Group Theory Homework 2

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1 Problem 1

Let $\mathrm{G} \times \mathrm{X} \to \mathrm{X}$ be a group action and let $s \,\epsilon\, \mathrm{X}$. Show that the stabilizer of the element s,

$$\mathrm{G}_s := \{g \,\epsilon\, \mathrm{G} \mid g \cdot s = s\}$$

is a subgroup of G.

G_s is by definition a subset of G.

(1) G_s must be non-empty:

$$\exists\, e \,\epsilon\, \mathrm{G}_s \text{ such that } \; e \cdot s = s$$

This is true by the defintion of G above.

(2) G_s must have closure: $\forall\, a, b \,\epsilon\, \mathrm{G}_s,\ (a \circ b) \ \epsilon\, \mathrm{G}_s$

$$g,\ h \,\epsilon\, \mathrm{G}_s \ \to\ g \cdot s = s,\ h \cdot s = s$$

$$g \cdot (h \cdot s) = s \ \ \Rightarrow\ (g \circ h)\, s = s$$
$$\Rightarrow\ \ (g \circ h)\ \epsilon\, \mathrm{G}_s$$

(3) G_s must contain the inverse of all of its elements: $\forall\, a \,\epsilon\, \mathrm{G}_s \,\exists\, b \,\epsilon \mathrm{G}_s$ such that $a \cdot b \ = \ e$