Extended Essay in Mathematics

Can the Monte Carlo Method Be Used to Aid Financials Decisions in The Stock Market?

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Abstract

This essay explores the applications of math in the financial field through the use of models to predict the probability of price movement outcomes of a stock or option. The focus of this research is going to be based on the Monte Carlo method (simulation), and its applications in other stochastic processes such as the Markov Chain and the extent of its utility or effectiveness in this area.

At the start of the essay, I introduce stochastic processes in general and then transition my focus to the Monte Carlo method in specific. After a brief history, the essay breaks down the method into its components, examining each in greater detail. Finally, this method is applied to Tesla's stock, using the closing prices from June of 2010 all the way until the end of 2019. This data is processed and used to produce a Brownian motion which models the price movement of the stock for the next month. The results from the simulation are then used further to create a frequency chart, depicting the most likely outcome at the end of the duration, as well as to create a Markov chain, which examines the probability of the price moving up or down given previous actions. In the evaluation, it was concluded that the Monte Carlo method has its limitations when predicting price movement of a single stock due to the many factors that aren't reflected in previous price action. As a result, it may be more useful when predicting the future value of an entire portfolio due to the lower volatility, or if used to test the performance of other models.

Most of the probability formulae included were taught in the HL math curriculum and furthered my understanding of the topic.

Word count: 291

Contents

Introduction

The life of an investor is an unpredictable and risky one. Those who have been successful with trading have risen to grand levels of wealth greater than many middle-class citizens could ever imagine. Establishing themselves as common household names such as Warren Buffet, Paul Tudor and more. Thus, many of the working class become blinded by these success stories and the idea of escaping from the 9-5 grind and decide to test their luck in the financial markets in an effort to make some money and effectively change their lives for the better. However, due to a lack of experience and knowledge, many are unable to beat the markets and end up losing all of their money.

Despite the fact that many beginner traders end up losing money, that does not mean that they will always be unsuccessful as investors. In many ways, trading is similar to sports, especially in the sense that it requires proper planning, experience, and most important practice in order to lessen the amount of losses one experiences. New traders won't have much practice nor experience thus, it's imperative that they create a sound game plan prior to entering trades to even the playing field as much as they can. In the context of the financial markets, these strategies can include the use of mathematical indicators which are used to calculate the probability of an asset moving in price a certain way which then determines entry and exit points.

An example of an effective tool is the Monte Carlo method and its many applications. This paper will be analyzing this method which includes its history, equations, and the effectiveness in the markets. Finally, we will dive into further applications of the Monte Carlo method with Markov chains to determine under what conditions it thrives in and where it falls short.

Monte Carlo Simulation

The Monte Carlo Simulation (also known as a multiple probability simulation) is a model used to determine the probability of different outcomes in a process which is plagued with unknown variables. The name "Monte Carlo Simulation" is derived from the numerous casinos in Monte Carlo, Monaco, as the model draws parallels to the random and unpredictable nature of the games whether it be roulette, slots, poker, and etc.[1] The simplest way to understand this model is to imagine "what is the chance of rolling a one on a six-sided die", although it is definitely possible to determine this result mathematically the probability we calculate, may not always work. Ideally, the chance to roll a one would be one out of six. But if we were to actually roll the die six times, how many times would the die actually land on a one? It could be one time, but it also could be two, or maybe even six times. With the Monte Carlo method, we are able to put this hypothesis to the test by simulating millions of dice rolls in an efficient manner that will tell us exactly the chance of rolling a one on a die.

History

This method was developed by a Polish mathematician involved in the Manhattan Project during the second world war by the name of Stanislaw Ulam. After the war, Ulam suffered from encephalitis and while in the hospital he entertained himself by playing many games of solitaire. Soon enough he became intrigued in plotting the outcomes of the games in order to observe the distribution and determine the probability of winning. Initially he attempted this through combinatorics, but the math became much too complex, resulting in him failing this first attempt. He then had an idea to simply play out many games and record the number of wins and divide by the total number of games played. Seeing as this task would take years to complete by hand, he decided to simulate the games on the computer instead. He shared this idea with another

1. Kenton, Will. "Monte Carlo Simulation." Investopedia. Investopedia, January 29, 2020. https://www.investopedia.com/terms/m/montecarlosimulation.asp.

mathematician, John von Neumann and together they developed the Monte Carlo Simulation that we know of today.[1]

Breakdown

The Monte Carlo simulation is based on the principles of inferential statistics, which is the use of a sample of data to make predictions about the entire population.[2] An example of this can be demonstrated using random walks. Consider a random walk of 5,000 steps, at each step the walk either moves forward or backwards. In order to calculate the result of the entire walk, or a prediction of the result, a sample of 100 walks (5,000 steps each) will be used to represent the entire population of possibilities. The distance from the origin from each of the 100 sample walks will then be averaged in order to predict the outcome of the model.

