Lecture 4:
1.3. Theory: Determinants of the Rate of Growth

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last revised: 2020-02-04

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<https://github.com/braddelong/public-files/blob/master/econ-135-lecture-4.pptx>

Lecture 4: Ideas Growth

Last of our three lectures on economic theory

Slides: <https://github.com/braddelong/public-files/blob/master/econ-135-lecture-4.pptx>
Read After: J. Bradford DeLong: Lecture Notes: Determinants of Progress <https://nbviewer.jupyter.org/github/braddelong/long-form-drafts/blob/master/solow-model-6-innovation.ipynb>
Files: datahub: <http://datahub.berkeley.edu/user-redirect/interact?account=braddelong&repo=long-form-drafts&branch=master&path=solow-model-6-innovation.ipynb>

1. **Assignment:** Malthusian economies paper <https://bcourses.berkeley.edu/courses/1487685/assignments/8065917>

MOAR References

- Michael Kremer (1993): Population Growth and Technological Change: One Million B.C. to 1990 <https://delong.typepad.com/files/kremer-million.pdf>...
- Charles I. Jones and Peter J. Klenow: Beyond GDP? Welfare across Countries and Time <https://web.stanford.edu/~chadj/JonesKlenowAER2016.pdf>...
- Chang-Tai Hsieh, Erik Hurst, Charles I. Jones, & Peter J. Klenow: The Allocation of Talent & U.S. Economic Growth <https://web.stanford.edu/~chadj/HHJK.pdf>...
- C.I. Jones: The Facts of Economic Growth <https://web.stanford.edu/~chadj/facts.pdf>...
- Philippe Aghion, Benjamin F. Jones, & Charles I. Jones: Artificial Intelligence & Economic Growth <https://web.stanford.edu/~chadj/AJJ-AIandGrowth.pdf>...
- Charles I. Jones: Paul Romer: Ideas, Nonrivalry, & Endogenous Growth <https://web.stanford.edu/~chadj/RomerNobel.pdf>...
- Richard R. Nelson & Howard Pack: The Asian Miracle & Modern Growth Theory <http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.977.8217&rep=rep1&type=pdf>...
- Charles I. Jones & Paul M. Romer: The New Kaldor Facts: Ideas, Institutions, Population, & Human Capital <https://web.stanford.edu/~chadj/JonesRomer2010.pdf>...

Long-Run Patterns: Global *h*, *g*, & *n*

Date	ideas Level H	Total Real World Income Y (billions)	Average Real Income per Capita y (per year)	Total Human Population L (millions)		Rate of Population and Labor Force Growth n	Rate of Efficiency-of-Labor Growth g	Rate of Ideas-Stock Growth h
-68000	1.0	$0	$1,200	0.1				
-8000	5.0	$3	$1,200	2.5		0.005%	0.000%	0.003%
-6000	6.3	$6	$900	7		0.051%	-0.014%	0.011%
-3000	9.2	$14	$900	15		0.025%	0.000%	0.013%
-1000	16.8	$45	$900	50		0.060%	0.000%	0.030%
0	30.9	$153	$900	170		0.122%	0.000%	0.061%
800	41.1	$270	$900	300		0.071%	0.000%	0.035%
1500	53.0	$450	$900	500		0.073%	0.000%	0.036%
1770	79.4	$825	$1,100	750		0.150%	0.074%	0.149%
1870	123.5	$1,690	$1,300	1300		0.550%	0.167%	0.442%
2020	2720.5	$90,000	$11,842	7600		1.177%	1.473%	2.061%

Long-Run Patterns: "Western" *h*, *g* & *n*

Global Growth: The Industrializing West (2019)

Date	ideas Level H	Total Real Income Y (billions)	Average Real Income per Capita y (per year)	Total "West" Population L (millions)	Rate of Population and Labor Force Growth n	Rate of Efficiency-of-Labor Growth g	Increasing Resources ρ	Rate of Ideas-Stock Growth h
-68000	1.0	$0.01	$1,200	0.005				
-8000	4.5	$0.12	$1,200	0.1	0.005%	0.000%	0.000%	0.002%
-6000	4.7	$0.18	$900	0.2	0.035%	-0.014%	0.000%	0.003%
-3000	7.5	$0.45	$900	0.5	0.031%	0.000%	0.000%	0.015%
-1000	15.0	$1.80	$900	2	0.069%	0.000%	0.000%	0.035%
0	23.7	$4.50	$900	5	0.092%	0.000%	0.000%	0.046%
800	30.0	$7.20	$900	8	0.059%	0.000%	0.000%	0.029%
1500	58.9	$25.00	$1,000	25	0.163%	0.015%	0.000%	0.096%
1770	101.0	$105.00	$1,400	75	0.407%	0.125%	0.257%	0.200%
1870	252.0	$490.00	$2,800	175	0.847%	0.693%	0.405%	0.914%
2020	8439.5	$40,000.00	$50,000	800	1.013%	1.922%	0.175%	2.341%

Where does the "ρ" come from?

