Chapter 3: Trees

Tree theory

See Alberto Montresor theory here: http://disi.unitn.it/~montreso/sp/slides/05-alberi.pdf (http://disi.unitn.it/~montreso/sp/slides/05-alberi.pdf)

See Trees on the book (https://interactivepython.org/runestone/static/pythonds/Trees/toctree.html)

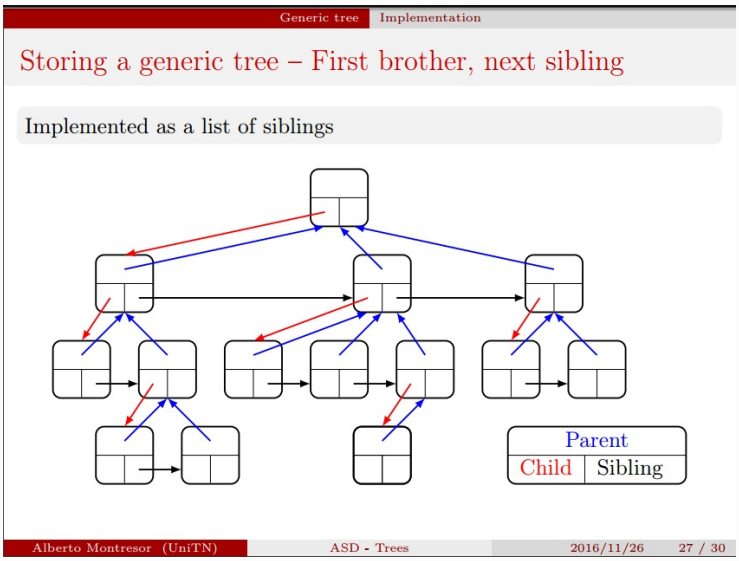
In particular, see :

- Vocabulary and definitions (https://interactivepython.org/runestone/static/pythonds/Trees/VocabularyandDefinitions.html)

GenericTree

GenericTree theory

See Alberto Montresor theory here (NOTE: currently they are being reworked): http://disi.unitn.it/~montreso/sp/slides/05-alberi.pdf (http://disi.unitn.it/~montreso/sp/slides/05-alberi.pdf) (slide 27 and following ones)



In this worksheet we are going to provide an implementation of a `GenericTree` class:

- Differently from the `UnorderedList`, which had actually two classes `Node` and `UnorderedList` that was pointing to the first node, in this case we just have one `GenericTree` class. So to grow a tree like the above one in the picture, for each of the boxes that you see we will need to createone instance of `GenericTree` and link it to the other instances.
- Ordinary simple trees just hold pointers to the children. In this case, we have an enriched tree which holds ponters also to up the *parent* and on the right to the *siblings*. Whenever we are going to manipulate the tree, we need to take good care of updating these pointers.

> **ROOT NODE: In this context, we call a node *root* if has no incoming edges *and* it has no parent nor sibling**

> **DETACHING A NODE: In this context, when we *detach* a node from a tree, the node becomes the *root* of a new tree, which means it will have no link anymore with the tree it was in.**

GenericTree exercises

You will implement the `GenericTree` class. You can start by copying the code skeleton and unit tests, then proceed reaading the following.

- A `GenericTree` class holds thus 3 pointers that link it to the other nodes: `_child`, `_sibling` and `_parent`. It also holds a value `data` which is provided by the user to store arbitrary data (could be ints, strings, lists, even other trees, we don't care):

  ```
  class GenericTree:

      def __init__(self, data):
          self._data = data
          self._child = None
          self._sibling = None
          self._parent = None
  ```

- To create a tree, just call the constructor passing whatever you want like this:

  ```
  tn = GenericTree(5)

  tblah = GenericTree("blah")
  ```

- To grow a `GenericTree`, as basic building block you will have to implement `insert_child`:

  ```
  def insert_child(self, new_child):
          """ Inserts new_child at the beginning of the children sequence. """

          raise Exception()
  ```

You can call it like this:

```
>>> ta = GenericTree('a')
>>> print ta
a

>>> tb = GenericTree('b')
>>> ta.insert_child(tb)
>>> print ta
a
\-b

>>> tc = GenericTree('c')
>>> ta.insert_child(tc)
>>> print ta
a
|-b
\-c

>>> td = GenericTree('d')
>>> tb.insert_child(td)
>>> print ta
a
|-b
| \-d
\-c
```