In the context of this essay, the population would be the possible price actions of the asset under inspection. The sample would be a number of different price actions, and this is then processed and used to calculate some statistics. Then these results are used to predict the outcome of the actual stock. The whole model is based on the premise that a random sample will likely reflect the behaviour of the entire population.

Next, I will be discussing the presence of variance in the model. Variance measures the extent to which a set of numbers are spread out from the average value. Variance is determined by the following formula:

$$variance\,(X) = \frac{\Sigma_{x \in X}(x-\mu)^2}{N}$$

X represents the list of items in the sample, and we can calculate variance by first finding the difference between each item in the list (x) and the mean (μ). The difference is then squared to

1. "Monte Carlo Method." Wikipedia. Wikimedia Foundation, February 13, 2020. https://en.wikipedia.org/wiki/Monte_Carlo_method#History.
2. Stephanie. "Inferential Statistics: Definition, Uses." Statistics How To, June 20, 2018. https://www.statisticshowto.datasciencecentral.com/inferential-statistics/.

ensure that all values are positive, as well as to make outliers in the data more prominent. This is then done for every item in the set, and the results are summed. Finally, the sum is divided by the cardinality of X because if this was not done, data with a larger number of values would have a greater variance which may lead to inaccurate results in many cases.

Greater variance coincides with a less precise result. Thus, the goal of the model is to reduce the variance as much as possible in order to provide an accurate representation of the price of an asset in the case of this essay. Luckily, this can be accomplished by using the law of large numbers which states that:

As the number of identically distributed, randomly generated variables increases, their sample mean approaches the expected mean of the population.[1]

This law can be demonstrated with the use of the die roll example. Imagine a situation where a die is rolled 30 times in an attempt to determine the probability of rolling a 6.

Let X = the number of sixes rolled

$$E\,(X) = 30 \times \frac{1}{6} = 5$$

The expected number of sixes to be rolled in a sample of 30 rolls is going to be five 6's. If this experiment were then to be tested for 3 separate trials, and the following results were produced:

X = {7, 5, 4}

The average of these results would be:

$$\overline{X_3} = \frac{7+5+4}{3} \cong 5.33$$

The law of large numbers states that as the number of trials n, approaches infinity, the mean of the trials will approach the theoretical mean.

1. "Law of Large Numbers (Video)." Khan Academy. Khan Academy. Accessed March 2, 2020. https://www.khanacademy.org/math/statistics-probability/random-variables-stats-library/expected-value-lib/v/law-of-large-numbers.

$$\overline{X_n} = \frac{X_1+X_2+X_3+\ldots+X_n}{n} \rightarrow 5,\ for\ n\rightarrow\infty$$

Thus, by running more simulations, the results will converge to the theoretical mean thus reducing the variance of the model. Following this law, in order to maximize the accuracy of the model, it is imperative that we use a large sample size. However, this idea can often be misinterpreted, leading to something that is known as the gambler's fallacy.[1]

Also known as the Monte Carlo fallacy, the gambler's fallacy is the belief that when a sample deviates from the theoretical probability, it will be evened out in the future. This most well-known case of the gambler's fallacy comes from a game of roulette at Monte Carlo. In roulette, players have the choice to bet on either red or black and that night at the casino, black was rolled 26 times in a row which is highly unlikely. The probability of this occurring is $\frac{1}{2^{26}}$ or $\frac{1}{67108864}$, and after hearing this, all of the gamblers gathered at the table and bet on red due to the belief that red would be rolled to balance this long streak of black. However, as the night went on the casino made record earnings that night as black kept coming up. This is because in actuality, the probability of black being drawn again (assuming that the game was not rigged) after the 26th occurrence is not $\frac{1}{2^{27}}$ but instead, only $\frac{1}{2}$ because each of these events are individual and are independent of the previous result.

