Final Project Of Advanced Dynamics

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PROBLEM DESCRIPTION

Consider a rigid body with moment of inertial $I = [7, 9, 12]kg - m^2$ moving in space. It's center of mass has an orbital speed of $v_G = 7.5km/s$ and tumbling at an angular velocity $\vec{\omega} = [\omega_1, \omega_2, \omega_3]rad/sec$.

(a) Let the initial $\vec{\omega} = [0, 3, 1], [1, 0, 3], and[3, 1, 0]$, and plot 3D trajectories of the angular momentum, $\vec{H} = I\vec{\omega}$, for each case on the sphere defined by body-fixed coordinates.
(b) Plot the rigid body as a rectangular box rotating along the orbit.

MATHEMATICS BACKGROUND

Rotation Matrix

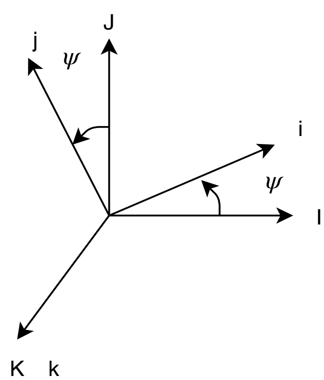

Fig. 1. ψ angle

According Fig1, we can find the following relationship between inertial frame(I, J, K) and body-fixed frame(i, j, k), and ψ is so-called the angle rotated about z-axis. As similar, we can define ϕ as well as θ to be the corresponding angles rotated about x-axis and y-axis as shown in Table I.

$$\begin{bmatrix} i \\ j \\ k \end{bmatrix} = \begin{bmatrix} cos\psi & sin\psi & 0 \\ -sin\psi & cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} I \\ J \\ K \end{bmatrix}$$

Rotate about x	Rotate about y	Rotate about z
$\begin{bmatrix} 1 & 0 & 0 \\ 0 & cos\phi & sin\phi \\ 0 & -sin\phi & cos\phi \end{bmatrix}$	$\begin{bmatrix} cos\theta & 0 & -sin\theta \\ 0 & 1 & 0 \\ sin\theta & 0 & cos\theta \end{bmatrix}$	$\begin{bmatrix} cos\psi & sin\psi & 0 \\ -sin\psi & cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}$
$R_1 \begin{bmatrix} \phi \end{bmatrix}$	$R_2 \begin{bmatrix} \theta \end{bmatrix}$	$R_3 \begin{bmatrix} \psi \end{bmatrix}$

TABLE I
ROTATION MATRIX

Angular Motion

For $\vec{\omega} = \omega_1 i + \omega_2 j + \omega_3 k$ is the instantaneous body angular rate, it cannot be integrated directly to obtain angular displacement. We need to convert $\vec{\omega}$ to Euler angle rate.
Here we take 3-2-1 rotation,

$$\vec{\omega} = \dot{\psi} z_I + \dot{\theta} y' + \dot{\phi} x_B \tag{1}$$

$$= \omega_1 x_B + \omega_2 y_B + \omega_3 z_B \tag{2}$$

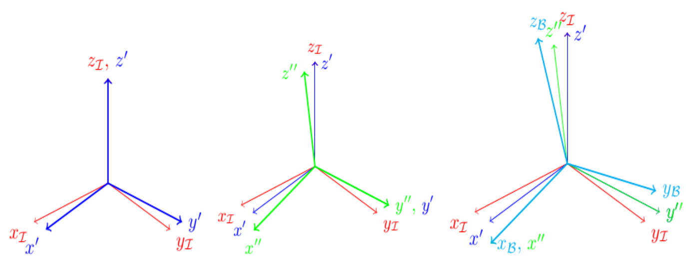

Fig. 2. Inertial to body frame
[1]

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = R_3 \begin{bmatrix} \psi \end{bmatrix} \begin{bmatrix} x_I \\ y_I \\ z_I \end{bmatrix}$$

$$\begin{bmatrix} x'' \\ y'' \\ z'' \end{bmatrix} = R_2 \begin{bmatrix} \theta \end{bmatrix} \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix}$$

$$\begin{bmatrix} x_B \\ y_B \\ z_B \end{bmatrix} = R_1 \begin{bmatrix} \phi \end{bmatrix} \begin{bmatrix} x'' \\ y'' \\ z'' \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = R_2^T \begin{bmatrix} \theta \end{bmatrix} R_1^T \begin{bmatrix} \phi \end{bmatrix} \begin{bmatrix} x_B \\ y_B \\ z_B \end{bmatrix} \tag{3}$$

$$\begin{bmatrix} x_I \\ y_I \\ z_I \end{bmatrix} = R_3^T \begin{bmatrix} \psi \end{bmatrix} R_2^T \begin{bmatrix} \theta \end{bmatrix} R_1^T \begin{bmatrix} \phi \end{bmatrix} \begin{bmatrix} x_B \\ y_B \\ z_B \end{bmatrix} \tag{4}$$