If you need to test your data structure, we provide you with this handy function `gt` that allows to easily construct trees from other trees

> **WARNING: DO NOT USE `gt` inside your implementation code !!!! `gt` is just meant for testing.**

```
def gt(data, children=[])
    """ Returns a GenericTree of which the root node is filled with provided
data
        and children. Children must be instances of GenericTree.
    """
```

NOTE: this function is *not* a class method, you can directly invoke it like this:

```
>>> print gt('a')
    a

    >>> print gt('a', gt('b'), gt('c'))
    a
    |-b
    \-c
```

GenericTree Code Skeleton

In [4]:

```
import unittest

class GenericTree:
    """ A tree in which each node can have any number of children.

        Each node is linked to its parent and to its immediate sibling on the right
    """

    def __init__(self, data):
        self. data = data
```

```
        self._child = None
        self._sibling = None
        self._parent = None

    def data(self):
        return self._data

    def child(self):
        return self._child

    def sibling(self):
        return self._sibling

    def parent(self):
        return self._parent

    def is_root(self):
        """ Return True if the node is a root of a tree, False otherwise

            A node is a root whenever it has no parent nor siblings.
        """
        return self._parent == None and self._sibling == None

    def is_subtree(self):
        """ Returns True if the node is a subtree of another tree

            A subtree always has a parent
        """
        return self._parent != None


    def children(self):
        """ Returns the children as a Python list """

        ret = []
        current = self._child
        while current != None:
            ret.append(current)
            current = current._sibling
        return ret

    def __str__(self):
        """ Returns a pretty string of the tree """

        def str_branches(node, branches):
            """ Returns a string with the tree pretty printed.

                branches: a list of characters representing the parent branches. Cha
racters can be either ` ` or '|'
            """
            strings = [str(node._data)]
            current = node._child
            while (current != None):
                if current._sibling == None:
                    # there are better end characters but let's not upset
                    # stupid Python with unicode problems
                    joint = '\-'
                else:
                    joint = '|-'


                strings.append('\n')
                for b in branches:
                     strings.append(b)
                strings.append(joint)
```

```
                strings.append(joint)
                if current._sibling == None:
                    branches.append('  ')
                else:
                    branches.append('| ')
                strings.append(str_branches(current, branches))
                branches.pop()
                current = current._sibling

            return "".join(strings)

        return str_branches(self, [])


    def insert_child(self, new_child):
        """ Inserts new_child at the beginning of the children sequence. """
        raise Exception("TODO Implement me !" )



    def insert_children(self, new_children):
        """ Takes a list of children and inserts them at the beginning of the curren
t children sequence,

            NOTE: in the new sequence new_children appear in the order they are pass
ed to the function!


            For example:
                >>> t = gt('a', gt('b'), gt('c))
                >>> print t
                a
                |-b
                \-c

                >>>  t.insert_children([gt('d'), gt('e')])
                >>> print t
                a
                |-d
                |-e
                |-b
                \-c
        """
        raise Exception("TODO Implement me !" )

    def insert_sibling(self, new_sibling):
        """ Inserts new_sibling as the immediate next sibling """
        raise Exception("TODO Implement me !" )

    def insert_siblings(self, new_siblings):
        """ Inserts new_siblings at the beginning of the siblings sequence,
            in the same order as they are passed.

            For example:

                >>> bt =  gt('b')
                >>> t = gt('a', bt , gt('c))
                >>> print t
                a
                |-b
                \-c

                >>>  bt.insert_children([gt('d'), gt('e')])
                >>> print t
                a
                |-b
```

```
                |-d
                |-e
                \-c
        """
        raise Exception("TODO Implement me !" )

    def has_child(self):
        """ Returns True if this node has a child, False otherwise """
        return self._child != None

    def detach_child(self):
        """ Detaches the first child.

            if there is no child, raises an Exception
        """
        raise Exception("TODO Implement me !" )


    def detach_sibling(self):
        """ Detaches the first sibling.

            If there is no sibling, raises an Exception
        """

        raise Exception("TODO Implement me !" )

    def detach(self, data):
        """ Detaches the first child that holds the provided data   """

        raise Exception("TODO Implement me !" )


def gt(*args):
    """ Shorthand function that returns a GenericTree containing the provided
        data and children. First parameter is the data, the following ones are the c
hildren.

        Usage examples:

        print gt('a')
        >>> a

        print gt('a', gt('b'), gt('c'))
        >>> a
            |-b
            \-c