Analysis

In order to test the accuracy and effectiveness of this model, I will be using python to program a simulation that predicts the price movement of a stock of Tesla Motors. The program gathers the closing price of Tesla's stock from every day from June 28th, 2010 to December 31st, 2019.[1] The following figure shows the price movement of Tesla's stock over the duration of time.

1. Kenton, Will. "Gambler's Fallacy Definition." Investopedia. Investopedia, January 29, 2020. https://www.investopedia.com/terms/g/gamblersfallacy.asp.

Figure 1: Tesla Stock Price Movement

From that, the percentage change from each day was found and this was used to calculate the log return of the stock. A lognormal distribution is used mainly because stock prices cannot be negative and are generally better represented by lognormal distribution instead of normal distribution.[2]

The closing price on the final day was taken from December 31st, 2019. I then set the conditions for the simulation which was 1000 simulations which predicted the movement of the price over the next trading month (23 days).

A Brownian motion will be used to model the daily returns of the stock. In order to model this random movement, there is both a deterministic component, which is the drift and a random stochastic component which is the volatility present in the function.

1. "Tesla, Inc. (TSLA) Stock Historical Prices & Data." Yahoo! Finance. Yahoo!, March 2, 2020. https://finance.yahoo.com/quote/TSLA/history?p=TSLA.
2. "Why Lognormal Distribution Is Used to Describe Stock Prices." Finance Train, July 1, 2013. https://financetrain.com/why-lognormal-distribution-is-used-to-describe-stock-prices/.

The drift represents the direction that the rates of return have been moving towards in the past. It's calculated by subtracting half of the variance from the mean.

$$Drift = -\frac{1}{2}\,{}^{2}$$

Next volatility is calculated by taking the standard deviation of Z and a random number from 0 to 1. Since there are 100 simulations, there will be 100 random numbers generated. These numbers act as a percentage while the Z value is the number of standard deviations away from the mean.

$$Volatility = \sigma Z[Rand\,(0; 1)]$$

Using these two components, the daily return function is now complete

$$daily\ return = \ e^{drift+volatility}$$

This function will be multiplied by each day's current price to obtain the following days price.

Some statistics produced from the simulation are the following:

Mean: $429.49

SD: 65.44940261609646

Max: $808.78

Min: $223.90

5th percentile: $329.79

25th percentile: $382.50

75th percentile: $469.97

95th percentile: $544.33

The following figures are the line chart produced from the movement of the price over time as well as a distribution curve showing the frequency each final price appears at.

