

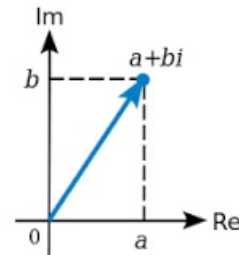
Chapter 3: Data Structures

class ComplexNumber

See theory here: <http://disi.unitn.it/~montreso/sp/slides/04-struttire.pdf> (<http://disi.unitn.it/~montreso/sp/slides/04-struttire.pdf>) (First slides until class Fraction)

Let's try to define a complex number:

A **complex number** is a **number** that can be expressed in the form $a + bi$, where a and b are real **numbers** and i is the imaginary unit which satisfies the equation $i^2 = -1$. In this expression, a is the real part and b is the imaginary part of the **complex number**.



Complex number - Wikipedia
https://en.wikipedia.org/wiki/Complex_number

As the Fraction class, the ComplexNumber holds two values, in this case one for the *real* part and one for the *imaginary* one.

- Note each method takes as first import `self` argument. `self` will always be a reference to the object itself, and allows accessing its fields and methods
- `self` is not a keyword of Python, you could use any name you want for the first parameter, but it is much better to follow conventions and stick using `self` !
- Methods beginning and ending with double underscore `'__'` have often special meaning in Python: if you see such a method around, it means it is overriding some default behaviour of Python

In [3]:

```
import unittest
import math

class ComplexNumber:

    def __init__(self, real, imaginary):
        self.real = real
        self.imaginary = imaginary

    def phase(self):
        """ Returns a float which is the phase (that is, the vector angle) of the complex number

        This method is something we introduce by ourselves, according to the definition:
        https://en.wikipedia.org/wiki/Complex_number#Absolute_value_and_argument
        """
        return math.atan2(self.imaginary, self.real)

    def log(self, base):
        """ Returns another ComplexNumber which is the logarithm of this complex number

        This method is something we introduce by ourselves, according to the definition:
```

(accomodated for generic base b)

https://en.wikipedia.org/wiki/Complex_number#Natural_logarithm

```
"""  
    return ComplexNumber(math.log(self.real) / math.log(base), self.phase() / ma  
th.log(base))
```

```
def __str__(self):  
    return str(self.real) + " + " + str(self.imaginary) + "i"
```

```
class ComplexNumberTest(unittest.TestCase):
```

```
    """ Test cases for ComplexNumber
```

```
        Note this is a *completely* separated class from ComplexNumber and  
        we declare it here just for testing purposes!  
        The 'self' you see here have nothing to do with the selfs from the  
        ComplexNumber methods!
```

```
    """
```

```
def test_init(self):  
    self.assertEqual(ComplexNumber(1,2).real, 1)  
    self.assertEqual(ComplexNumber(1,2).imaginary, 2)
```

```
def test_phase(self):  
    """  
        NOTE: we can't use assertEquals, as the result of phase() is a  
        float number which may have floating point rounding errors. So it's  
        necessary to use assertAlmostEqual  
        As an option with the delta you can declare the precision you require.  
        For more info see Python docs:  
        https://docs.python.org/2/library/unittest.html#unittest.TestCase.assert  
AlmostEqual
```

```
        NOTE: assertEquals might still work on your machine but just DO NOT use i  
t  
        for float numbers!!!
```

```
    """
```

```
    self.assertAlmostEqual(ComplexNumber(0.0,1.0).phase(), math.pi / 2, delta=0.  
001)
```

```
def test_str(self):  
    self.assertEqual(str(ComplexNumber(1,2)), "1 + 2i")  
    #self.assertEqual(str(ComplexNumber(1,0)), "1")  
    #self.assertEqual(str(ComplexNumber(1.0,0)), "1.0")  
    #self.assertEqual(str(ComplexNumber(0,1)), "i")  
    #self.assertEqual(str(ComplexNumber(0,0)), "0")
```

```
def test_log(self):  
    c = ComplexNumber(1.0,1.0)  
    l = c.log(math.e)  
    self.assertAlmostEqual(l.real, 0.0, delta=0.001)  
    self.assertAlmostEqual(l.imaginary, c.phase(), delta=0.001)
```

In [4]:

```
algolab.run(ComplexNumberTest)
```

```
.....  
-----  
Ran 4 tests in 0.009s
```