From equation 3&4, we can get the following equations.

$$y' = cos(\phi)y_B - sin(\phi)z_B \tag{5}$$

$$z_I = -sin(\theta)x_B + cos(\theta)sin(\phi)y_B + cos(\theta)cos(\phi)z_B \tag{6}$$

Substitute equation 5&6 into equation 1&2,

$$\begin{bmatrix} \dot{\psi} \\ \dot{\theta} \\ \dot{\phi} \end{bmatrix} = \frac{1}{cos\theta} \begin{bmatrix} 0 & sin\phi & cos\phi \\ 0 & cos\theta cos\phi & -cos\theta sin\phi \\ cos\theta & sin\theta sin\phi & sin\theta cos\phi \end{bmatrix} \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix} \tag{7}$$

Angular Momentum

From lecture notes, we know that the dynamic equation of angular velocity as following.

$$\begin{aligned} \dot{\omega_1} &= \Delta_{23}\omega_2\omega_3 \\ \dot{\omega_2} &= \Delta_{31}\omega_1\omega_3 \\ \dot{\omega_3} &= \Delta_{12}\omega_1\omega_2 \end{aligned} \tag{8}$$

where $\Delta_{23} = (I_2 - I_3)/I_1$; $\Delta_{31} = (I_3 - I_1)/I_2$; $\Delta_{12} = (I_1 - I_2)/I_3$, and I is the moment of inertial at body-fixed frame

SOLUTION

Sol (a) can be obtained by equation 8

- $\omega_0 = [0, 3, 1]$, $t = [0, 10]$

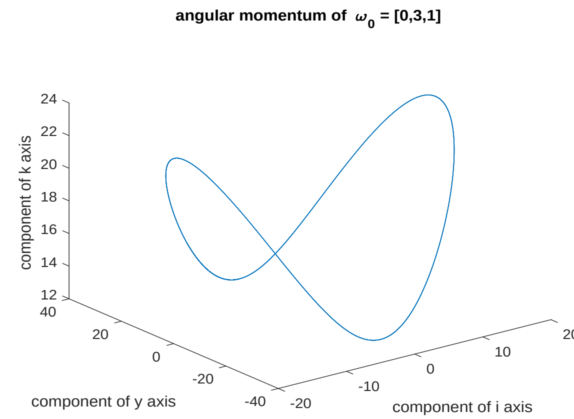

Fig. 3. angular momentum case 1

- $\omega_0 = [1, 0, 3]$, $t = [0, 10]$

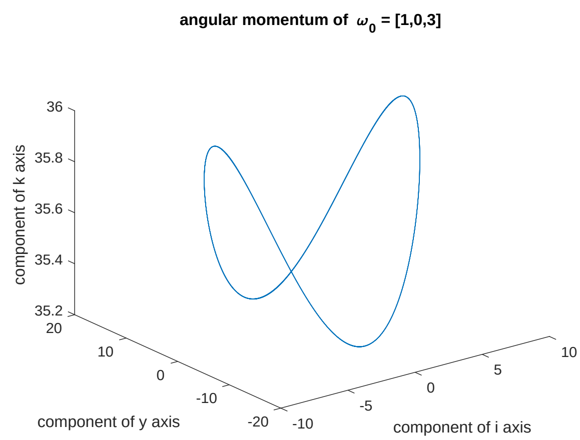

Fig. 4. angular momentum case 1

- $\omega_0 = [3,1,0]$, $t = [0,10]$

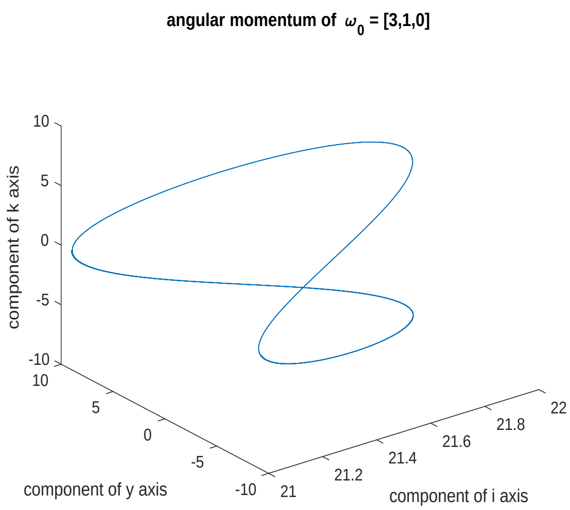

Fig. 5. angular momentum case 1

Sol (b) can be obtained by equation 7 with initial condition of $[\psi_0, \theta_0, \phi_0] = [0,0,0]$; $\omega_0 = [0,3,1]$ rad/sec