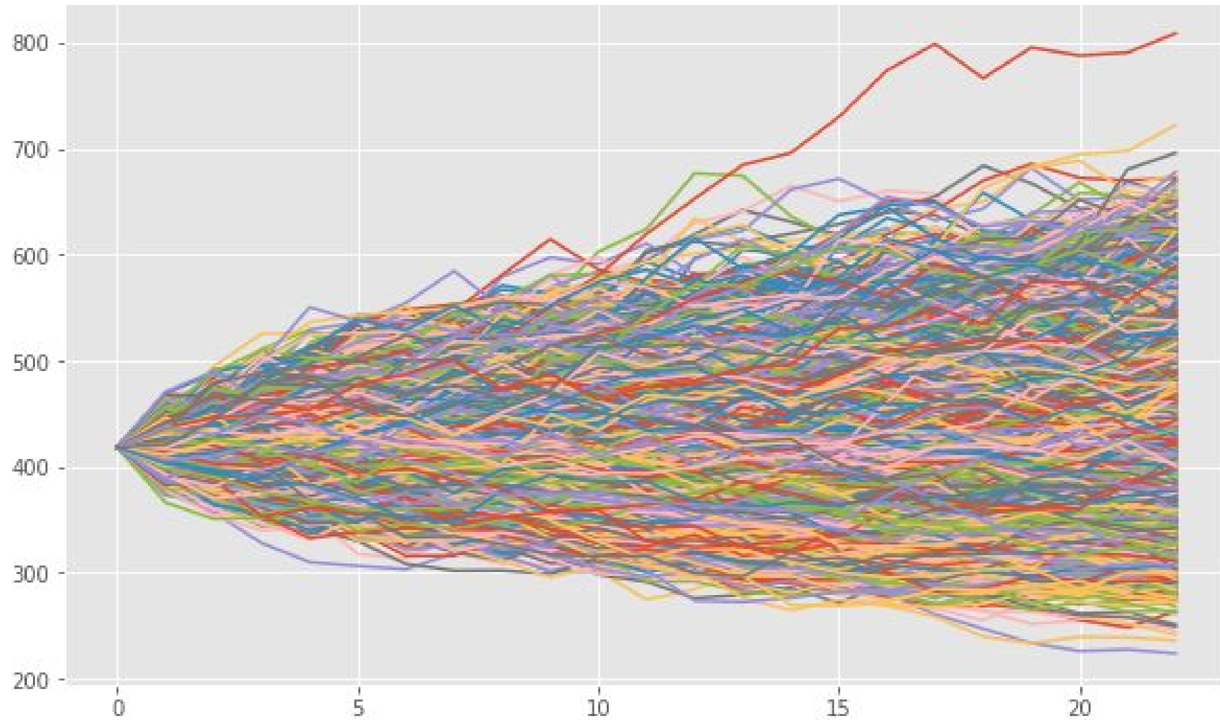


Figure 2: Monte Carlo Simulation (TSLA)

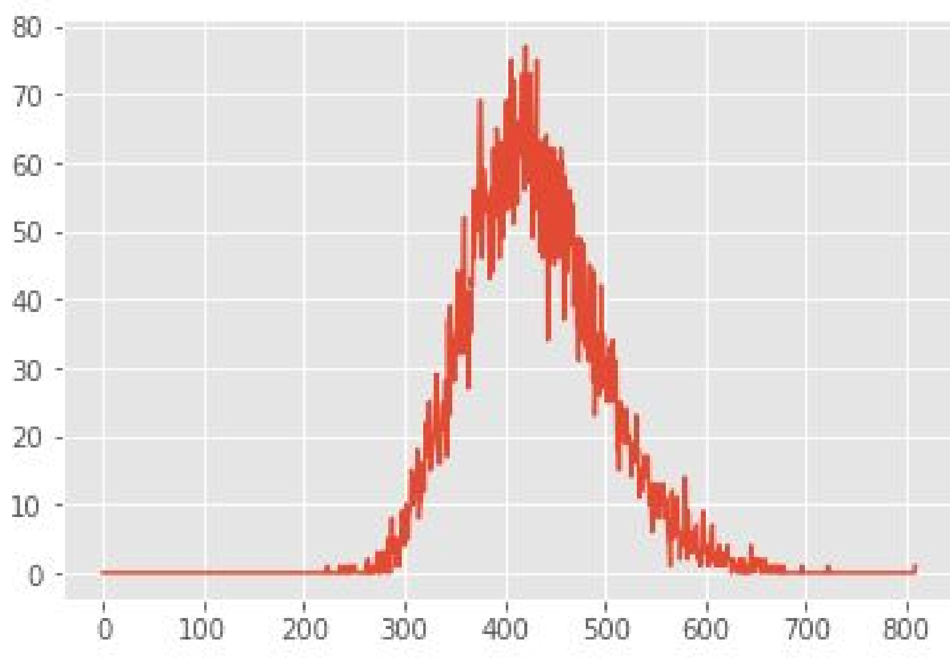


Figure 3: Frequency Chart of Final Price

At the end of the simulation, the most frequent result for the closing price on Friday January 31st was about $432 which appeared 75 times of the 10000 (entire frequency chart in appendix). Which is much less than the actual closing price which was $650.57. This large difference is primarily due to the fact that in the month of December, Tesla received a $1.62 billion loan from Chinese banks to open their second assembly plant which will be based in Shanghai. This news has promoted confidence amongst buyers causing more to purchase the stock leading to an increase of 26.8% throughout December,[1] which continued in January with a 53% increase.[2] However, because the simulation only considers the previous price action, it doesn't take into account the news and recent hype that was created around the stock and as a result does not show the same optimism.