OK

Once the `__init__` method is defined, we can create a `ComplexNumber` with a call like `'ComplexNumber(3,5)'`

Notice in the constructor call we *do not* pass anything as `self` parameter (after all, we are creating the object)

In [5]:

```
my_complex = ComplexNumber(3,5)
```

We can now try to use one of the methods we defined:

In [6]:

```
phase = my_complex.phase()  
print phase
```

```
1.03037682652
```

We can also pretty print the whole complex number. Internally, `print` function will look if the `ComplexNumber` has defined an `__str__` method. If so, it will pass to the method the instance `my_complex` as the first argument, which in our methods will end up in the `self` parameter:

In [7]:

```
print my_complex
```

```
3 + 5i
```

We can also call methods that require a parameter like `log(base)`. Notice that `log` function returns a `ComplexNumber`, and Python will automatically pretty print it for us.

In [8]:

```
logarithm = my_complex.log(math.e)  
print logarithm
```

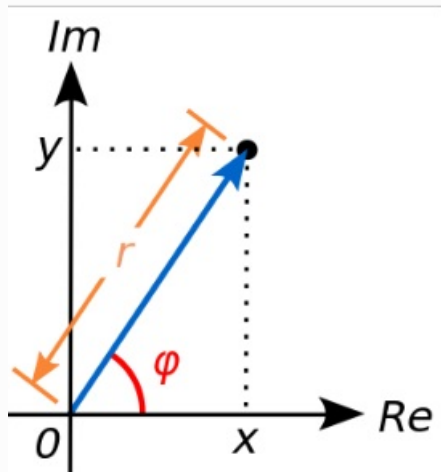
```
1.09861228867 + 1.03037682652i
```

Ok, now we are ready to define our own stuff.

Complex numbers magnitude

The *absolute value* (or *modulus* or *magnitude*) of a complex number $z = x + yi$ is

$$r = |z| = \sqrt{x^2 + y^2}.$$



Implement the magnitude method, using this signature:

```
def magnitude(self):  
    """ Returns a float which is the magnitude (that is, the absolute value) of the complex number  
  
        This method is something we introduce by ourselves, according to the definition:  
        https://en.wikipedia.org/wiki/Complex\_number#Absolute\_value\_and\_argument  
    """  
    raise Exception("TODO implement me!")
```

To test it, add this test case to ComplexNumberTest class (notice the almost in assertAlmostEquals !!!):

```
def test_magnitude(self):  
    self.assertAlmostEqual(ComplexNumber(3.0,4.0).magnitude(),5, delta=0.001)
```

Complex numbers equality

Here we will try to give you a glimpse of some aspects related to Python equality, and trying to respect interfaces when overriding methods. Equality can be a nasty subject, here we will treat it in a simplified form.

Equality [\[edit\]](#)

Two complex numbers are equal **if and only if** both their real and imaginary parts are equal. In symbols:

$$z_1 = z_2 \leftrightarrow (\text{Re}(z_1) = \text{Re}(z_2) \wedge \text{Im}(z_1) = \text{Im}(z_2)).$$

- Implement equality for `ComplexNumber` more or less as it was done for `Fraction`

Use this method signature:

```
def __eq__(self, other):
```

and use this simple test case to check for equality:

```
def test_integer_equality(self):
    """
        Note all other tests depend on this test !

        We want also to test the constructor, so in c we set stuff by han
d
    """
    c = ComplexNumber(0,0)
    c.real = 1
    c.imaginary = 2
    self.assertEqual(c, ComplexNumber(1,2))
```

- Beware 'equality' is tricky in Python for float numbers! Rule of thumb: when overriding `__eq__`, use 'dumb' equality, two things are the same only if their parts are literally equal
- If instead you need to determine if two objects are similar, define other 'closeness' functions.
- (Non mandatory read) if you are interested in the gory details of equality, see
 - [How to Override comparison operators in Python \(http://jcalderone.livejournal.com/32837.html\)](http://jcalderone.livejournal.com/32837.html)
 - [Messing with hashing \(http://www.asmeurer.com/blog/posts/what-happens-when-you-mess-with-hashing-in-python/\)](http://www.asmeurer.com/blog/posts/what-happens-when-you-mess-with-hashing-in-python/)

Complex numbers isclose

Complex numbers can be represented as vectors, so intuitively we can determine if a complex number is close to another by checking that the distance between its vector tip and the the other tip is less than a given delta. There are more precise ways to calculate it, but here we prefer keeping the example simple.

