Assignment 6

SCP8082721 - QUANTUM INFORMATION AND COMPUTING

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Intro and theory

- dealing with N body non interacting pure state
- a general state can be written as

$$|\psi\rangle = \sum_{\alpha_1,\alpha_2,...,,\alpha_N} C_{\alpha_1} C_{\alpha_2} ... C_{\alpha_N} |\alpha_1\rangle \otimes |\alpha_2\rangle \otimes ... \otimes |\alpha_N\rangle \,,$$

  - with d^N different coefficients α_i
- if the state is separable we can instead write it as

$$|\psi\rangle = \sum_{\alpha} C_{\alpha_1} C_{\alpha_2} ... C_{\alpha_N} |\alpha_1\rangle \otimes |\alpha_2\rangle \otimes ... \otimes |\alpha_N\rangle \,,$$

  - where we have $d \cdot N$ different coefficients
  - a non-separable (entangled state) is e.g. a Bell state $|\psi\rangle = 1/\sqrt{2}\left(|0\rangle_A \otimes |0\rangle_B + |1\rangle_A \otimes |1\rangle_B\right)$

Density matrix

- $\rho = |\psi\rangle \langle\psi|$
  1. $\mathrm{Tr}(\rho^2) = \mathrm{Tr}(\rho) = 1$
  2. $\rho^2 = \rho$
  3. pure state: when diagonalizing ρ, only one $p_\alpha \neq 0$
- the reduced density matrix is defined as

$$\rho_A = \mathrm{Tr}_B(\rho_{AB}) = \sum_n \langle n|_B \, \rho_{AB} \, |n\rangle_B$$

- and to get the specific elements of the matrix

$$(\rho_A)_{a,b} = \sum_n \langle n|_B \, \langle a|_A \, \rho_{AB} \, |b\rangle_A \, |n\rangle_B$$

Code development

- using a trick to get a different one number label for all states: $\alpha = 2i + j$
- then the subroutine computes for each element (example 2 qubits $\Rightarrow N = d = 2$)
- for i in d^{N-1}
  - for j in d^{N-1}
    - for n in d

$$(\rho_A)_{i,j} \mathrel{+}= (\rho_{AB})_{2i+n,2j+n}$$
$$(\rho_B)_{i,j} \mathrel{+}= (\rho_{AB})_{2n+i,2n+j}$$

Example - Bell state

- As a test of the developed code I use the state

$$|\psi\rangle = \frac{1}{\sqrt{2}} \left(|0\rangle_A \otimes |0\rangle_B + |1\rangle_A \otimes |1\rangle_B \right)$$

- giving a density matrix

$$\rho = \begin{bmatrix} 0.5 & 0 & 0 & 0.5 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0.5 & 0 & 0 & 0.5 \end{bmatrix}$$

- and reduced density matrices are

$$\rho_A = \rho_B = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}$$

Example - Generic state

- On the other hand for a generic (non-separable) state

$$|\psi\rangle = \begin{bmatrix} 0.569 - 0.108j \\ 0.056 + 0.161j \\ -0.473 - 0.192j \\ 0.57 - 0.223j \end{bmatrix}$$

- giving a density matrix

$$\rho = \begin{bmatrix} 0.336 & 0.015 - 0.097j & -0.249 + 0.160 & 0.349 + 0.065j \\ 0.015 + 0.097j & 0.029 & -0.057 - 0.065j & -0.004 + 0.104j \\ -0.249 - 0.160j & -0.057 + 0.065j & 0.261 & -0.227 - 0.215j \\ 0.349 - 0.065j & -0.004 - 0.104j & -0.227 + 0.215j & 0.375 \end{bmatrix}$$

- and reduced density matrices are

$$\rho_L = \begin{bmatrix} 0.585 & 0.157 - -0.239j \\ 0.157 + 0.239j & 0.415 \end{bmatrix}, \rho_R = \begin{bmatrix} 0.907 & -0.087 + 0.083j \\ -0.087 - 0.083j & 0.093 \end{bmatrix}$$