An Analytical Model of Covid-19 Lockdowns

Łukasz Rachel*

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PRELIMINARY - COMMENTS WELCOME

Abstract

This paper studies the policy of locking down and re-opening an economy during an epidemic. The model nests the common SIR epidemiological model, but with infection rates that depend on the economic choices of individuals and with an explicit trade-off between the fiscal cost of lockdowns and their health benefits. Unlike other models in this area it is analytically tractable, with equilibrium lockdowns and optimal lockdown policies described using phase diagrams. The decentralized equilibrium features two externalities: individual choices drive aggregate infection rates and also determine fiscal costs that future taxes must pay for. The *fiscal externality* always prolongs the equilibrium lockdown relative to the optimum and the *infection externality* can be of either sign. Both results illustrate that the equilibrium lockdown might start earlier than the optimal lockdown. I show that if a lockdown is very effective in controlling the spread of the disease then it will paradoxically lead to a second wave unless the lockdown is enacted on or after a threshold date that I characterize. I prove that the optimal policy is to start the lockdown on that date, and that this policy minimizes deaths from the virus. I consider optimal lockdown policies of different types, including broad and targeted measures. These all reduce total deaths by a similar amount (between 20-33% depending on the lockdown effectiveness) but have dramatically different output and fiscal costs. However, policy is time-inconsistent, as authorities are tempted to remove the lockdown too early, and ex-post this may lead to a worse outcome than no lockdown at all. Track and trace strategies, vaccine discovery and healthcare systems capacity constraints all lead to an earlier and lengthier lockdown compared to the baseline.

Keywords: coronavirus, lockdown, containment policies

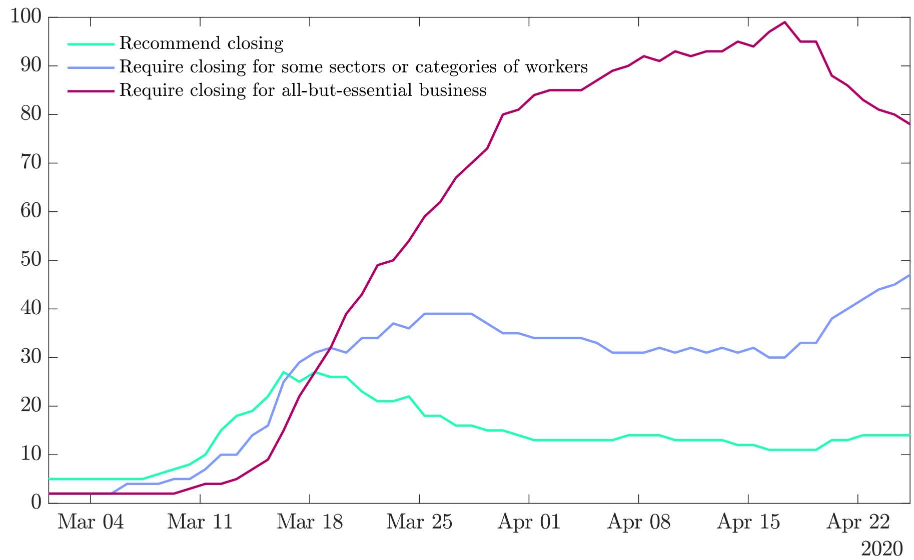
JEL Classification: E1, I1, H0.

*Centre for Macroeconomics, LSE. Email: l.p.rachel@lse.ac.uk. I am grateful to Ricardo Reis and Ben Moll for helpful comments. All the views are solely of the author. *Disclaimer*: I am an economist, not an epidemiologist. This paper uses the basic epidemiological model together with the tools of modern macroeconomics to better understand the current crisis.

1 Introduction

The Covid-19 pandemic is an enormous shock that has already generated a policy response at an unprecedented scale. Most strikingly, authorities in 140 countries ordered some form of shut down in a space of a few weeks (Figure 1). Still, some governments were deemed slow to act, and many individual consumers and firms responded earlier: data on visits to restaurants, hotels or gyms show large declines that preceded the official lockdown.[1]

Figure 1: Number of Countries Which Implemented Workplace Shutdown Policies



Source: Oxford Coronavirus Government Response Tracker (Hale et al., 2020). Of the total of 152 countries in the database, 140 implemented some form of shut-down.

In this paper I study the decisions to enter and exit a lockdown using continuous time methods. My framework is analytically tractable and allows for analysis of the epidemic paths using phase diagrams. This makes the model straightforward to understand and leads to several interesting results.

I define a lockdown as a situation when infected and/or susceptible individuals refrain from certain activities because of the disease. Lockdown can arise as a result of decentralized choices or be imposed by the planner. I define *lockdown effectiveness* ε as a percentage reduction in the basic reproduction number $\mathcal{R}_0$ that the lockdown achieves. It

[1]Data from SafeGraph, OpenTable and others similar sources show this pattern occurred in the US and many other jurisdictions. There are some high profile examples too. For example, in the UK the English Premier League put all games on hold on 13 March, 11 days before the British government announced nationwide lockdown.

is a simple and intuitive measure of how powerful lockdown is; and it can be matched directly to the emerging data and empirical evidence on lockdowns imposed in the Covid-19 pandemic so far.

The first important result is that there exists a threshold $\bar{\varepsilon}$ such that if a lockdown is *more* powerful than this threshold, an immediate and arbitrarily long lockdown results in an *unstable* resting point: if and when lockdown is lifted, the epidemic re-emerges and there is a second wave of the disease. Moreover, I show that the severity of the second wave is greater the more effective the lockdown is: that is, there is a trade-off between the initial suppression and the return of the disease further down the line.

The second result is that it is possible to avoid the second wave of the disease when lockdown is highly effective through delaying its start date. I provide analytical expression for the trigger point that avoids the second wave and minimizes the overall death toll.

I then proceed to analyze the decentralized equilibrium. I show that, given perfect knowledge about own health status and no altruism, individuals who are currently infected or recovered never choose to lock down in equilibrium. As to the susceptible individuals, I set up their expected utility maximization problem and I derive easily-interpretable and intuitive optimality conditions. I then prove that it is never optimal for these individuals to start lockdown at the very onset of an epidemic.

The next set of results clarify the nature of externalities present in the decentralized equilibrium. There are two major externalities in my baseline model: an *infection externality* and a *fiscal externality*.

The infection externality is well known: it comes from the fact that, in equilibrium, individual decisions to isolate have an external impact on economy-wide infection rates. I go further and make several contributions to our understanding of this externality. First, noting that there is no externality in the behavior of the recovered, I split the infection externality into the part driven by the infected and the part driven by the susceptible. I confirm the obvious result that the former always works in the expected direction: the behavior of the infected boosts infection rates, an effect that is external to their individual problem. I then further split the externality of the susceptibles into the intra- and intertemporal components, which consider the impact of today's behavior on today's and on future's infection rates, respectively. I show that in the model of discrete choice between lockdown and no-lockdown there is no intratemporal externality. Finally and most importantly, I prove that it is impossible to sign the intertemporal component in general:

the external effect of today's behavior can either raise or lower future infection rates. This means that the equilibrium lockdown can happen earlier, and/or last longer, than the socially optimal lockdown of the susceptible population.

As to the fiscal externality, I impose that the government's role in the epidemic is to provide economic disaster relief in the form of income support: specifically, I assume that the government covers a certain proportion of income of those who are in lockdown, financing this expenditure and the loss of tax revenues associated with lockdowns with borrowing. In equilibrium, individuals take government transfers and future taxes as given, but their choices ultimately determine the level of debt incurred by the government and thus the level of future taxes. This externality tends to prolong equilibrium lockdown relative to the social optimum. In a sense it can be thought of as a stand-in for negative macroeconomic effects of lockdowns more generally.

Having developed the understanding of the externalities embedded in the decentralized equilibrium I turn to the analysis of optimal lockdown policy. Specifically I analyze the time-0 problem of a utilitarian social planner who maximizes expected lifetime welfare of a representative individual. I consider four alternative assumptions with regards to what instrument is at the planner's disposal: a lockdown of the susceptibles only; a lockdown of infected only; a broad lockdown of the entire population; and a variant of the broad lockdown in which the recovered patients receive immunity passports. I prove that, fixing a start and end-date across these policies, all of them have exactly identical effect on the epidemic dynamics. But while the benefits side of the tally is the same, the costs differ significantly across these tools.

I am able to characterize the optimal lockdown policies analytically. Optimal lockdown never starts at the onset of the epidemic. For the case when lockdown is not very effective, policy optimally trades off extra deaths for lockdown duration. For the case where lockdown is very effective, the start date is determined in a way that guarantees that, to first order, policy minimizes the total death toll. In that latter case – which is probably more empirically relevant – I prove that an all-or-nothing lockdown policy is optimal even if the government can finely control the stringency of the lockdown.

I then verify these analytical predictions by computing optimal lockdown policies in a calibrated version of my baseline model. In order to do so I develop fast and reliable algorithms for computation of equilibrium and optimal lockdowns. I illustrate the case where the equilibrium lockdown starts before the socially optimal lockdown, but overall in my baseline model equilibrium and socially optimal lockdowns are not dramatically different. I calculate that optimal lockdowns bring a reduction of total deaths relative to no lockdown of around a fifth when the lockdown effectiveness is below the threshold $\bar{\varepsilon}$ and of around a third when it is above it. These reductions in mortality mean that optimal lockdown policies can significantly reduce the welfare cost of the Coronavirus (by around a sixth), despite having significant macroeconomic and fiscal costs. For example, my model suggests that the optimal broad lockdown of the type observed across many countries today is associated with a fall in GDP of 34% and the rise of government debt to potential GDP of 20 percentage points in the first year, and yet such policy cuts the welfare cost of Covid-19 by 13% relative to no-lockdown.

I evaluate the social welfare function for all possible lockdown start- and end-dates to study not just the good but also the bad ideas on how to implement a lockdown. This is a useful contribution because it emphasizes that while lockdowns *can* bring substantial benefits, not all lockdowns *do so*. One particularly poorly performing strategy is a lockdown that starts early and is lifted too soon.

I then ask whether the time-0 optimal lockdown policies are time consistent. For the lockdown policies that resemble those in the real world – i.e. those that include everyone no matter their health status – I answer this question in the negative: as the health-status structure of the population changes through the epidemic, the authorities are tempted to shorten the lockdown, relative to the time-0 optimal strategies. Combined, the last two results highlight the risks of lifting restrictions too early.

In much of the paper I work with a simple extension of the workhorse epidemiology model of William and McKendrick (1927) in which the disease spreads, in part, through economic activity and in which the government picks up a share of the lockdown bill. This baseline model lacks several important and realistic features of the current situation: I leave out ICU capacity constraints, I do not model heterogeneity of any kind, I ignore the possibility that a treatment or a vaccine will be developed, I assume it is impossible to fully suppress the virus, and I focus on a single, continuous lockdown episode of a given effectiveness. Basing my analysis on the simple model is useful because the mechanisms I explore lie at the center of many richer models and the hope is that lessons learnt here will be useful for understanding those more elaborate frameworks. I show how to use a graphical representation of the dynamic system – a basic phase diagram – to study the results and build intuition.[2] In the final section of the paper I show how simple extensions

[2]Both of these features are inspired by a classic paper by Werning (2012) who studies macroeconomics of the liquidity trap in a basic New Keynesian model in continuous time.

of my framework can be helpful in studying many of the aforementioned features. Specifically I explore whether and how the model developed here can be used to think about the possibility of full suppression of the virus, healthcare capacity constraints and vaccine or treatment discovery. All of these elements put a premium on avoiding the strategies where the rate of infection is allowed to climb high, thus tilting optimal policies towards earlier and more lengthy lockdowns relative to the baseline. Hence the lesson from this paper ought not to be that the optimal policies prescribed by the analytical model below should be recommended to the policymakers. Instead, my model elucidates which considerations drive which characteristics of optimal policies, thus building more thorough understanding of the trade-offs and through that enhancing the policy debate.

Literature. This paper relates to a fast growing literature on the economics of epidemics applied to the Covid-19 pandemic.[3] What is different about my paper is the nature of the lockdown policy: I focus on the binary lockdown-on / lockdown-off choice, studying when the equilibrium lockdown starts and when it ends, whereas the aforementioned papers study the choices along the intensive margin. My approach is arguably more realistic from the perspective of an individual worker who decides whether or not to go out and work given the virus, and is also perhaps more closely aligned with the perspective of the government officials who face discrete choices on when to start and end the lockdown. More importantly, it makes the model very transparent and allows for intuitive graphical analysis. Furthermore, I show that in the context of my baseline model the planner always chooses the all-or-nothing lockdown, even if more flexible lockdown policies are available. This serves to pin down precisely the conceptual reasons for gradualism in mitigation policies (all of which are absent from my baseline model, but many of which I discuss in the final section of the paper).

Several recent papers made progress studying mitigation policies and deriving the socially optimal responses to the pandemic.[4] Compared to these studies my analytical

[3]Early papers by Atkeson (2020) and Stock (2020) provide an economist's perspective on the baseline SIR epidemiology models. Closely related to the present paper is the work by Eichenbaum et al. (2020) who study a competitive equilibrium of an economy populated by hand-to-mouth agents whose actions affect the rates of transmission of the disease and compare it to the socially optimal mitigation policies. Two concurrent papers analyze the equilibrium and optimal mitigation policies: Jones et al. (2020) focus on the working-from-home aspect and healthcare system capacity constraints; Farboodi et al. (2020) study decentralized allocation and optimal mitigation policies in continuous time.

[4]Feng (2007) provides an epidemiological perspective on quarantine and isolation policies. Eichenbaum et al. (2020) study the optimal tax on consumption during an epidemic, and Alvarez et al. (2020) and Piguillem and Shi (2020) set up and solve a dynamic planning problem where the planner chooses the timing

model allows me to derive optimal policy in closed form, which helps in building the intuition and understanding the model's mechanics. One additional advantage of my formulation relative to existing work is that lockdown policy in my model has naturally adverse fiscal implications (whereas for example mitigation policies in Eichenbaum et al. (2020) raise government's revenue), which allows me to calculate the fiscal footprint of the pandemic depending on the lockdown policy chosen by the authorities. Alvarez et al. (2020) and Acemoglu et al. (2020) only consider the planning problem whereas I write down and solve the decentralized equilibrium of the economy. This allows for an explicit analysis of the potential conflicts between policymakers and atomistic individuals making decisions and for a systematic analysis of welfare implications.

The analysis of the fiscal footprint of lockdowns relates to the broader strand of work on policy implications of the Covid shock. Guerrieri et al. (2020) study whether the supply shock associated with the lockdown can lead to aggregate demand deficiency and thus warrant monetary and fiscal loosening. Jordà et al. (2020) provide a long-term historical perspective and find that the natural rate is significantly lower in the years following a pandemic. Focusing on the most recent history, Bahaj and Reis (2020) describe how the swap lines arrangements by the Fed impacted the funding markets. Kaplan et al. (2020) build a HANK model of the pandemics and evaluate a range of policies to form a pandemic policy frontier. Glover et al. (2020) consider heterogeneity along the age and workplace dimensions to point out where the major disagreements on the severity and duration of mitigation policies lie.

The study also links to the short paper by Hall et al. (2020) who consider how much consumption should the society be willing to give up to save lives. In the benchmark model I follow the literature and assume that the loss associated with death is equal to the foregone value of future lifetime utility (Hall and Jones (2007)). Given the discount rate of 4% per annum and annual income of around \$60,000, this value is roughly \$10 million in terms of consumption today. This is in line with the estimates in the literature on the value of statistical life used for example in transportation policy (Andersson and Treich (2011), Kniesner and Viscusi (2019)). The conclusions I reach are largely independent of this value however – for instance, I derive the optimal policy paths analytically and only confirm the results using the numerical simulations of the calibrated model.[5]

and intensity of a lockdown policy. Acemoglu et al. (2020) consider optimal policy in a model with multiple risk groups, highlighting that targeted mitigation policies improve the trade-off between economic activity and deaths.

[5]The paper also highlights the importance of testing: the result that track and trace policies bring about

Roadmap. The paper is structured as follows. Section 2 outlines the analytical model. 3 defines and studies the decentralized equilibrium. Section 4 analyzes the externalities, and Section 5 studies the optimal lockdown policy. Section 6 shows that the time-0 plans are not time consistent. Section 7 discusses how some of the key extensions of the baseline model can be incorporated in the framework and whether they change the results. Section 8 concludes.

2 Analytical model

2.1 Pre-epidemic environment

The economy consists of identical individuals of measure 1 with preferences:

$$U = \int_0^\infty e^{-\rho t} u(c, n) dt$$

where in general $u = \log c - \theta v(n)$ and n is labor supply. In the rest of the paper I assume for simplicity that $\theta = 0$, but all the results easily generalize to the case with elastic labor supply. Crucially, the focus of this paper is on the extensive margin so that labor supply can take only two possible values: $n \in \{0, \bar{n}\}$. That is, individuals can work or stay at home, but cannot reduce their hours worked along the intensive margin. I denote the share of individuals that work with λ. If individuals work they earn after-tax wage w.[6] Income of an individual who stays at home is h denominated in terms of the units of the final good. Income h is the sum of three components: income from market activities such as working from home (share ψ_{WFH}), home production (share ψ_{HPR}) and government transfer (share ψ_{GOV}). The shares ψ are constant and sum to one. There is no capital or any other form of saving, and so individual's budget constraint is:

$$c_W = w\bar{n}$$

when individuals work and

$$c_U = h$$

significant welfare benefits relative to other lockdown measures resonates with the findings of Berger et al. (2020) who consider conditional quarantine policies and show that a given reduction in death rates can be achieved with looser mitigation measures if more information is available.

[6] Government collects proportional taxes on gross labor income in normal times. The role of the government and its budget constraint are described in detail below.

when they stay at home.

Production technology is linear in labor:

$$Y = AN$$

and markets are competitive, implying that $w = A$. Equilibrium requires that consumers maximize utility, firms maximize profits and markets clear:

$$C = \lambda A\bar{n} + (1-\lambda)h.$$

2.2 Epidemic

To model infection I use a well-known SIR model with 4 population groups: susceptible, infected, recovered and dead. I denote by λ^i the share of individuals in group i that work, for example the number of susceptibles that work is $S_W = \lambda^S S$. I also assume that a fraction $1-\phi \leq 1$ of the infected must stay out of work due to ill health, so that $I_W := \lambda^I \phi I$.