Given two complex numbers

$$z_1 = a + bi$$

and

$$z_2 = c + di$$

We can consider them as close if they satisfy this condition:

$$\sqrt{(a-c)^2 + (b-d)^2} < \delta$$

- Implement the method, adding it to ComplexNumber class:

```
def isclose(self, c, delta):  
    """ Returns True if the complex number is within a delta distance from  
    complex number c.  
    """  
    raise Exception("TODO Implement me!")
```

and add this test case to ComplexNumberTest class:

```
def test_isclose(self):  
    """ Notice we use `assertTrue` because we expect `isclose` to return  
    a `bool` value, and  
    we also test a case where we expect `False`  
    """  
    self.assertTrue(ComplexNumber(1.0,1.0).isclose(ComplexNumber(1.0,1.1)  
    , 0.2))  
    self.assertFalse(ComplexNumber(1.0,1.0).isclose(ComplexNumber(10.0,10  
    .0), 0.2))
```

REMEMBER: Equality with `__eq__` and closeness functions like `isclose` are very different things. Equality should check if two objects have the same memory address or, alternatively, if they contain the same things, while closeness functions should check if two objects are similar. You should never use functions like `isclose` inside `__eq__` methods.

Complex numbers addition

Complex numbers are **added** by separately adding the real and imaginary parts of the summands.

That is to say:

$$(a + bi) + (c + di) = (a + c) + (b + d)i.$$

Similarly, **subtraction** is defined by

$$(a + bi) - (c + di) = (a - c) + (b - d)i.$$

- a and c correspond to real, b and d correspond to imaginary
- implement addition for ComplexNumber more or less as it was done for Fraction in theory slides
- write some tests as well!

Use this definition:

```
def __add__(self, other):  
    raise Exception("TODO implement me!")
```

And add this to the ComplexNumberTest class:

```
def test_add_zero(self):  
    self.assertEqual(ComplexNumber(1,2) + ComplexNumber(0,0), ComplexNum  
ber(1,2));  
  
    def test_add_numbers(self):  
        self.assertEqual(ComplexNumber(1,2) + ComplexNumber(3,4), ComplexNum  
ber(4,6));
```

Adding a scalar

We defined addition among ComplexNumbers, but what about addition among a ComplexNumber and an int or a float?

Will this work?

```
ComplexNumber(3,4) + 5
```

What about this?

```
ComplexNumber(3,4) + 5.0
```

Try to add the following method to your class, and check if it does work with the scalar:

In [10]:

```
def __add__(self, other):  
    # checks other object is instance of the class ComplexNumber  
    if isinstance(other, ComplexNumber):  
        return ComplexNumber(self.real + other.real, self.imaginary + other.imagi  
nary)  
  
    # else checks the basic type of other is int or float  
    elif type(other) is int or type(other) is float:  
        return ComplexNumber(self.real + other, self.imaginary)  
  
    # other is of some type we don't know how to process.  
    # In this case the Python specs say we MUST return 'NotImplemented'  
    else:  
        return NotImplemented
```

Hopefully now you have a better add. But what about this? Will this work?

```
5 + ComplexNumber(3,4)
```

Answer: it won't, Python needs further instructions. Usually Python tries to see if the class of the object on left of the expression defines addition for operands *to the right* of it. In this case on the left we have a float number, and float numbers don't define any way to deal to the right with your very own ComplexNumber class. So as a last resort Python tries to see if your ComplexNumber class has defined also a way to deal with operands *to the left* of the ComplexNumber, by looking for the method `__radd__`, which means *reverse addition*. Here we implement it :

```
def __radd__(self, other):
    """ Returns the result of expressions like    other + self    """
    if (type(other) is int or type(other) is float):
        return ComplexNumber(self.real + other, self.imaginary)
    else:
        return NotImplemented
```

To check it is working and everything is in order for addition, add these test cases:

```
def test_add_zero(self):
    self.assertEqual(ComplexNumber(1,2) + ComplexNumber(0,0), ComplexNumber(1,2));

    def test_add_numbers(self):
        self.assertEqual(ComplexNumber(1,2) + ComplexNumber(3,4), ComplexNumber(4,6));

    def test_add_scalar_right(self):
        self.assertEqual(ComplexNumber(1,2) + 3, ComplexNumber(4,2));

    def test_add_scalar_left(self):
        self.assertEqual(3 + ComplexNumber(1,2), ComplexNumber(4,2));

    def test_add_negative(self):
        self.assertEqual(ComplexNumber(-1,0) + ComplexNumber(0,-1), ComplexNumber(-1,-1));
```