Limitations of the model

Now one major limitation of the Monte Carlo simulation and stochastic processes in general is the size of the sample that one must use in order to obtain an accurate or reasonable prediction. In my test of the model I was only able to simulate 1,000 simulations because it was beginning to take too long to run a greater number of simulations. By implementing a timer into the program, I found that for every 100 simulations it took about 20 seconds to complete, and for 1000 simulations it took about 200 seconds. Following this trend, 10000 simulations would take roughly 2000 seconds which is the longest I can realistically run this simulation for as the next interval after that would be 100000 simulations at 20000 seconds or roughly 5.5 hours. However, this limitation can be bypassed to some extent with improved technology. Those who may use this model in big firms most likely have access to better computers than the one I have at home,

1. Whiteman, Lou. "Why Shares of Tesla Soared 26% in December." The Motley Fool. The Motley Fool, January 5, 2020. https://www.fool.com/investing/2020/01/05/why-shares-of-tesla-soared-26-in-december.aspx.
2. Ari Levy, Lora Kolodny. "Tesla Stock Wraps up Its Best Month since 2013, Adding $40 Billion in Value in January." CNBC. CNBC, January 31, 2020. https://www.cnbc.com/2020/01/31/tesla-tsla-shares-best-month-since-2013-and-third-best-ever.html.

and as a result can complete a much greater number of processes in less time than I can. This will lead them to more data which in turn provides a more accurate result that investors can be confident in.

Another limitation to the model which was mentioned previously is that it only takes it doesn't take into account factors that aren't included in the price history such as company leadership, the economy, news, or hype.[1]

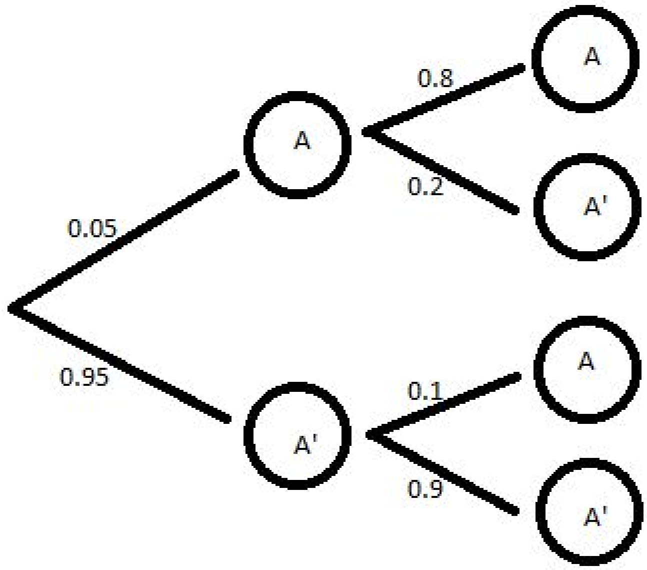
Further Applications

Now we will be examining an application of the monte Carlo method, as the strength of this method is that it can be used as part of other models for calculating stock prices and this can improve the results we obtain. In this paper, the monte Carlo method will be used to create a Markov Chain which can be used to determine conditional probabilities of price movement.

Markov Chains

So, what exactly is a Markov chain? Essentially, it's a stochastic process that determines the probability that a mathematical system will transition from one state to another.[1] For example, if we consider company "A" that sells children's toys and, in this scenario, we will say they control 5% of the toy market, meaning that 5% of all toys purchased come from company "A". Now imagine they are running an ad campaign which ensures that 80% of their existing customers return and 10% of customers who bought from other companies now choose to buy from company "A". We can use a simple tree diagram to model this probability as shown below:

1. Kenton, Will. "Monte Carlo Simulation." Investopedia. Investopedia, January 29, 2020. https://www.investopedia.com/terms/m/montecarlosimulation.asp.



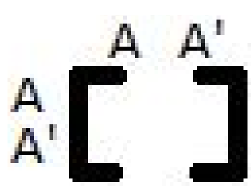
A represents the toys purchased from company “A” while *A’* represents the toys purchased from every other company in the market. If we wanted to determine how many people would purchase toys from company “A” next year after this ad campaign is implemented, we can simply multiply the branches and add the results.

$$P\ (A) = 0.05 * 0.8 + 0.95 * 0.1$$

$$P\ (A) = 0.135$$

As we can see, in the following year company “A” will control 13.5% of the toy market assuming that the campaign did exactly as it was supposed to, and everything went perfectly.

There is another way to represent a Markov chain and this is with transition matrices where the probabilities of each state are put into the cells.



1. “Markov Chains.” Brilliant Math & Science Wiki. Accessed March 2, 2020. https://brilliant.org/wiki/markov-chains/.