- "Ghost acreage"—conquest and resource utilization (sugar islands, timberlands, cottonlands, etc.)
- Cultural expansion—Australia, Canada, New Zealand, & U.S.; Spain & Italy & Scandinavia; plus Japan, Korea, Taiwan, Hong Kong, & Singapore

“West”?









Non-Rivalry

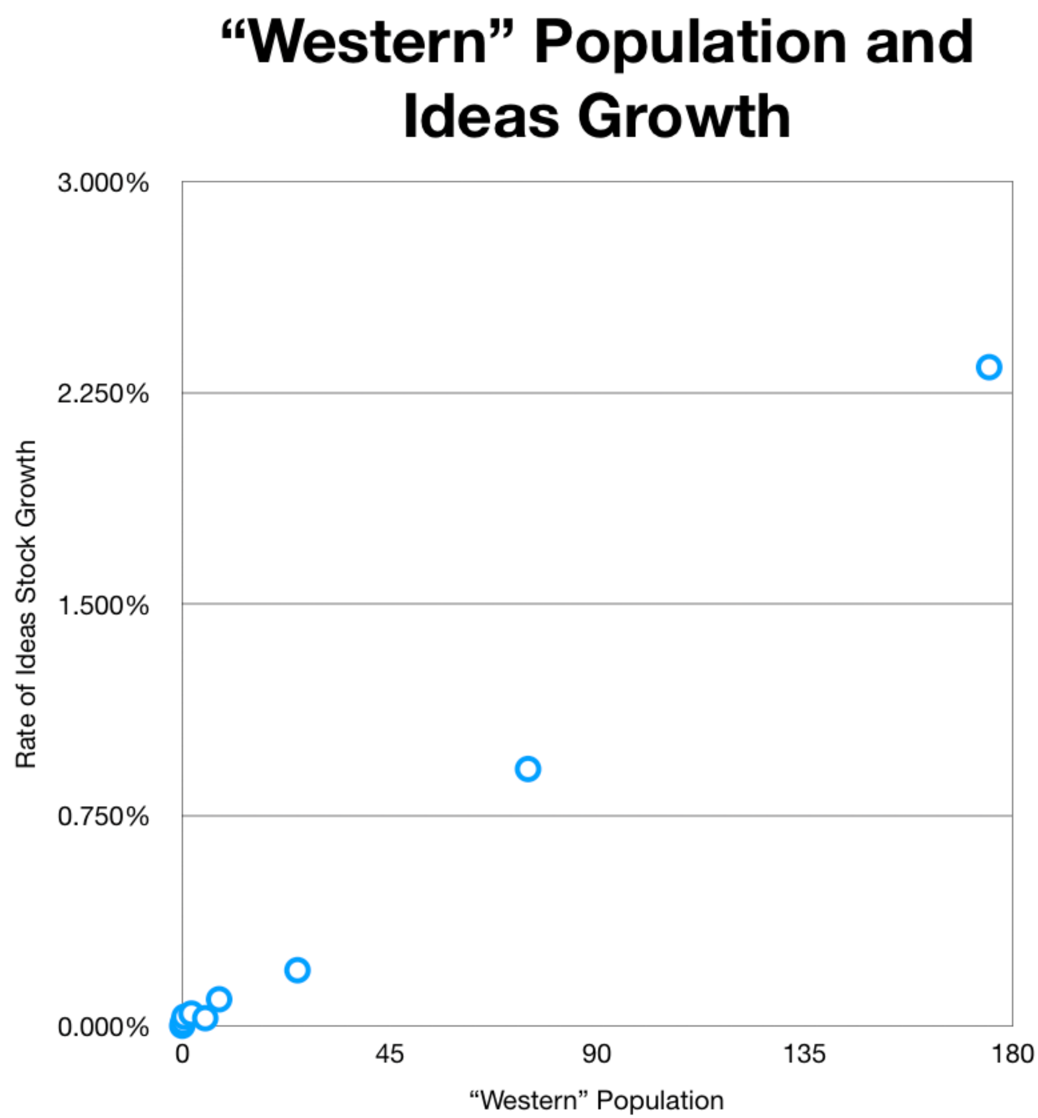
One person's work in adding to *H* can rapidly benefit all—if it is allowed to spread:

- Attempts to keep it from spreading—to limit knowledge's distribution by somehow charging those using it a price—must violate the optimality condition that the costs imposed on people for making use of commodities reflect and match the burden that their withdrawal of the commodities from the common stock imposes on the rest of the community
- **Friedrich Engels** (1843): *Outlines of a Critique of Political Economy* <https://www.marxists.org/archive/marx/works/1844/df-jahrbucher/outlines.htm>:
    - "According to the economists, the production costs of a commodity consist of three elements: the rent for the piece of land required to produce the raw material; the capital with its profit, and the wages for the labour required for production and manufacture.... [Since] capital is "stored-up labour"... two sides–the natural, objective side, land; and the human, subjective side, labour, which includes capital and, besides capital, a third factor which the economist does not think about–I mean the mental element of invention, of thought, alongside the physical element of sheer labour.
    - "What has the economist to do with inventiveness? Have not all inventions fallen into his lap without any effort on his part? Has one of them cost him anything? Why then should he bother about them in the calculation of production costs? Land, capital and labour are for him the conditions of wealth, and he requires nothing else. Science is no concern of his. What does it matter to him that he has received its gifts through Berthollet, Davy, Liebig, Watt, Cartwright, etc.–gifts which have benefited him and his production immeasurably?…"

“Western” *h* & Population

Michael Kremer (1993): *Population Growth & Technological Change: One Million B.C. to 1990:*

- <https://delong.typepad.com/files/kremer-million.pdf>
- Two heads are better than one
- Invention is proportional to the human population
    - Or to the human population in effective intellectual contact—the “West”
    - Or to the leading-edge population of the “West”
    - Or to the “West’s” STEM workforce
- Cf.: Jared Diamond: *Guns, Germs, and Steel* <https://delong.typepad.com/files/diamond.pdf>
- A race—between “the West”, “Greater China”, South Asia, the Middle East, Sub-Saharan Africa, Pre-Latin America, Australasia, Polynesia, Tasmania, Flinders Island?



Implications of THABtO

Michael Kremer (1993): *Population Growth & Technological Change: One Million B.C. to 1990:*

- We expect to see superexponential growth:

Assume that output is given by

> $$Y = Ap^{\alpha} R^{1-\alpha}$$

where A is the level of technology, p is population, and $R = 1$ is land, henceforth normalized to one. Per capita income y therefore equals $Ap^{\alpha-1}$. opulation increases above the steady-state equilibrium level of per capita income y^* and decreases below it. Diminishing returns to labor imply that a unique level of population, p^* generates income y^* :

> $$p^* = \left(\frac{A}{y^*}\right)^{(1/(1-\alpha))}$$

In a larger population there will be proportionally more people lucky or smart enough to come up with new ideas.

Implications of THABtO II

If research productivity per person is independent of population and if A affects research output the same way it affects output of goods (linearly, by definition), then the rate of change of technology will be:

> $$\frac{dA}{dt} = \pi A p$$

Take the log derivative of the population determination equation:

> $$\frac{dln(p)}{dt} = \left(\frac{1}{1-\alpha}\right)\frac{dln(A)}{dt}$$

and substitute in the expression for the growth rate of technology:

> $$\frac{dp}{dt} = \frac{\pi p^2}{1-\alpha}$$

to get superexponential growth of population (and total income)—as long as the Malthusian régime lasts, there is no demographic transition, .

After the demographic transition (if one occurs), things will be different…

Computational Experiment

 # fitting the Kremer model to historical population

- There were 2.5 million people 10,000 years ago, at the invention of agriculture. There were 15 million people 5,000 years ago, at the invention of writing
- datahub: <http://datahub.berkeley.edu/user-redirect/interact?account=braddelong&repo=long-form-drafts&branch=master&path=solow-model-6-innovation.ipynb>

A value of $\pi/(1-\alpha) = 0.00006666$ serves. But that value predicts that human population would cross 170 million heading upwards not in the year 1 but in the year -2080: early in the Bronze Age.

If we want to fit our three pre-1 benchmarks, we cannot have two heads being fully as good as one. So, instead, let us assume not that a 1% increase in the STEM workforce raises the rate of technological progress by 1% but rather by λ% for some parameter λ. So the dynamics for population then become:

> $$\frac{dp}{dt} = \frac{\pi p^{1+\lambda}}{1-\alpha}$$

$\alpha = 0.5, \pi = 0.00003264, \lambda = 0.8529$ fit the pre-1 benchmarks well.

