Assignment 7

SCP8082721 - QUANTUM INFORMATION AND COMPUTING
2022-2023

David Lange

December 20, 2022

Intro and Theory

- we are dealing with the ising model

$$\hat{H} = \lambda \sum_i^N \sigma_i^z + \sum_i^{N-1} \sigma_i^x \sigma_{i+1}^x,$$

- with $\sigma^x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ and $\sigma^z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$
- both parts can be written explicitly as Tensor products
  1. $\sigma_i^z = \mathbb{1}_1 \otimes \mathbb{1}_2 \otimes ... \otimes \sigma^z \otimes \mathbb{1}_{i+1} \otimes ... \otimes \mathbb{1}_N$
  2. $\sigma_i^x \sigma_{i+1}^x = \mathbb{1}_1 \otimes \mathbb{1}_2 \otimes ... \otimes \sigma^x \otimes \sigma^x \otimes \mathbb{1}_{i+2} \otimes ... \otimes \mathbb{1}_N$
- The general tensor product can be written as (A is a k×l matrix and B a m×n)

$$A \otimes B = \begin{bmatrix} A_{11}B & A_{12}B & \dots & A_{1l}B \\ \vdots & & & \vdots \\ A_{k1}B & A_{k2}B & \dots & A_{kl}B \end{bmatrix}$$

Spectrum as a function of interaction strength λ

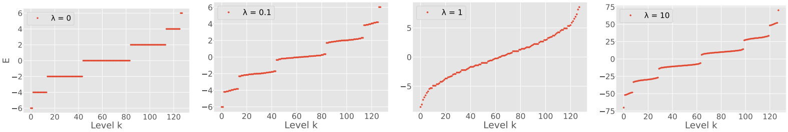

Figure: Here, number of sides $N = 7$, so $2^7 = 128$ energy levels.

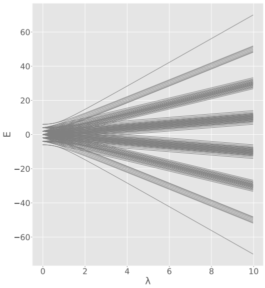

Figure: Another way to see the same effect is by plotting all eigenvalues as a function of λ.

- We can see that 7 energy levels are degenerate at first but the degeneracy is lost by increasing the interaction strength.
- in particular when $\lambda = 0$ (interaction dominated), then the ground state is two fold degenerate (all spins align with the x-axis) while the first excited state is 2N fold degenerate (one spin aligned opposite to the others)
- when λ becomes large (field dominated) the ground state is not degenerate (all spins aligned with the field), while the first excited state is now N fold degenerate, since the spins have a preferred direction. The highest possible energy level is then when all spins are anti-aligned with the field.

Quantum phase transition

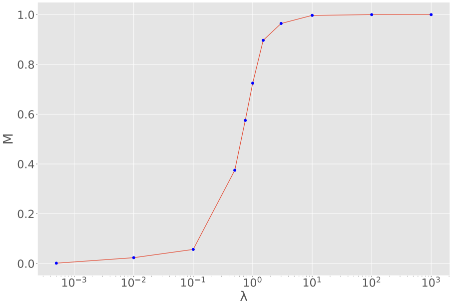

Figure: Calculating the average magnetization of the ground state $\hat{M} = 1/N \sum_{i=1}^{N} \sigma_i^z \rightarrow M = \langle\psi_{gs}| \hat{M} |\psi_{gs}\rangle$ for $N = 7$ again. We can see that the phase transition occurs at around $\lambda = 1$, as expected. Since we are dealing with the zero temperature Ising model we have a quantum phase transition driven by quantum fluctuations, in this case the presence of a magnetic field.

Some thoughts on improvement

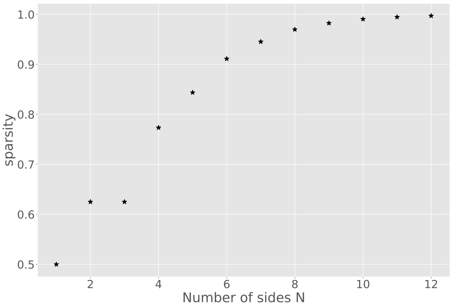

Figure: The ratio between number of zero elements and total elements of the Hamiltonian.

As we can see by increasing the number of sides, the ratio converges towards one, meaning the sparsity of the matrix increases. Since so many elements are zero my next idea would be to compile an algorithm that uses a sparse format to store and work with the matrix. One package in Fortran that can do diagonalization with sparse formatted matrices is for example Arpack (https://en.wikipedia.org/wiki/ARPACK). This is an important note since the highest possible sides that can be evaluated with the code now are only around $N = 14$.