Complex numbers multiplication

Multiplication and division [\[edit\]](#)

The multiplication of two complex numbers is defined by the following formula:

$$(a + bi)(c + di) = (ac - bd) + (bc + ad)i.$$

In particular, the [square](#) of the imaginary unit is -1 :

$$i^2 = i \times i = -1.$$

- Implement multiplication for ComplexNumber, using inspiration from previous `__add__` implementation
- Can you extend multiplication to work with scalars (both left and right) as well?

To implement `__mul__`, copy this definition into ComplexNumber class:

```
def __mul__(self, other):  
    raise Exception("TODO Implement me!")
```

and add test cases to ComplexNumberTest class:

```
def test_mul_by_zero(self):  
    self.assertEqual(ComplexNumber(0,0) * ComplexNumber(1,2), ComplexNumber(0,0));  
  
    def test_mul_just_real(self):  
        self.assertEqual(ComplexNumber(1,0) * ComplexNumber(2,0), ComplexNumber(2,0));  
  
    def test_mul_just_imaginary(self):  
        self.assertEqual(ComplexNumber(0,1) * ComplexNumber(0,2), ComplexNumber(-2,0));  
  
    def test_mul_scalar_right(self):  
        self.assertEqual(ComplexNumber(1,2) * 3, ComplexNumber(3,6));  
  
    def test_mul_scalar_left(self):  
        self.assertEqual(3 * ComplexNumber(1,2), ComplexNumber(3,6));
```

Stack

Stack theory

See theory here: <http://disi.unitn.it/~montreso/sp/slides/04-struttura.pdf> (<http://disi.unitn.it/~montreso/sp/slides/04-struttura.pdf>) (Slide 46)

See [stack definition on the book](http://interactivepython.org/runestone/static/pythonds/BasicDS/WhatisaStack.html) (<http://interactivepython.org/runestone/static/pythonds/BasicDS/WhatisaStack.html>)

and following sections :

- [Stack Abstract Data Type](http://interactivepython.org/runestone/static/pythonds/BasicDS/TheStackAbstractDataType.html) (<http://interactivepython.org/runestone/static/pythonds/BasicDS/TheStackAbstractDataType.html>)
- [Implementing a Stack in Python](http://interactivepython.org/runestone/static/pythonds/BasicDS/ImplementingaStackinPython.html) (<http://interactivepython.org/runestone/static/pythonds/BasicDS/ImplementingaStackinPython.html>)
- [Simple Balanced Parenthesis](http://interactivepython.org/runestone/static/pythonds/BasicDS/SimpleBalancedParentheses.html) (<http://interactivepython.org/runestone/static/pythonds/BasicDS/SimpleBalancedParentheses.html>)
- [Balanced Symbols - a General Case](http://interactivepython.org/runestone/static/pythonds/BasicDS/BalancedSymbols(AGeneralCase).html) ([http://interactivepython.org/runestone/static/pythonds/BasicDS/BalancedSymbols\(AGeneralCase\).html](http://interactivepython.org/runestone/static/pythonds/BasicDS/BalancedSymbols(AGeneralCase).html))

Stack exercises

On slide 46 of [theory](http://disi.unitn.it/~montreso/sp/slides/04-struttura.pdf) (<http://disi.unitn.it/~montreso/sp/slides/04-struttura.pdf> (Slide 46)) there is the pseudo code for a version of stack we will call CappedStack:

- Implement the pseudo code using the following skeleton and unit tests
- Name internal variables that you don't want to expose to class users by prepending them with one underscore ' _ ', like `_elements` or `_cap`.
 - The underscore is just a convention, class users will still be able to get internal variables by accessing them with field accessors like `mystack._elements`
 - If users manipulate private fields and complain something is not working, you can tell them it's their fault!
- This time, we will try to write a little more robust code. In general, when implementing pseudocode you might need to think more about boundary cases. In this case, we add the additional constraint that if you pass to the stack a negative or zero cap, your class initialization is expected to fail and raise an `Exception`

Notice that:

