LUMPED PARAMETER LINE MODELING
Part 2: Cable Models

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Cable Overview

Cables are insulated conductors that can be used for underground construction or for routing in trays or ducts. A typical layout for a cable is shown in Fig. 1. The phase conductors are enclosed in an electrical insulator. The conductors are typically composed of copper or aluminum which are stranded for the larger diameters. The sizes are the same as those used for overhead conductors. Multiple conductors can be enclosed in a common sheath for three-phase feeders. The insulation consists of either a special plastic or an oil-impregnated paper wrapping.

For underground applications, the cable must have a sheath (jacket) for protecting the cable against mechanical stress, water and corrosives in the air or soil. Lead used to be the primary material used for sheathing but more recently special plastics are being substituted instead. Sometimes for underground applications the cable is also armored with an additional layer of protection around the sheath.

Underground construction can make use of ducts or direct burial. The ducts are made out of concrete, plastic, fiber or iron pipe. When installing multiple cable in ducts, the heat dissipated by the cables must be computed ahead of time to prevent thermal problems. Underground services are made accessible via manholes commonly seen in urban streets. The interface between overhead and underground construction is made with a ‘pothead’ using an assembly referred to as a riser. These can be observed where the overhead is tapped off to make connections to customers.

This section will describe how we can develop a cable circuit model for network analysis. The input to this calculation will be the cable construction parameters. An example of these parameters is shown in a Milsoft WindMil cable equipment table printed below in Fig. 2. Once we have the equivalent series resistance, series reactance and shunt capacitance, then we can perform the same types of calculations that we did before for overhead feeder circuits.

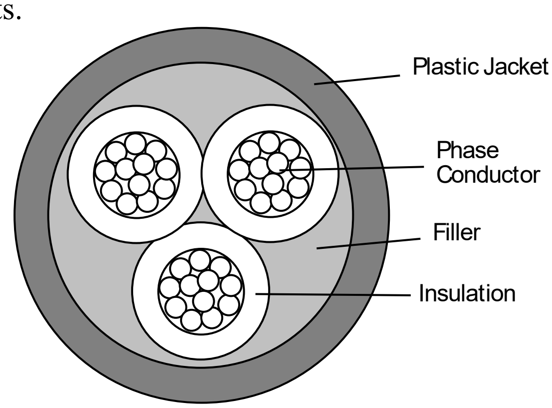

Fig. 1 Three-Phase Cable

UG Conductor Data

Description	Cable Type	Ampacity	Insul	Jacket	Neutrl	ExtJkt	Cond	Dielect	# Neut Strands	GMR Phase (feet)	GMR Neutrl (feet)	Resistance Phase (ohms/mile)	Resistance Neutrl (ohms/mile)	Distance to Cn (feet) / Capactive Charging Factor
1/0 AL 15kV 1/3 Neut	CONC	228	0.0712	0.0712	0.0819	0.0819	0.0311	2.4000	6	0.0118	0.0318	1.1088	2.9621	0.0383 0
1000 AL 15KV 1/3	CONC	575	0.1442	0.1442	0.1612	0.1612	0.0960	2.4000	20	0.0371	0.0318	0.1162	0.3538	0.0383 0
4/0 AL 15KV 1/3 N	CONC	326	0.0783	0.0783	0.0948	0.0948	0.0044	2.4000	11	0.0167	0.0318	0.5544	1.6157	0.0383 0
500 AL 15KV 1/3	CONC	350	0.1029	0.1117	0.1252	0.0000	0.0677	2.4000	16	0.0260	0.0318	0.2323	0.7022	0.0383 0

Notes from Milsoft WindMil Help Screen:

Conductor Name shows the name of the currently selected conductor.

Current Carrying Capacity of Conductor (Amps) defines the current carrying capacity of the selected conductor.

Phase Conductor Resistance (Ohms/Mile) defines the phase conductor resistance of the conductor.

