

**ELEC 4700**  
**Assignment - 1**  
**Monte-Carlo Modeling of Electron Transport**

**Due: Sunday, Feb. 4, 2018 23:59**

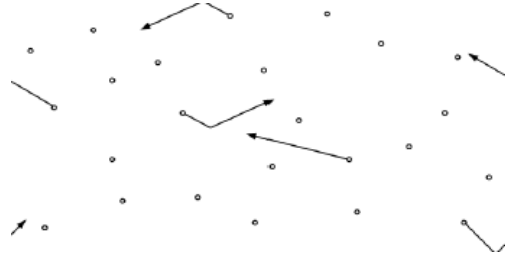


Figure 1: Semiconductor with a population of electrons.

**Modeling Drift Current In a Semiconductor**

Consider the semiconductor region in Figure 1. You will be modeling the carriers as a population of electrons in an N-type Si semiconductor crystal.

Use the following values:

- Effective mass of electrons  $m_n = 0.26m_0$  where  $m_0$  is the rest mass.
- The nominal size of the region is  $200\text{nm} \times 100\text{nm}$ .

## 1 Electron Modelling (40)

Model the electrons in the silicon as particles with the effective mass above using a simplistic Monte-Carlo model.

1. What is the thermal velocity  $v_{th}$ ? Assume  $T = 300K$ .
2. If the mean time between collisions is  $\tau_{mn} = 0.2ps$  what is the mean free path?
3. Write a program that will model the random motion of the electrons. The program should do the following:
  - Assign each particle a random location in the  $x-y$  plane within the region defined by the extent of the Silicon. For simplicity you may use a small number of particles (1000-10000 works well) but you can start much smaller initially if you like.
  - Assign each particle with the fixed velocity given by  $v_{th}$  but give each one a random direction.

- At a fixed time interval of  $\Delta t$ , update the particle location using Newton's laws of motion. You will need to pick a time step size that takes into account the velocity of your particles and the size of the region. Typically the spacial step should be smaller than 1/100 of the region size. Simulate for nominally 1000 timesteps. This should allow each particle to bounce around quite a bit inside the region.
- For a few of the particles trace out their trajectories using the 'plot' command in Matlab. To plot the trajectories you should keep the previous  $x$  and  $y$  positions. *Hint: use breakpoints to step through the trajectory and see what is "going on"; use the 'pause' command in Matlab to have the plot update in a loop.*
- Show a 2-D plot of all (or a subset) of the particles that updates with each time step. *Hint: use the 'pause' command in Matlab to have the plot update in a loop.*
- For the  $y$  direction use a boundary condition where the particle reflects at the same angle (specular) and retains its velocity.
- For the  $x$  direction use a periodic boundary condition where the particle jumps to the opposite edge. i.e. if it reaches the right side it appears at the left with the same velocity.
- Calculate and display the semiconductor temperature on the plot at a fixed time interval and verify that it stays constant.
- Your program should plot trace trajectories producing something like Figure 2.
- Hint: if you use arrays of positions and velocities ( $P_x, P_y, V_x, V_y$ ) in Matlab many of your calculations can be done using array math which will be much quicker!

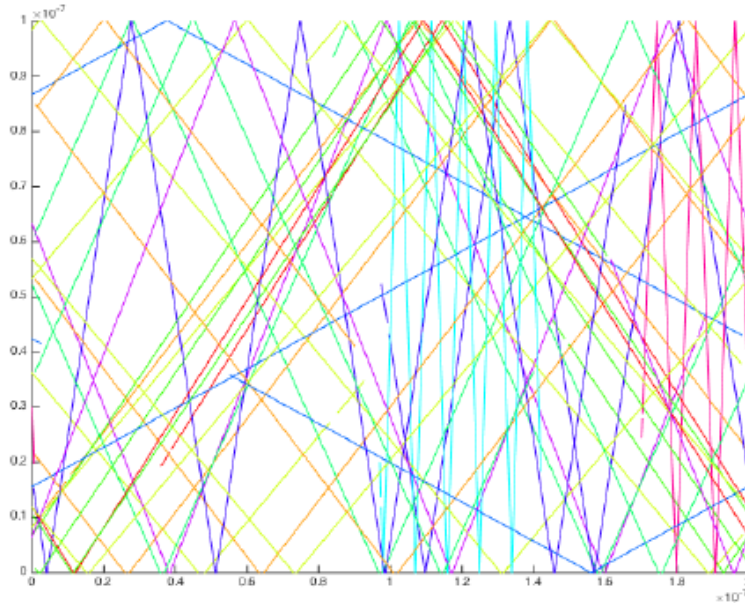


Figure 2: Sample particle trajectories.

**Include in Report** a)  $v_{th}$  b) MFP c) i) 2-D plot of particle trajectories ii) Temperature plot

## 2 Collisions with Mean Free Path (MFP) (25)

1. Assign a random velocity to each of the particles at the start. To do this you can use a Maxwell-Boltzmann distribution for each velocity component. Ensure that the average of all the speeds will be  $v_{th}$ . Plot the distribution in a histogram (Matlab `'hist'` function).
2. Model the scattering of the electrons using an exponential scattering probability:  $P_{scat} = 1 - e^{-\frac{dt}{\tau_{mn}}}$ , where  $dt$  is the time since the last timestep (and  $P_{scat}$  calculation), and  $\tau_{mn}$  is the mean time between collisions. Use the  $\tau_{mn}$  given above. At every time step (for every electron) use something like this: if  $P_{scat} > rand()$  then the particle scatters. When the electron scatters re-thermalize its velocities and assign new velocities  $V_x$ ,  $V_y$  from Maxwell-Boltzmann distributions. Refer to Figure 3 for a sample plot.
3. What happens to the average temperature over time?
4. Measure the actual Mean Free Path and mean time between collisions to verify your model.