- In this case, when this stack reaches cap size, successive push requests silently exit without raising errors. Other implementations might raise an error and stop execution when trying to push over on already filled stack.
- In this case, when this stack is required to pop or peek, if it is empty the functions will not return anything. During the Python translation, we might not return anything as well and relying on Python implicitly returning `None`.
- `pop` will both modify the stack *and* return a value

In [11]:

```
import unittest

class CappedStack:

    def __init__(self, cap):
        """ Creates a CappedStack capped at cap. Cap must be > 0, otherwise an AssertionError is thrown """
        raise Exception("TODO Implement me!")

    def size(self):
        raise Exception("TODO Implement me!")

    def isEmpty(self):
```

```

def is_empty(self):
    raise Exception("TODO Implement me!")

def pop(self):
    raise Exception("TODO Implement me!")

def peek(self):
    raise Exception("TODO Implement me!")

def push(self, item):
    raise Exception("TODO Implement me!")

def cap(self):
    """ Returns the cap of the stack
    """
    raise Exception("TODO Implement me!")

class CappedStackTest(unittest.TestCase):

    """ Test cases for CappedStackTest

        Note this is a completely separated class from CappedStack and
        we declare it here just for testing purposes!
        The 'self' you see here have nothing to do with the selfs from the
        CappedStack methods!
    """

    def test_init_wrong_cap(self):
        with self.assertRaises(AssertionError):
            CappedStack(0)
        with self.assertRaises(AssertionError):
            CappedStack(-1)

    def test_cap(self):
        self.assertEqual(CappedStack(1).cap(), 1)
        self.assertEqual(CappedStack(2).cap(), 2)

    def test_size(self):
        s = CappedStack(5)
        self.assertEqual(s.size(), 0)
        s.push("a")
        self.assertEqual(s.size(), 1)
        s.pop()
        self.assertEqual(s.size(), 0)

    def test_isEmpty(self):
        s = CappedStack(5)
        self.assertTrue(s.isEmpty())
        s.push("a")
        self.assertFalse(s.isEmpty())

    def test_pop(self):
        s = CappedStack(5)
        self.assertEqual(s.pop(), None)
        s.push("a")
        self.assertEqual(s.pop(), "a")
        self.assertEqual(s.pop(), None)

    def test_peek(self):
        s = CappedStack(5)
        self.assertEqual(s.peek(), None)
        s.push("a")

```

```
s.push("a")
self.assertEqual(s.peek(), "a")
self.assertEqual(s.peek(), "a") # testing peek is not changing the stack
self.assertEqual(s.size(), 1)
```

```
def test_push(self):
    s = CappedStack(2)
    self.assertEqual(s.size(), 0)
    s.push("a")
    self.assertEqual(s.size(), 1)
    s.push("b")
    self.assertEqual(s.size(), 2)
    self.assertEqual(s.peek(), "b")
    s.push("c") # capped, pushing should do nothing now!
    self.assertEqual(s.size(), 2)
    self.assertEqual(s.peek(), "b")
```