There are three potential ways people get infected:

1. Through consumption, at rate:

$$\pi_c \left(\lambda^S C_W^S + (1-\lambda^S) C_U^S\right) \left(\lambda^I \left(\phi C_W^I + (1-\phi) C_U^I\right) + (1-\lambda^I) C_U^I\right) SI,$$

   where C_W^S stands for aggregate consumption of the susceptible workers, and so on.

2. Through work, at rate:

$$\pi_n \lambda^S \lambda^I \phi \bar{n}^2 SI,$$

3. Through random encounters not related to work or consumption, at rate:

$$\pi_o SI.$$

To keep the model as simple as possible, I set $\pi_c = 0$: infection can happen only through work and through random encounters, and not through consumption, but this is without much loss of generality.[7] The evolution of the epidemic is then described by the following

[7]The reason is that, given the hand-to-mouth consumers populating this economy, labor supply and consumption are linked one-to-one.

system of differential equations:

$$\dot{S} = -\left(\pi_n \lambda^S \lambda^I \phi \bar{n}^2 + \pi_o\right) SI \tag{1}$$

$$\dot{I} = -\dot{S} - \pi_r I - \pi_d I \tag{2}$$

$$\dot{R} = \pi_r I \tag{3}$$

$$\dot{D} = \pi_d I \tag{4}$$

with initial conditions: $S_0 = 1 - \epsilon$, $I_0 = \epsilon$, $R_0 = 0$ and $D_0 = 0$.

Several simplifying assumptions are embedded in (1) - (4). Fist, there are only four states. The most obvious omissions are exposed (people who caught the virus but do not yet have the symptoms) and critical condition states. While these are useful extensions that generate more realistic dynamics, they are not required for the points I make in this paper. Another simplifying assumption – one that is present in all economics papers on Covid19 – is that the period of infectiveness is distributed exponentially, with density $(\pi_r + \pi_d)\, e^{-(\pi_r+\pi_d)t}$ and mean value of $\frac{1}{\pi_r+\pi_d}$. A large literature in epidemiology as well as the early data available for Covid19 suggest that the exponential assumption is wide off the mark, both for past epidemics as well as the present one (Wearing et al. (2005), Feng et al. (2007), Flaxman et al. (2020), Verity et al. (2020)). A standard practice in more advanced epidemiology modeling is to approximate the time spent in the infectious stage with a Gamma distribution. This however makes the model more involved, and I only pursue this in a separate forthcoming paper. Relatedly, equations (1) - (4) imply that death and recovery rates are constant and do not depend on the age structure or the capacity of the healthcare system. This buys transparency and analytical tractability; the implications of a more realistic assumptions are explored in Section 7. Finally, the horizontal incidence takes the form of what Hethcote (2000) calls "mass action law": equation 1 features SI and not $\frac{SI}{N}$ on the right hand side. This is a ubiquitous assumption in the literature. It simplifies the calculations a little relative to the alternative, and given that I normalize initial population to unity, the two approaches yield identical outcomes as long as the number of deaths is not very large. I explore how varying the degree of increasing returns in the infection matching technology changes the conclusions in Section 7 (the answer is that this does not matter much).

Note also that this simple model nests the standard SIR setting with no macro feedback which obtains if we set $\pi_n = 0$.

Analytical results familiar from the epidemiology literature. I now collect some useful concepts and insights into how the epidemic develops according to this simple model. To fix ideas, whenever this is informative, I show a rough number that corresponds to the case of Covid-19.

Let $\hat{\pi} := \pi_n \lambda^S \lambda^I \phi \bar{n}^2 + \pi_o$ denote the contact rate. Begin by noting that the number of infected will grow if and only if

$$I > 0 \text{ and } S > \bar{S} := \frac{\pi_r + \pi_d}{\hat{\pi}}. \tag{5}$$

Condition (5) defines the threshold $\bar{S}$, which is the *herd immunity threshold:* if $S < \bar{S}$, the epidemic is past its peak and eventually dies out. This threshold will play a key role in the analysis. This threshold is related to the *basic reproductive rate of infection* defined as $\mathcal{R}_0 := \frac{S_0 \hat{\pi}}{\pi_r + \pi_d}$: if the seed of infection is small, which I assume throughout, then $\mathcal{R}_0 = \frac{1}{\bar{S}}$, and for the epidemic to develop we must have $\mathcal{R}_0 > 1$. I assume that for Covid19, $\mathcal{R}_0 = 2.5$ which implies $\bar{S} = 0.4$.[8] That is, the herd immunity threshold for Covid19 is at around 60% cumulative infections.

It is useful to define the time-varying equivalent to $\mathcal{R}_0$ as

$$\mathcal{R}(t) := \frac{S(t)}{\bar{S}}. \tag{6}$$

If $\mathcal{R}(t) > 1$, or equivalently if $S > \bar{S}$, the number of infected drifts up over time.

The model admits an analytical solution. Dividing equation (2) by (1) we obtain a first-order ODE:

$$\frac{dI}{dS} = -1 + \frac{\bar{S}}{S}, \tag{7}$$

which, given the initial conditions S_0 and I_0, has the solution:

$$I(t) = -S(t) + \bar{S} \log S(t) + I_0 + S_0 - \bar{S} \log S_0. \tag{8}$$

Assuming a small initial seed of infection, the solution is

$$I(t) = -S(t) + \bar{S} \log S(t) + 1. \tag{9}$$

[8] $\mathcal{R}_0$ of 2.5 is in line with the early estimates although perhaps towards the lower range of the most recent ones (Ferguson et al. (2020),Flaxman et al. (2020)). In any case, one advantage of my analytical framework is that one can trace the impact of an alternative assumptions (e.g. a higher $\mathcal{R}_0$) very easily.

Taking the limit as $t \to \infty$ and noting that $I(\infty) = 0$ shows that the initial basic reproduction number and the eventual share of the population that will have encountered the virus $S(\infty)$ are tightly linked:

$$\mathcal{R}_0 = \frac{\log S(\infty)}{S(\infty) - 1}. \tag{10}$$

This equations implies that for Covid-19 $S(\infty) = .11$ – absent any intervention, nearly 90% of the population contract the disease. Moreover, condition (5) and equation (9) imply that at the peak of the epidemic is:

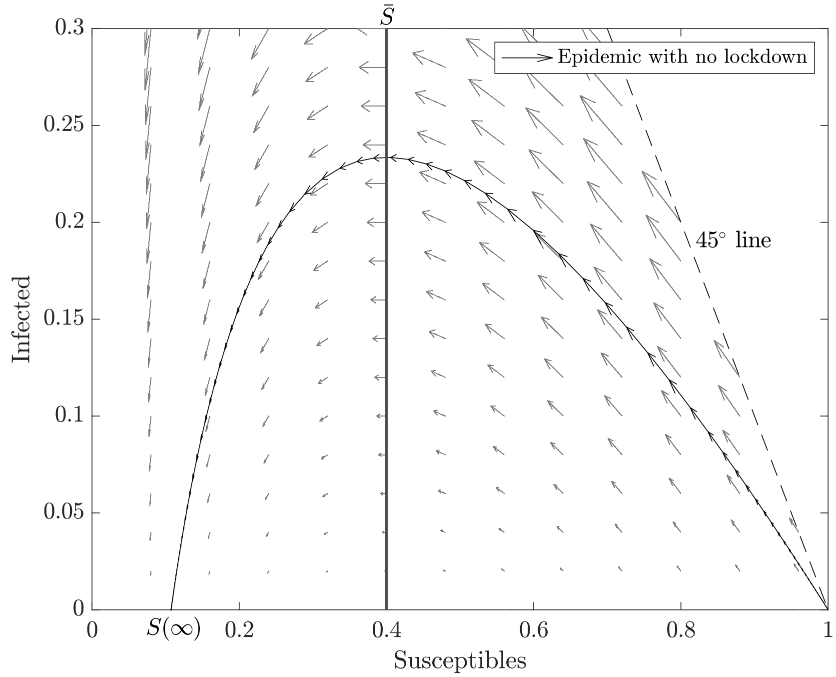
$$I_{max} = -\bar{S} + \bar{S} \log \bar{S} + 1.$$

For Covid-19 with no intervention, $I_{max} = .23$.

Graphical representation. One of my contributions is that I study this baseline macro-SIR model using a phase diagram (Figure 2).[9] The number of susceptible is on the horizontal axis; the number of infected on the vertical axis (recall that initial population is normalized to 1). The solid vertical line denotes the herd immunity threshold $\bar{S}$. The arrows depict the dynamics of the system at any feasible point in the plane; the length of the arrows denotes the velocity of the changes; and the arched solid line with arrows depicts the dynamics of the disease when the seed of infection is small and there is no lockdown of any kind. There are several things to note. First, the starting point is close to the bottom-right corner of the Figure given the small seed of initially infected people. Second, the dynamics of the system are such that the number of infected individuals is constant when $S = \bar{S}$ – indeed, this follows straight from the definition of $\bar{S}$. Third, the disease eventually infects many more people than $\bar{S}$: $S(\infty) < \bar{S}$. Fourth, the system travels faster the further away it is from the x-axis. Fifth, the x-axis is the line of singularities: any point on the x-axis is an equilibrium of the system. However, there is a fundamental difference between the two segments demarcated by the vertical line at $\bar{S}$: all the points on the x-axis to the left of $\bar{S}$ represent stable equilibria, whereas all the points to the right are unstable: any small perturbation away from the axis will put in motion powerful dynamics that will see both S and I increase. This final observation will be very important for policy, as it will dictate what outcomes are feasible under aggressive suppression strategies. Given its importance, I formally state this result in the following lemma:

[9] I discuss the calibration later on, but to fix ideas the parametrization of the model shown is an accurate description of the Covid19 dynamics with no mitigation policies in place.

Figure 2: Dynamics of the epidemic in the analytical model



Lemma 1. *Steady states of the system with $I = 0$ and $S > \bar{S}$ are unstable, in the sense that an infinitely small perturbation to I triggers dynamics that take the system away from that steady state. Conversely, the steady states with $I = 0$ and $S < \bar{S}$ are stable.*

Proof. Follows directly from equation (2) and the definition of $\bar{S}$. □

2.3 Lockdown

In this Section I outline how lockdown affects the system's dynamics. I begin with a precise definition:

Definition 1. The economy is in *lockdown* at t if either:

(i) none of the susceptible work: $\lambda^S(t) = 0$; or

(ii) none of the infected work $\lambda^I(t) = 0$; or

(iii) both.

Lockdown lasts for a single continuous period of time. It starts at $T_0 \geq 0$ and ends at $T_1 \geq T_0$.

Importantly, the above definition restricts the meaning of a lockdown to the situation where none of the susceptible or infected work. This restriction is without loss of generality when it comes to the competitive equilibrium: because all individuals of a given health status are identical, and none can adjust the intensive margin of their labour supply, the equilibrium is fully symmetric, and either all the susceptible people work or everyone stays home. If lockdown is the result of a policy choice, I assume that the government policy tool is blunt in the sense that it can only set $\lambda^i = \{0,1\}$ for some combination of $i \in S, I, R$. I discuss the details of the lockdown tools available to the planner later on. I will also prove that using the blunt tool is always the optimal thing to do.

Lockdown of any type – be it a self-imposed decision to isolate or a policy implemented by the government on the whole or on the subset of the population – can be represented as a reduction in the contact rate $\hat{\pi}$ and a rise in $\bar{S}$, as follows:

$$\hat{\pi}(t) = \begin{cases} \pi_n \lambda^S \lambda^I \phi \bar{n}^2 + \pi_o & \text{no lockdown: } t \notin [T_0, T_1] \\ \pi_o & \text{lockdown: } t \in [T_0, T_1] \end{cases}$$

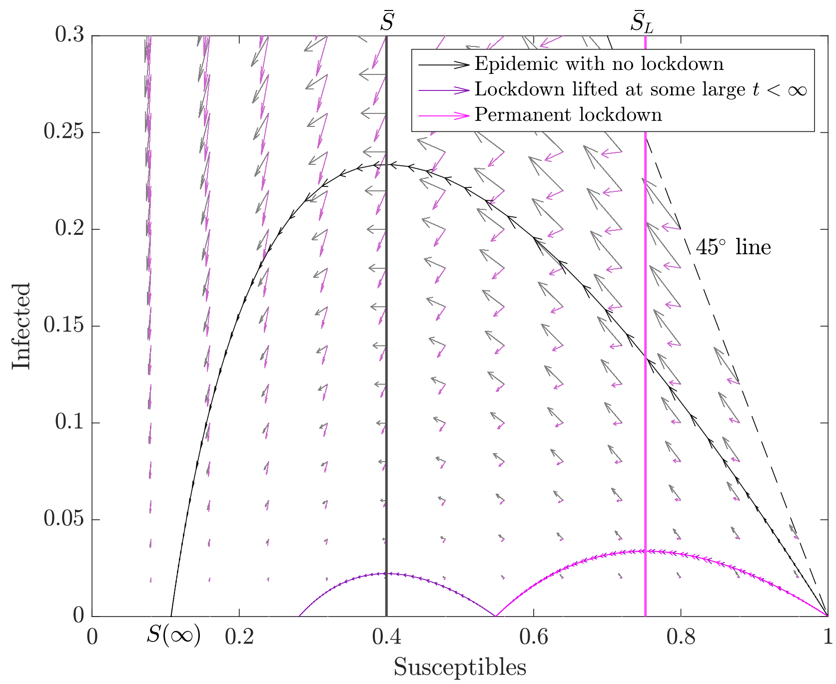
Graphically lockdown is represented by a rightward shift of the $\bar{S}$ line (recall that $\bar{S} := \frac{\pi_r + \pi_d}{\hat{\pi}}$). During lockdown the system is characterized by the $\bar{S}_L$ line and the associated dynamics shown in pink arrows in Figure 3. With a *permanent* lockdown in place, the dynamics of the epidemic are as depicted by the pink arched curve: relative to the no-lockdown curve in black, the virus proceeds more slowly, it peaks at a much lower level, and its resting point is at a significantly higher level of susceptibility.

Of course a permanent lockdown is neither realistic nor desirable, so it is useful to think what happens when it ends. For the case shown in the Figure, the resting point of the system under permanent lockdown is in the unstable segment of the x-axis (it is to the right of the $\bar{S}$ line). If at any $T_1 < \infty$ the lockdown was lifted, the dynamic system would again be described by the black $\bar{S}$ line and the associated vector field. The result in Lemma 1 means that the epidemic would be set in motion again (dark pink trajectory in the Figure). It would feature a second peak (at $S = \bar{S}$), before eventually settling down at a stable steady state with $S(\infty) < \bar{S}$.

An alternative case could be that the resting point of the system under lockdown is on the left of $\bar{S}$, and the steady state is stable. In that case relaxing the lockdown at some large but finite T_1 would not result in the return of the epidemic.

In Figure 3 the lockdown starts immediately at the onset of the epidemic: $T_0 = 0$.

Figure 3: Dynamics of the epidemic with lockdown in place



Of course, lockdown can start at any $T_0 \geq 0$, in which case the epidemic tracks the "no lockdown" trajectory until time T_0 (and the level of susceptible share $S(T_0)$) is reached, and then departs from it as the lockdown dynamics take over. The important thing to understand is that the system dynamics under lockdown are not affected by lockdown timing, and only depend on the lockdown effectiveness: that is, by how much the $\dot{I} = 0$ curve shifts.

The following proposition introduces a concept of effectiveness and shows how it drives the dynamic response of the economy to a lockdown.

Proposition 1. *Lockdown effectiveness ε satisfies:*

$$\varepsilon := \frac{\mathcal{R}_0 - \mathcal{R}_0^L}{\mathcal{R}_0} = \frac{\mathcal{R}(t) - \mathcal{R}^L(t)}{\mathcal{R}(t)} = \frac{\bar{S}_L - \bar{S}}{\bar{S}_L} = \frac{\pi_n \lambda^S \lambda^I \phi \bar{n}^2}{\pi_n \lambda^S \lambda^I \phi \bar{n}^2 + \pi_o}. \tag{11}$$

There exists a threshold $\bar{\varepsilon}$ such that if $\varepsilon > \bar{\varepsilon}$ an immediate lockdown leads to unstable suppression of the disease: lifting the lockdown results in a second wave of infections. The threshold $\bar{\varepsilon}$ is given by:

$$\bar{\varepsilon} = \frac{\bar{S}}{\bar{S} - 1}\left(1 - \log \bar{S} - \frac{1}{\bar{S}}\right) \quad (= 0.39 \textit{ for Covid-19}).$$

There exists another threshold $\hat{\varepsilon} > \bar{\varepsilon}$ such that if $\varepsilon > \hat{\varepsilon}$ an immediate lockdown suppresses the virus completely, preventing the epidemic. But once the lockdown is lifted the epidemic starts over and follows the no-lockdown trajectory. Threshold $\hat{\varepsilon}$ is given by:

$$\hat{\varepsilon} = 1 - \frac{1}{\mathcal{R}_0} = 1 - \bar{S} \;\; (= 0.6 \textit{ for Covid-19}).$$

Proof. See Appendix. □

The first part of Proposition 1 defines and computes the key characteristic of a lockdown: its effectiveness ε. A more effective lockdown – one with higher ε – results in a greater compression of the initial reproduction number. The proposed measure of effectiveness is simply a percentage change in $\mathcal{R}_0$ (or equivalently $\mathcal{R}(t)$) that results from imposing a lockdown. The proposition shows that this is the same as measuring effectiveness with the share of the infectious activities that are eliminated in a lockdown. It is thus a natural and a very intuitive measure.

The second part of the Proposition computes the threshold beyond which the lockdown is sufficiently effective that the suppression of the virus is rapid and the resting point of the system under lockdown is below the herd immunity level $\bar{S}$ (this is the case shown in Figure 3). Lifting the restrictions at some point results in a second wave of infections in this case.

The third part of the Proposition characterizes an even higher threshold for lockdown effectiveness $\hat{\varepsilon}$ beyond which lockdown is so effective that, if imposed immediately at the onset of the epidemic, it suppresses the disease completely so that the number of the infected cases declines and the epidemic is stopped in its tracks. This might sound appealing, but the Proposition shows that it is an extreme version of a "kicking the can down the road" strategy: once the lockdown is lifted, the epidemic re-emerges and develops along the original no-lockdown trajectory. This starkly highlights the critical feature of this model: that it is never possible to suppress the virus completely at a level of susceptibility that is above $\bar{S}$. I discuss the reasons why this is a good working assumption and what are its main drawbacks, as well as how the alternatives can affect the conclusions, in Section 7 below.