    """
    if (len(args) == 0):
        raise Exception("You need to provide at least one argument for the data!")

    data = args[0]
    children = args[1:]

    r = GenericTree(data)
    for c in reversed(children):
        r.insert_child(c)
    return r

def str_trees(t1, t2, error_row=-1):
    """ Returns a string version of the two trees side by side

        If error_row is given, the line in error is marked.
        If error_row == -1 it is ignored
    """
```

```
    """

    s1 = str(t1)
    s2 = str(t2)

    lines1 = s1.split("\n")
    lines2 = s2.split("\n")

    max_len1 = 0
    for line in lines1:
        max_len1 = max(len(line.rstrip().decode("utf-8")), max_len1)

    max_len2 = 0
    for line in lines2:
        max_len2 = max(len(line.rstrip().decode("utf-8")), max_len2)

    strings = []

    i = 0

    while i < len(lines1) or i < len(lines2):

        if i < len(lines1):
            strings.append(lines1[i].rstrip())
            len1 = len(lines1[i].rstrip().decode("utf-8"))
        else:
            len1 = 0

        if (i < len(lines2)):
            len2 = len(lines2[i].rstrip().decode("utf-8"))

            pad_len1 = 4 + max_len1 - len1
            strings.append((" " * pad_len1) + lines2[i].rstrip())
        else:
            len2 = 0

        if (error_row == i):
            pad_len2 = 2 + max_len1 + max_len2 - len1 - len2
            strings.append((" " * pad_len2) + "<--- DIFFERENT ! ")

        strings.append("\n")

        i += 1

    return "".join(strings)


class GenericTreeTest(unittest.TestCase):

    def assertNotNone(self, ret, function_name):
        """ Asserts method result ret equals None """
        self.assertEquals(None, ret,
                          function_name
                          + " specs say nothing about returning objects! Instead you
 are returning " + str(ret))


    def assertTreeEqual(self, t1, t2):
        """ Asserts the trees t1 and t2 are equal """

        def rec_assert(c1, c2, row):

            if c1.data() != c2.data():
                raise Exception("data() is different!\n\n "
                                + str_trees(t1,t2,row))
            i = 0
```

```
            i = 0

            cs1 = c1.children()
            cs2 = c2.children()
            if (len(cs1) != len(cs2)):
                raise Exception("Children sizes are different !\n\n"
                                + str_trees(t1, t2, row + min(len(cs1), len(cs2))) )
            while (i < len(cs1) ):
                rec_assert(cs1[i], cs2[i], row + 1)
                i += 1

        rec_assert(t1, t2, 0)

    def assertRoot(self, t):
        """ Checks provided node t is a root, if not raises Exception """

        self.assertTrue(t.is_root(), "Detached node " + t.data() + " is not a root,
does it have still the _parent or _sibling set to something ?")

    def test_str_trees(self):
        self.assertTrue('a' in str_trees(gt('a'), gt('b')))
        self.assertTrue('b' in str_trees(gt('a'), gt('b')))

        self.assertTrue('a' in str_trees(gt('a', gt('b')), gt('b', gt('c'))))
        self.assertTrue('c' in str_trees(gt('a', gt('b')), gt('b', gt('c'))))


    def test_assert_trees_equal(self):
        self.assertTreeEqual(gt('a'), gt('a'))
        self.assertTreeEqual(gt('a', gt('b')), gt('a', gt('b')))

        with self.assertRaises(Exception):
            self.assertTreeEqual(gt('a'), gt('b'))
        with self.assertRaises(Exception):
            self.assertTreeEqual(gt('a', gt('b')), gt('a', gt('c')))

        # different structure
        with self.assertRaises(Exception):
            self.assertTreeEqual(gt('a', gt('b')), gt('a', gt('b',gt('c'))))

        with self.assertRaises(Exception):
            self.assertTreeEqual(gt('a', gt('b',gt('c'))), gt('a', gt('b')))


    def test_insert_child(self):
        ta = GenericTree('a')
        self.assertEqual(ta.child(), None)
        tb = GenericTree('b')
        ret = ta.insert_child(tb)
        self.assertEqual(ret, None, self.assertNotNone(ret, "insert_child"))
        self.assertEqual(ta.child(), tb)
        self.assertEqual(tb.parent(), ta)
        self.assertEqual(tb.sibling(), None)
        self.assertEqual(tb.child(), None)

        tc = GenericTree('c')
        ta.insert_child(tc)
        self.assertEqual(ta.child(), tc)
        self.assertEqual(tc.sibling(), tb)
        self.assertEqual(tc.parent(), ta)
        self.assertEqual(tb.sibling(), None)

    def test_insert_children(self):

        t = gt('a')
        t.insert_children([gt('d'), gt('e')])
```

```
        t.insert_children([gt('d'), gt('e')])
        self.assertTreeEqual(t, gt('a', gt('d'), gt('e')))
        t.insert_children([gt('b'), gt('c')])
        self.assertTreeEqual(t, gt('a', gt('b'), gt('c'), gt('d'), gt('e')))


    def test_detach_child(self):

        tb = gt('b')
        tc = gt('c')

        t = gt('a', tb, tc)

        ret = t.detach_child()
        self.assertNotNone(ret, "detach_child")

        self.assertTreeEqual(t, gt('a', gt('c')))
        self.assertRoot(tb)

        ret = t.detach_child()
        self.assertTreeEqual(t, gt('a'))
        self.assertRoot(tc)


        with self.assertRaises(Exception):
            ret = t.detach_child()

    def test_detach_one_node(self):
        t = gt('a')

        with self.assertRaises(Exception):
            t.detach('a')

        self.assertTreeEqual(t, gt('a'))

    def test_detach_two_nodes(self):
        b1 = gt('b')
        t = gt('a', b1)
        t.detach('b')
        self.assertRoot(b1)


    def test_detach_duplicates(self):
        b1 = gt('b')
        t = gt('a', b1, gt('b'))
        t.detach('b')
        self.assertTreeEqual(t, gt('a', gt('b')))
        self.assertRoot(b1)
```