Solutions

ComplexNumber Solution

In [12]:

```
import unittest
import math

class ComplexNumber:

    def __init__(self, real, imaginary):
        self.real = real
        self.imaginary = imaginary

    def __str__(self):
        return str(self.real) + " + " + str(self.imaginary) + "i"

    def phase(self):
        """ Returns a float which is the phase (that is, the vector angle) of the complex number

        This method is something we introduce by ourselves, according to the definition:
        https://en.wikipedia.org/wiki/Complex_number#Absolute_value_and_argument
        """
        return math.atan2(self.imaginary, self.real)

    def log(self, base):
        """ Returns another ComplexNumber which is the logarithm of this complex number

        This method is something we introduce by ourselves, according to the definition:
        (accomodated for generic base b)
        https://en.wikipedia.org/wiki/Complex_number#Natural_logarithm
        """
        return ComplexNumber(math.log(self.real) / math.log(base), self.phase() / math.log(base))

    def magnitude(self):
        """ Returns a float which is the magnitude (that is, the absolute value) of the complex number

        This method is something we introduce by ourselves, according to the def
```

```

initiation:
    """ https://en.wikipedia.org/wiki/Complex\_number#Absolute\_value\_and\_argument
    """
    return math.sqrt(self.real**2 + self.imaginary**2)

def __eq__(self, other):
    return self.real == other.real and self.imaginary == other.imaginary

def isclose(self, c, delta):
    """ Returns True if the complex number is within a delta distance from compl
    ex number c.
    """
    return math.sqrt((self.real-c.real)**2 + (self.imaginary-c.imaginary)**2) <
delta

def __add__(self, other):
    if isinstance(other, ComplexNumber):
        return ComplexNumber(self.real + other.real, self.imaginary + other.imagi
nary)
    elif type(other) is int or type(other) is float:
        return ComplexNumber(self.real + other, self.imaginary)
    else:
        return NotImplemented

def __radd__(self, other):
    if (type(other) is int or type(other) is float):
        return ComplexNumber(self.real + other, self.imaginary)
    else:
        return NotImplemented

def __mul__(self, other):
    if isinstance(other, ComplexNumber):
        return ComplexNumber(self.real * other.real - self.imaginary * other.ima
ginary,
                             self.imaginary * other.real + self.real * other.ima
ginary)
    elif type(other) is int or type(other) is float:
        return ComplexNumber(self.real * other, self.imaginary * other)
    else:
        return NotImplemented

def __rmul__(self, other):
    if (type(other) is int or type(other) is float):
        return ComplexNumber(self.real * other, self.imaginary * other)
    else:
        return NotImplemented

class ComplexNumberTest(unittest.TestCase):
    """ Test cases for ComplexNumber

    Note this is a *completely* separated class from ComplexNumber and
    we declare it here just for testing purposes!
    The 'self' you see here have nothing to do with the selfs from the
    ComplexNumber methods!
    """

    def test_init(self):
        self.assertEqual(ComplexNumber(1,2).real, 1)
        self.assertEqual(ComplexNumber(1,2).imaginary, 2)

```

```

self.assertEqual(ComplexNumber(1,2).imaginary, 2)

def test_phase(self):
    """
        NOTE: we can't use assertEquals, as the result of phase() is a
        float number which may have floating point rounding errors. So it's
        necessary to use assertAlmostEqual
        As an option with the delta you can declare the precision you require.
        For more info see Python docs:
        https://docs.python.org/2/library/unittest.html#unittest.TestCase.assert
AlmostEqual

        NOTE: assertEquals might still work on your machine but just DO NOT use i
t
        for float numbers!!!
    """
    self.assertAlmostEqual(ComplexNumber(0.0,1.0).phase(), math.pi / 2, delta=0.
001)

def test_str(self):
    self.assertEqual(str(ComplexNumber(1,2)), "1 + 2i")
    #self.assertEqual(str(ComplexNumber(1,0)), "1")
    #self.assertEqual(str(ComplexNumber(1.0,0)), "1.0")
    #self.assertEqual(str(ComplexNumber(0,1)), "i")
    #self.assertEqual(str(ComplexNumber(0,0)), "0")

def test_log(self):
    c = ComplexNumber(1.0,1.0)
    l = c.log(math.e)
    self.assertAlmostEqual(l.real, 0.0, delta=0.001)
    self.assertAlmostEqual(l.imaginary, c.phase(), delta=0.001)

def test_magnitude(self):
    self.assertAlmostEqual(ComplexNumber(3.0,4.0).magnitude(),5, delta=0.001)

def test_integer_equality(self):
    """
        Note all other tests depend on this test !

        We want also to test the constructor, so in c we set stuff by hand
    """
    c = ComplexNumber(0,0)
    c.real = 1
    c.imaginary = 2
    self.assertEqual(c, ComplexNumber(1,2))

def test_isclose(self):
    """ Notice we use `assertTrue` because we expect `isclose` to return a `boo
l` value, and
        we also test a case where we expect `False`
    """
    self.assertTrue(ComplexNumber(1.0,1.0).isclose(ComplexNumber(1.0,1.1), 0.2))
    self.assertFalse(ComplexNumber(1.0,1.0).isclose(ComplexNumber(10.0,10.0), 0.
2))

def test_add_zero(self):
    self.assertEqual(ComplexNumber(1,2) + ComplexNumber(0,0), ComplexNumber(1,2
));

def test_add_numbers(self):
    self.assertEqual(ComplexNumber(1,2) + ComplexNumber(3,4), ComplexNumber(4,6
))