One implication of Proposition 1 is that there is a trade-off between the size of the first and the second wave of the epidemic:

Lemma 2. *Consider an immediate and arbitrarily long but finite lockdown with $\bar{\varepsilon} < \varepsilon < \hat{\varepsilon}$. The peak of the second wave of the epidemic and the proportion of people who at some point contract* *the virus* $1 - S(\infty)$ *are both increasing in* ε.

Proof. See Appendix. $\square$

This Lemma is a straightforward and yet striking implication of Proposition 1. The more effective the lockdown is initially, the more the epidemic bites back.

Proposition 1 and Lemma 2 focused on immediate lockdowns. The following result considers how the effects of the lockdown depend on when it starts:

Proposition 2. *For any lockdown with effectiveness* $\varepsilon > \bar{\varepsilon}$*, there exists a time threshold* T_0^* *and the associated threshold of susceptibility* S^* *such that if the lockdown is implemented after* T_0^* *it achieves stable suppression of the disease. The threshold is given by:*

$$S^* = S(T_0^*) = \exp \frac{1 + \bar{S}_L \log \bar{S} - \bar{S}}{\bar{S}_L - \bar{S}}.$$

Any arbitrarily long lockdown with effectiveness $\varepsilon > \bar{\varepsilon}$ *achieves the minimum feasible number of deaths in the long-run if and only if it starts exactly at* T_0^*.

Proof. See Appendix. $\square$

Proposition 1 contained some mixed news with regards to the effects of a very effective lockdown: if deployed immediately, the lockdown does a very good job at suppressing the virus in the near term, but beyond that will lead to a second outbreak of the epidemic if and when the restrictions are lifted. Proposition 2 contains some good news: it states that the second peak outcome can be avoided by implementing arbitrarily effective lockdown later in the epidemic. In particular, for any lockdown effectiveness above the threshold $\bar{\varepsilon}$, the proposition calculates the minimum starting time – in terms of the level of susceptibility – that is required for the lockdown to achieve a stable suppression of the disease. Indeed, if the lockdown is implemented exactly at T_0^* and is in place for an arbitrarily long period, the epidemic settles as the point where $S(\infty) = \bar{S}$, minimizing the cumulative infection rate and the death toll $\pi_r(1 - \bar{S})$.

2.4 The government's role in the epidemic

Beyond its ability to order a lockdown, the government has two roles. First, in normal times the government collects labor income taxes and spends the proceeds on some public good or social welfare programs G. This spending is not modeled explicitly except that the government is committed to purchases of G every period, including during the epidemic. Second, the government finances a certain part of income h that all workers who are in lockdown receive (irrespectively of whether the lockdown is voluntary or imposed by the government). These two roles of the government allow me to think about the dual impact of Coronavirus on government finances: an epidemic drives up spending on income support while lower activity leads to a decline in tax receipts. This creates a hole in government budget. To keep the modeling of the fiscal side simple but reflective of current circumstances, I assume that, at the start of the pandemic, the government borrows the required funds and procures the required output directly in the international market, paying some constant interest rate $\bar{r}$.[10] This means that the government can become indebted despite no internal saving in the economy. The government then finances its debt by levying constant lump-sum tax τ every period on those who survived the virus, starting from some date $\hat{T}$ (after the epidemic has ran its course). Given these assumptions, the government's intertemporal budget constraint is:

$$B_0 \leq e^{-\bar{r}\hat{T}} \int_{\hat{T}}^{\infty} e^{-\bar{r}(t-\hat{T})} \left(S(t) + R(t)\right) \tau dt \tag{12}$$

where B_0 is the amount (in units of output) borrowed by the government at time-0 and τ is the lump-sum tax paid by each surviving individual each period. Consequently, from date $\hat{T}$, the budget constraints for the susceptible and the recovered individuals are

$$c_W^R = c_W^S = A\bar{n} - \tau. \tag{13}$$

I assume (and verify) that in equilibrium $A\bar{n} - \tau > h$ so that all survivors work in the post-epidemic steady state.

The amount of fiscal support B_0 is obviously endogenous to the macro- and epidemiological-environment and to the potential lockdown policies that are adopted during the epi-

[10] A natural – and fair – objection to this assumption is that Covid-19 is a global shock that affects all countries. The assumption here does not need be taken too literally for that reason, and there are other ways to rationalize it. One is to think of the governments as borrowing from the rich (those towards the top of the income and wealth distributions) who are outside of my model (Mian et al. (2019a,b)). Another is to think of the central bank as providing the necessary liquidity to the government during the pandemic. Economically what matters is that the government is allowed to borrow today against taxes tomorrow.

demic. Given the assumptions above, the government debt at time-0 is:

$$B_0 = \left(\psi_{GOV} h + (1 - \psi_{WFH}) A \bar{n} \frac{\tau_n}{1 - \tau_n}\right) \cdot$$
$$\cdot \int_0^\infty \left((1 - \lambda_S(t))S(t) + (1 - \phi\lambda_I(t))\, I(t) + (1 - \lambda_R(t))R(t)\right) dt. \quad (14)$$

where τ_n is the average tax rate on labor income – an exogenous parameter. Government borrowing consists of two parts: income paid to those in lockdown, and cover for the shortfall in revenues associated with declining tax receipts. Since the government is prohibited from austerity measures – G is fixed an non-negotiable – it needs to borrow to cover this shortfall. Clearly, a longer and broader lockdown will increase the burden on the fiscal authority.

3 Equilibrium lockdown

The purpose of this Section is to find the equilibrium values of T_0 and T_1 and analyze the associated effects on the epidemic and the economy. I begin with the formal definition of the competitive equilibrium.

Definition 2. A *perfect-foresight competitive equilibrium* is a sequence of macro variables Y and C, sequence of epidemic variables S, I, R, D, sequence of labor supply choices $\{\bar{n}_i\}_{i\in\{S,I,R\}}$ and the associated sequence of lockdown indicators $\{\lambda_i\}_{i\in\{S,I,R\}}$, sequence of taxes $\{\tau\}$ and the level of government borrowing at time-0 B_0 such that: (i) households maximize their expected lifetime utility at time-0 taking the trajectory of the epidemic, behavior of other individuals, wages, government transfers and taxes as given; (ii) firms maximize profits taking wages as given; (iii) government adjusts borrowing and taxes to satisfy demand for transfers, its spending commitments and its intertemporal budget constraint; (iv) the trajectory of the epidemic is consistent with the individual lockdown decisions; (v) goods and labor markets clear.

Having defined the equilibrium concept, I now move on the characterization.

3.1 Preliminaries

To begin, note that in equilibrium the infected and the recovered individuals never choose to lock down. Since there is no altruism, individuals care only about maximizing their expected utility. For infected or recovered individuals there is no further risk of infection, and because $w\bar{n} - \tau > h$ and there is no disutility of labor, the optimal choice for these individuals is to always work.

Lemma 3. *In equilibrium, fraction ϕ of the infected and all of the recovered individuals work and do not lock down: $\lambda^I = \lambda^R = 1$.*

Proof. Follows immediately from $c_W > c_U$ and the fact that the currently infected and the recovered cannot become infected again. □

The interesting part is the problem of the susceptible individuals. Denote the time-varying infection rates of a susceptible person who goes to work with π_W and the corresponding probability for the person who locks down with π_U:

$$\pi_W := \pi_o I + \pi_n \phi I \bar{n}^2 \tag{15}$$

$$\pi_U := \pi_o I \tag{16}$$

Note that $\pi_W \geq \pi_U \forall t$: going to work is always riskier than being locked down.

To formulate the problem of a susceptible individual choosing the optimal lockdown, it is first useful to specify the hypothetical value functions of individuals that are stuck in a given employment status during the epidemic. These satisfy the following Bellman equations:

$$\rho U^S = u_U^S + \pi_U \left(U^I - U^S \right) + \dot{U}^S \tag{17}$$

$$\rho W^S = u_W^S + \pi_W \left(\phi W^I + (1-\phi) U^I - W^S \right) + \dot{W}^S \tag{18}$$

$$\rho U^I = u_U^I + \pi_r \left(W^R - U^I \right) + \pi_d (0 - U^I) + \dot{U}^I \tag{19}$$

$$\rho W^I = u_W^I + \pi_r \left(W^R - W^I \right) + \pi_d (0 - W^I) + \dot{W}^I \tag{20}$$

$$\rho U^R = u_U^R + \dot{U}^R \tag{21}$$

$$\rho W^R = u_W^R + \dot{W}^R \tag{22}$$

The corresponding boundary conditions are:

$$W^S(\hat{T}) = W^I(\hat{T}) = W^R(\hat{T}) = \frac{\log(Ah - \tau)}{\rho},\ U^S(\hat{T}) = U^I(\hat{T}) = U^R(\hat{T}) = \frac{\log h}{\rho}. \tag{23}$$

Note that the value of death is zero, meaning that the cost of dying is the foregone utility of regaining health.

3.2 The lockdown problem of an individual agent

To save on notation let $V(t) := \phi W^I + (1-\phi)U^I$ denote the expected continuation value of contracting the virus at t. At period zero any susceptible individual chooses the pair $\{T_0, T_1\}$ pinning down the timing of the lockdown to maximize her utility:

$$\mathcal{U}^S = \max_{\{T_0 \geq 0, T_1 \geq T_0\}} \left\{ \begin{array}{r} \int_0^{T_0} e^{-\rho t - \int_0^t \pi_W(s)ds} \left(u(c_W) + \pi_W(t)V(t)\right) dt + \\ +e^{-\int_0^{T_0} \pi_W(s)ds} \int_{T_0}^{T_1} e^{-\rho t - \int_{T_0}^t \pi_U(s)ds} \left(u(c_U) + \pi_U(t)V(t)\right) dt + \\ +e^{-\rho T_1 - \int_0^{T_0} \pi_W(s)ds - \int_{T_0}^{T_1} \pi_U(s)ds} W^S(T_1) \end{array} \right\}. \quad (24)$$

3.3 Results

The following Lemma characterizes the solution to this problem:

Lemma 4. *The start and end dates of the lockdown of the susceptibles in the decentralized equilibrium, $\{T_0, T_1\}$, satisfy:*

$$\begin{aligned} & u(c_W) - u(c_U) + \\ & \qquad + \left(\pi_W(T_0) - \pi_U(T_0)\right) \cdot (V(T_0) - \\ & \qquad - \int_{T_0}^{T_1} e^{-\rho(t-T_0) - \int_{T_0}^t \pi_U(s)ds} \left(u(c_U) + \pi_U(t)V(t)\right) dt - \\ & \qquad\qquad - e^{-\rho(T_1-T_0) - \int_{T_0}^{T_1} \pi_U(s)ds} W^S(T_1)) = 0 \quad (25) \end{aligned}$$

$$u(c_U) - \rho W^S(T_1) + \pi_U(T_1)\left(V(T_1) - W^S(T_1)\right) + \dot{W}^S(T_1) = 0 \quad (26)$$

Proof. These follow directly from the first order optimality conditions in problem (24). □

Expressions in this Proposition tell us exactly what trade-offs individuals face when deciding on their personal lockdown strategy. Consider first postponing the start of a lockdown by a marginal amount (going to work for an extra day, say). The first line in equation (25) states that one benefit of doing so is the difference between the instantaneous utility of going to work versus being locked down. The remaining lines consider the fact that an extra day of going to work increases the probability of becoming sick relative to staying at home. Upon becoming sick, the agent receives value V (the second line) but foregoes the stream of utility that a healthy person in a lockdown receives (the third line) and also foregoes the discounted value of returning to work as a healthy person once the lockdown is over (the fourth line).

The agent also decides when to end the lockdown and return to work (equation (26)). The benefit of spending an extra day in a lockdown is the associated utility flow minus the utility of returning to work, plus the change in the value of returning to work. The cost is that one becomes sick and gains V but loses the value of returning to work healthy.

This framing of the problem leads to several further results:

Lemma 5. *Any strategy in which $T_0 = T_1 = T_a$ is equivalent to any other strategy $T_0 = T_1 = T_b$. All such strategies correspond to no lockdown.*

Proof. See Appendix.

Lemma 5 simply states that deciding one morning that it is time to start the lockdown and then deciding the same morning to return to work results in an individual working through the epidemic, i.e. no lockdown taking place. □

The following Proposition puts a lower bound on the start of equilibrium lockdown:

Proposition 3. *In a competitive equilibrium, lockdown is never implemented in the initial period t=0.*

Proof. At $t = 0$, $\pi_W(0) - \pi_U(0) \approx 0$ but $u(c_W) > u(c_U)$. Thus it is beneficial to postpone the lockdown until later date. □

Thus individuals will never lock down straight away after the epidemic breaks out since at that point the probability of getting infected is very small (and the term $\pi_W(t) - \pi_U(t)$ is close to zero). As long as utility from going to work is greater than the utility of staying at home, individuals will decide to go to work as normal. As the disease spreads the chances of getting infected at work relative to staying at home increases, and at some point locking down is the preferred option.

Lockdown effectiveness has also important implications for the existence of the decentralized equilibrium, as highlighted in the following proposition:

Proposition 4. *Equilibrium in pure strategies in which individuals choose a single continuous lockdown period might cease to exist if $\varepsilon > \bar{\varepsilon}$.*

Proof. See Appendix. $\square$

When lockdown is very effective and $\varepsilon > \bar{\varepsilon}$, a double-peaked epidemic path might emerge. Faced with such path, individuals might be indifferent between locking down during the first or during the second peak – either strategy could be optimal. But a convex mix of such strategies is almost certainly not optimal, since it would mean locking down in between the two waves. Such non-convexity means that a fixed point between individual lockdown policies and optimal policies given the epidemic path may not exist. A lesson here is that, to the extent that people think in terms of a single lockdown episode, the equilibrium behavior is hard to predict, to the notion of prevention paradox This situation mixed strategies and/or multiple lockdown episodes the equilibrium could still exist – I pursue this extensions in ongoing work, but for this paper I focus on the single-lockdown-episode only.

4 Externalities in the analytical model

There are two externalities present in the decentralized equilibrium. The first is the *infection externality*: each individual takes the current and future economy-wide rates of infection as given, and yet the individual decisions do, in equilibrium, drive the economy-wide infection rates. This is most obvious when it comes to the infected individuals: for them, going to work carries no risk anymore, but it introduces high risk for all the susceptibles around them. But the externality is also present in the context of the susceptible individuals, who do not internalize the impact of their behavior on aggregate infection rates today and in the future. This latter component is a little more subtle, and I analyze it carefully below.

The second externality is the *fiscal externality*. When making lockdown decisions, individuals take government transfers and future taxes as given. But as individuals decide to lock down, they draw on government resources to finance consumption through lockdown and they diminish government revenues if they stop working and paying taxes. Clearly, individual lockdown decisions have external impact on government finances. More widespread lockdown leads to higher borrowing and thus higher future taxes.

4.1 Infection Externality

To analyze the first of these externalities in detail, let us assume for a moment that $\psi_{GOV} = \tau_n = G = 0$, so that government does not engage in any borrowing, and spending and taxes are zero at all times. The fiscal externality disappears and only the infection externality remains.

The following lemma characterizes the externalities in case of the recovered and infected individuals:

Lemma 6. *Suppose the planner knows the health status of each individual, and individual income in lockdown is above some extremely low level. Then it is never optimal to lock down the recovered, and it is always optimal to lock down the infected.*

Proof. The recovered do not affect infection probabilities; not being locked down yields higher utility than being locked down which proves the first part. A permanent lockdown of the infected has first-order beneficial impact on the spread of the virus and has only short-run second-order costs (since the period of time any individual spends in the infected state is short and in the long-run $I = 0$). Hence it is always optimal to lock down the infected. □

Lemma 6 implies that there is no externality in the behavior of the recovered individuals – they always choose to work and the planner is content with that choice. This is certainly not the case for the infected – in equilibrium they choose to work, but the planner always locks them down. This is the part of the infection externality that is obvious and often discussed in the literature.

Consider now the decisions of the susceptibles: to fix ideas, suppose for a moment that the planner can only enact a lockdown of the susceptible part of the population. Mathematically, the infection externality shows up in that a susceptible individual takes π_W and π_U as given when solving maximization problem (24). The planner instead internalizes the effect of susceptibles' lockdown decisions on infection rates.

Let $\pi_I := \hat{\pi} \cdot I$ denote the infection rate faced by a susceptible individual. Consider the planner's decision to postpone the start of the susceptible-only lockdown marginally. This has two kind of effects on the infection rates. The *intratemporal* component describes what happens to infection rates today:

$$\frac{\partial \pi_I(T_0)}{\partial T_0},$$

while the *intertemporal* component captures the effect on future infection rates:

$$\partial \frac{\{\pi_I(t)\}_{t>T_0}}{\partial T_0}.$$

The following Proposition characterizes these externalities:

Proposition 5. *The intratemporal component is internalized in the extensive-margin choices of the susceptible individuals. The intertemporal component is of ambiguous sign:*

$$\partial \frac{\{\pi_I(t)\}_{t>T_0}}{\partial T_0} \lesseqgtr 0.$$

Overall, it is impossible to sign the infection externality in general.

Proof. Recall equations (15) and (16). They imply that the marginal effect of postponing lockdown on today's infection rates is:

$$\frac{\partial \pi_I(T_0)}{\partial T_0} = \pi_W(T_0) - \pi_U(T_0) = \pi_n \phi \bar{n}^2 I(T_0). \tag{27}$$

The marginal impact is given by the difference in the probabilities of infection under no lockdown vs. lockdown. The probabilities themselves do not change, since the number of infected at a given point in time is a state variable (it is determined by the past choices and associated dynamics). The key to note is that equation (27) features the same trade-off as in the decentralized problem. The reason is that, with discrete choice of whether to lock down or not, individuals take the probabilities as given but they still effectively choose over these probabilities. Thus the intratemporal externality vanishes.

