MAU34405 - Homework 1: due date: Monday, 23.09.24

All questions carry equal weight.

Exercise 1: Consider a classical gas undergoing a free expansion (expansion in the vaccum). The gas is initially confined to $\frac{1}{3}$ of the total volume V_0 of a thermaly and chemicaly isolated container, and is in thermal equiibrium at temperaure $T = T_0$. At a given time, the internal divider of the container is removed, allowing the gas to expand into the rest of the container eventually attaining the total volume V_0. If the equation of state for this monoatomic gas is:

$$P(T, V) = \frac{kNT}{V - bN} - a\frac{N^2}{V^2} \,,$$

and its heat capacity at constant volume is

$$C_V \equiv \left(\frac{\partial E}{\partial T}\right)_V = \frac{3}{2}kN \,,$$

where (a, b) are positive constants and k denotes Boltzman's constant, determine:

1. The thermodynamic quantities conserved in the process, and
2. The temperature of the gas, T_F, once it has reached thermal equilibrium in the container.

Exercise 2: Consider a single component system (*i.e.* a system containing only one type of particles), which obeys the following equations of state:

$$T = \frac{4AS^3}{NV^2}, \qquad P = \frac{2AS^4}{NV^3} \,,$$

with A a constant coefficient.

1. Find the internal energy of the system $E(S, V, N)$ by integration. Note that the energy of the system can be determined up to an arbitrary constant.
2. The Gibbs-Duhem relation

   $$d\mu = -\frac{S}{N}dT + \frac{V}{N}dP$$

   is an explicit manifestation of the fact that the intensive, thermodynamic parameters of a singe component system are not independent. Use the Gibbs-Duhem relation for the system above to compute the chemical potential μ. Then use your answer to compute the internal energy $E(S, V, N)$ again. Can you determine the value of the arbitrary constant of integration you found in part 1 of the exercise?

Exercise 3: A thermodynamic system obeys the following equations:

$$P = \frac{E}{V}, \qquad T = BE^{\frac{2}{3}}V^{-\frac{1}{3}}N^{-\frac{1}{3}}$$

with B a given positive constant. When the temperature and pressure of this system are (T_i, P_i), it starts undergoing an adiabatic process in mechanical isolation. The pressure of the system when the final equilibrium state is reached is P_f. Express the final temperature T_f in terms of (T_i, P_i, P_f).

Exercise 4: Consider a single-component, chemically isolated system undergoing an adiabatic, quasi-static and irreversible process. Assume that the pressure and the volume of the system during this process satisfy: $PV^k = a$ with (k, a) positive constants. Show then that:

$$E = \frac{PV}{k-1} + Nf\left(\frac{PV^k}{N^k}\right)$$

Manuela Kulaxizi