PLANCKS Lecture I: Hydrodynamics/Fluid Mechanics
Based on lectures given by Dr. Chaolun Wu

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1 Introduction

Fluid Mechanics, or Hydrodynamics, can be defined as a **low-energy effective description of a many-body dynamical system**. This system could be governed by either classical or quantum mechanics. From first principles, we have conservation laws for Noether charges. We have two equations of motion:

$$\partial_\mu J^\mu = 0 \quad \partial_\mu T^{\mu\nu} = 0 \quad (Rel/NR) \tag{1}$$

where J^ν is the charge current, and $T^{\mu\nu}$ is the energy-stress tensor.

The system itself has 4 variables: $u^\mu(x)$ or $\vec{v}(\vec{x},t), \epsilon(\vec{x},t), P(\vec{x},t), \rho(\vec{x},t)$. These correspond to four-velocity or velocity, energy density, pressure and mass density respectively.

We aim to find '**constituent relations**', which are $J^\nu, T^{\mu\nu}$ as functions of the system variables $u^\mu, P, \ldots$. The way in which we find these is why we call this an **effective** description. Since any terms allowed by the symmetries of the system will exist, we know that certain terms are contained in each of the relations, e.g.

$$J^\nu \sim u^\mu, \quad T^{\mu\nu} \sim u^\mu u^\nu, g^{\mu\nu} \tag{2}$$

index notation implies the relations are manifestly covariant.

Since our description is **low-energy**, we organise our constituent relations using a derivative expansion as a Taylor series, where we can charecterise our spacial/temporal variation/fluctuation by

$$\vec{p} \sim \vec{\partial}, \quad E \sim \partial_t \tag{3}$$

and our charecteristic scales of length, time are the mean free path l_{mfp} and relaxation time T_{rel} respectively. In a **low-energy** description, we can say $l_{\mathrm{mfp}}|\vec{\partial}|, T_{\mathrm{rel}} \ll 1$