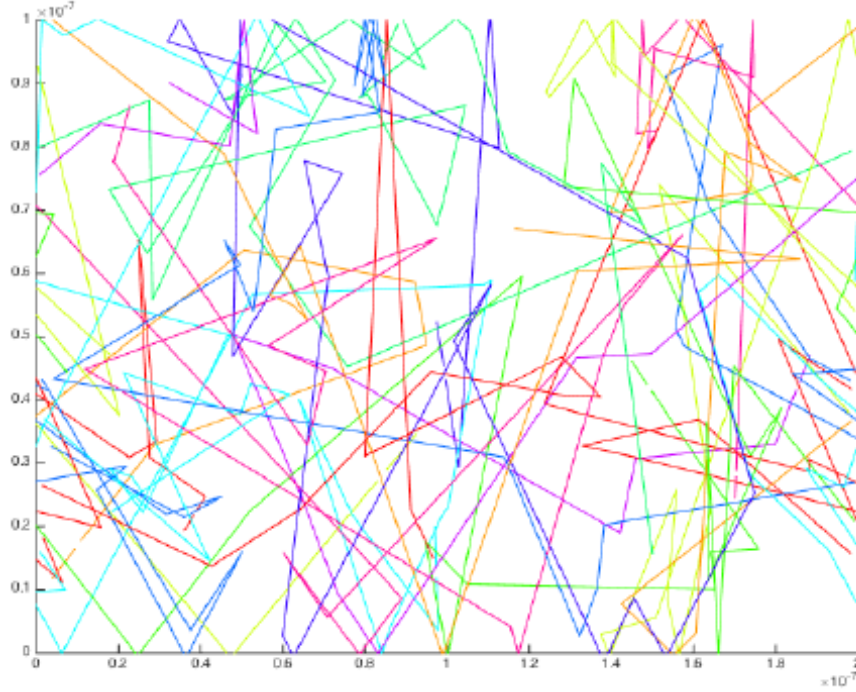


Figure 3: Sample trajectories with scattering.

**Include in Report** a) Histogram b) 2-D plot of particle trajectories c) Temperature plot d) MFP and  $\tau_{mn}$

### 3 Enhancements (35)

1. Add in the inner rectangle "bottle neck" boundary as in Figure 4. There are a number of ways to do this. I did it by defining "boxes" that reflect particles and then adding a number of "boxes" to the region. You will need to be careful not to put electrons in the "boxes" at the start. You should also check that no electrons are leaking through the "boxes".
2. Make all boundaries capable to be either specular or diffusive (ie re-thermalized).
3. Calculate an electron density map from the final electron positions.
4. Calculate a temperature map and display with colors.

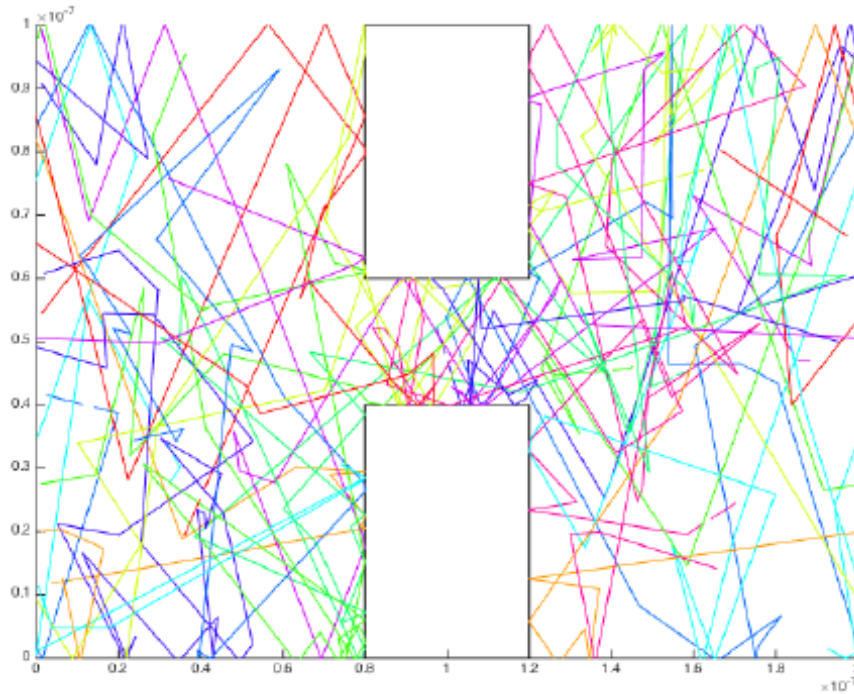


Figure 4: Sample trajectories with "bottle-neck".

**Include in Report** a) 2-D plot of particle trajectories c) Electron density map d) Temperature map