GenericTree Solution

In [5]:

```
import unittest

class GenericTree:
    """ A tree in which each node can have any number of children.

        Each node is linked to its parent and to its immediate sibling on the right
    """

    def __init__(self, data):
```

```
        self._data = data

        self._child = None
        self._sibling = None
        self._parent = None

    def data(self):
        return self._data

    def child(self):
        return self._child

    def sibling(self):
        return self._sibling

    def parent(self):
        return self._parent

    def is_root(self):
        """ Return True if the node is a root of a tree, False otherwise

            A node is a root whenever it has no parent nor siblings.
        """
        return self._parent == None and self._sibling == None

    def is_subtree(self):
        """ Returns True if the node is a subtree of another tree

            A subtree always has a parent
        """
        return self._parent != None


    def children(self):
        """ Returns the children as a Python list """

        ret = []
        current = self._child
        while current != None:
            ret.append(current)
            current = current._sibling
        return ret

    def __str__(self):
        """ Returns a pretty string of the tree """

        def str_branches(node, branches):
            """ Returns a string with the tree pretty printed.

                branches: a list of characters representing the parent branches. Cha
racters can be either ` ` or '|'
            """
            strings = [str(node._data)]
            current = node._child
            while (current != None):
                if current._sibling == None:
                    # there are better end characters but let's not upset
                    # stupid Python with unicode problems
                    joint = '\-'
                else:
                    joint = '|-'


                strings.append('\n')
                for b in branches:
                    strings.append(b)
```

```
                    strings.append(b)
                strings.append(joint)
                if current._sibling == None:
                    branches.append('  ')
                else:
                    branches.append('| ')
                strings.append(str_branches(current, branches))
                branches.pop()
                current = current._sibling

            return "".join(strings)

        return str_branches(self, [])


    def insert_child(self, new_child):
        """ Inserts new_child at the beginning of the children sequence. """

        new_child._sibling = self._child
        new_child._parent = self
        self._child = new_child


    def insert_children(self, new_children):
        """ Takes a list of children and inserts them at the beginning of the curren
t children sequence,

            NOTE: in the new sequence new_children appear in the order they are pass
ed to the function!


            For example:
                >>> t = gt('a', gt('b'), gt('c))
                >>> print t
                a
                |-b
                \-c

                >>>  t.insert_children([gt('d'), gt('e')])
                >>> print t
                a
                |-d
                |-e
                |-b
                \-c
        """
        for c in reversed(new_children):
            self.insert_child(c)

    def insert_sibling(self, new_sibling):
        """ Inserts new_sibling as the immediate next sibling """
        new_sibling.parent = self.parent
        new_sibling._sibling = self._sibling
        self._sibling = new_sibling

    def insert_siblings(self, new_siblings):
        """ Inserts new_siblings at the beginning of the siblings sequence,
            in the same order as they are passed.

            For example:

                >>> bt =  gt('b')
                >>> t = gt('a', bt , gt('c))
                >>> print t
                a
                |-b
```

```
            |-b
            |-d
            \-c

