r}_e$ at time t due to a source which *at the retarded time*

$$t - r/c \tag{5.2}$$

had position, velocity, and acceleration

$$[\mathbf{r}_N], [\mathbf{v}], [\mathbf{A}].$$

Because of the appearance of r in the retarded time (5.2), which is itself needed to calculate $\mathbf{r}$, these equations are less explicit than could be desired.

However, if one starts with a situation in which the source has been at rest for some time, r is initially just the instantaneous distance to the source. One can keep track of it subsequently by integrating the differential equation

$$\frac{dr}{dt} = s^{-1}\mathbf{r}\cdot(\dot{\mathbf{r}}_e - [\mathbf{v}]) \tag{5.3}$$

which follows from

$$\mathbf{r}^2 = (\mathbf{r}_e - [\mathbf{r}_N])\cdot(\mathbf{r}_e - [\mathbf{r}_N])$$

on differentiating with respect to time, noting that

$$\frac{d}{dt}[\mathbf{r}_N] = [\mathbf{v}]\left(1 - \frac{dr}{c\,dt}\right)$$

In the particular case of uniform motion, $\mathbf{A} = 0$, the retarded quantities can be expressed in terms of unretarded ones:

$$\left.\begin{aligned} &[\mathbf{A}] = \mathbf{A} = 0 \\ &[\mathbf{v}] = \mathbf{v} = \text{constant} \\ &[\mathbf{r}_N] = \mathbf{r}_N - \mathbf{v}r/c \\ &r = \frac{c^{-1}\mathbf{v}\cdot(\mathbf{r}_e - \mathbf{r}_N) + \sqrt{(c^{-1}\mathbf{v}\cdot(\mathbf{r}_e - \mathbf{r}_N))^2 + (\mathbf{r}_e - \mathbf{r}_N)^2(1 - v^2/c^2)}}{(1 - v^2/c^2)} \end{aligned}\right\} \tag{5.4}$$

the last expression being the solution of

$$r^2 = (\mathbf{r}_e - \mathbf{r}_N + c^{-1}r\mathbf{v})^2$$

With these expressions (5.1) reduces to (1).

6 To verify this for the hydrogen atom ($Z = 1$) with a realistic orbit radius, e.g., the Bohr radius

$$\hbar(mcZ\alpha)^{-1}\sqrt{1 - (Z\alpha)^2}$$

where α is the fine structure constant, $\sim 1/137$, might require much computing time. The acceleration has to be very gentle, because the internal forces are weak, and because the orbit is close to an 'integral resonance instability' (in the language of particle accelerator theory). Taking a larger value of Z, e.g. $Z \sim 70$, much larger accelerations are possible and a modest computing time suffices. The idea of obtaining the Fitzgerald and Larmor effects in such a system, by straightforward integration of equations of motion, was perhaps suggested to me by a remark of J. Larmor[10].

7 Conceivably the motion of the earth relative to the sun, and the motion of the sun itself relative to whatever inertial frame we adopt, could conspire to make the earth itself momentarily at rest. But this situation would not persist as the earth continues round the sun, assuming the latter to move rather uniformly. By the way, the orbital velocity of the earth is about 3×10^5 cm/sec. The velocity of the earth's surface relative to the centre, due to the daily rotation, is about one hundredth of this.

8 The only modern text-book taking essentially this road, among those with which I am acquainted, seems to be that of L. Janossy: *Theory of Relativity Based on Physical Reality*, Acadèmiaia Kiado, Budapest (1971).

9 These fields follow from the point-source retarded potentials of Lienard (1898) and Wiechert (1900). See, for example, W. K. H. Panofsky and M. Phillips: *Classical Electricity and Magnetism*. Addison-Wesley (1964), Eqs. 20-13, 20-15. Unfortunately, for our purpose, in modern textbooks this material is usually presented after chapters on relativity. But the incidental reference to relativity, which can then appear, can be disregarded; the business at hand is just the writing down of certain solutions of Maxwell's equations.

10 J. Larmor, *Aether and Matter*. Cambridge (1900) p. 179. The example is used by Larmor to illustrate a very general correspondence between stationary and moving systems, based on what is now called the Lorentz invariance of the Maxwell equations, which Larmor establishes to second order in v/c. Note that he does not write separate equations for the motion of sources, like our (3) and (5). He seems to have in mind a model in which the motion of singularities is dictated somehow by the field equations, in analogy with the motion of vortex lines in hydrodynamics. Larmor summarizes his general conclusions on p. 176:

'We derive the result, correct to the second order, that if the internal forces of a material system arise wholly from electrodynamic actions between the systems of electrons which constitute the atoms, then an effect of imparting to a steady material system a uniform velocity of translation is to produce a uniform contraction of the system in the direction of the motion, of amount $\varepsilon^{-1/2}$ or $1 - 1/2v^2/C^2$. The electrons will occupy corresponding positions in this contracted system, but the aethereal displacements in the space around them will not correspond: if (f, g, h) and (a, b, c) are those of the moving system, then the electric and magnetic displacements at corresponding points of the fixed systems will be the values that the vectors

$$\varepsilon^{1/2}\left(\varepsilon^{-1/2} f, g - \frac{v}{4\pi G^2} c, h + \frac{v}{4\pi C^2} b\right)$$

and

$$\varepsilon^{1/2}(\varepsilon^{-1/2} a, b + 4\pi v h, c - 4\pi v g)$$

had at a time const. $+vx/C^2$ before the instant considered when the scale of time is enlarged in the ratio $\varepsilon^{1/2}$'.

The special example is described on p. 179:

'As a simple illustration of the general molecular theory, let us consider the group formed of a pair of electrons of opposite signs describing steady circular orbits round each other in a position of rest. (The orbital velocities are in this illustration supposed so small that radiation is not important): we can assert from the correlation, that when this pair is moving through the aether with velocity v in a direction lying in the plane of their orbits, these orbits relative to the translatory motion will be flattened along the direction of v to ellipticity $1 - 1/2v^2/C^2$, while there will be a first-order retardation of phase in each orbital motion when the electron is in front of the mean position combined with acceleration when behind it so that on the whole the period will be changed only in the second-order ratio $1 + 1/2v^2/C^2$. The specification of the orbital modification produced by the translatory motion, for the general case when the direction of that motion is inclined to the plane of the orbit, may be made similarly: it can also be extended to an ideal molecule constituted of any orbital system of electrons however complex'.

I think it may be pedagogically useful to start with the example, integrating the equations in some pedestrian way, for example by numerical computation. The general argument, involving as it does a change of variables, can (I fear) set off premature philosophizing about space and time.

Note that W. Rindler, *Am. J. Phys.* **38**(1970), 1111, finds Larmor insufficiently explicit about time dilation:

'Apparently *no one* before Einstein in 1905 voiced the slightest suspicion that all moving clocks might go slow'.