ELEC 4700 Assignment-2 Finite Difference Method

Due: Sunday, Feb. 25, 2018 11:59PM

Laplace's equation by FD can be used to solve electrostatic potential problems $\nabla^2 V = 0$, or current flow problems in inhomogeneous solids $\nabla(\sigma_{x,y}\nabla V) = 0$. Either case can be modeled as an orthogonal resistor network with resistors of value 1 or $\frac{1}{\sigma}$. The conception of the problem as an orthogonal 2-D mesh of resistors is very useful for setting up the boundary conditions (BC) and the interface conditions where two regions have different σ.

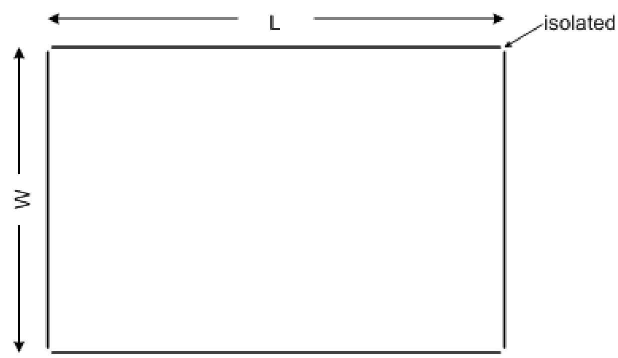

Figure 1: Rectangular region with isolated conducting sides

1. Use the Finite Difference Method (**I strongly suggest you use the matrix form of the problem** $GV = F$) to solve for the electrostatic potential in the rectangular region $L \times W$ shown in Figure 1 using $\nabla^2 V = 0$. In this exercise we'll experiment to see how accurate our model is.

   (a) Solve the simple case where $V = V_0$ at $x = 0$ and $V = 0$ at $x = L$. Note that in this case the top/bottom BC are not fixed. You could use $\frac{dV}{dy} = 0$ for the BC or treat this as a 1-D case.

   (b) Solve the case where $V = V_0$ at $x = 0$, $x = L$ and $V = 0$ at $y = 0$, $y = W$.

   Compare the solution of a bunch of mesh sizes to the analytical series solution:

$$V(x,y) = \frac{4V_0}{\pi} \sum_{n=1,3,5...}^{\infty} \frac{1}{n} \frac{\cosh\left(\frac{n\pi x}{a}\right)}{\cosh\left(\frac{n\pi b}{a}\right)} \sin\left(\frac{n\pi y}{a}\right)$$

   A typical plot is given in Figure 2.

   Draw some conclusions about meshing. *(Hint: does the solution approach the analytical solution?).* Note that both solutions have some explicit error. It is useful to plot a "movie" of the analytical series solution as it converges to the "real" solution. How do you know/decide when to stop the analytical series? Griffiths "Intro to Electrodynamics 3e" contains a derivation of the analytical solution in Example 3.4. Comment on the advantages and disadvantages of the numerical versus analytical solution.

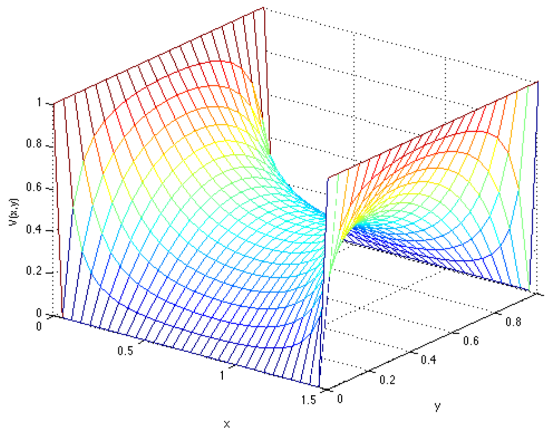

Figure 2: Typical potential plot for case where ends are at same potential and sides at same potential.

Note: For plots use a ratio of 3/2 for L/W.

Include in Report

a) 2-D plot of $V(x)$ b) Matching surface plots of $V(x, y)$. Only include one plot for each. Conclusions on meshing and comments on numerical vs analytical

2. Use the Finite Difference Method to solve for the current flow in the rectangular region $L \times W$ shown in Figure 3 using $\nabla\left(\sigma_{x,y}\nabla V\right) = 0$. To model the boxes as highly resistive we will use a low $\sigma_{x,y}$ inside the boxes. Note that $\sigma_{x,y}$ needs to remain finite. Start with $\sigma = 1$ outside the boxes and $\sigma = 10^{-2}$ inside. In this exercise we'll experiment to see how the current flow is affected by the "bottle-neck".

   (a) Calculate the current flow at the two contacts. Generate plots of $\sigma(x, y)$, $V(x, y)$, $\vec{E}_x$, $\vec{E}_y$, $\vec{J}(x, y)$

   (b) Investigate mesh density.

   (c) Investigate narrowing of "bottle-neck".

   (d) Investigate varying the σ of the box.

   Include in Report

   a) Current , plots: $\sigma(x, y)$, $V(x, y)$, $\vec{E}(x, y)$, $\vec{J}(x, y)$ b) Graph of current vs mesh size c) Graph or table of current vs various bottle-necks. d) Graph of current vs σ .

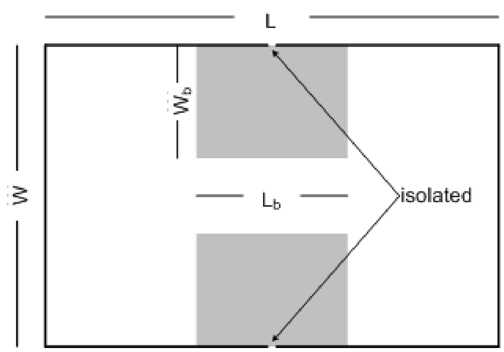

Figure 3: Rectangular region with isolated conducting sides and "bottle-neck".