

Advanced Classical Mechanics HW

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1 Question 1

$S = \int_{t_A}^{t_B} L(t) dt$ is the definition of Action w.r.t. the time-dependent Lagrangian. dx can be substituted by $\sqrt{dx^2 + dy^2}$, and $L(t)$ can be substituted by $\rho g \cdot y(x)$, the expression for potential energy.

$$S = \rho g \int_{x_A}^{x_B} y(x) \sqrt{dx^2 + dy^2}$$

$$S = \rho g \int_{x_A}^{x_B} y(x) \sqrt{1 + \frac{dy^2}{dx^2}} dx$$

$$S = \rho g \int_{x_A}^{x_B} y(x) \sqrt{1 + \dot{y}^2(x)} dx$$

Here the differential equation of y in the integral is the Lagrangian as a function of x instead of t .

$$L = y \sqrt{1 + \dot{y}^2}$$

We then input this into the $\varepsilon - L$ equation, $\frac{d}{dx}[\partial_{\dot{y}} L] = \partial_y L$,

$$\frac{d}{dx}[y \dot{y} (1 + \dot{y}^2)^{-1/2}] = (1 + \dot{y}^2)^{-1/2}$$