Advanced Classical Mechanics HW

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1 Question 1

$S = \int_{t_A}^{t_B} L(t)\mathrm{d}s$ is the definition of Action w.r.t. the time-dependent Lagrangian. $\mathrm{d}x$ can be substituted by $\sqrt{\mathrm{d}x^2 + \mathrm{d}y^2}$, and $L(t)$ can be substituded by $\rho g \cdot y(x)$, the expression for potential energy.

$$S = \rho g \int_{x_A}^{x_A} y(x)\sqrt{\mathrm{d}x^2 + \mathrm{d}y^2}$$

$$S = \rho g \int_{x_A}^{x_B} y(x)\sqrt{1 + \frac{\mathrm{d}y^2}{\mathrm{d}x^2}}\mathrm{d}x$$

$$S = \rho g \int_{x_A}^{x_B} y(x)\sqrt{1 + \dot{y}^2(x)}\mathrm{d}x$$

Here the differential equation of y in the integral is the Lagrangian as a function o x instead of t.

$$L = y\sqrt{1 + \dot{y}^2}$$

We then input this into the $\varepsilon - L$ equation, $\frac{\mathrm{d}}{\mathrm{d}x}[\partial_{\dot{y}} L] = \partial_y L$,

$$\frac{\mathrm{d}}{\mathrm{d}x}[y\dot{y}(1 + \dot{y}^2)^{-1/2}] = (1 + \dot{y}^2)^{-1/2}$$