Assignment 8

SCP8082721 - QUANTUM INFORMATION AND COMPUTING

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David Lange

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Intro and theory

- we are again dealing with the Ising model

$$\hat{H} = \lambda \sum_{i}^{N} \sigma_i^z + \sum_{i}^{N-1} \sigma_i^x \sigma_{i+1}^x,$$

- with $\sigma^x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ and $\sigma^z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$
- both parts can be written explicitly as Tensor products
  1. $\sigma_i^z = \mathbb{1}_1 \otimes \mathbb{1}_2 \otimes ... \otimes \sigma^z \otimes \mathbb{1}_{i+1} \otimes ... \otimes \mathbb{1}_N$
  2. $\sigma_i^x \sigma_{i+1}^x = \mathbb{1}_1 \otimes \mathbb{1}_2 \otimes ... \otimes \sigma^x \otimes \sigma^x \otimes \mathbb{1}_{i+2} \otimes ... \otimes \mathbb{1}_N$
- The general tensor product can be written as (A is a k×l matrix and B a m×n)

$$A \otimes B = \begin{bmatrix} A_{11}B & A_{12}B & \dots & A_{1l}B \\ \vdots & & & \vdots \\ A_{k1}B & A_{k2}B & \dots & A_{kl}B \end{bmatrix}$$

RSRG Algorithm

1. consider system with size N described by H_N
2. build system of size 2N described by
   $H_{2N} = H_N \otimes \mathbb{1} + \mathbb{1} \otimes H_N + H_{int}$
   with $H_{int} = \left[\bigotimes_{J=1}^{N-1} \mathbb{1}(\otimes\sigma^x)\right] \otimes \left[(\sigma^x \otimes) \bigotimes_{J=1}^{N-1} \mathbb{1}\right] = H_{int}^L \otimes H_{int}^R$
3. diagonalize H_{2N} and use projector to calculate truncated version $\Rightarrow \tilde{H}_{2N} = P^\dagger H_{2N} P$
4. build system of size 4N
   $H_{4N} = \tilde{H}_{2N} \otimes \left[\bigotimes_{J=1}^{N} \mathbb{1}\right] + \left[\bigotimes_{J=1}^{N} \mathbb{1}\right] \otimes \tilde{H}_{2N} + \tilde{H}_{int}$ with
   $H_{int} = P^\dagger \left[\bigotimes_{J=1}^{N} \mathbb{1}(\otimes H_{int}^L)\right] P \otimes P^\dagger \left[(H_{int}^R \otimes) \bigotimes_{J=1}^{N} \mathbb{1}\right] P$
5. diagonalize again H_{4N} and calculate the truncated version again by projecting it onto the first d^N eigenstates
6. iterate through steps 3., 4. and 5. until sufficient convergence is reached

RSRG: Results

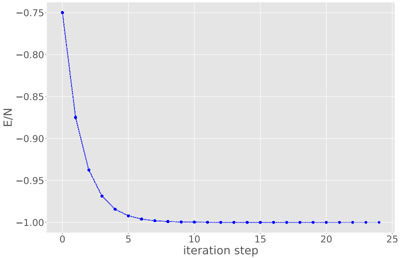

Figure: Ground state energy in terms of iteration stage of the RSRG algorithm with no field $\lambda = 0$.

- The ground state energy with $\lambda = 0$ is when all spins are aligned so $E/N = -(N-1)/N$ which converges towards -1 as can be seen in the figure
  - Note: The exact calculation of the Ising Hamiltonian (Hand-in 7) took 68.3 seconds for $N = 11$, while the RSRG takes 0.048 seconds and ends up at a system size of $N = 67108864$

RSRG: More results

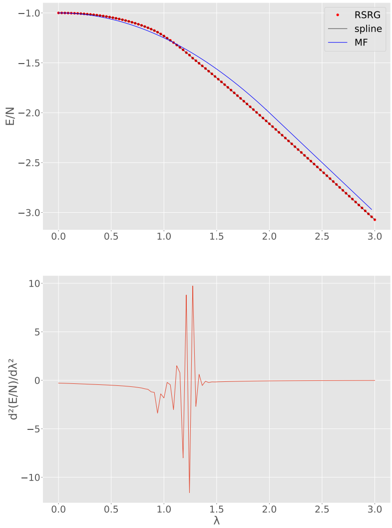

Figure: Energy density of different values of λ for the RSRG algorithm and the MF result for comparison (top) and the second derivative of the spline created using the points from the RSRG algorithm.

- The mean field solution is given by $E/N = -1 - \lambda^2/4$ for $\lambda \in [-2, 2]$ and $-|\lambda|$ otherwise, it differs from the RSRG result for small values of λ
- we also know there is a quantum phase transition around $\lambda = 1$, which can be seen by calculating the second derivative of the curve which has a discontinuity around that value