Geometric Mean Radius of Phase Cond (Feet) defines the geometric mean radius of the phase conductor.

Type Neutral select either Concentric or Copper Tape Shield as the neutral. Concentric is used for all concentric strand neutrals.

Concentric Neutral Resistance (Ohms/Mile) specifies the concentric neutral resistance. This value is used only if Concentric is selected for Type Neutral and it is not required for a tape shield neutral. The resistance of tape shield neutrals is calculated using the thickness of the shield and the basic resistivity of copper.

Number of Individual Strands in Concentric Neutral specifies the number of strands in the concentric neutral. This value is used only if Concentric is selected for Type Neutral and is not required for a tape shield neutral.

OD of Cable Including Neutral Conductor (Feet) specifies the outside diameter of the cable including the concentric neutral or tape shield but not any jacket external of the concentric neutral or tape shield.

Diameter Under Neutral (Over Screen) specifies the diameter of the circle just under the concentric neutral or tape shield. This diameter will include any semi-conductive jacket or screen over the insulation. The difference between OD of Cable Including Neutral

Conductor and Diameter Under Neutral, divided by two, should be the diameter of a concentric neutral strand or the thickness of the tape shield. Analysis warnings and errors may occur, if Diameter Under Neutral is equal to or greater than OD of Cable Including Neutral Conductor.

OD Of Cable Insulation (Feet) is the outside diameter of the cable insulation not including any semi-conductive jacket or screen. This value should be less than or equal to Diameter Under Neutral.

Diameter of Conductor (Feet) specifies the diameter of the phase conductor. This value is used to calculate charging on underground line when Capacitive Charging Current Factor is zero. If used this value should be less than OD Of Cable Insulation.

Dielectric Constant of Insulation provides the dielectric constant of the insulation. It is used to calculate charging and is not required if Capacitive Charging Current Factor is not zero.

Fig. 2 Cable Construction Data

Cable Impedance Calculations
We will focus on two common types of cables for our calculations. These are the concentric neutral illustrated in Fig. 3 and the tape shield cable shown in Fig. 4. We will want to follow a set of calculation steps similar to those used for overhead feeders, and this will involve calculating phase and neutral resistances, GMRs and equivalent spacings. This will allow us to compute models for three-phase cable systems, such as that shown in Fig. 5.