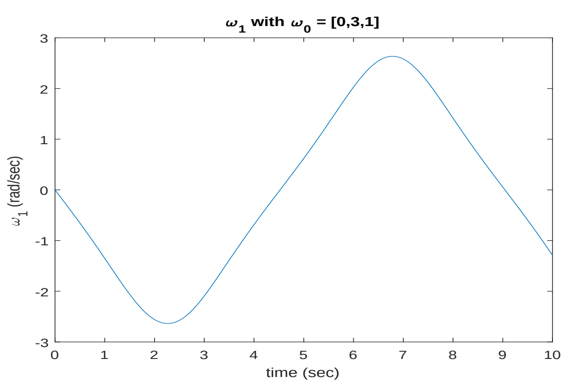

Fig. 6. x_B component of angular velocity

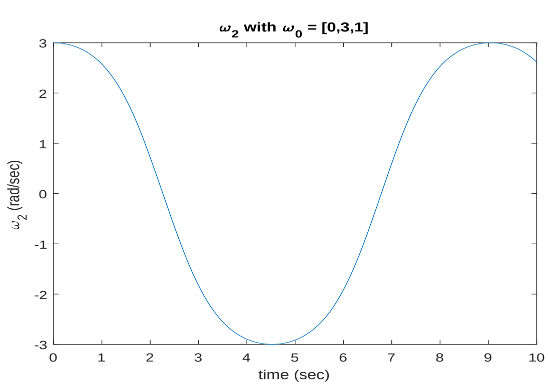

Fig. 7. y_B component of angular velocity

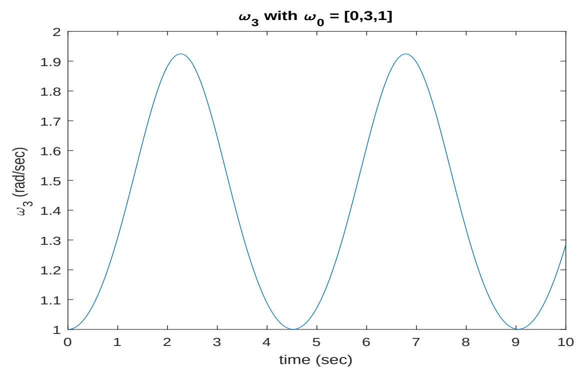

Fig. 8. z_B component of angular velocity

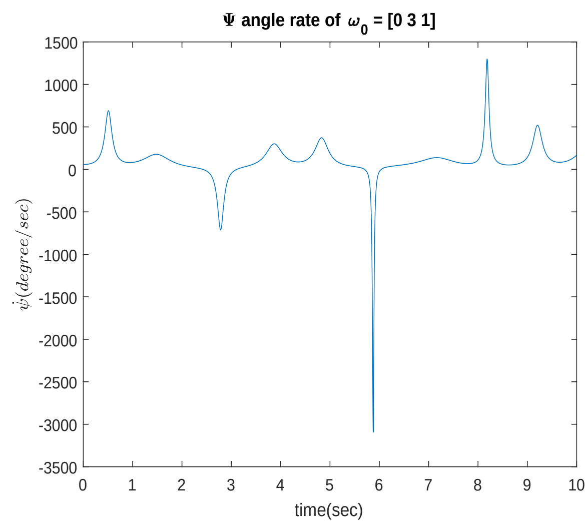

Fig. 9. z_I component of angular rate

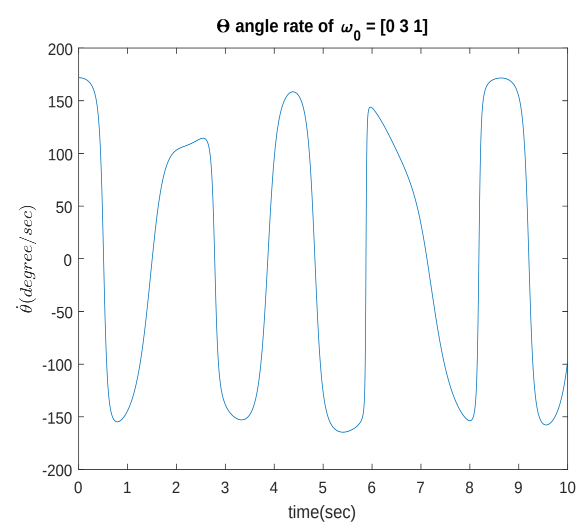

Fig. 10. y_I component of angular rate

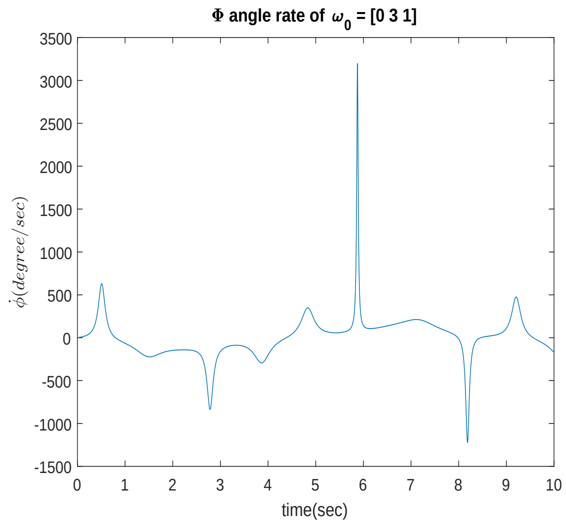

Fig. 11. x_B component of angular rate

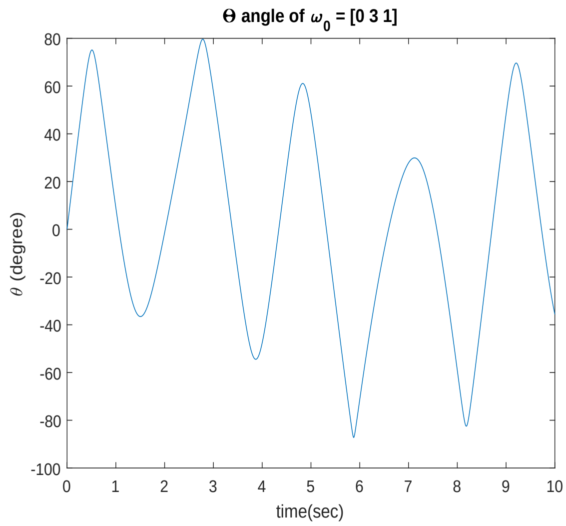

Fig. 13. θ euler angle

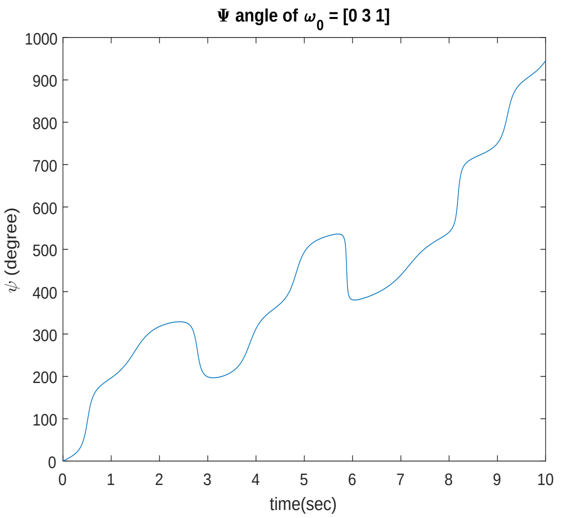

Fig. 12. ψ euler angle

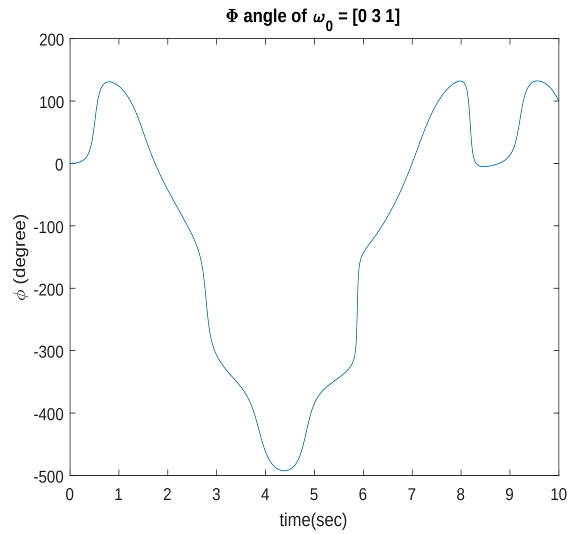

Fig. 14. ϕ euler angle

TRANSLATION

If the rigid body moves along the x-axis $7.5km/s$, which means that it would increase $7.5km$ per second along x axis. The following equation 9 is the translation matrix.

$$\begin{bmatrix} x\prime \\ y\prime \\ z\prime \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 7.5 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \tag{9}$$

CONCLUSION

With the above information, we know each euler angle varying with time. If we also figure out the path of $C.G$, then we can plot the animation of the rigid body. Animation is in the $advanced_dynamics_final.m$ attached in the folder.

APPENDIX

LIST OF FIGURES

LIST OF TABLES

REFERENCES

[1] *https://www.google.com.tw/search?q =euler+angle&source=lnms&tbm=isch&sa =X&ved=0ahUKEwiojeWg2-vbAhWJA4 gKHWZkAHAQ_AUICigB&biw=1309&bih=697#imgrc=_*