Computational Experiment II

 # fitting the Kremer model to historical population

- There were 2.5 million people 10,000 years ago, at the invention of agriculture. There were 15 million people 5,000 years ago, at the invention of writing
- datahub: <http://datahub.berkeley.edu/user-redirect/interact?account=braddelong&repo=long-form-drafts&branch=master&path=solow-model-6-innovation.ipynb>

Even if two heads are not quite as good as one—are only 1.85 times as good as one—there need to be other sources of drag in order to have kept the world from an Industrial Revolution-class breakthrough late in the Later Han, and under the late Antonine and Severian dynasties:

Surely the effective STEM labor force depends on means of knowledge recording and communication. And it is not foolish to expect *ex ante* that there would be some diminishing returns from exhaustion of low-hanging fruit at some point. We do seem to see a jump up in growth with the invention of writing, and cities. Shouldn't we also see a jump up with the alphabet? Shouldn't we also see a jump up with the invention of printing? Perhaps the effects of the picking of the low-hanging fruit in exhausting opportunities and slowing growth civilization-wide are visible in the slowdown after the year one. Perhaps the effective STEM labor force gets big bumps up with the alphabet and with printing that together, in the large, offset this exhaustion.

Clearly, however, two heads are better than one will not suffice to understand the relative constancy of global economic growth rates since the coming of modern economic growth

Looking for Constant *h*…

Charles I. Jones (1995): *R&D-Based Models of Economic Growth:*

- <https://delong.typepad.com/files/jones-r--d.pdf>
- "The prediction of permanent scale effects on growth from the R&D equation means that the models of Romer/Grossman-Helpman/Aghion-Howitt and others are all easily rejected.... However, the R&D-based models [remain] intuitively very appealing.... [Is there] a way to maintain the basic structure of these models while eliminating the prediction of [permanent] scale effects [on the rate of growth?]…"
- Can we preserve the insights that ideas are non-rival and that technology is the ballgame and still understand why growth did not accelerate faster and bring us an Industrial Revolution early in the first millennium, and, in fact, has not further accelerated since the late 1800s? Chad Jones's answer is: "Yes!"
    - Crowding of researchers, and
    - Picking of low-hanging technological fruit

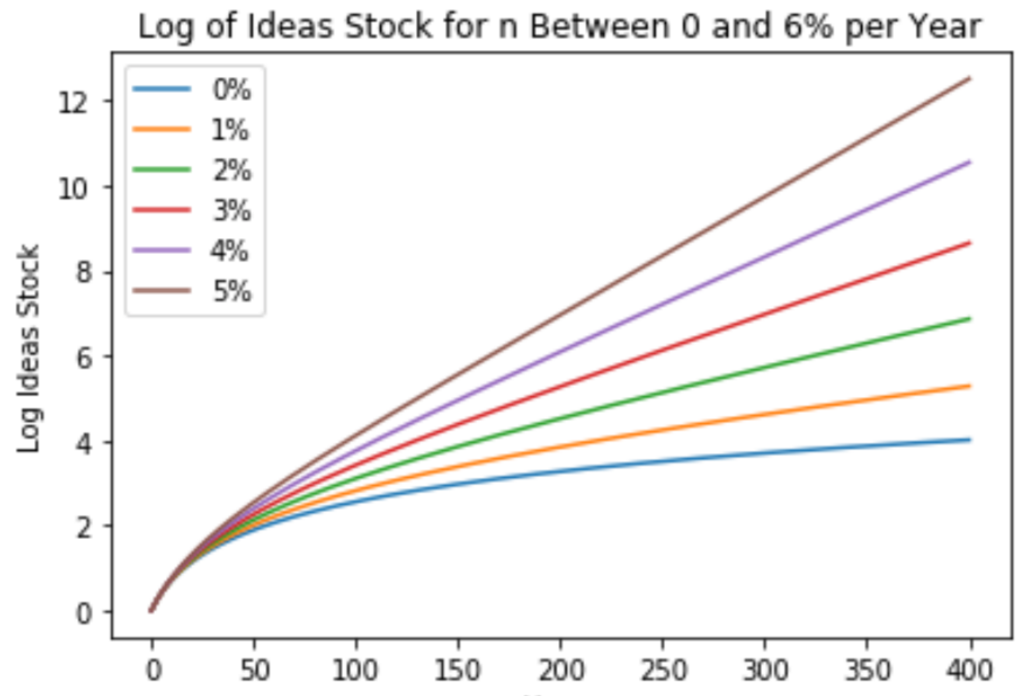
$$\frac{dH}{dt} = \delta L_{stem}^{\lambda} H^{\phi}$$

$$\frac{dH/dt}{H} = \delta L_{stem}^{\lambda} H^{\phi-1}$$

More Computational Experiments

Set the initial ideas stock H_0 and the initial STEM labor force L_0 at 1:

- Set the R&D researcher crowding parameter λ=0.5
- For striking results, set the low-hanging-fruit parameter ϕ=0.1
- The rate of growth n of the STEM labor force varies from 0 to 6% per year



Understanding the Jones Model

Indeed: a steady-state in *h*:

- For that convergence to a constant growth rate to happen, in the long run the increase in the effective STEM labor force $L^{\lambda}{}_{STEM}$ has to be exactly offset by diminishing returns to innovative effort $\delta H^{\phi-1}$
- There will be a level of ideas H* at which that growth rate h* would be attained
- The bigger is φ—the more low-hanging fruit is picked—the lower will be H* and the higher will be h*

$$\lambda \frac{1}{L_{stem}} \frac{dL_{stem}}{dt} = (1-\phi)\frac{dH/dt}{H}$$

$$\lambda n = (1-\phi)h^*$$

$$h^* = \frac{\lambda n}{1-\phi}$$

$$\frac{\lambda n}{1-\phi} = \delta L_{stem}^{\lambda}(H^*)^{\phi-1}$$

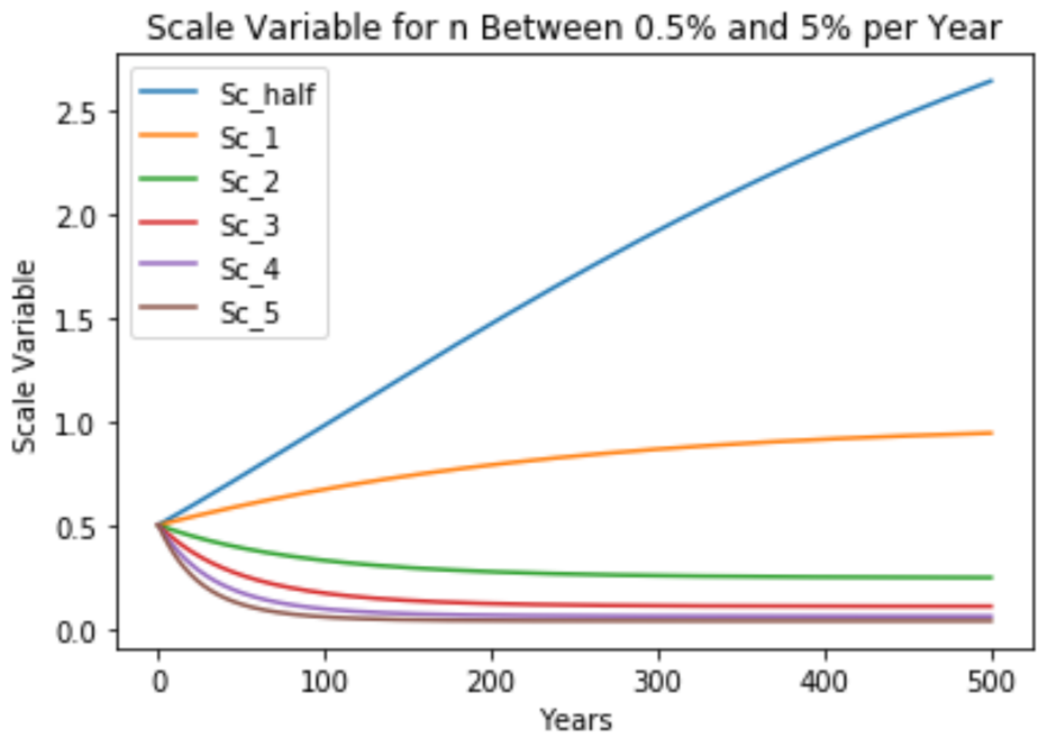
$$H^* = \left(\frac{\delta(1-\phi)}{\lambda}\right)^{1/(1-\phi)} \left(\frac{1}{n}\right)^{1/(1-\phi)} L_{stem}^{\lambda/(1-\phi)}$$

$$HL_{stem}^{-\lambda/(1-\phi)} = (\delta(1-\phi)/\lambda)^{1/(1-\phi)} n^{-1/(1-\phi)}$$

Superexponential Convergence

But over a long time span—not generations, longer:

- H grows more rapidly than h^* until it closes in on H^* on the steady-state balanced-growth path
- First, initial superexponential growth; that growth rate then declines until the growth rate asymptotes to merely exponential growth at the rate $h^* = \lambda n/(1-\phi)$
- We have an h* of 2%/year
- We seem to have had an n_{stem} of 6%/year
- Looks like: $\lambda/(1-\phi) = 1/3$
- $\phi = 0.5 \Rightarrow \lambda = 0.17$
- $\phi = 0.1 \Rightarrow \lambda = 0.3$



Review: Solow-Malthus Model Basics

How do we make sense of the fact that people were ingenious and inventive back before 1500, and yet standards of living did not increase?

- Although population did increase—slowly
- Other parts of the model
- Balanced-growth equilibrium
- Convergence to equilibrium
- Lecture notes: <https://nbviewer.jupyter.org/github/braddelong/long-form-drafts/blob/master/solow-model-5-pre-industrial.ipynb>
    - datahub: <http://datahub.berkeley.edu/user-redirect/interact?account=braddelong&repo=long-form-drafts&branch=master&path=solow-model-5-pre-industrial.ipynb>

Solow-Malthus Model Basics: Efficiency of Labor

How do we make sense of the fact that people were ingenious and inventive back before 1500, and yet standards of living did not increase?

- We make efficiency of labor a function of available natural resources per worker. by setting the rate of efficiency of labor growth $\mathbf{g = h - n/\gamma}$
- Thus g = 0 if and only if: $\mathbf{n = n^{*mal} = h\gamma}$.

Back before 1500—and even later—people are anxious to have children:

- $\mathbf{1/\Phi}$ is the fraction of production devoted to necessities
- $\mathbf{y^{sub}}$ is the "subsistence" standard of necessities consumption at which population growth averages zero:
- Then, back before the demographic transition: $\mathbf{n = \beta(y/(\Phi y^{sub}) - 1)}$

Understanding the Solow-Malthus Equilibrium: Population and Labor Force

The salience of capital in determining productivity

The extent to which population depresses productivity

The ratio of knowledge to subsistence income

Nuisance terms

$$L_t^{*mal} = \left[\left(\frac{H_t}{y^{sub}} \right) \left(\frac{s}{\delta} \right)^{\theta} \left(\frac{1}{\phi} \right) \left[\frac{1}{(1+\gamma h/\delta)^{\theta}} \frac{1}{(1+\gamma h/\beta)} \right] \right]^{\gamma}$$

The Malthusian equilibrium population

The ratio of savings to depreciation

The inverse of the taste for luxury

Notes:

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Understanding the Solow-Mathus Equilibrium: Prosperity

Sensitivity of productivity to population

Malthisian equilibrium income level

True zpg subsistence

Rate of useful ideas creation

$$y^{*mal} = \phi y^{sub} \left(1 + \frac{n^{*mal}}{\beta}\right) = \phi y^{sub} \left(1 + \frac{\gamma h}{\beta}\right)$$

Taste for luxuries

Responsiveness of population growth to prosperity

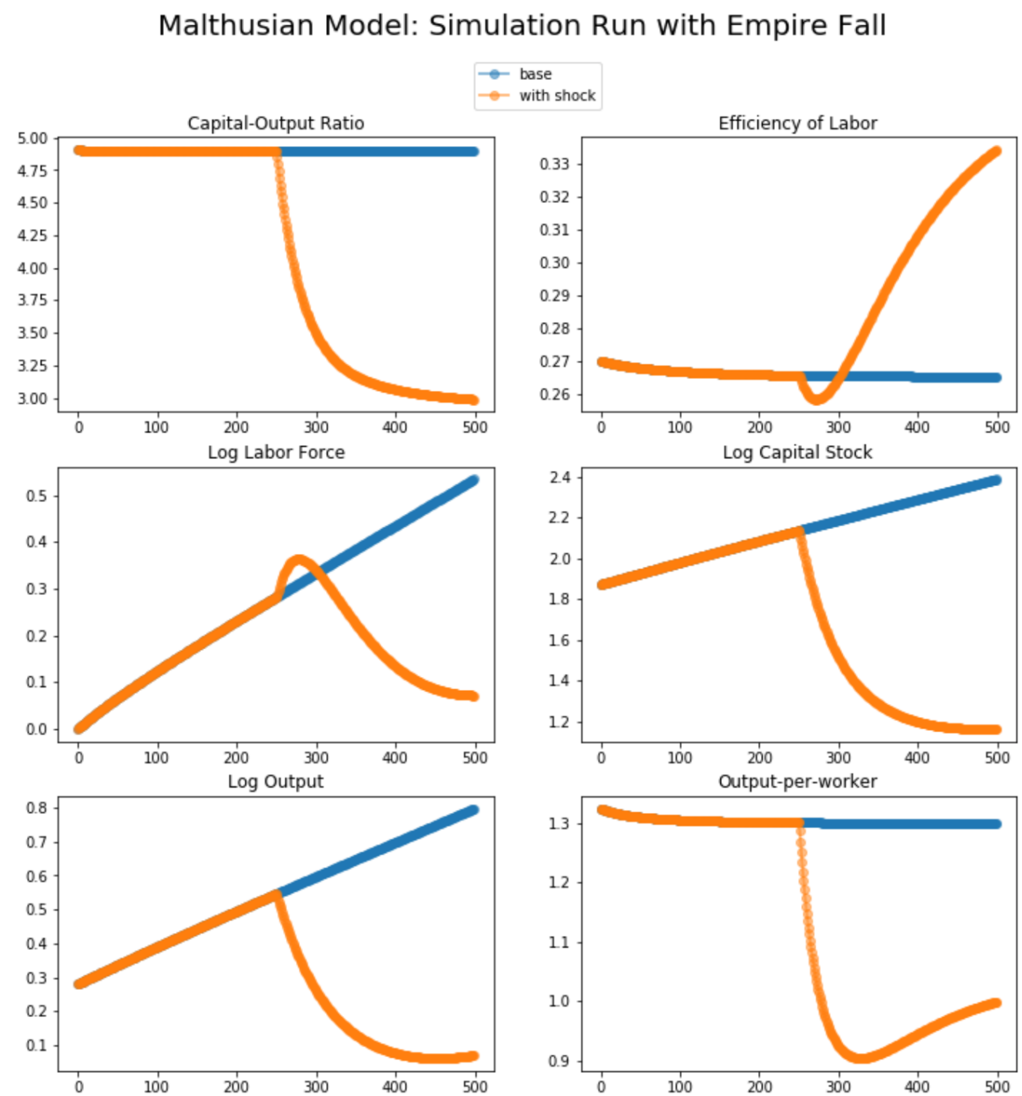
Notes:

-

Steady-State and Along the Transition Path

The fall of an empire:

- <https://nbviewer.jupyter.org/github/braddelong/LS2019/blob/master/2019-10-14-Ancient_Economies.ipynb>
  - A decline in inequality, taste for luxuries, and taste for urban living: $\Delta\varphi = -0.25$
  - A decline in law-and-order that produces a sharp fall in the savings rate: $\Delta s = -0.10$



Review: Solow Model Basics

Lecture Notes: <https://www.bradford-delong.com/2020/01/lecture-notes-the-solow-growth-model-the-history-of-economic-growth-econ-135.html>

$$(2.1.2)\ Y = \kappa^{\theta} E L\ ;\ (2.1.3)\ y = \kappa^{\theta} E\ ;\ (2.1.1)\ \kappa = \frac{K}{Y}$$

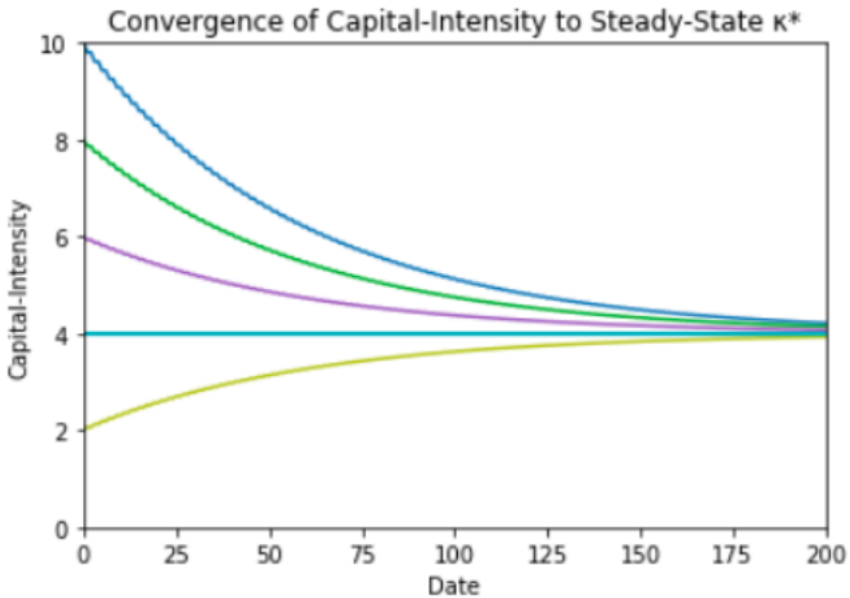
$$\frac{dE}{dt} = gE \qquad \frac{dL}{dt} = g_L L = nL \qquad \frac{dK}{dt} = sY - \delta K = \left(\frac{s}{\kappa} - \delta\right) K$$

$$(1.16)\ \kappa^* = \frac{s}{n+g+\delta}$$

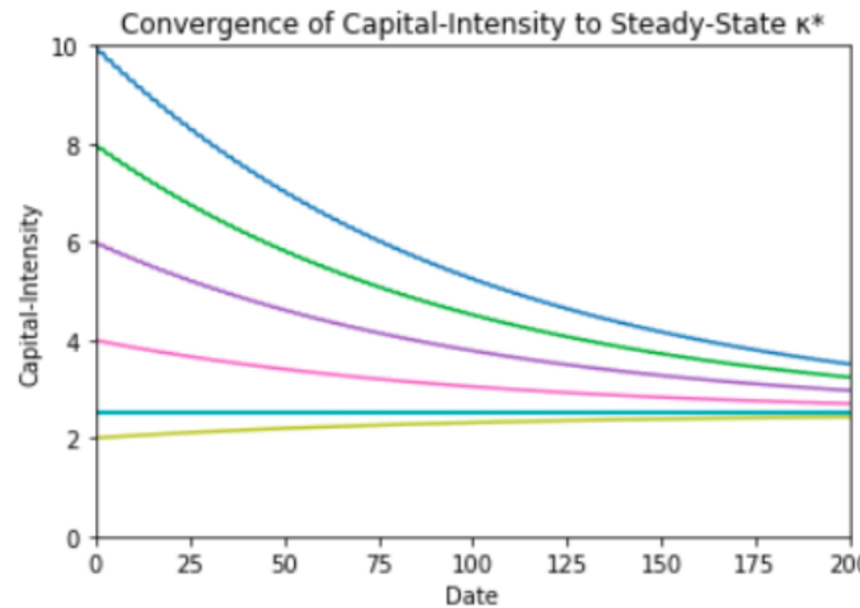
This κ^* we define as the *steady-state balanced-growth equilibrium* value of capital-intensity in the Solow growth model. If the capital-intensity $\kappa = \kappa^*$, then it is constant, and the economy is in balanced growth, with Y and K growing at the rate n+g, E and y growing at the rate g, and L growing at the rate n.

$$(1.18)\ \frac{d\kappa}{dt} = -\frac{n+g+\delta}{1+\theta}(\kappa - \kappa^*)$$

Solving the Model




```
κ_max = 10
κ = κ_max
for i in range(5):
    cg = κ_convergence_graph(κ_0=κ, s = 0.20, n = 0.01,
          g = 0.015, δ = 0.025, θ = 1/2, T = 200)
    cg.draw()
    κ = κ-2
```




```
κ_max = 10
κ = κ_max
for i in range(5):
    cg = κ_convergence_graph(κ_0=κ, s = 0.15, n = 0.02,
          g = 0.015, δ = 0.025, θ = 2, T = 200)
    cg.draw()
    κ = κ-2
```

Along the Balanced-Growth Path

Everything except κ—which is constant—grows at a constant proportional rate: either n, or g, or n+g;

- Labor force L grows at n
- Income per worker y and the efficiency of labor E grow at g
- Total income Y and the capital stock K grow at n+g

$$E_t^* = e^{gt} E_0$$

$$L_t^* = e^{nt} L_0$$

$$Y_t^* = (\kappa^*)^\theta E_t L_t = (\kappa^*)^\theta e^{gt} E_0 e^{nt} L_0 = (s/(n + g + \delta))^\theta e^{gt} E_0 e^{nt} L_0$$

$$K_t^* = \kappa^* Y_t^* = (s/(n + g + \delta))^{(1+\theta)} e^{gt} E_0 e^{nt} L_0$$

$$y_t^* = (\kappa^*)^\theta E_t = (\kappa^*)^\theta e^{gt} E_0 = (s/(n + g + \delta))^\theta e^{gt} E_0$$

Big Ideas: Lecture 4: Idea Stock Growth

Takeaways from this lecture:

- People were ingenious and inventive back before 1500, yet standards of living did not increase
- Populations, however, did: slowly
- THEN WE GET AN EXPLOSION
  - Two heads are better than one at R&D. Does not quite work…
    - Explosion too soon…
    - Even with research stepping on its toes…
    - And writing, cities, printing, &c. should have boosted research…
  - Picking the low-hanging fruit?
- Agrarian age: h^* = 0.035%/year; n_{stem} = 0.07%/year; $\lambda/(1-\phi) = 1/2$
- MEG age: h^* = 2%/year; n_{stem} = 6%/year; $\lambda/(1-\phi) = 1/3$
- What causes the increase in L_{stem}?
- What institutions make it profitable for n_{stem} to be higher?

Catch Our Breath…

- Ask a couple of questions?
- Make a couple of comments?
- Any more readings to recommend?