The intertemporal externality comes from the fact that today's decision whether to lock down or not affects the entire future path of infection rates $\{\pi_I(t)\}_{t=T_0}^{\infty}$ that a representative agent will face. Consider the choice of T_0 in the planning problem. For any $t > T_0$ we have:

$$\frac{\partial \pi_I(t)}{\partial T_0} = \begin{cases} \frac{\partial \pi_U(t)}{\partial T_0} = \pi_0 \frac{\partial I(t)}{\partial T_0} & \text{if } t \in [T_0, T_1] \\ \frac{\partial \pi_W(t)}{\partial T_0} = \left(\pi_o + \pi_n \phi \bar{n}^2\right) \frac{\partial I(t)}{\partial T_0} & \text{if } t > T_1 \end{cases}$$

Thus the lockdown decision affects future infection rates only via the (future) number of the infected. For any $t \in (T_0, T_1]$, equation (8) implies:

$$I(t) = -S(t) + \bar{S}_L \log S(t) + I(T_0) + S(T_0) - \bar{S}_L \log S(T_0).$$

Differentiating, we obtain:

$$\frac{\partial I(t)}{\partial T_0} = \underbrace{\frac{\partial S(t)}{\partial T_0}}_{<0} \underbrace{\left(\frac{\bar{S}_L}{S(t)} - 1\right)}_{\lesseqgtr 0} + \underbrace{\frac{\partial I(T_0)}{\partial T_0}}_{\lesseqgtr 0} + \underbrace{\frac{\partial S(T_0)}{\partial T_0}}_{<0} \underbrace{\left(1 - \frac{\bar{S}_L}{S(T_0)}\right)}_{\lesseqgtr 0}$$

The result in the Proposition follows from the fact that the sign of the right hand side is ambiguous. In particular, marginally postponing the lockdown can lower the infection rate late in the epidemic. To see this note that for t high enough the term in parentheses $\left(\frac{\bar{S}_L}{S(t)} - 1\right)$ is positive, and thus the first term on the right-hand-side is negative. The rest of the proof, based on similar logic, can be found in the Appendix. □

Proposition 5 clarifies the nature of infection externality present in epidemiology models. It shows that the prevailing one-way view of the externalities in the context of an epidemic is incomplete. It is true that the behavior of the infected always generates negative externality in the form of higher infection rates. But externalities that emerge from the behavior of the susceptibles are more subtle. The effects that individual decisions have on infection probabilities *today* are effectively internalized in the discrete choice model of the lockdown, because individuals cannot affect – but effectively do choose – the probability of infection when they decide whether or not to lock down. This is why there is no intratemporal externality. Most interestingly, the impact of today's decisions on future infection rates can be of either sign. What is the intuition? It is that an individual decision to lock down early ignores the fact that this can store trouble for the longer-term dynamics of the epidemic, through prolonging it and/or making the second wave of the epidemic more severe or more likely. This logic is the natural consequence of the trade-off highlighted in Lemma 2. Consequently, lockdown in the decentralized equilibrium can start before the socially optimal lockdown, and can last for a longer period of time. This theoretical result is helpful when trying to understand the recent phenomena observed in the United States and in other countries, where the private sector reacted to the news of the pandemic before governments did. And it is important for thinking about the exit strategy out of a lockdown: it highlights that it is not guaranteed that individuals will be comfortable going back to the pre-pandemic level of activity even when the governments tell them it is time to do so.[11]

4.2 Fiscal externality

To analyze the fiscal externality, revert back to the case with $\psi_{GOV} > 0$ and $\tau_n > 0$. We have the following result:

Proposition 6. *Fiscal externality unambiguously prolongs the equilibrium lockdown relative to the social optimum.*

Proof. Equation (14) implies that government debt B_0 is increasing in the duration of the lockdown. Government's intertemporal budget constraint (12) implies that future taxes τ are increasing in B_0. The effects of higher taxes are external to the individuals' problem because individuals take taxes as given. But higher taxes unambiguously lower long-run continuation values (equation (23)), which reduce the optimal duration of the lockdown, all else equal. $\square$

Fiscal externality is macroeconomic in nature. Given the government's commitment to finance a given share of lockdown income up to some fixed replacement rate, each additional day that any single individual spends in lockdown carries fiscal consequences. Yet those are not reflected in the problem of atomistic individuals.

This logic behind fiscal externality highlights that it can be thought in more general terms as capturing negative macroeconomic effects of lockdowns. For example, it is probable that a lockdown will lead to some deterioration in the stock of social, human, organizational and ordinary capital. Or, if the economy is at the zero lower bound and an extended lockdown weighs on demand after lockdown,[12] there could be aggregate demand effects as in Farhi and Werning (2016). Such effects, not internalized by individual consumers, workers and firms, would lead the planner to apply a shorter lockdown, relative to the decentralized outcome.

[11] Indeed, numerous surveys suggest that a very significant fraction of people would continue to refrain from certain activities even if the official lockdown was lifted. See, for example April 2020 YouGov Survey for the UK and April 2020 Pew Research Center Survey for the US.

[12] Note that it is far from certain whether lockdown will drag or boost demand post-lockdown (see Guerrieri et al. (2020) for the excellent discussion of the topic). But *if* lockdown boosts demand down the line, then monetary policy in the form of higher interest rates can bring demand back to a sustainable level, diffusing the externality. The asymmetry of the situation is driven by the zero lower bound on interest rates, as the central bank struggle to offset the effects of lower propensity to spend driven by the lockdown.

5 Optimal lockdown

Having discussed externalities in the previous Section, I now study the optimal lockdown strategy that a benevolent social planner would implement at the onset of the epidemic (I discuss the time-consistency properties of optimal time-0 policies in Section 6). As in any social planner problem, it is important to specify the objective and the tools that the planner has at her disposal.

5.1 Planner's objective and tools

I assume that the objective of the planner is to maximize lifetime utility of the susceptibles at time-0:

$$\max_{\{T_0, T_1\}} \mathcal{U}^{\mathcal{S}}(0)$$

This is a natural objective, not least because at the onset of the epidemic all people (except a vanishingly small fraction) are susceptible.

In terms of the tools at the planner's disposal I consider four possibilities:

Definition 3. I define four lockdown instruments / types as follows:

Type 1: *lockdown of susceptibles only:* government sets $\{T_0 \geq 0, T_1 \geq T_0\}$. $\lambda_S(t) = 0$ for $t \in [T_0, T_1]$, and $= 1$ otherwise. $\lambda_I(t) = \lambda_R(t) = 1 \forall t$.

Type 2: *lockdown of infected only:* government sets $\{T_0 \geq 0, T_1 \geq T_0\}$. $\lambda_I(t) = 0$ for $t \in [T_0, T_1]$, and $= 1$ otherwise. $\lambda_S(t) = \lambda_R(t) = 1 \forall t$.

Type 3: *broad lockdown:* government sets $\{T_0 \geq 0, T_1 \geq T_0\}$. $\lambda_S(t) = \lambda_I(t) = \lambda_R(t) = 0$ for $t \in [T_0, T_1]$, and $= 1$ otherwise.

Type 4: *lockdown with immunity passports:* government sets $\{T_0 \geq 0, T_1 \geq T_0\}$. $\lambda_S(t) = \lambda_I(t) = 0$ for $t \in [T_0, T_1]$, and $= 1$ otherwise. $\lambda_R(t) = 1 \forall t$.

The first tool lets the planner lock down only the susceptible population. I choose this tool not for the realism but because it serves as a useful comparison to the competitive equilibrium outcome, in which only the susceptible isolate. The comparison between the equilibrium outcome and the optimal type-1 lockdown is thus informative of the externality present in the behavior of the susceptibles. Type-2 lockdown isolates infected individuals only.[13] This again is not a very realistic policy given the information frictions

[13] I assume that under type-2 lockdown policy, the susceptibles are left with no choice (i.e. the planner forces the susceptibles to work). This is a simplifying assumption and it can easily be relaxed; but it is also

present in the real world, but it again is a useful benchmark. The third possibility – broad lockdown – is perhaps the most relevant in the real world to date. Finally, the fourth case is the lockdown combined with immunity passports which let the recovered return to work. This might become the realistic instrument of choice once large-scale antibody testing becomes available.

The following result highlights that the impact of these different lockdown variations on the dynamic of the epidemic is the same.

Proposition 7. *Consider lockdown policies of the four types defined above and of fixed timing $\{T_0, T_1\}$. In the model with $\pi_c = 0$, these four policies have identical effect on the epidemic dynamics.*

Proof. Under all four policies

$$\hat{\pi}(t) = \begin{cases} \pi_n \phi \bar{n}^2 + \pi_o & \text{if } t \notin [T_0, T_1] \\ \pi_o & \text{if } t \in [T_0, T_1] \end{cases}.$$

□

The implication of Proposition 7 is that, while the four lockdown types appear very different, if implemented at exactly the same time and for identical duration, the benefits that these lockdowns will yield in terms of slowing the dynamics of the epidemic are the same across the four policies. This perhaps surprising result is intuitive once we note that, as long as *either* the infected or the susceptible are locked down, the transmission rate through economic activity falls to zero in any lockdown.

Of course the fact that benefits of these policies are the same does not mean that the optimal lockdown timing and duration will be the same because costs can vary enormously across the four policy types. Nonetheless, if the costs are identically zero across the policies, optimal lockdowns will be the same across the policies:

Lemma 7. *If the replacement rate is 100% and the share of government transfer in income under lockdown is zero ($\psi_{GOV} = 0$), then lockdown of any type is costless. If $\varepsilon > \bar{\varepsilon}$, optimal lockdown of any type is immediate and permanent: $T_0 = 0$ and $T_1 = \infty$. If $\varepsilon < \bar{\varepsilon}$ then there are infinitely many optimal lockdown policies, with $T_0 = 0$ and T_1 arbitrarily large. Thus optimal lockdown timing is identical across the four lockdown instruments.*

without the loss of generality as the susceptibles will always choose to work when infected-only lockdown is implemented. This is because working does not introduce any additional infection risk.

Proof. If lockdown is costless, it is optimal to use the lockdown to the full extent to minimize the number of deaths. □

Moving away from the benchmark of costless lockdown, I now study optimal lockdown policies in a more realistic setting where the replacement rate is less than one and thus lockdown is costly. The following Proposition characterizes the optimal lockdown policy analytically:

Proposition 8. *If $\varepsilon \geq \bar{\varepsilon}$ the optimal lockdown starts at T_0^* defined in Proposition 2, and ends at T_1^*, where T_1^* is a known deterministic function of the model parameters. The optimal policy achieves close to the lowest feasible death toll as long as the replacement rate is not too small and the fiscal cost is not too large.*

If $\varepsilon < \bar{\varepsilon}$ an immediate and sufficiently long lockdown achieves the minimum feasible number of total deaths, but is never optimal. Instead, it is always optimal to start the lockdown at a date $T_0^* > 0$ and end it at $T_1^* < \infty$, where both T_0^* and T_1^* are known functions of the model parameters.

Proof. See Appendix. □

Proposition 8 fully characterizes the optimal lockdown policy. If lockdown is very effective, the optimal strategy is to hold nerve and only activate the lockdown when the epidemic's trajectory post lockdown is approaching $\bar{S}$ as a resting point. That point is never reached, however, as this would take a suboptimally long time. At some point the economy is close enough to this threshold, so that additional infections when the lockdown is lifted are limited. It is thus optimal to lift the lockdown at that time. The proof of the Proposition contains the details.

When lockdown is less effective than threshold $\bar{\varepsilon}$, the Proposition shows that the optimal lockdown also never starts at $t = 0$, because at that point the trade-off between extra infections and shorter lockdown is very favorable. This characteristic of the optimal lockdown is the same as in the competitive equilibrium.

Proposition 8, and this paper more generally, have so far considered a binary full lockdown / no lockdown decisions and strategies. The following result shows that these all-or-nothing strategies are optimal, even if the planner can choose an arbitrary stringency of a lockdown:

Proposition 9. *Suppose the planner can control how strict a lockdown policy of a given type is, that is, the planner can choose any $\lambda \in [0,1]$. With this added flexibility, the optimal lockdown policy in the case with $\varepsilon \geq \bar{\varepsilon}$ is still as described by Proposition 8.*

Proof. For the case with $\varepsilon > \bar{\varepsilon}$, the lockdown strategy described in Proposition 8 achieves, to first order, the lowest feasible number of deaths. Thus choosing $\lambda \in (0,1)$ cannot lower the number of deaths. But it must increase the time spent in lockdown. Since lockdown is costly, such strategy cannot be optimal. □

Proposition 9 is a striking result. It says that the optimal policy described in the Proposition 8 is also the optimal choice out of a much larger class of instruments, where the planner can choose not only the start and end date of a lockdown but also adjust its stringency continuously and in any way she wishes. The proposition shows that in the baseline analytical model the flexibility to make lockdown a gradual process brings no benefits but raises the costs, and therefore is never exercised by the planner.

One important corollary from this is that the results presented in this paper are therefore more general than they first appear. The motivation to focus on the lockdown-on / lockdown-off policies was, in part, that it simplified the exposition and allowed for analytical characterization of the solution. But Proposition 9 shows that in this setting the blunt tool is indeed the optimal one. In this sense the restriction to $\lambda \in \{0,1\}$ that has been the working assumption of the paper is thus without loss of generality.

I now illustrate these results using numerical simulations. To do so, I first outline the calibration of the model.

5.2 Calibration of the analytical model

Macro Parameters. I calibrate the model to a weekly frequency, with the discount rate $\rho = 0.96^{\frac{1}{52}} - 1$. The discount rate in this model is important only in so far as it determines the continuation value of staying alive; the annual interest rate of 4% translates into value of statistical life equal to \$10 million, in line with the literature (Andersson and Treich (2011), Kniesner and Viscusi (2019)). I assume that the government can borrow in the international markets at 1%, broadly matching the low borrowing costs observed across the industrial economies today (Rachel and Summers, 2019). I set $\bar{n} = \frac{1}{5} \cdot 24 \cdot 7$ to match the average weekly hours worked across advanced economies (Rachel (2020)). Calibrating A to 34 means that the per-capita annual income in steady state is $24 \cdot 7 \cdot 52 \cdot \bar{n} \cdot A = \$60,000$, matching the pre-Covid GDP per capita in the United States. I set τ_n to 25%, matching the average labor income tax rate in the OECD economies.

The key parameter that guides the trade-off between the misery of death and the misery of the lockdown is h – the value of home production.[14] In the baseline calibration I set h to hit the replacement rate of 80%, that is I set h such that:

$$r := \frac{h}{A\bar{n}} = 80\%.$$

This choice is motivated by the income support measures introduced by several countries including the UK which offer furloughed workers up to around 80% of their pre-shock salary. Unless stated otherwise, I assume that one-half of h is market income, while home production and government transfer are each a quarter: $\psi_{WFH} = \frac{1}{2}$, $\psi_{HPR} = \psi_{GOV} = \frac{1}{4}$. This is motivated by the early estimates on the proportion of people that can work from home plus those who work in the essential sectors, as well as the initial estimates of the share of workers who have been furloughed (Dingel and Neiman (2020), Tomer and Kane (2020), Davies (2020)).

Epidemiology Parameters. I assume that the initial seed of infection represents 0.001 of the total population.

I set the parameter ϕ which guides the proportion of individuals that are infected but asymptomatic to 50%, which corresponds to the shares of asymptomatic infected in the Italian town of Vo' Euganeo where all individuals where tested twice ((Zingales, 2020), Lavezzo et al. (2020)).

The recovery and death parameters π_r and π_d are determined by the mortality rate and the average length of time it takes to either recover or pass away after contracting the virus. Since the model is weekly, for a baseline case fatality ratio of 1% and an average disease duration of 18 days, we have that $\pi_d = 0.01 \cdot \frac{7}{18}$ and $\pi_r = \frac{7}{18} - \pi_d$.

I fix the initial basic reproductive number to $\mathcal{R}_0 = 2.5$ and I consider two parametrizations of lockdown effectiveness by setting $\varepsilon = 0.36$ and $\varepsilon = 0.60$ (recall that the threshold $\bar{\varepsilon}$ equals .39, so that these two calibrations lie on the two sides of this important threshold). In the first of these the lockdown reduces $\mathcal{R}_0$ by 36% to 1.6. In the second, lockdown reduces $\mathcal{R}_0$ by 60%, to 1. These values are representative of the confidence ranges across the epidemiological estimates of the effects of lockdowns on the transmission of Covid19 (Flaxman et al. (2020), Lavezzo et al. (2020)). This early evidence suggests that the lockdown policy has led to a substantial decrease in $\mathcal{R}_0$. Nonetheless, it also points to the

[14] If h is high and close to the wage income obtained while working, then lockdown is a relatively painless experience and we might expect the self-imposed lockdown to occur earlier and last for longer. Conversely, when h is low, the trade-off between material well-being and possibility of illness and death is much steer.

wide confidence bands around the estimates: for example, the 50% central tendency in the estimates for the UK puts the values of $\mathcal{R}_0$ post-lockdown at between 0.8 and 1.8.

Actual vs. model implied deaths for the UK. I now illustrate how the calibrated model stacks up against the death toll data for the United Kingdom. The UK has implemented a national lockdown on 24 March. Whether this is early or late in the epidemic crucially depends on the proportion of the population that had contracted the virus by that point. The share of the population who had the virus will not be observable until accurate large-scale antibody testing is developed and deployed. For now, the evidence is scarce and paints a very mixed picture. For example, WHO suggested that by around mid-April most places would have seen cumulative infection rates of around 3%[15], while another study suggested that 21% of New Yorkers have had the virus by mid April.[16] While appreciating the extreme levels of uncertainty surrounding this figure, one way to resolve this conundrum is to calibrate the share of cumulative infections at the time when lockdown was introduced to match the dynamics of the reported deaths. Based on this exercise, I assume that by 24 March around 1% of the UK population have had the virus. This is broadly consistent with the WHO estimates for mid-April.

Figure 4 shows the data on cumulative deaths in the UK[17] against three model scenarios: no lockdown, a highly effective lockdown, and a less effective lockdown. For illustrative purposes I assume that lockdown lasts for 12 weeks and so all the restrictions are lifted in mid-June. The Figure reveals that the exact reduction in $\mathcal{R}_0$ makes a first order difference for the expected paths of deaths, at least up until the point when the restrictions are lifted. It shows that the UK data supports the range of $\mathcal{R}_0$ reduction considered in my calibration.[18] It also illustrates that if all restrictions were suddenly lifted

[15]https://www.theguardian.com/society/2020/apr/20/studies-suggest-very-few-have-had-covid-19-without-symptoms

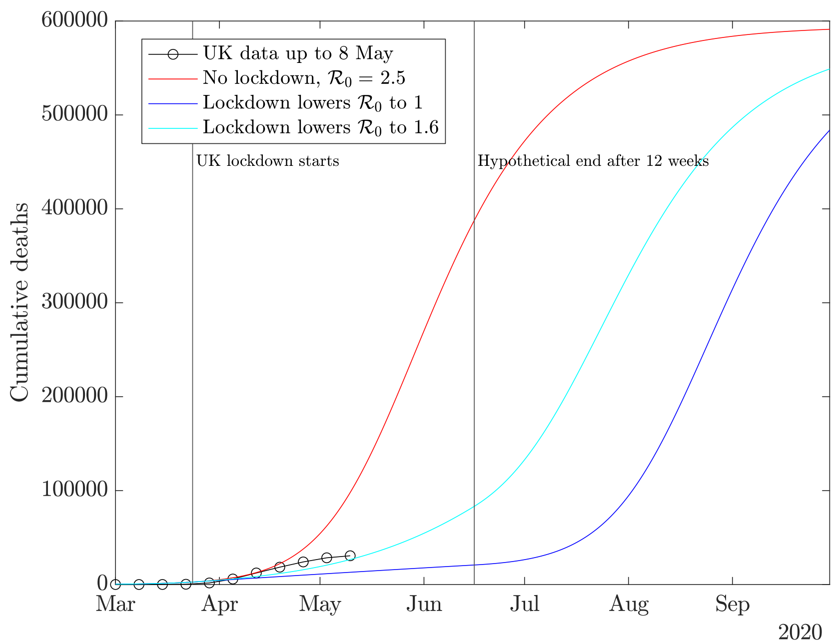
[16]https://www.nytimes.com/2020/04/23/nyregion/coronavirus-new-york-update.html

[17]Financial Times reports that excess deaths in the UK are twice the official reported deaths. Many of the excess deaths might be indirect (e.g. caused by constraints on medical care for non-covid patients). Since I do not model those I err on the side of caution and focus on the official data. But in future work I plan to explore using the excess deaths as well.

[18]Matching the deaths data with my simple SIR model should be taken as illustrative only and must not be over-interpreted. The model's simplicity means that it is not very accurate in reflecting the short-term dynamics. For instance, the exponential distribution of the infected state is clearly counterfactual. The Figure shows that the trajectory of deaths first followed the $\mathcal{R}_0 = 1.6$ more closely, but then lately paralleled $\mathcal{R}_0 = 1$. The right interpretation is that the model provides a reasonable approximation to the data and supports the ranges for $\mathcal{R}_0$ estimated by epidemiologists, not that it has any strong implications for the actual basic reproduction number in the UK today.

in June, the model predicts a dramatic return of the epidemic. This is a similar result to what Jones and Fernández-Villaverde (2020) find for the United States. Note that the second wave in this scenario would be sharper for the if the lockdown was highly effective to begin with. This is because there is less build-up of immunity by June in that case – illustrating the result in Lemma 2.

Figure 4: UK deaths data against model trajectories



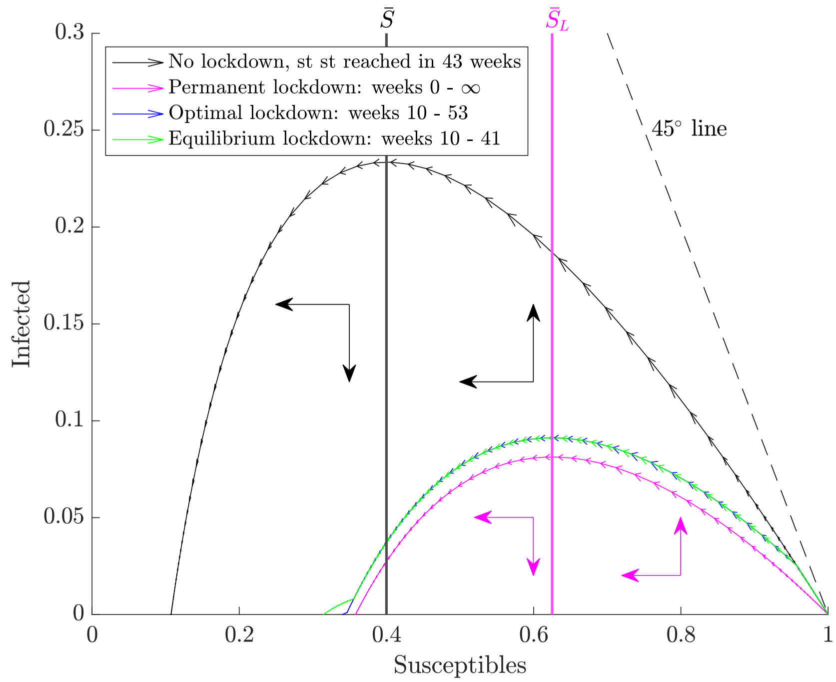
5.3 Type 1: Lockdown for the susceptibles only

I now compute the optimal policy when the planner who can lock down only the susceptible part of the population.[19] This is the case that is useful for thinking about the infection externality due to the behavior of the susceptibles. To focus attention on this externality, in this subsection I set $\psi_{GOV} = 0$ so that there is no fiscal externality.

Lockdown not very effective, $\varepsilon < \bar{\varepsilon}$. Figure 5 considers the calibration where the lockdown is not powerful enough bring the basic reproductive number below unity. By the definition of $\bar{\varepsilon}$, in this case a permanent lockdown takes the system to a stable resting point to the left of the $\bar{S}$ line. Competitive equilibrium exists in this case; equilibrium lockdown

[19] I provide the description of the algorithms used to solve the model and find the optimum in the Appendix.

Figure 5: Equilibrium and optimal lockdown when $\varepsilon < \bar{\varepsilon}$, with no fiscal externality ($\psi_{GOV} = 0$)

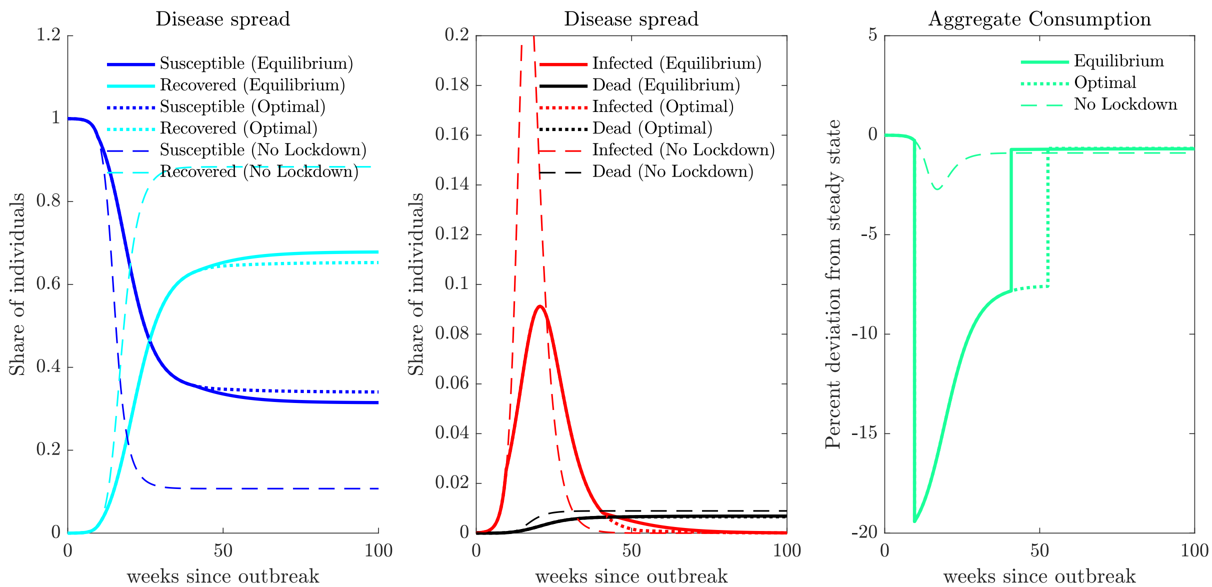


starts in week 10 and ends in week 41 (see legend in the Figure). Optimal lockdown starts slightly *later*, illustrating the possibility highlighted by Proposition 5 and matching some of the facts and data discussed in the introduction. The optimal lockdown lasts three months longer, extending to over a year. That longer duration means that eventually it achieves a lower cumulative level of infection (a higher $S(\infty)$) and thus a lower level of deaths. The difference in the death toll between the competitive equilibrium outcome and under optimal lockdown is 0.04% of the population.

Why is it not optimal to start lockdown earlier? It is because this prolongs the lockdown and that is economically costly. In particular, if the lockdown starts immediately it takes over 60 weeks to reach herd immunity; under optimal lockdown it only takes 36 weeks (of which only 26 weeks in lockdown). This is a reminder that the system's dynamics are such that the epidemic proceeds faster the further north in the phase diagram. Thus any path that lies to the south of another path is also one that travels more slowly and thus takes more time.

Figure 6 illustrates what these different lockdowns mean for the time series of the epidemic and for macroeconomic aggregates. The epidemic dynamics intuitively match the story in the phase plane. Aggregate consumption declines by up to 20% at the start

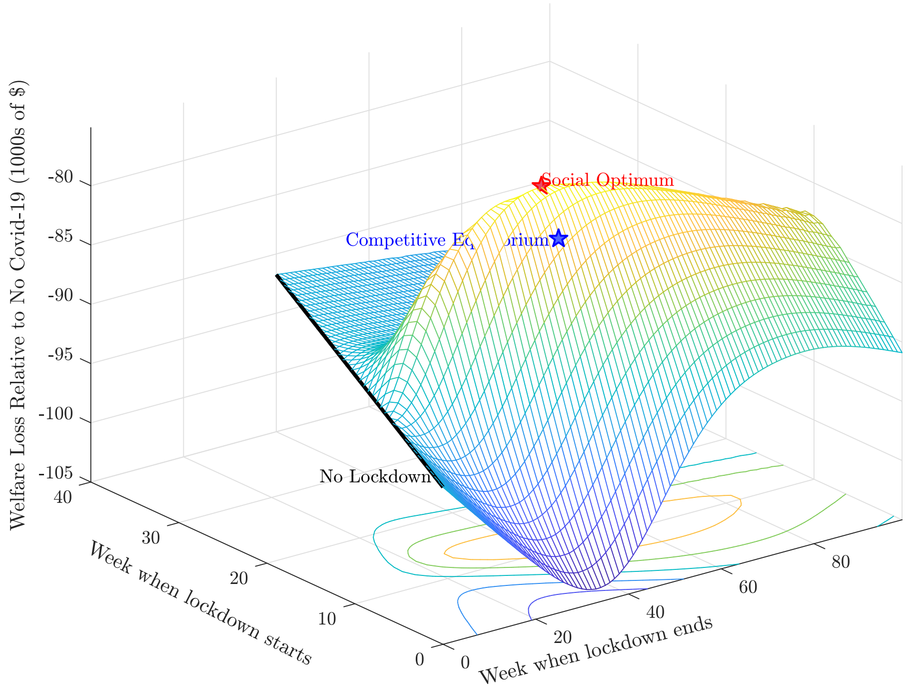
Figure 6: Epidemic in equilibrium and under the optimal type-1 (S-only) lockdown



of the lockdown and then recovers as the number of susceptibles declines. This path illuminates the trade-off struck by optimal policy: to achieve a smaller rise in deaths compared to the equilibrium outcome, planner implements a longer lockdown, which is costly in terms of output and consumption. How much longer will depend on the assumed value of statistical life and the cost of the lockdown.

To further explore the welfare impact of the different configurations of the start and the end date of a lockdown of type-1, Figure 7 shows the social welfare function – i.e. the lifetime utility of the susceptible persons at date-0 – as a function of the timing of the susceptible-only lockdown that is put in place. The welfare measure is transformed into units that correspond to dollar value loss relative to no Covid. The Figure also marks the competitive equilibrium and the social optimum, and the black 45-degree line marks the "no lockdown" scenario (recall Lemma 5). There are several interesting things to note. First, implementing the lockdown at any date beyond 40 weeks is pointless – by that point, the epidemic has already passed. The social welfare function in that region is flat and welfare is at low level. Second, the Figure verifies that the social optimum attains a greater level of welfare than the competitive equilibrium or the "no lockdown" scenario. Third, the Figure reveals just how bad is the idea of lifting the lockdown too early, especially if it is implemented early on in the epidemic cycle. The reason is that the early lift-off causes a large spike in cases further down the line, while the economic cost is borne out early when the risk is not that high.

Figure 7: The Social Welfare Function (S-only lockdown)



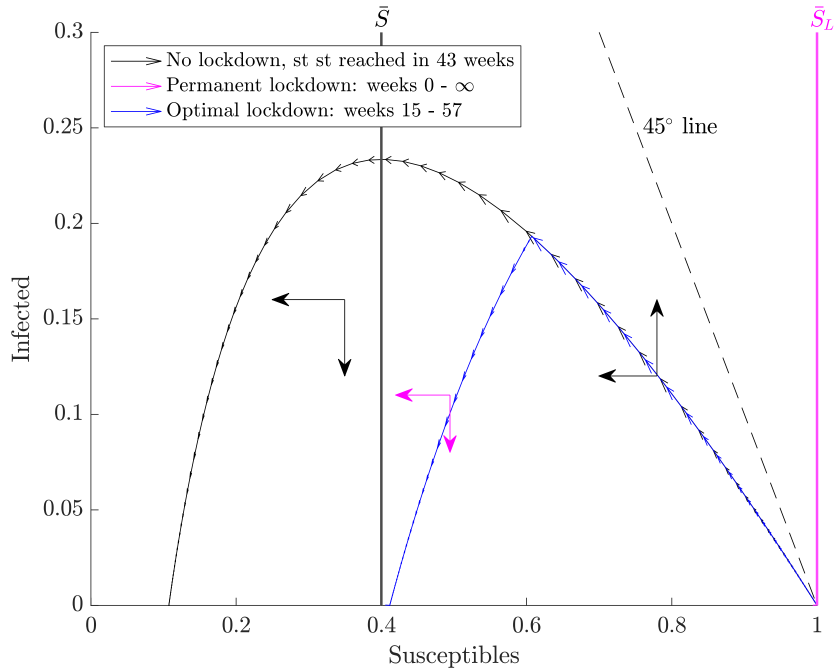
Lockdown very effective, $\varepsilon > \bar{\varepsilon}$ **.** In this case the competitive equilibrium in pure strategies cannot be found (Proposition 4). Immediate suppression is extremely effective but is unsustainable unless the lockdown lasts forever. In that environment the optimal strategy involves starting the lockdown at T^*, which allows for the optimal path to converge to around point $\bar{S}$ (Figure 8). This ensures that when the planner lifts lockdown there is only a minimal re-emergence of the epidemic: effectively the planner roughly minimizes the number of deaths given the available instruments.

5.4 Optimal use of other lockdown instruments

I now briefly analyze the optimal policy when one of the other three lockdown instruments are available, focusing on the case with $\varepsilon < \bar{\varepsilon}$ so that decentralized equilibrium exists and $\psi_{GOV} = 0.25$ so that government picks up a quarter of the lockdown bill.

Figure 9 summarizes the welfare effects of pursuing different lockdown strategies. Qualitatively, the social welfare function looks similar across types 1, 3 and 4, but is different when the instrument involves lockdown of the infected only. In that latter case it is optimal to implement an immediate and permanent lockdown. This is intuitive and in line with Lemma 7, because lockdown of the infected only is costless in the long-run

Figure 8: Optimal lockdown when $\varepsilon > \bar{\varepsilon}$



Notes: The Figure shows the type-1 (susceptible-only) lockdown, but to first order this is the shape of all optimal policies under types 3 and 4 as well.

(since $I(\infty) = 0$).

The other three panels of the Figure illustrate that, across these policies, it is a poor idea to implement the lockdown early and lift it too soon: welfare in that case can be significantly lower than in the no-lockdown scenario.

Figure 9: The Social Welfare Function for the four lockdown instruments when lockdown lowers $\mathcal{R}_0$ to 1.6

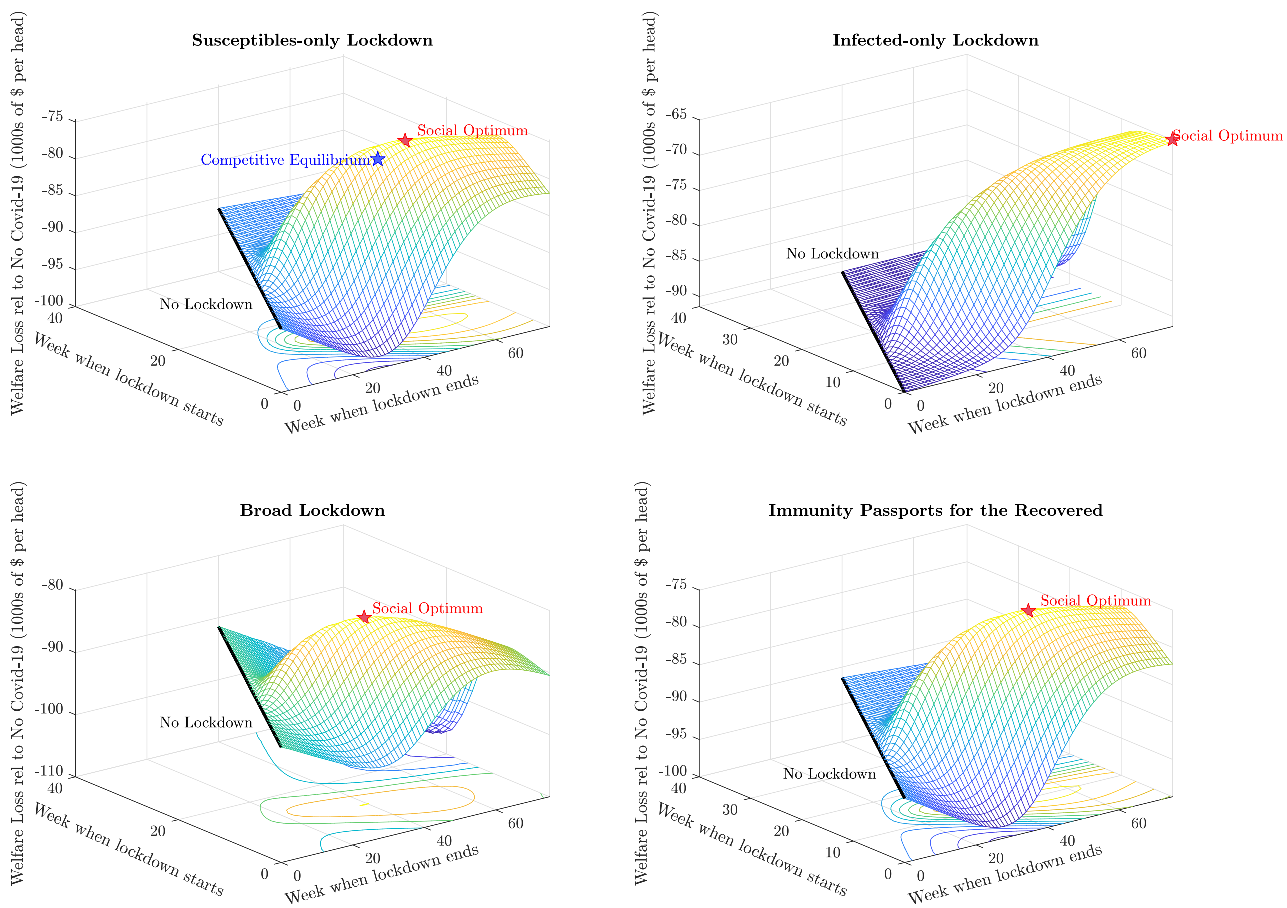
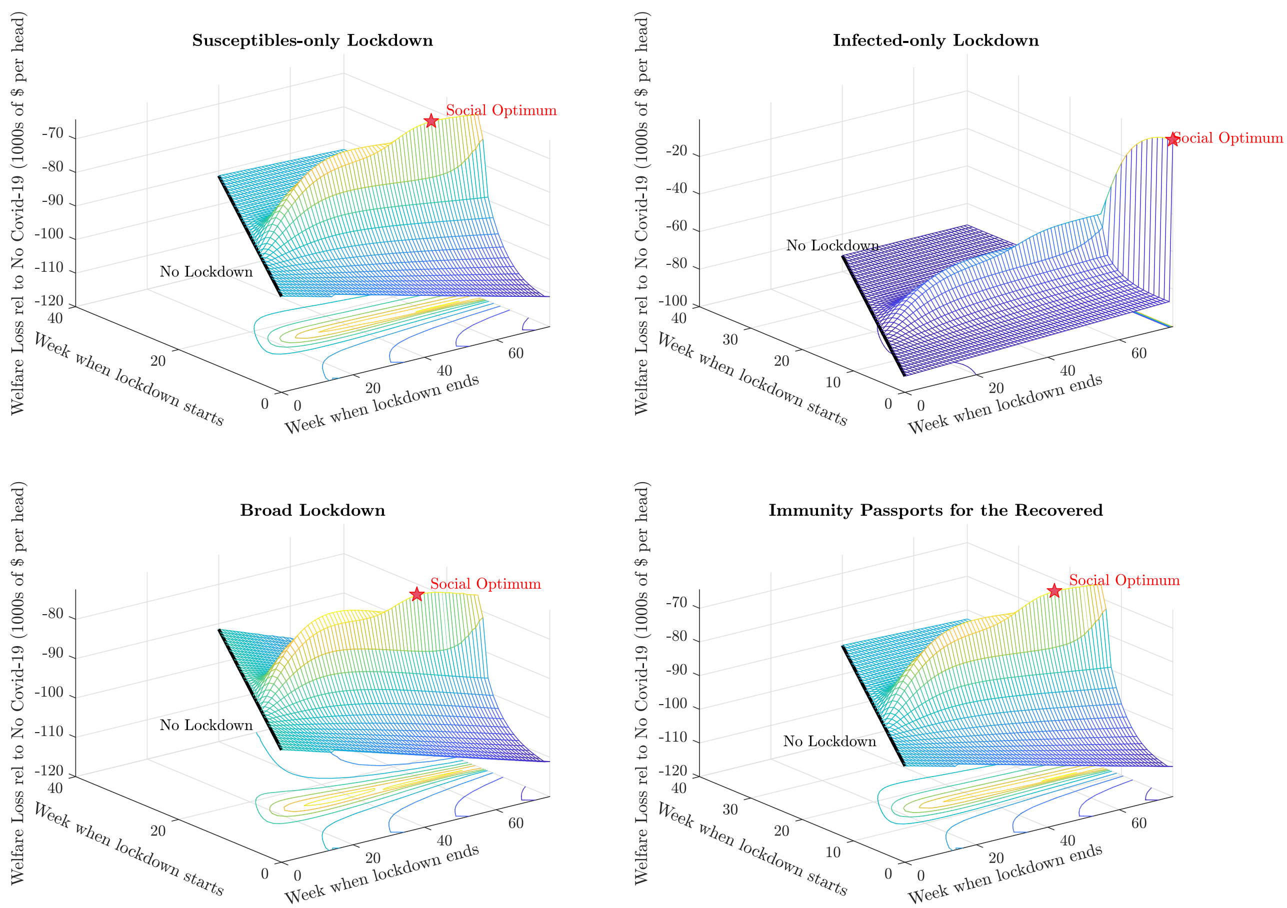


Figure 10 computes equivalent social welfare functions for the case when lockdown is very effective. The change relative to the less effective lockdown is substantial. The effective lockdown is much more demanding of the policymaker, in the sense that it is important to implement the lockdown at the right time (around 3 months into the epidemic). Do it too early and you do not build any immunity and face a powerful second wave when the lockdown is lifted. Do it too late and you hardly affected the epidemic dynamics at all. The lesson here is that wielding a powerful instrument is also more difficult.

Again, for the type-2 lockdown, the optimal strategy is to make this lockdown permanent (the final row along the "when lockdown ends" axis). This is essentially an application of Lemma 7, since this form of lockdown is almost costless (and exactly costless in the long-run, once $I \to 0$).

Figure 10: The Social Welfare Function for the four lockdown instruments when lockdown lowers $\mathcal{R}_0$ to 1



Recall that in this section I assumed that the government foots a quarter of the lockdown bill and at the same time it faces declining tax revenues due to lockdown. How important is the fiscal cost for determining optimal lockdown policy? Each week of lockdown leads to higher government borrowing and so higher future taxes. This prospect reduces the duration of the optimal lockdown. The effect is significant and similar across the two calibrations of ε. Relative to the case where there is no fiscal cost of lockdowns, the optimal lockdown is about 15-20% (5 to 6 weeks) shorter in the case with fiscal cost of lockdowns. Fiscal externality is thus a powerful mechanisms that should not be overlooked.[20]

[20] A natural question is what is the optimal level of income support during lockdown, and how does that interact with the lockdown policy itself. I hope to be able to study both of these questions in future work.

5.5 Social and economic impact of lockdown policies

Table 1 compares the four optimal lockdown strategies to the no lockdown scenario and the equilibrium lockdown, in terms of their epidemic, macro and welfare implications.

Table 1: Social and Economic Effects of the Decentralized Equilibrium Lockdown and Optimal Lockdown Policies, $\varepsilon < \bar{\varepsilon}$

			Optimal Lockdown Policies			
	No lockdown	Equilibrium	S-only	I-only	Broad	Passports
Lockdown						
Week lockdown begins	-	10	10	0	11	10
Week lockdown ends	-	41	49	-	39	49
Duration (weeks)	-	31	38	∞	28	38
Epidemic (% of population)						
Contracted virus	89.3	68.6	66.8	64.2	68.9	66.8
Peak infected	23.4	9.1	9.6	8.1	10.2	9.6
Total deaths	0.893	0.686	0.668	0.642	0.689	0.668
Economic Impact in Year 1						
Agg C loss (%)	-1.1	-8.5	-8.8	-1.2	-11.8	-9.1
GDP loss (%)	-1.9	-22.8	-23.9	-2.4	-33.7	-24.9
Fiscal support (rise in Debt/GDP*, pp)	0.8	13.1	13.9	1.2	20.0	14.4
Lifetime utility rel to no virus						
Welfare Loss (% of life utility)	-0.88	-0.73	-0.72	-0.63	-0.77	-0.72
Welfare Loss ($ per capita)	-91000	-76000	-75000	-65000	-80000	-75000

The first three rows compare the timing and the duration of the lockdown. Equilibrium lockdown starts in week 10 and lasts for 31 weeks. This is 7 weeks shorter than the optimal lockdown of the susceptibles only. The infected-only lockdown is implemented straight away and is permanent. Broad lockdown starts at a similar time, but lasts a shorter period because it is much more costly. This cost can be reduced by the introduction of immunity passports, which increases the optimal duration back to 38 weeks. What these results illustrate is that the start-date of the lockdown is basically independent of the type of lockdown one considers. This conclusion is even more true when $\varepsilon > \bar{\varepsilon}$.

The next three rows show how the epidemic develops under these different policies. Overall, the different optimal lockdown policies yield a similar picture in terms of the development of the disease, in line with the result in Proposition 7. Relative to no lockdown, policies can reduce the share of people who contract the disease by over 20 percentage points. Peak infections are reduced by a factor of two-and-a-half, even though peak infections are not in the planner's objective function in the baseline model (recall that the death rate is exogenous and constant). Given the constant death rate, deaths decline in line with the share of the population that contracted the virus.

The next three rows focus on the macroeconomic impact in the first year following the outbreak. I consider three headline macro variables: loss of aggregate consumption relative to no-Covid-19, aggregate GDP loss, and the implied fiscal support needed to sustain spending and transfers. The macro impact varies widely across the scenarios. As highlighted above, the no lockdown scenario is associated with only a minor recession. But once we allow for endogenous behavioral response of the private sector, the equilibrium lockdown leads to a very large contraction in GDP of 23% in the first year; aggregate consumption declines by less as home production and government transfers provide a cushion. The required direct fiscal transfer is of the order of 13 percentage points of GDP (of which 7 percentage points is the direct income support and 6 percentage points is due to lost revenues). These numbers rise further in magnitude in the case of the optimal susceptible-only lockdown, simply due to its longer duration.[21] The infected-only lockdown emerges as a very cheap way to limit the spread of the virus, highlighting the huge economic benefit that can be brought by massive testing programs. The final two columns show that the lockdown instruments that are most likely to be used absent these programs are also the most expensive. The associated GDP losses are large (between 25-34%) and the required direct fiscal support is also substantial, in the region of 15-20% of GDP.

The final two rows of the table provide a summary measure that weighs the benefits and costs of the lockdown strategies in terms of saved lives and lost economic activity. When the virus first emerges and no mitigation strategies are possible, representative consumer's *lifetime* welfare drops by around -0.9%, equivalent to $91,000 *per capita* in terms of current income and consumption (in other words, one year and a half worth of income). It is thus a very large hit.

The ability to lock down can help: the welfare loss is reduced by a sixth with the equilibrium lockdown. Interestingly, optimal lockdown policies can improve on the equilib-

[21]Note that the macroeconomic costs rise less-than-proportionately with lockdown duration. This is because the additional weeks happen at the end of the epidemic, when the number of susceptibles (who are in lockdown) is much lower than at the beginning. In other words in the case of S-only lockdown the first week of lockdown is always the most expensive (see also Figure 6).

rium only marginally, except if the authorities can identify and lock down the infected. If the government can only lock up the susceptibles it will do so for longer than in equilibrium, but the gains from fewer deaths are rather small (recall that at this point the epidemic moves slowly, so the extra 7 weeks of lockdown does not change the overall infection path all that much). Broadly speaking, the infection externality roughly offsets the fiscal externality in this case, leaving the decentralized equilibrium close to the optimum.[22]

6 Time consistency

Are the optimal lockdown policies discussed so far time consistent? There are two possible sources of time-inconsistency. The first is that as time passes, what has initially been optimal is no longer optimal from the perspective of a susceptible agent. The second is that the health-status composition of the population changes as the epidemic proceeds. This latter concern is particularly important for broad lockdown policies. At the outset of an epidemic, the planner sets out the lockdown policy to maximize the welfare of the susceptibles, who at that point are the population. But as the epidemic progresses, larger and larger fraction of the population are recovered. Thus if the planner cares about the entire population, the initially optimal strategy may not be optimal ex-post.

In this Section I show that only the latter of these two can be a source of time inconsistency.

Consider the value function of the susceptible agent at time some time z, given lockdown start and end dates of T_0 and T_1:

$$V_S(z) = \begin{cases} \int_z^{T_0} e^{-\rho t - \int_z^t \pi_W(s)ds} \left(u(c_W) + \pi_W(t)V(t)\right) dt + \\ + e^{-\rho(T_0 - z) - \int_z^{T_0} \pi_W(s)ds} \int_{T_0}^{T_1} e^{-\rho(t-T_0) - \int_{T_0}^t \pi_U(s)ds} \left(u(c_U) + \pi_U(t)V(t)\right) dt + \\ + e^{-\rho(T_1 - z) - \int_z^{T_0} \pi_W(s)ds - \int_{T_0}^{T_1} \pi_U(s)ds} W^S(T_1) & z < T_0 \\ \int_z^{T_1} e^{-\rho(t-z) - \int_z^t \pi_U(s)ds} \left(u(c_U) + \pi_U(t)V(t)\right) dt + e^{-\rho(T_1 - z) - \int_z^{T_1} \pi_U(s)ds} W^S(T_1) & T_0 < z < T_1 \\ W^S(z) & z > T_1 \end{cases}$$

Does the planner have an incentive to deviate from the optimal $t = 0$ plan at any time $z > t$? To be specific, consider the following deviation: at some $z \in (T_0, T_1)$ the planner

[22] It is important to note however that the equilibrium assumes perfect knowledge of the underlying health status, and as such is not feasible in the real world.

considers marginally bringing forward the end date of the lockdown. Such deviation would be beneficial if:

$$u(c_U) + \pi_U(T_1)V(T_1) - (\rho + \pi_U(T_1))W^S(T_1) + \frac{\partial W^S}{\partial t}\ |_{t=T_1} > 0 \tag{28}$$

The crucial thing about this condition is that the left-hand side is independent of z. But that implies that the left hand side is the same for all z, including $z = 0$. The optimality condition at time-0 thus implies that the left hand side of (28) is equal to zero, and so the deviation is not profitable. Similar logic shows that the time-0 optimality condition with respect to T_0 implies that no deviation is beneficial for $z < T_0$. We thus have the following proposition:

Proposition 10. *The planner who maximizes the welfare of the susceptible agent only has no incentive to deviate from the time-0 optimal strategy.*

Proof. See the argument above. □

The problem is that the utilitarian planner does not maximize the welfare of the susceptible only. Instead, her objective is to maximize welfare of the representative individual, which at time z is given by:

$$\mathcal{W}(z) = S(z) \cdot V_s(z) + I(z) \cdot V_I(z) + R(z) \cdot V_R(z).$$

Assuming that the seed of infection is vanishingly small at $t = 0$, we have:

$$\mathcal{W}(0) = V_S(0).$$

Thus the time-0 optimal strategy is to maximize the time-0 lifetime utility of the susceptible agent, and the resulting strategy satisfies the optimality condition that sets the left-hand side of equation (28) to zero. Consider again the deviation in which at some $z \in (T_0, T_1)$ the planner marginally brings forward the end date of the lockdown. The marginal benefit to the susceptibles is zero by the optimality condition of the original time-0 plan. But the marginal benefit to the infected and the recovered are both positive because:

$$u_I - u_I^L > 0$$
$$u_R - u_R^L > 0$$

We thus have the following result:

Proposition 11. *The time-0 optimal lockdown strategy of the utilitarian social planner is time-inconsistent. As the shares of the infected and the recovered in the population increase, bringing forward the end-date of the lockdown, relative to the time-0 optimal plan, is welfare improving.*

Proof. See the argument above. □

This Proposition shows that because the underlying objective of the social planner changes through time, the utilitarian planner's time-0 strategy is not time consistent. The practical lesson for the real world is that the broad lockdown measures announced at the outset of the epidemic may not be followed through eventually.

7 Game-changers?

So far I have studied the workings of the baseline analytical model of the Covid-19 epidemic, and the implications for the equilibrium and for optimal policy. The model has been kept deliberately simple, abstracting from many features of the real world by design. The question I tackle in this section is how some of these features change the workings of the model and the conclusions of the paper thus far. I discuss three such elements: feasible suppression of the virus, healthcare capacity constraints, and the possibility of vaccine or treatment discovery. I ask whether these features can have game-changing implications for the conclusions about optimal policy. I also briefly discuss how an alternative assumption on the infections matching technology changes the picture.

7.1 Feasible full suppression

The key implication of the baseline model has been that full suppression of the virus below the herd immunity level $\bar{S}$ is not feasible: mathematically, any resting point to the right of $\bar{S}$ was assumed to be unstable (Lemma 1). This might be a reasonable implication for several reasons. First, even the most sophisticated track-and-trace strategies may struggle to identify and isolate all existing cases, given the widespread community transmission already in place in case of Covid-19. Second, even if such strategies were successful domestically in a given country, virus is likely to make a re-entry from abroad once borders open, especially that, to date, 212 countries and territories recorded cases of

Coronavirus. Finally, even in the unlikely event that the entire world managed to identify and suppress all human cases, the virus could re-emerge if and when it is transmitted, once again, from animals to humans.

Nonetheless, it is useful and straightforward to discuss briefly how an alternative assumption would change the workings of the model. Feasible full suppression means that there exists a threshold of infected $\underline{I}$ below which the virus is eliminated. A natural number for such threshold is 1 over the population size of the given city, province, country or the world, depending on at which level the model is considered: since the smallest positive number of people that can have the virus is one, a number below one indicates that the last patient has recovered or died, and no-one in the given area is a carrier any more. A sufficiently long use of an effective-enough lockdown (with $\varepsilon > \bar{\varepsilon}$) can then fully suppress the virus. This strategy will indeed be optimal if the effectiveness of the lockdown is significantly larger than the threshold $\bar{\varepsilon}$ and if the utility cost of the lockdown is not prohibitively high.

A related question is how long does the lockdown need to last in order to deliver full suppression. Taking the full suppression threshold of $\frac{1}{Population}$, if the starting point in terms of the infected is only 0.001% of the population (e.g. 6600 infected in the UK), and if the lockdown lowers $\mathcal{R}_0$ to $\frac{1}{2}$, it takes 46 weeks to suppress the virus completely. It is likely that lockdown of this stringency which covers broad swathes of the population for so long is simply not enforceable in many jurisdictions, not least in western democracies. It is also all-but-guaranteed to be associated with large economic costs. A solution would be to deliver the required reduction in $\mathcal{R}_0$ via a type-2 lockdown of the infected-only. This *could* be feasible with the use of widespread testing and Bluetooth tracking or similar, although the efficacy of these technologies remains untested on mass scale. If such technologies were less effective, the required length of a lockdown increases exponentially. For example, with a looser but still highly effective lockdown that lowers $\mathcal{R}_0$ to 1, it takes a staggering 1869 weeks – or 36 years – to suppress the virus.[23] This highlights that full suppression strategy quickly runs into practical feasibility constraints.

