The hillslope diffustion equation is:

$$\frac{\partial z}{\partial t} = D\nabla^2 z = D\left(\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2}\right) \tag{1}$$

By taking the Fourier transform of equation (1) in both x and y directions, we obtain the spectral form of this problem for wavenumbers k_x and k_y:

$$\frac{\partial \hat{\eta}_{k_x,k_y}}{\partial t} = D\big(- k_x^2 \hat{\eta}_{k_x,k_y} - k_y^2 \hat{\eta}_{k_x,k_y}\big) \tag{2}$$

where the wavenumber k_x is:

$$k_x = \frac{2\pi n_x}{N_x \Delta x}, \qquad n_x = [-N_x/2, N_x/2] \tag{3}$$

where N_x is the dimension of the grid in the x direction.

The **explicit form** of this equation considers the right hand side of the equation only at time step n:

$$\frac{\hat{\eta}^{n+1}_{k_x,k_y} - \hat{\eta}^{n}_{k_x,k_y}}{\Delta t} = D\big(- k_x^2 \hat{\eta}^{n}_{k_x,k_y} - k_y^2 \hat{\eta}^{n}_{k_x,k_y}\big) \tag{4}$$

The **implicit (Crank-Nicholson) form** of this equation uses both time n and time $n+1$ on the right side, and does not have the stability constraint of the explicit form:

$$\frac{\hat{\eta}^{n+1}_{k_x,k_y} - \hat{\eta}^{n}_{k_x,k_y}}{\Delta t} = \frac{D}{2}\big(- k_x^2 \hat{\eta}^{n}_{k_x,k_y} - k_y^2 \hat{\eta}^{n}_{k_x,k_y} - k_x^2 \hat{\eta}^{n+1}_{k_x,k_y} - k_y^2 \hat{\eta}^{n+1}_{k_x,k_y}\big) \tag{5}$$

For comparison, a simple explicit method often used to solve the diffusion equation is:

$$\frac{\eta^{n+1}_{i,j} - \eta^{n}_{i,j}}{\Delta t} = D\left(\frac{\eta^{n}_{i-1,j} - 2\eta^{n}_{i,j} + \eta^{n}_{i+1,j}}{\Delta x^2} + \frac{\eta^{n}_{i,j-1} - 2\eta^{n}_{i,j} + \eta^{n}_{i,j+1}}{\Delta y^2}\right) \tag{6}$$