```

```

    ));

    def test_add_scalar_right(self):
        self.assertEqual(ComplexNumber(1,2) + 3, ComplexNumber(4,2));

    def test_add_scalar_left(self):
        self.assertEqual(3 + ComplexNumber(1,2), ComplexNumber(4,2));

    def test_add_negative(self):
        self.assertEqual(ComplexNumber(-1,0) + ComplexNumber(0,-1), ComplexNumber(-
1,-1));

    def test_mul_by_zero(self):
        self.assertEqual(ComplexNumber(0,0) * ComplexNumber(1,2), ComplexNumber(0,0
));

    def test_mul_just_real(self):
        self.assertEqual(ComplexNumber(1,0) * ComplexNumber(2,0), ComplexNumber(2,0
));

    def test_mul_just_imaginary(self):
        self.assertEqual(ComplexNumber(0,1) * ComplexNumber(0,2), ComplexNumber(-2,
0));

    def test_mul_scalar_right(self):
        self.assertEqual(ComplexNumber(1,2) * 3, ComplexNumber(3,6));

    def test_mul_scalar_left(self):
        self.assertEqual(3 * ComplexNumber(1,2), ComplexNumber(3,6));

```

In [13]:

```

algolab.run(ComplexNumberTest)

```

```

.....
-----
Ran 17 tests in 0.031s

OK

```

Stack Solution

In [14]:

```

import unittest

class CappedStack:

    def __init__(self, cap):
        """ Creates a CappedStack capped at cap.

        Cap must be > 0, otherwise an AssertionError is thrown
        """
        assert cap > 0
        # notice we assign to variables with underscore to respect Python convention
        self._cap = cap
        # notice with use _elements instead of the A in the pseudocode, because it i
        # clearer, starts with underscore, and capital letters are usual reserved
        # for classes or constants
        self._elements = []

```

```

self._elements = []

def size(self):
    return len(self._elements)

def isEmpty(self):
    return len(self._elements) == 0

def pop(self):
    if (len(self._elements) > 0):
        return self._elements.pop()
    # else: implicitly, Python will return None

def peek(self):
    if (len(self._elements) > 0):
        return self._elements[-1]
    # else: implicitly, Python will return None

def push(self, item):
    if (len(self._elements) < self._cap):
        self._elements.append(item)
    # else fail silently

def cap(self):
    """ Returns the cap of the stack
    """
    return self._cap

```

```

class CappedStackTest(unittest.TestCase):

```

```

    """ Test cases for CappedStackTest

    Note this is a *completely* separated class from CappedStack and
    we declare it here just for testing purposes!
    The 'self' you see here have nothing to do with the selfs from the
    CappedStack methods!
    """

```

```

def test_init_wrong_cap(self):
    with self.assertRaises(AssertionError):
        CappedStack(0)
    with self.assertRaises(AssertionError):
        CappedStack(-1)

def test_cap(self):
    self.assertEqual(CappedStack(1).cap(), 1)
    self.assertEqual(CappedStack(2).cap(), 2)

def test_size(self):
    s = CappedStack(5)
    self.assertEqual(s.size(), 0)
    s.push("a")
    self.assertEqual(s.size(), 1)
    s.pop()
    self.assertEqual(s.size(), 0)

def test_isEmpty(self):
    s = CappedStack(5)
    self.assertTrue(s.isEmpty())
    s.push("a")
    self.assertFalse(s.isEmpty())

```



```
def test_pop(self):
    s = CappedStack(5)
    self.assertEqual(s.pop(), None)
    s.push("a")
    self.assertEqual(s.pop(), "a")
    self.assertEqual(s.pop(), None)

def test_peek(self):
    s = CappedStack(5)
    self.assertEqual(s.peak(), None)
    s.push("a")
    self.assertEqual(s.peak(), "a")
    self.assertEqual(s.peak(), "a") # testing peek is not changing the stack
    self.assertEqual(s.size(), 1)

def test_push(self):
    s = CappedStack(2)
    self.assertEqual(s.size(), 0)
    s.push("a")
    self.assertEqual(s.size(), 1)
    s.push("b")
    self.assertEqual(s.size(), 2)
    self.assertEqual(s.peak(), "b")
    s.push("c") # capped, pushing should do nothing now!
    self.assertEqual(s.size(), 2)
    self.assertEqual(s.peak(), "b")
```

In [15]:

```
algolab.run(CappedStackTest)
```

.....

Ran 7 tests in 0.015s

OK

In [16]: