Assignment 5

SCP8082721 - QUANTUM INFORMATION AND COMPUTING
2022-2023

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Intro and theory

- solving time-dependent one-dimensional harmonic oscillator

$$\hat{H} = \hat{T} + \hat{V} = \hat{p}^2/2m + \frac{\omega^2(\hat{q} - q_0(t))^2}{2m}$$
$$\Rightarrow \hat{H} = -\frac{\hbar^2}{2m}\nabla^2\psi(x) + \frac{\omega^2(x - t/T)^2}{2m}$$

- with $\hat{p} = -i\hbar\partial_x$ and $\hat{q} = x$
- the initial state at t = 0 is $\psi_0 = \left(\frac{a}{\pi}\right)^{1/4} \exp\left\{\left(-x^2/2\right)\right\}$, the ground state of the time-independent harmonic oscillator
- we use split-operator method to solve the Hamiltonian

$$\psi(t + dt) = \left(e^{-i\frac{\hat{V}}{2}dt} e^{-i\hat{T}dt} e^{-i\frac{\hat{V}}{2}dt} + \mathcal{O}(dt^3)\right)\psi(t)$$
$$\Rightarrow \psi(t + dt) = e^{-i\frac{\hat{V}}{2}dt}\mathcal{F}^{-1}\left[e^{-i\hat{T}dt}\mathcal{F}\left[e^{-i\frac{\hat{V}}{2}dt}\psi(t)\right]\right] + \mathcal{O}(dt^3)$$

Results

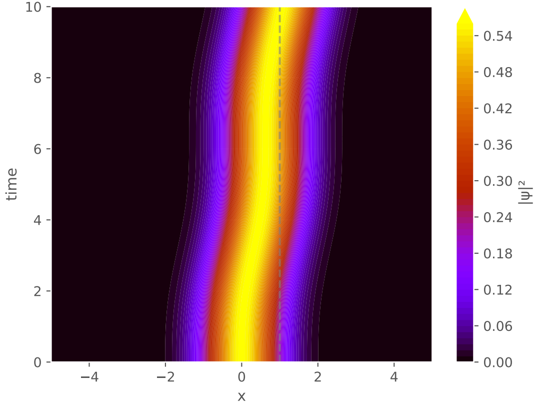
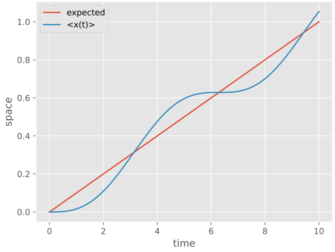


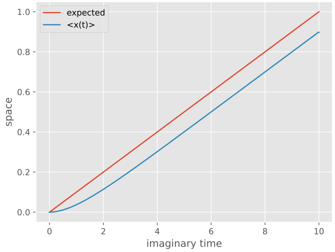
Figure: Square norm of $|\psi$(t)> as a function of x and t. ($T_{max} = 10, xmin = -10, xmax = 10, n_{mesh} = 1000, n_t = 1000$)

- Basically in the code, I have as inputs both the spatial and then time grid, potential parameters and the initial state of the wave function, here a Gaussian centered around $x = 0$.
- in terms of flexibility, it is nice that I can change both initial wave packet and also the potential quite easily in the program.
- Also, setting up the momentum grid ... (needed for the evolution in Fourier space), gladly python has this nice function that does it automatically: $\underline{\text{k}_{space} = \text{np.fft.fftfreq}(\text{n}_{mesh}, \Delta x) \cdot 2\pi}$ (it essentially creates f = [0, 1, ..., n/2-1, -n/2, ..., -1] / $(\Delta x \cdot n)$)
- In all subsequent results I use $xmin = -10, xmax = 10, n_{mesh} = 1000, n_t = 1000$.

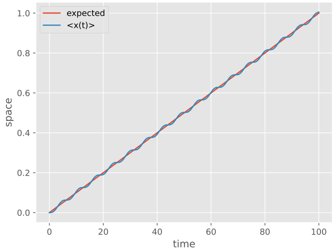
More results



(a) $T_{max} = 10$



(b) $T_{max} = 10$, $\tau = it$



(c) $T_{max} = 100$

Figure: Basically, (a) and (c) show what happens to <x(t)> when T_{max} is increased, while (b) is just a (not very certain about it) try what happens if time is replaced with imaginary time.
In that case my (very vague) idea is that since we are exponentially "killing" higher order states, the system should evolve only through the ground state, like mentioned in the lectures.

Even more results

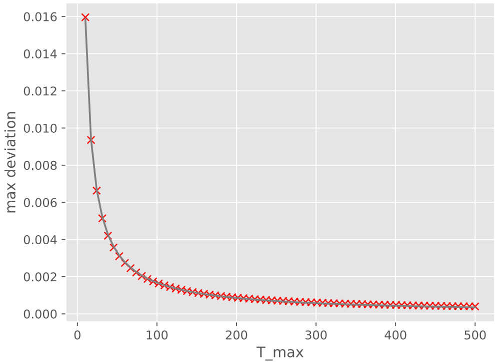


Figure: Shows the maximum deviation between expected and $< x(t) >$ for different T_{max}, as expected the error decreases with increasing time. The fit gives a behaviour very close to $\Delta x = 1/T$.

- going back to plots (a) and (c):
- Expected behaviour is that the Ground state evolves linearly like $x = t/T$, while in reality the expectation value $< x(t) >= \int |\psi|^2 x dx$ oscillates around the $x = t/T$ line.
- to assume that the wave function stays in the ground state of the moving potential (the $t = 0$ instantaneous eigenstate) is an adiabatic approximation and is assuming a slowly varying potential. Thus, I expect the error to decrease when increasing the T_{max}. This behaviour can be verified by the plot on the left, which shows that the maximum deviation decreases as $1/T$.
- Another (maybe) interesting property to check could be the frequency of oscillations. Preliminary check shows they increase slightly but monotonously with increasing T_{max}.