The probabilities will be filled into their respective spaces. The top row and bottom row correspond to the top and bottom branches in the tree diagram respectively.

$$P = [0.8\ 0.2\ 0.1\ 0.9\ ]$$

The initial state matrix is defined as:

$$S_0 = [0.05\ \ 0.95]$$

This is because initially company "A" controls 5% of the toy market.

Finding the next state can also be done with matrix multiplication using the initial state and the transition matrix.

$$S_1 = [0.05\ \ \ 0.95] * [0.8\ 0.2\ 0.1\ 0.9\ ]$$

$$S_1 = [0.05 * 0.8 + 0.95 * 0.1\ \ \ 0.05 * 0.2 + 0.95 * 0.9]$$

$$S_1 = [0.135\ \ \ 0.865]$$

As shown above, we get the same results with both the transition matrix and the tree diagrams. Now what if we wanted to find the effects of the campaign after 2 years, how about 3 years? Assuming the ads still have the same impact on the company's sales (transition matrix stays the same) then we can simply keep multiplying the current state matrix by the transition matrix. By doing so we obtain the following results:

Matrix 1: [0.135 0.865]

Matrix 2: [0.1945 0.8055]

Matrix 3: [0.23615 0.76385]

…

Matrix 19: [0.33301036463641465 0.6669896353635859]

1. Kathan, Maurie. "Markov Chain Monte Carlo." Medium. Towards Data Science, March 15, 2019. https://towardsdatascience.com/markov-chain-monte-carlo-291d8a5975ae.

Matrix 20: [0.33310725524549034 0.6668927447545102]

And eventually we see that the results approach 0.333… or 1/3. This shows that after about 19 years, the advertisement campaign will no longer have any effect on the company's sales. We can check this result by multiplying this current state matrix by the transition matrix.[1]

$$\left[\frac{1}{3}\ \ \frac{2}{3}\right] * [0.8\ 0.2\ 0.1\ 0.9\ ]$$

$$[\tfrac{1}{3} * 0.8 + \tfrac{2}{3} * 0.1\ \ \tfrac{1}{3} * 0.2 + \tfrac{2}{3} * 0.9]$$

$$[\tfrac{1}{3}\ \ \tfrac{2}{3}]$$

Markov Chain Monte Carlo (MCMC)

Next, we will be applying the Monte Carlo method with the Markov chains with the same stock data used in the last simulation to see how the results may differ. In this simulation, I will be using the prices generated from the example above to generate the chains needed to determine the probability of the price going up or down depending on its last movement.

To do this, a program was written that first calculates the difference between the closing price of consecutive days for each simulation. Then it records whether the price went up or down the day before and what happened on the current day. A probability is then produced from the ratio of these results and it is printed out.[1]

The data from the simulation produced the following results:

P (B | A) = 0.5093

P (B' | A) = 0.4907

P (B | A') = 0.5088

P (B' | A') = 0.4912

1. patrickJMT. "Markov Chains – Part 1". *YouTube* video, 12:18, January 13, 2010. https://www.youtube.com/watch?v=uvYTGEZQTEs

In the probabilities above, "A" means that the price went up the previous day, therefore A prime represents the price going down the previous day. "B" is representative of the same information except it pertains to the current day.

As a result, the transition matrix can be written as:

$$P = [0.5093\ 0.4907\ 0.5088\ 0.4912\ ]$$

Thus, from our Markov Chain, we have found that if the price of Tesla's stock increased then the probability that it will increase again is 0.5093, and if it decreased then the probability that it will increase is 0.4907. As we can see, these probabilities are very close to 0.5 and this is due to the large amount of simulations this data is made from. As a result of the law of large numbers mentioned earlier, with a larger number of simulations the probability of positive and negative price movements are both very close to the average which is 0.5.

Conclusion

Throughout this paper, I have analyzed and tested the Monte Carlo method and identified some of its strengths and weaknesses. Then to expand, I began looking into specific applications of the Monte Carlo method in different models and once again assessed where the method was effective in providing more accurate data and where it did not. Overall, I believe that the Monte Carlo method can be an amazing tool due to it being such a broad process that it can be found useful in nearly any field ranging from the sciences all the way to finance. This method lends itself to many calculations and various applications, and as a result is able to evolve into many different forms, each of which have their own circumstances where they shine the most. In the case of predicting the prices of individual stocks, because the price movement can be influenced greatly by other external factors such as consumer optimism, and the company's performance,

1. Kathan, Maurie. "Markov Chain Monte Carlo." Medium. Towards Data Science, March 15, 2019. https://towardsdatascience.com/markov-chain-monte-carlo-291d8a5975ae.