            >>>  bt.insert_children([gt('d'), gt('e')])
            >>> print t
            a
            |-b
            |-d
            |-e
            \-c
        """
        for s in reversed(new_siblings):
            self.insert_sibling(s)

    def has_child(self):
        """ Returns True if this node has a child, False otherwise """

        return self._child != None

    def detach_child(self):
        """ Detaches the first child.

            if there is no child, raises an Exception
        """

        if (self._child == None):
            raise Exception("There is no child !")
        else:

            detached = self._child
            self._child = self._child._sibling
            detached._parent = None
            detached._sibling = None


    def detach_sibling(self):
        """ Detaches the first sibling.

            If there is no sibling, raises an Exception
        """

        if (self._sibling == None):
            raise Exception("There is no sibling !")
        else:
            self._sibling._parent = None
            self._sibling = self._sibling._sibling

    def detach(self, data):
        """ Detaches the first child that holds the provided data   """

        if (self._child != None):
            current = self._child
            prev = None
            while current != None:
                if (current._data == data):
                    if prev == None: # first element list
                        self.detach_child()
                    else:
                        current._parent = None
                        current._sibling = None
                        prev._sibling = current._sibling
                    return
                else:
                    prev = current
                    current = current._sibling
        raise Exception("Couldn't find any children holding this data:" + str(data))
```

```
        raise Exception("Couldn't find any children holding this data:" + str(data))


def gt(*args):
    """ Shorthand function that returns a GenericTree containing the provided
        data and children. First parameter is the data, the following ones are the c
hildren.

        Usage examples:

        print gt('a')
        >>> a

        print gt('a', gt('b'), gt('c'))
        >>> a
            |-b
            \-c

    """
    if (len(args) == 0):
        raise Exception("You need to provide at least one argument for the data!")

    data = args[0]
    children = args[1:]

    r = GenericTree(data)
    for c in reversed(children):
        r.insert_child(c)
    return r

def str_trees(t1, t2, error_row=-1):
    """ Returns a string version of the two trees side by side

        If error_row is given, the line in error is marked.
        If error_row == -1 it is ignored
    """

    s1 = str(t1)
    s2 = str(t2)

    lines1 = s1.split("\n")
    lines2 = s2.split("\n")

    max_len1 = 0
    for line in lines1:
        max_len1 = max(len(line.rstrip().decode("utf-8")), max_len1)

    max_len2 = 0
    for line in lines2:
        max_len2 = max(len(line.rstrip().decode("utf-8")), max_len2)

    strings = []

    i = 0

    while i < len(lines1) or i < len(lines2):

        if i < len(lines1):
            strings.append(lines1[i].rstrip())
            len1 = len(lines1[i].rstrip().decode("utf-8"))
        else:
            len1 = 0

        if (i < len(lines2)):
            len2 = len(lines2[i].rstrip().decode("utf-8"))
```

```
            pad_len1 = 4 + max_len1 - len1
            strings.append((" " * pad_len1) + lines2[i].rstrip())
        else:
            len2 = 0

        if (error_row == i):
            pad_len2 = 2 + max_len1 + max_len2 - len1 - len2
            strings.append((" " * pad_len2) + "<--- DIFFERENT ! ")

        strings.append("\n")

        i += 1

    return "".join(strings)


class GenericTreeTest(unittest.TestCase):

    def assertNotNone(self, ret, function_name):
        """ Asserts method result ret equals None """
        self.assertEquals(None, ret,
                          function_name
                          + " specs say nothing about returning objects! Instead you
 are returning " + str(ret))


    def assertTreeEqual(self, t1, t2):
        """ Asserts the trees t1 and t2 are equal """

        def rec_assert(c1, c2, row):

            if c1.data() != c2.data():
                raise Exception("data() is different!\n\n "
                                + str_trees(t1,t2,row))
            i = 0

            cs1 = c1.children()
            cs2 = c2.children()
            if (len(cs1) != len(cs2)):
                raise Exception("Children sizes are different !\n\n"
                                + str_trees(t1, t2, row + min(len(cs1), len(cs2))) )
            while (i < len(cs1) ):
                rec_assert(cs1[i], cs2[i], row + 1)
                i += 1

        rec_assert(t1, t2, 0)

    def assertRoot(self, t):
        """ Checks provided node t is a root, if not raises Exception """

        self.assertTrue(t.is_root(), "Detached node " + t.data() + " is not a root,
does it have still the _parent or _sibling set to something ?")

    def test_str_trees(self):
        self.assertTrue('a' in str_trees(gt('a'), gt('b')))
        self.assertTrue('b' in str_trees(gt('a'), gt('b')))

        self.assertTrue('a' in str_trees(gt('a', gt('b')), gt('b', gt('c'))))
        self.assertTrue('c' in str_trees(gt('a', gt('b')), gt('b', gt('c'))))


    def test_assert_trees_equal(self):
        self.assertTreeEqual(gt('a'), gt('a'))
        self.assertTreeEqual(gt('a', gt('b')), gt('a', gt('b')))
```

```
        with self.assertRaises(Exception):
            self.assertTreeEqual(gt('a'), gt('b'))
        with self.assertRaises(Exception):
            self.assertTreeEqual(gt('a', gt('b')), gt('a', gt('c')))

        # different structure
        with self.assertRaises(Exception):
            self.assertTreeEqual(gt('a', gt('b')), gt('a', gt('b',gt('c'))))

        with self.assertRaises(Exception):
            self.assertTreeEqual(gt('a', gt('b',gt('c'))), gt('a', gt('b')))


    def test_insert_child(self):
        ta = GenericTree('a')
        self.assertEqual(ta.child(), None)
        tb = GenericTree('b')
        ret = ta.insert_child(tb)
        self.assertEqual(ret, None, self.assertNotNone(ret, "insert_child"))
        self.assertEqual(ta.child(), tb)
        self.assertEqual(tb.parent(), ta)
        self.assertEqual(tb.sibling(), None)
        self.assertEqual(tb.child(), None)

        tc = GenericTree('c')
        ta.insert_child(tc)
        self.assertEqual(ta.child(), tc)
        self.assertEqual(tc.sibling(), tb)
        self.assertEqual(tc.parent(), ta)
        self.assertEqual(tb.sibling(), None)

    def test_insert_children(self):

        t = gt('a')
        t.insert_children([gt('d'), gt('e')])
        self.assertTreeEqual(t, gt('a', gt('d'), gt('e')))
        t.insert_children([gt('b'), gt('c')])
        self.assertTreeEqual(t, gt('a', gt('b'), gt('c'), gt('d'), gt('e')))


    def test_detach_child(self):

        tb = gt('b')
        tc = gt('c')

        t = gt('a', tb, tc)

        ret = t.detach_child()
        self.assertNotNone(ret, "detach_child")

        self.assertTreeEqual(t, gt('a', gt('c')))
        self.assertRoot(tb)

        ret = t.detach_child()
        self.assertTreeEqual(t, gt('a'))
        self.assertRoot(tc)


        with self.assertRaises(Exception):
            ret = t.detach_child()

    def test_detach_one_node(self):
        t = gt('a')

        with self.assertRaises(Exception):
```

```
        with self.assertRaises(Exception):
            t.detach('a')

        self.assertTreeEqual(t, gt('a'))

    def test_detach_two_nodes(self):
        b1 = gt('b')
        t = gt('a', b1)
        t.detach('b')
        self.assertRoot(b1)


    def test_detach_duplicates(self):
        b1 = gt('b')
        t = gt('a', b1, gt('b'))
        t.detach('b')
        self.assertTreeEqual(t, gt('a', gt('b')))
        self.assertRoot(b1)
```

In [6]:

```
algolab.run(GenericTreeTest)
```

```
........
----------------------------------------------------------------------
Ran 8 tests in 0.007s

OK
```

In [7]: