Problem Set 1

Problem 1

A long cylinder of radius R carries a uniform volume charge density ρ.
a) Use Gauss's law to determine the electric field $\mathbf{E}$ and the scalar potential Φ due to this cylinder:

1. Inside the cylinder (i.e., for $r < R$) where r is the radial distance from the cylinder's axis.

2. Outside the cylinder (i.e., for $r > R$).

b) Use Poisson equation for the scalar potential to verify that your solution indeed corresponds to the charge distribution specified in part a).

Problem 2

Consider three concentric metal spheres with radii $r_1 < r_2 < r_3$. The middle sphere (radius r_2) carries a charge Q, while the inner and outer spheres are initially uncharged.

Subsequently, the inner (sphere 1) and the outer sphere (sphere 3) are connected by a conducting wire, which is isolated from the middle sphere. Determine the resulting charges on sphere 1 and sphere 3 after equilibrium is reached.

Problem 3

Recall

$$\Phi(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\int_V \frac{\rho(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}d^3r' + \frac{1}{4\pi}\oint_S \left(\Phi\frac{\partial}{\partial n}\left(\frac{1}{|\mathbf{r}-\mathbf{r}'|}\right) - \frac{1}{|\mathbf{r}-\mathbf{r}'|}\frac{\partial\Phi}{\partial n}\right) da'$$

Consider the case of a charge-free space enclosed by a sphere S of radius R_0 centered on the point $\vec{x}_0$. Prove the mean value theorem of electrostatics:

$$\Phi(\vec{x}_0) = \frac{1}{4\pi R_0^2}\oint_S \Phi(\vec{x}')da'$$

or, equivalently, $\Phi(\vec{x}_0) = \langle\Phi(\vec{x})\rangle_{sphere}$, the averaged value of the scalar potential on the sphere.