Monte Carlo methods may be more useful when modelling an entire portfolio.[1] This is because portfolios typically have a lower volatility and also is not as likely to rise or fall as drastically as the individual stocks within it.

As time goes on, this model will only become more effective due to improvements in our hardware, but it is likely that there will also be more sophisticated technology to be developed which may provide more accurate results. However, the stock market is a wild beast in itself and predicting its movements consistently is no easy feat and may never be accomplished even with the most complex mathematics.

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Appendix

Frequency Chart Information, prices with a frequency of 0 were omitted to minimize size of data as much as possible.

Price	**Freq.**

224	1

236	1

242	1

245	1

249	1
250	1
251	1
262	1
264	2
268	1
272	1
273	3
274	1
276	1
278	3
279	1
280	3
281	1
283	2
284	5
285	3
287	5
288	8
289	4
290	3
291	5
292	1
293	4
294	5
295	1
296	4
297	4
298	9
299	5
300	7
301	4
302	10
303	6
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495	26
496	42
497	27
498	35
499	34
500	30
501	31
502	25
503	25
504	29
505	33
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507	25
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525	18
526	14
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531	23
532	16
533	18
534	11
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536	15
537	12
538	13

539	17
540	16
541	17
542	14
543	17
544	10
545	11
546	10
547	6
548	13
549	12
550	10
551	13
552	9
553	8
554	12
555	10
556	13
557	10
558	13
559	8

560	11
561	8
562	8
563	5
564	4
565	1
566	6
567	12
568	5
569	7
570	8
571	7
572	11
573	6
574	6
575	2
576	6
577	6
578	4
579	5
580	14

581	7
582	2
583	9
584	5
585	4
586	3
587	6
588	4
589	4
590	3
591	7
592	2
593	3
594	2
595	1
596	3
597	8
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600	3
601	4

602	1
603	1
604	3
605	2
606	5
607	7
608	2
609	2
610	1
611	3
612	1
613	2
614	4
615	1
616	3
617	2
618	1
619	2
620	1
621	3
622	4

623	2
624	2
625	2
627	1
628	2
629	1
630	1
631	1
634	2
635	2
638	2
640	1
642	1
645	4
646	1
647	1
649	2
650	1
651	1
652	2

653	1
654	2
656	1
657	1
659	2
660	1
661	1
664	1
665	1
668	1
669	1
671	1
672	1
675	1
678	1
696	1
722	1
809	1

Price list of first 8 simulations, for 20 days.

Day	1	2	3	4	5	6	7	8
1	418.33	418.33	418.33	418.33	418.33	418.33	418.33	418.33
2	399.804	400.831	429.244	402.807	392.836	432.246	434.283	421.882
3	419.269	392.369	443.272	406.214	393.711	455.796	417.14	420.689
4	420.915	392.976	455.883	389.338	369.476	446.948	416.583	413.877
5	407.627	389.159	477.364	381.4	374.653	464.921	407.088	398.819
6	429.435	378.156	491.095	386.036	366.527	468.405	413.495	383.139
7	438.433	397.222	485.461	381.689	351.072	463.278	405.881	372.125
8	448.33	400.836	484.048	367.555	357.415	502.363	404.746	384.869
9	446.901	388.17	476.964	371.581	374.164	507.169	397.6	395.573
10	454.481	394.351	501.299	375.42	374.015	480.35	415.826	419.175
11	414.892	429.787	495.327	372.839	369.18	485.572	416.804	415.084
12	394.667	448.404	480.75	365.857	379.649	488.307	417.2	423.692
13	405.54	427.378	486.079	378.924	397.478	505.401	408.953	431.709
14	396.218	417.335	494.384	389.887	409.355	529.161	403.023	409.259
15	399.389	418.758	464.117	368.806	391.525	525.248	396.649	414.659
16	422.666	401.301	453.113	369.992	385.957	520.014	376.876	411.81
17	419.377	407.885	466.426	346.571	370.6	506.203	371.038	413.504
18	403.239	380.18	458.426	335.212	387.61	483.408	372.503	409.422

19	401.517	381.005	453.432	334.692	378.403	477.144	379.825	412.335
20	412.344	377.03	452.063	329.748	376.751	491.769	370.013	439.505