MAU22101: Exercises Week 2

Problem 1 Let $\sigma = (12\cdots l) \in S_n$ be an l-cycle where $l = 2k$ is even. Find the cycle decomposition of σ^k.

Problem 2 Prove that the order of an element in S_n equals the least common multiple of the lengths of the cycles in its cycle decomposition.

Problem 3 Let G be a group. Show that the three maps $\rho_l, \rho_r, \rho_{\mathrm{ad}} \colon G \times G \to G$ defined by

$$\begin{aligned} \rho_l(g,x) &= gx, \\ \rho_r(g,x) &= xg^{-1}, \\ \rho_{\mathrm{ad}}(g,x) &= gxg^{-1}, \end{aligned}$$

define groups actions of G on G.

Problem 4 Let $g \in G$ and define the map $\varphi \colon G \to G$ by $\varphi(x) = gxg^{-1}$. Show that ϕ is a group-isomorphism.

Problem 5 Let G be a group. Show that the formula

$$\begin{aligned} \mathbb{Z}/2\mathbb{Z} \times G &\longrightarrow G \\ (0,g) &\mapsto g \\ (1,g) &\mapsto g^{-1} \end{aligned}$$

defines a group action of $\mathbb{Z}/2\mathbb{Z}$ on G.