Nonetheless, assuming that full suppression through track and trace strategy is feasible (from the epidemiological, political and economic perspectives), it might be preferable to the lockdown strategy outlined earlier in the paper. The calculus will depend on the efficacy of the track and trace technologies and hence the length of the complete lockdown

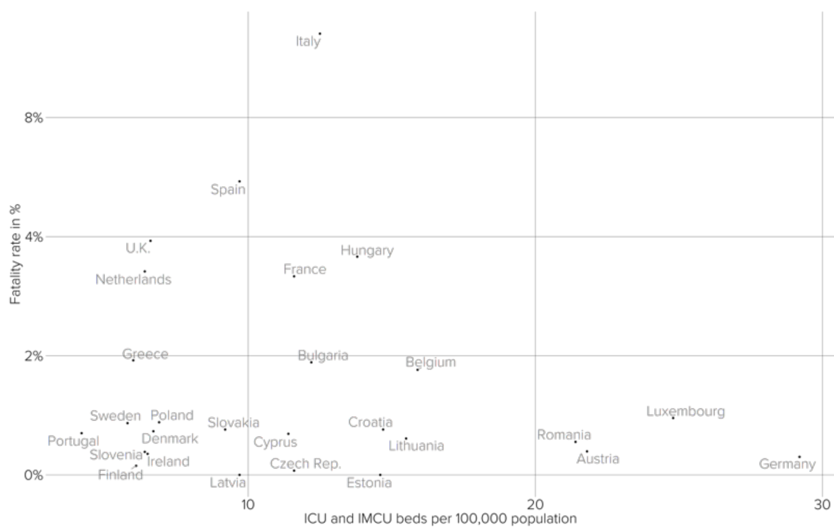
[23] Note that the virus does dissipate in this case because $\mathcal{R}(t) < 1$ at all times. Of course my simple model – which excludes vital dynamics, i.e. births and deaths – is not well suited for analysis of long-horizons. This figure should not be read literally, but highlights the orders of magnitude involved.

that is required to suppress the virus. But perhaps a more important lesson from this is that the lockdown strategies analyzed in this paper and the track and trace suppression strategies are substitutes, not complements. Complete suppression stops the progress towards a stable steady state to the left of the herd immunity threshold. This seems to run counter to some of the popular commentary about the epidemic.

7.2 Healthcare capacity constraints: ICU beds and PPE

One of the main concerns at the onset of an epidemic was that the sudden increase in the number of infections will overwhelm the healthcare systems, leading to higher death rate among patients and putting the medical and care staff at risk. The decisions to implement full lockdowns early have been driven, in large part, by this concern. Indeed, even given this endogenous policy response, there is clearly a correlation between the death rate and ICU capacity across countries (Figure 11).

Figure 11: Coronavirus deaths and ICU capacity



Source: Politico Research, John Hopkins University and Rhodes et al. (2012).

A natural way to model this phenomenon is to depart from the assumption of an exogenous and constant death rate and instead make it a function of the currently infected (this is the approach followed by Eichenbaum et al. (2020); Alvarez et al. (2020); Kaplan et al. (2020) and others). Of course the precise analysis of optimal lockdown policy is more complex in this case. But the qualitative impact these constraints on optimal policy is clear. Consider two lockdown policies that resulted in two infection paths that end up in the same steady state $S(\infty)$. In the analysis up to now both of these paths lead to identical death toll from the virus. Crucially, the planner strictly prefers the path with higher infection rates – the one that lies further north on the phase diagram. This is because along such a path the disease progresses faster, implying a shorter time spent in lockdown.

ICU capacity constraints introduce a countervailing force: the planner now dislikes high infection rates since they are associated with rising mortality. This is a first order effect that for sure leads to an earlier and lengthier lockdown.

7.3 Possibility of a vaccine or treatment

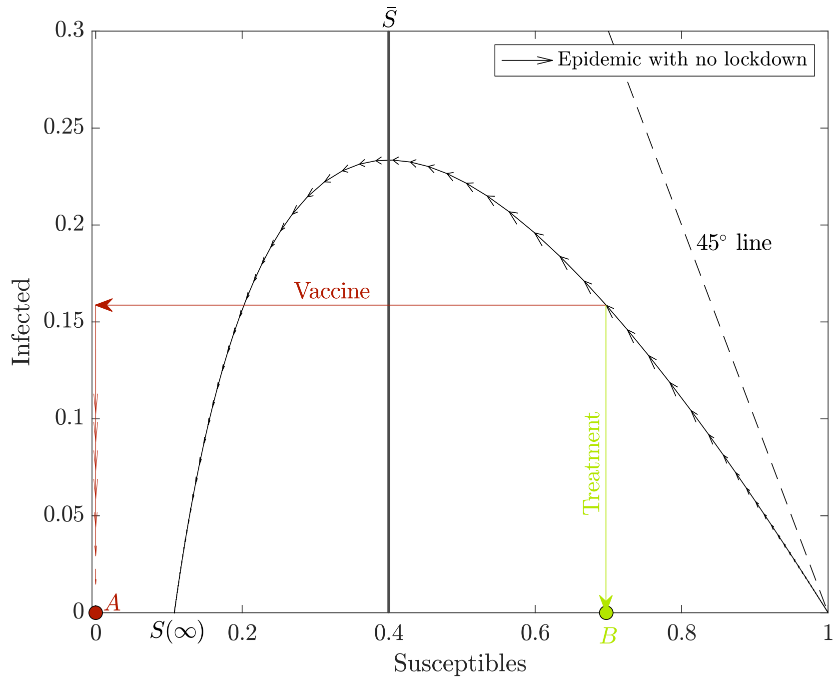
A similar effect is present if there is a possibility of a vaccine or treatment discovery. Positive chance of such an event effectively shortens the planning horizon, which makes the planner dislike the paths of the epidemic that are in the northern part of the phase diagram. The progression of the epidemic in these regions is very rapid; all else equal a higher probability of a discovery of a cure makes these paths less appealing. A possibility of a vaccine introduces an option value to waiting and thus leads to more front-loaded lockdowns that last for longer.

Graphically, an effective vaccine removes the susceptibility out of the population, instantly shifting the system horizontally to the y-axis. The infection is then eradicated as those with the virus recover or die. The new steady state A is at the origin (Figure 12). An effective treatment does the polar opposite: it instantly removes the infected part of the population, shifting the system vertically down to the x-axis, and crucially making the resting point stable (since any newly infected patients can be treated). The new steady state of the system is at point B. If discoveries of this kind are possible, the planner adjusts the optimal policy so that the resulting epidemic path is always closer to the starting point for each t.

7.4 More general infection matching process

The baseline model assumed that infections spread according to the quadratic matching, which is a standard assumption in epidemiology with long tradition in economics (popularized in the coconut model of Diamond (1982)). But as pointed out in the recent paper by Acemoglu et al. (2020), the form of the matching technology that generates infections might matter for the model's dynamics and is not obvious a-priori. For example,

Figure 12: Dynamics of the epidemic when a vaccine or a treatment are found



infections in the context of a workplace could potentially be characterized by increasing returns of lesser degree than 2, or perhaps could be of constant returns. How do these alternative assumptions change the analysis?

To stay close to notation of Acemoglu et al. (2020), let $\alpha \in [1, 2]$ be a parameter controlling the degree of increasing returns (with $\alpha = 2$ corresponding to the baseline model and $\alpha = 1$ to a matching function with constant returns to scale) and assume that the first two differential equations of the model are now given by:

$$\dot{S} = \frac{-\hat{\pi} S I}{(S + I + R)^{2-\alpha}} \tag{29}$$

$$\dot{I} = -\dot{S} - \pi_r I - \pi_d I \tag{30}$$

The herd immunity threshold in this more general case, denoted $\bar{S}_M$ (for "matching") solves:

$$\frac{\hat{\pi} \bar{S}_M}{(1 - \pi_d(1 - \bar{S}_M))^{2-\alpha}} = (\pi_r + \pi_d) \tag{31}$$

where I used the fact that in the model with constant death rate, $S + I + R = 1 - D = 1 - \pi_d(1 - S)$. Equation (31) implies that $\bar{S}_M$ is constant and independent of I, just as in the baseline model, and because $(1-\pi_d(1-\bar{S}_M))^{2-\alpha} \leq 1$, we know that $\bar{S}_M \leq \bar{S}$: the immunity threshold with less-than-quadratic returns to scale in the infection function is lower than in the baseline quadratic case. How much lower? We can get a hint by looking at the expression $(1-\pi_d(1-\bar{S}_M))^{2-\alpha}$. Since π_d is small, the expression is likely to be very close to 1 for any calibration. For example, in my baseline calibration $\bar{S} = 0.4$, and $\bar{S}_M = 0.3991$ even for $\alpha = 1$. This signals that the returns to scale, at least when formulated in this way in the context of the baseline model, might not be a first order issue.

We can go further to see the impact of constant returns assumption more explicitly. Equations (29) and (30) imply:

$$\frac{dI}{dS} = -1 + \frac{\pi_r + \pi_d}{\hat{\pi}} \frac{(1-\pi_d(1-S))^{2-\alpha}}{S}. \tag{32}$$

We can solve this ODE easily for the two limit cases. The limit case $\alpha = 2$ corresponds exactly to the baseline model; for the case where $\alpha = 1$ the solution is:

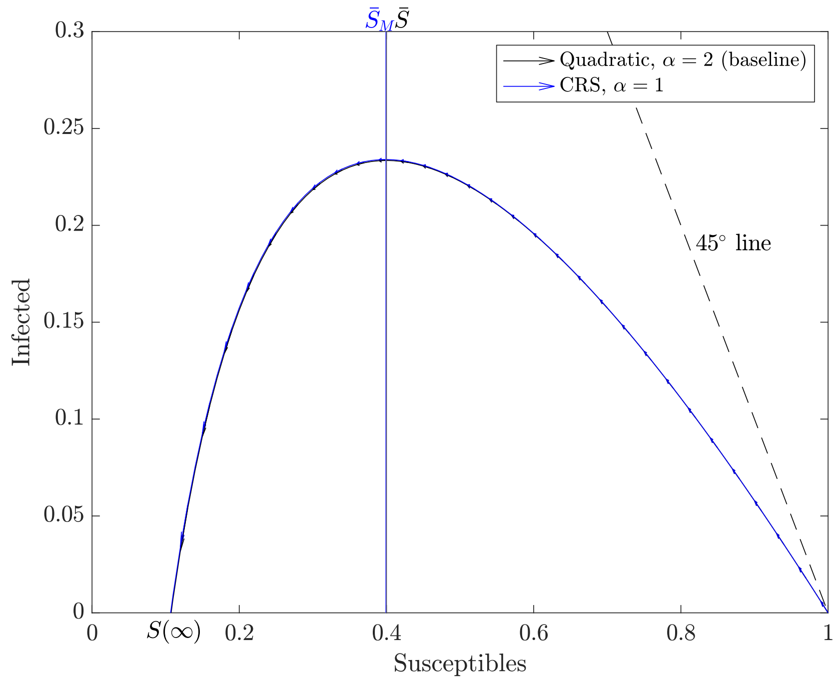
$$I(t) = -\left(1 - \frac{(\pi_r + \pi_d)\,\pi_d}{\hat{\pi}}\right) S(t) + \frac{(\pi_r + \pi_d)\,(1-\pi_d)}{\hat{\pi}} \log S(t) + 1 - \frac{(\pi_r + \pi_d)\,\pi_d}{\hat{\pi}}.$$

Figure 13 illustrates the impact of the constant returns to scale assumption on the model. The bottom line is that the constant returns assumption makes practically no difference in the baseline model with a single group and a low death rate.

8 Conclusion

In this paper I showed how to set up and solve an analytical model of the Covid-19 pandemic and how to use simple graphical apparatus to understand the workings of the model and to gain intuition. The model is tractable enough to yield analytical solution to the optimal lockdown policy problem. Because of its simplicity, it is quite revealing: for example, I showed how a careful analysis using the model leads to a more nuanced view of the externalities compared to what is often discussed in the literature. Furthermore I demonstrated that some of the more important complex features of the real world can be usefully analyzed within the framework. I believe that appreciating the results and the intuition of the simple model can be helpful when processing the data, news and policy announcements over the coming weeks and months, as well as for considering the implications of richer and larger modeling frameworks.

Figure 13: Dynamics of the epidemic with different assumptions about returns to scale in the infection technology



One other way to deploy the tools and results of this paper is in teaching the economics of Covid-19 as well as explaining the associated concepts, epidemic dynamics and feasibility and trade-offs of the various policy strategies to the wider public, beyond the economics or the epidemiology professions. Take a simple case of the phase diagram: at the most basic level it is a device that forces us to think through the entire epidemic until a new stable steady state is reached. Instead, the numerical time series analysis popular right now might push the second wave of the epidemic "off the chart", possibly leading to misleading conclusions.

Plenty of extensions of the current framework are possible. The most pressing one is the analysis of multiple recurrent lockdown episodes. While these are never optimal in the baseline model, they are almost certainly useful in the context of ICU capacity constraints. Examples of other extensions include: adding more states to the SIR model to allow for a richer analysis and better match the incoming data (e.g. the exposed and in-intensive-care states); modeling the distribution of exposed and latent periods with Gamma distribution; allowing for a realistic possibility that efficiency of the track and trace strategies is decreasing in the number of infected in the population; and including the utility, social, psychological and productivity costs of lockdown. I am actively pursuing these directions in ongoing work.

Appendix

Proof of Proposition 1

Proof. Equation (11) follows from the definitions of $\mathcal{R}_0$ and $\bar{S}$ and simple algebra. Immediate lockdown (i.e one with $T_0 = 0$) results in stable suppression if and only if

$$S_L(\infty) < \bar{S}.$$

Because $\bar{S}$ is fixed, to prove the result it suffices to show that $S_L(\infty)$ is increasing in ε. Equation (9) implies that:

$$S_L(\infty) - \bar{S}_L \log S_L(\infty) - 1 = 0$$

Combining this with the result that $\bar{S}_L = \frac{\bar{S}}{1-\varepsilon}$ (implied by equation (11)) yields:

$$S_L(\infty) - \frac{\bar{S}}{1-\varepsilon} \log S_L(\infty) - 1 = 0. \tag{33}$$

Differentiating equation (33) with respect to ε and rearranging we get:

$$\frac{\partial S_L(\infty)}{\partial \varepsilon} = \frac{\bar{S}(1-\varepsilon)^{-2} \log S_L(\infty)}{1 - \frac{\bar{S}_L}{S_L(\infty)}} \geq 0$$

where the inequality follows from the fact that $S_L(\infty) \leq 1$ and $\bar{S}_L > S_L(\infty)$.

To find the value of the threshold note that it is pinned down by the condition $S_L(\infty) = \bar{S}$. Equation (11) implies that $\varepsilon = 1 - \frac{\bar{S}}{\bar{S}_L}$. And so:

$$-\bar{S} + \bar{S}_L \log \bar{S} + 1 = 0.$$

Dividing by $\bar{S}_L$ yields

$$-\frac{\bar{S}}{\bar{S}_L} + \log \bar{S} + \frac{1}{\bar{S}_L} = 0.$$

Therefore:

$$\bar{\varepsilon} - 1 + \log \bar{S} + \frac{1-\bar{\varepsilon}}{\bar{S}} = 0.$$

Rearranging:

$$\bar{\varepsilon} = \frac{\bar{S}}{\bar{S}-1}\left(1 - \log \bar{S} - \frac{1}{\bar{S}}\right).$$

The final part of the Proposition follows from the fact that $\hat{\varepsilon}$ is defined by $\mathcal{R}_0^L = \bar{S}_L = 1$. The trajectory post-lockdown is the same as in the no lockdown scenario because the initial seed of infection is assumed to be infinitesimally small. □

Proof of Lemma 2

Proof. Consider the trajectory of the epidemic from the end of the lockdown T_1 onwards, which is characterized by:

$$I(t) = -S(t) + \bar{S} \log S(t) + I_R + S_R - \bar{S} \log S_R$$

where (S_R, I_R) is the state of the system at T_1, and so $I_R = -S_R + \bar{S}_L \log S_R + 1$. For T_1 sufficiently large, $I_R \approx 0$. Thus:

$$I(t) = -S(t) + \bar{S} \log S(t) + S_R - \bar{S} \log S_R.$$

The peak occurs at $S(t) = \bar{S}$:

$$I_{max} = -\bar{S} + \bar{S} \log \bar{S} + S_R - \bar{S} \log S_R.$$

Differentiating with respect to ε :

$$\frac{\partial I_{max}}{\partial \varepsilon} = \left(1 - \frac{\bar{S}}{S_R}\right)\frac{\partial S_R}{\partial \varepsilon} > 0$$

where the inequality follows from the fact that $\bar{\varepsilon} < \varepsilon < \hat{\varepsilon}$ and $\frac{\partial S_R}{\partial \varepsilon} > 0$.

The final resting point is given by:

$$0 = -S(\infty) + \bar{S} \log S(\infty) + S_R - \bar{S} \log S_R.$$

Differentiating:

$$\frac{\partial S(\infty)}{\partial \varepsilon} = \frac{S(\infty)}{S(\infty) - \bar{S}}\left(1 - \frac{\bar{S}}{S_R}\right)\frac{\partial S_R}{\partial \varepsilon} > 0$$

which proves the result. □

Proof of Proposition 2

Proof. Recall equation (8). The two equations that pin down S^* are, first, the equation under the lockdown dynamics that has the resting point exactly at $\bar{S}$ and the initial condition at (S^*, I^*):

$$0 = -\bar{S} + \bar{S}_L \log \bar{S} + I^* + S^* - \bar{S}_L \log S^*$$

and, second, the equation that characterizes the position of the no-lockdown system at T_0^* with the initial conditions $(1-\epsilon, \epsilon)$. With ϵ small:

$$I^* = -S^* + \bar{S} \log S^* + 1 + \bar{S} \log(1-\epsilon).$$

Combining these two equations yields:

$$0 = -\bar{S} + \bar{S}_L \log \bar{S} + \bar{S} \log S^* + 1 - \bar{S}_L \log S^*.$$

Rearranging and solving for S^* yields the result. □

Proof of Lemma 5

Proof. Let $T_0 = T_1 = T_a$ and $T_0 = T_1 = T_b > T_a$ be two equivalent policies. Then

$$U(a) = \int_0^{T_a} e^{-\rho t} \left(e^{-\int_0^t \pi_W(s) ds} \left(u(c_W) + \pi_W(t) V(t) \right) \right) dt + e^{-\rho T_a - \int_0^{T_a} \pi_W(s) ds} W(T_a)$$

and so

$$\begin{aligned} U(b) - U(a) = \\ & e^{-\rho T_a - \int_0^{T_a} \pi_W(s) ds} \int_{T_a}^{T_b} e^{-\rho t} \left(e^{-\int_{T_a}^t \pi_W(s) ds} \left(u(c_W) + \pi_W(t) V(t) \right) \right) dt + \\ & + e^{-\rho T_b - \int_0^{T_b} \pi_W(s) ds} W(T_b) - e^{-\rho T_a - \int_0^{T_a} \pi_W(s) ds} W(T_a) \end{aligned}$$

Let:

$$\begin{aligned} Flow(b) - Flow(a) &:= e^{-\rho T_a - \int_0^{T_a} \pi_W(s) ds} \int_{T_a}^{T_b} e^{-\rho t} \left(e^{-\int_{T_a}^t \pi_W(s) ds} \left(u(c_W) + \pi_W(t) V(t) \right) \right) dt \\ Scrap(b) - Scrap(a) &:= e^{-\rho T_b - \int_0^{T_b} \pi_W(s) ds} W(T_b) - e^{-\rho T_a - \int_0^{T_a} \pi_W(s) ds} W(T_a) \end{aligned}$$

where $U(b) - U(a) = (Flow(b) - Flow(a)) + (Scrap(b) - Scrap(a))$. So to prove equivalence it suffices to show that:

$$Flow(b) - Flow(a) = -\left(Scrap(b) - Scrap(a)\right)$$

We have:

$$\begin{aligned} W(T_a) = \int_{T_a}^{\infty} e^{-\rho(t-T_a)-\int_{T_a}^{t}\pi_W(s)ds}\left(u(c_W)+\pi_W(t)V(t)\right)dt = \\ \int_{T_a}^{T_b} e^{-\rho(t-T_a)-\int_{T_a}^{t}\pi_W(s)ds}\left(u(c_W)+\pi_W(t)V(t)\right) + e^{-\rho(T_b-T_a)-\int_{T_a}^{T_b}\pi_W(s)ds}W(T_b) \end{aligned}$$

Thus:

$$\begin{aligned} &Scrap(b) - Scrap(a) = \\ &\qquad e^{-\rho T_b-\int_0^{T_b}\pi_W(s)ds}W(T_b) - e^{-\rho T_a-\int_0^{T_a}\pi_W(s)ds}\times \\ &\left[\int_{T_a}^{T_b} e^{-\rho t-\int_{T_a}^{t}\pi_W(s)ds}\left(u(c_W)+\pi_W(t)V(t)\right) + e^{-\rho(T_b-T_a)-\int_{T_a}^{T_b}\pi_W(s)ds}W(T_b)\right] = \\ &\qquad -e^{-\rho T_a-\int_0^{T_a}\pi_W(s)ds}\left[\int_{T_a}^{T_b} e^{-\rho t-\int_{T_a}^{t}\pi_W(s)ds}\left(u(c_W)+\pi_W(t)V(t)\right)\right] \\ &\qquad = -(Flow(b) - Flow(a)) \end{aligned}$$

which proves that $U(b) - U(a) = 0$. □

Proof of Proposition 4

Proof. Let $\mathcal{L} \subset \mathbb{R}_+^2$ denote the set of feasible lockdown strategies of a given effectiveness ε. Each element of the set is a vector of two positive real numbers, $\{T_0, T_1\}$, with $T_0 \leq T_1$. This set is closed. As long as lockdown is not costless it is also bounded, since lockdown that lasts forever cannot be optimal. This imposes a condition on the end of the lockdown: $T_1 \leq T^V$ where T^V is arbitrarily large but finite. Bounded in this way $\mathcal{L}$ is a compact set. Let $\mathcal{E}$ denote the set of sequences fully characterizing the epidemic trajectory: $\mathcal{E} := \{I, \pi_W, \pi_U\}_{t=0}^{\infty}$. Note that $\mathcal{E}$ contains all the epidemic information that is relevant for individual problem in (24). Let $f : \mathcal{L} \to \mathcal{E}$ be the correspondence that maps a given lockdown strategy to the resulting epidemic trajectory, and let $g : \mathcal{E} \to \mathcal{L}$ be the correspondence that maps the epidemic to optimal lockdown strategy. Finally, let

$\mathcal{F} := f \circ g$, $\mathcal{F} : \mathcal{L} \to \mathcal{L}$ be the correspondence that maps a given lockdown strategy to optimal lockdown strategies. An equilibrium is a fixed point of $\mathcal{F}$. Kakutani's Fixed Point Theorem guarantees that $\mathcal{F}$ has a fixed point if set $\mathcal{L}$ is nonempty, compact and convex and $\mathcal{F}$ is upper hemicontinuous and convex-valued, that is the set $\mathcal{F}(x) \subset \mathcal{L}$ is nonempty and convex for every $x \in \mathcal{L}$. Clearly, the first four requirements are satisfied. But convex-valuedness of $\mathcal{F}$ fails when $\varepsilon > \bar{\varepsilon}$ because g is not convex-valued when the infection curve is double-peaked. Loosely, a susceptible individual who faces a double peak of infections can use the lockdown to avoid the risk of one or the other. A convex combination of such strategies will result in result in a lockdown that happens in between the two peaks, which is clearly suboptimal. This proves that g is not convex-valued and conditions for the existence of a fixed point are not satisfied. □

Proof of Proposition 5

Proof. Here I present the full proof for the intertemporal externality. The intertemporal externality comes from the fact that today's decision to lock down or not will affect the entire future path of infection probabilities $\{\hat{\pi}(t)\}_{t=T_0}^{\infty}$. Consider first the decision to start the lockdown (i.e. the choice of T_0). For any $t > T_0$ we have:

$$\frac{\partial \hat{\pi}(t)}{\partial T_0} = \begin{cases} \frac{\partial \pi_U(t)}{\partial T_0} = \pi_0 \frac{\partial I(t)}{\partial T_0} & \text{if } t \in [T_0, T_1] \\ \frac{\partial \pi_W(t)}{\partial T_0} = \left(\pi_o + \pi_n \phi I \bar{n}^2\right) \frac{\partial I(t)}{\partial T_0} & \text{if } t > T_1 \end{cases}$$

Thus the problem has two parts. For any $t \in (T_0, T_1]$, equation (8) implies:

$$I(t) = -S(t) + \bar{S}_L \log S(t) + I(T_0) + S(T_0) - \bar{S}_L \log S(T_0).$$

Differentiating:

$$\frac{\partial I(t)}{\partial T_0} = \underbrace{\frac{\partial S(t)}{\partial T_0}}_{<0} \underbrace{\left(\frac{\bar{S}_L}{S(t)} - 1\right)}_{\lesseqgtr 0} + \underbrace{\frac{\partial I(T_0)}{\partial T_0}}_{\lesseqgtr 0} + \underbrace{\frac{\partial S(T_0)}{\partial T_0}}_{<0} \underbrace{\left(1 - \frac{\bar{S}_L}{S(T_0)}\right)}_{\lesseqgtr 0}$$

The sign of the right hand side is ambiguous. In particular, marginally postponing the lockdown can have external effects to lower the infection rate late in the epidemic. To see this note that for t high enough the second term in braces is positive, and thus the first term on the right-hand-side is negative.

Similarly, for $t > T_1$

$$I(t) = -S(t) + \bar{S}\log S(t) + I(T_1) + S(T_1) - \bar{S}\log S(T_1)$$

And thus:

$$\frac{\partial I(t)}{\partial T_0} = \underbrace{\frac{\partial S(t)}{\partial T_0}}_{<0}\underbrace{\left(\frac{\bar{S}}{S(t)} - 1\right)}_{\lesseqgtr 0} + \underbrace{\frac{\partial I(T_1)}{\partial T_0}}_{=0} + \underbrace{\frac{\partial S(T_1)}{\partial T_0}}_{=0}\underbrace{\left(1 - \frac{\bar{S}}{S(T_0)}\right)}_{\lesseqgtr 0}$$

Consider now the decision to prolong the lockdown. For any $t > T_1$ we have:

$$\frac{\partial \hat{\pi}(t)}{\partial T_1} = \frac{\partial \pi_W(t)}{\partial T_1} = \left(\pi_o + \pi_n \phi I \bar{n}^2\right)\frac{\partial I(t)}{\partial T_1}.$$

where now:

$$I(t) = -S(t) + \bar{S}\log S(t) + I(T_1) + S(T_1) - \bar{S}\log S(T_1).$$

Differentiating:

$$\frac{\partial I(t)}{\partial T_1} = \underbrace{\frac{\partial S(t)}{\partial T_1}}_{>0}\underbrace{\left(\frac{\bar{S}}{S(t)} - 1\right)}_{\lesseqgtr 0} + \underbrace{\frac{\partial I(T_1)}{\partial T_1}}_{\lesseqgtr 0} + \underbrace{\frac{\partial S(T_1)}{\partial T_1}}_{<0}\underbrace{\left(1 - \frac{\bar{S}}{S(T_1)}\right)}_{\lesseqgtr 0}$$

The first term is now positive, as longer lockdown means the number of susceptibles is higher down the line. The sign of the overall effect is again ambiguous. □

Proof of Proposition 8

Proof. Start with the case with $\varepsilon \geq \bar{\varepsilon}$. Consider first the decision to end the lockdown, and fix the lockdown start date at T_0^*, when the state of the epidemic is (S^*, I^*). Specifically, consider a strategy of ending lockdown at $\tilde{T}_1$, with associated state $(\tilde{S}_1, \tilde{I}_1)$. The marginal cost of ending the lockdown a little sooner is the higher rate of infection that results. To pin down this cost, note that the ultimate resting point, denoted x, is satisfies equation (8):

$$0 = -x + \bar{S}\log x + \tilde{I}_1 + \tilde{S}_1 - \bar{S}\log \tilde{S}_1$$

where $\tilde{I}_1 = -\tilde{S}_1 + \bar{S}_L \log \tilde{S}_1 + I^* + S^* - \bar{S}_L \log S^*$. Combining these two equations yields:

$$0 = -x + \bar{S}\log x + \bar{S}_L \log \tilde{S}_1 + I^* + S^* - \bar{S}_L \log S^* - \bar{S}\log \tilde{S}_1 \tag{34}$$

Differentiating with respect to $\tilde{T}_1$:

$$0 = \frac{\partial x}{\partial \tilde{T}_1}\left(\frac{\bar{S}}{x} - 1\right) + \frac{\bar{S}_L - \bar{S}}{\tilde{S}_1}\frac{\partial \tilde{S}_1}{\partial \tilde{T}_1}$$

which gives

$$\frac{\partial x}{\partial \tilde{T}_1} = \frac{x}{\bar{S} - x}\frac{\bar{S} - \bar{S}_L}{\tilde{S}_1}\pi_o \tilde{I}_1.$$

All quantities on the right hand side are known (x is the solution to (34)). Moreover, the right hand side is negative (since $\bar{S}_L > \bar{S}$) and is increasing in $\tilde{T}_1$. Furthermore,

$$\lim_{\tilde{T}_1 \to \infty} \frac{\partial x}{\partial \tilde{T}_1} = 0.$$

Thus the longer the lockdown lasts, the smaller is the cost, in terms of extra infections, of releasing the lockdown; in the limit, the cost becomes nil.

The benefit of a shorter lockdown is:

$$(1 - D(\tilde{T}_1))\left(\log(c_W) - \log(c_U)\right)$$

because $\dot{D} \geq 0$, this is also decreasing in $\tilde{T}_1$. However,

$$\lim_{\tilde{T}_1 \to \infty}(1 - D(\tilde{T}_1))\left(\log(c_W) - \log(c_U)\right) = (1 - \bar{S}\pi_d)\left(\log(c_W) - \log(c_U)\right) > 0.$$

Thus there exists a $\tilde{T}_1 < \infty$ at which marginal cost of ending the lockdown is equal to the marginal benefit. It is at that point when optimal lockdown is lifted.

Consider now the start of a lockdown. Starting lockdown before T_0^* always leads to a longer lockdown and more deaths so cannot be optimal. Consider postponing the start of the lockdown marginally. Using equation (8) twice we can characterize the resting point. Let y denote this resting point, and assume for simplicity that T_1 is large enough so that $y \approx \bar{S}$. Then:

$$0 = -y + \bar{S}_L \log y + \left[-S^* + \bar{S}\log S^* + 1\right] + S^* - \bar{S}_L \log S^*.$$

Rearranging:

$$0 = -y + \bar{S}_L \log y + (\bar{S} - \bar{S}_L)\log S^* + 1$$

Differentiating with respect to T_0^*:

$$0 = -\frac{\partial y}{\partial T_0^*} + \frac{\bar{S}_L}{y}\frac{\partial y}{\partial T_0^*} + (\bar{S} - \bar{S}_L)\frac{\frac{\partial S^*}{\partial T_0^*}}{S^*}$$

Rearranging:

$$\frac{\partial y}{\partial T_0^*} = \frac{y}{y - \bar{S}_L}(\bar{S} - \bar{S}_L)\frac{\frac{\partial S^*}{\partial T_0^*}}{S^*}$$

We are concerned with the point where $y \approx \bar{S}$ so finally:

$$\frac{\partial y}{\partial T_0^*} = \bar{S}\frac{\frac{\partial S^*}{\partial T_0^*}}{S^*} = \bar{S}I^*\left(\pi_n\phi\bar{n}^2 + \pi_o\right)$$

For the calibration used here, this equals 0.075 – that is, an extra week of no lockdown results in 7.5% of population contracting the virus. The utility cost of this is at least as great as that associated with extra deaths. If $\mathcal{C}$ denotes the cost, then:

$$\mathcal{C} \geq \bar{S}I^*\left(\pi_n\phi\bar{n}^2 + \pi_o\right)\pi_d\mathcal{U}^S(T^*) \geq \bar{S}I^*\left(\pi_n\phi\bar{n}^2 + \pi_o\right)\pi_d\frac{\log(c_U)}{\rho + \pi_d}$$

where the second inequality comes from the fact that lifetime utility of a susceptible agent at T^* is at least as great as lifetime utility of an infected agent with symptoms at T^*.

This needs to be compared with the benefit of a shorter lockdown, which depends on the type of lockdown, and are largest for type-3 lockdown where all agents are locked down. Thus:

$$\mathcal{B} \leq (1 - D(T^*))\left(\log(c_W) - \log(c_U)\right) \leq \log(c_W) - \log(c_U)$$

This implies that

$$\mathcal{C} \geq \mathcal{B} \text{ if } \bar{S}I^*\left(\pi_n\phi\bar{n}^2 + \pi_o\right)\pi_d\frac{\log(c_U)}{\rho + \pi_d} \geq \log(c_W) - \log(c_U)$$

that is if

$$\log\left(\frac{c_W}{c_U}\right) = \log\left(\frac{1}{h}\right) \leq 1 + \frac{\left(\pi_n\phi\bar{n}^2 + \pi_o\right)\pi_d}{\rho + \pi_d}$$

This condition is sufficient, although not necessary, for the postponement of the lockdown to be suboptimal. This condition puts a lower bound on h; it is satisfied for any reasonable parametrization. For the current calibration, the right-hand side is equal to 1.8, an order of magnitude greater than the left-hand side (0.22).

Turning now to the case where $\varepsilon < \bar{\varepsilon}$. Consider a decision to start the lockdown at some T_0, when the state of the epidemic is (S_0, I_0). Fix the lockdown end date T_1 and the associated state (S_1, I_1). The cost of postponing the start of the lockdown marginally is the associated rise in infections; the benefit is the shorter duration of the lockdown. Consider the cost. The cumulative infections z satisfy:

$$0 = -z + \bar{S}\log z + I_1 + S_1 - \bar{S}\log S_1 \tag{35}$$

where $I_1 = -S_1 + \bar{S}_L \log S_1 + I_0 + S_0 - \bar{S}_L \log S_0$. We also know that $I_0 = -S_0 + \bar{S}\log S_0 + 1$ since the initial seed of infection is assumed to be small. Combining these equations yields:

$$0 = -z + \bar{S}\log z + \bar{S}_L \log S_1 + (\bar{S} - \bar{S}_L)\log S_0 - \bar{S}\log S_1 + 1. \tag{36}$$

Differentiating with respect to T_0 :

$$0 = \frac{\partial z}{\partial T_0}\left(\frac{\bar{S}}{z} - 1\right) + \frac{\bar{S} - \bar{S}_L}{S_0}\frac{\partial S_0}{\partial T_0}$$

Thus

$$\frac{\partial z}{\partial T_0} = \frac{z}{\bar{S} - z}\frac{\bar{S}_L - \bar{S}}{S_0}\left(\pi_n \phi \bar{n}^2 + \pi_o\right) I_0 = \frac{z}{\bar{S} - z}\frac{\bar{S}_L - \bar{S}}{S_0}\left(\pi_n \phi \bar{n}^2 + \pi_o\right)\left(-S_0 + \bar{S}\log S_0 + 1\right)$$

which implies

$$\lim_{T_0 \to 0}\frac{\partial z}{\partial T_0} = 0$$

since $S_0 \to 1$ in the limit. That is, in the limit the cost of postponing lockdown till the later date is zero. But the benefit is positive as long as $c^W > c^U$. It is thus optimal to postpone the start of the lockdown. If we ignore the discounting, optimal T_0 is pinned down by:

$$\frac{z}{\bar{S} - z}\frac{\bar{S}_L - \bar{S}}{S_0}\left(\pi_n \phi \bar{n}^2 + \pi_o\right)\left(-S_0 + \bar{S}\log S_0 + 1\right)\mathcal{U}^S(T_1) = \log c^W - \log c^U$$

where z is the solution to equation (35).

Finally, consider the decision to end the lockdown at T_1, with the associated state (S_1, I_1). Cumulative infections satisfy equation (36). Differentiating that equation with respect to T_1 we get

$$0 = \frac{\partial z}{\partial T_1}\left(\frac{\bar{S}}{z} - 1\right) + \frac{\bar{S}_L - \bar{S}}{S_1}\frac{\partial S_1}{\partial T_1}$$

which yields

$$\frac{\partial z}{\partial T_1} = \frac{z}{\bar{S} - z}\frac{\bar{S}_L - \bar{S}}{S_1}\left(\pi_n \phi \bar{n}^2 + \pi_o\right) I_1$$

This is negative, since longer lockdown saves infections and lives. Ignoring discounting, the optimal lockdown end date is pinned down by:

$$\frac{z}{\bar{S} - z}\frac{\bar{S}_L - \bar{S}}{S_1}\left(\pi_n \phi \bar{n}^2 + \pi_o\right) I_1 \mathcal{U}^S(T_1) = (1 - D(T_1))\left(\log c^U - \log c^W\right).$$

□

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