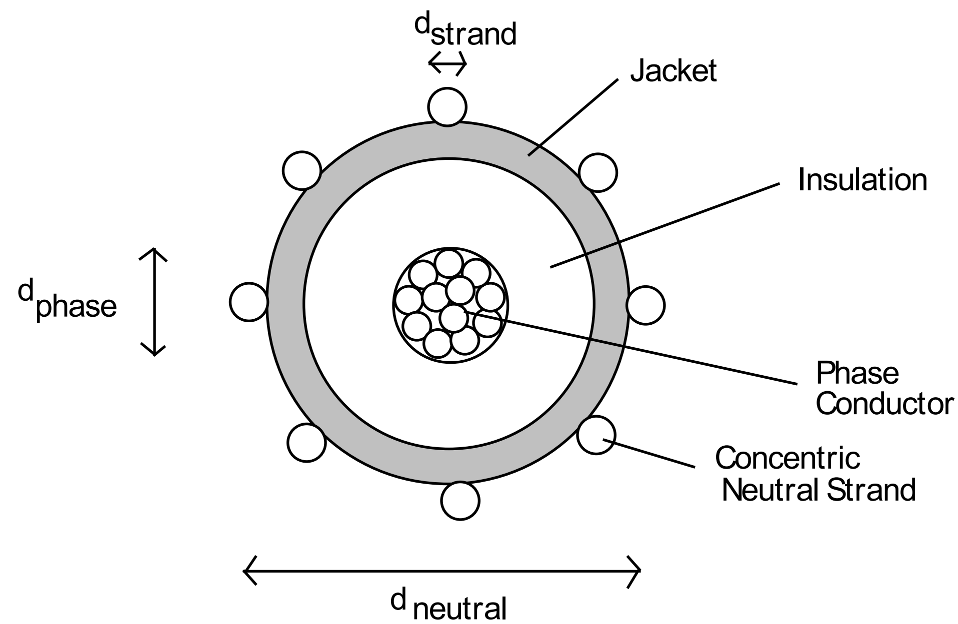

Fig. 3 Concentric Neutral Cable

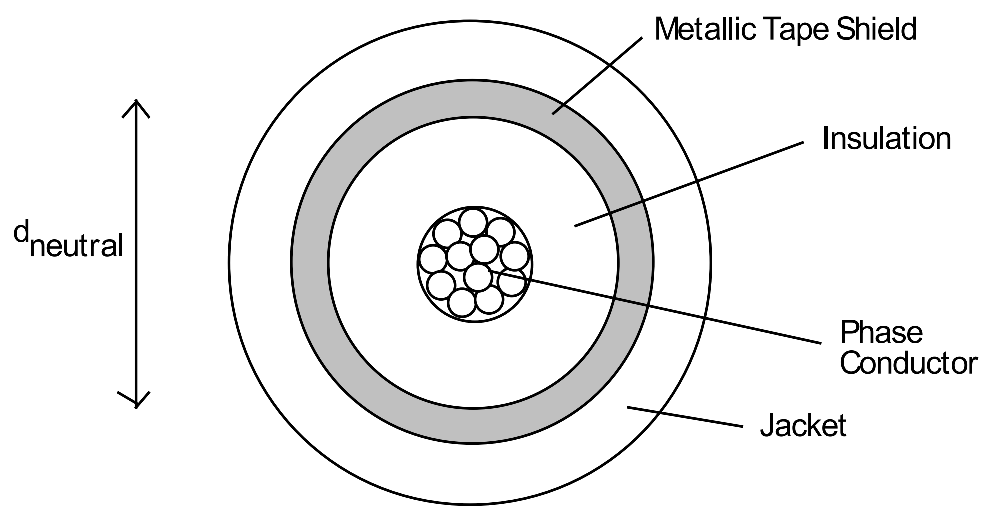

Fig. 4 Tape Shield Cable

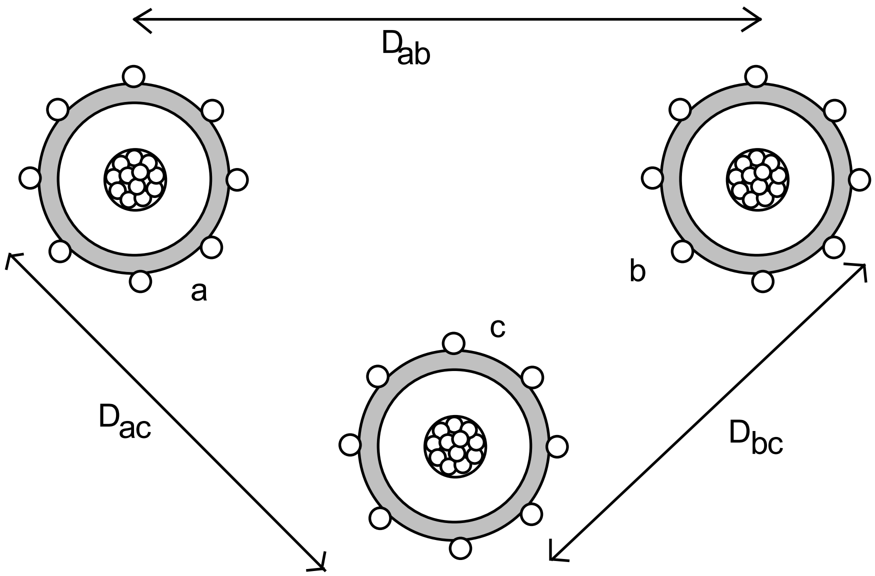

Fig. 5 Three-Phase Cable System

Concentric Neutral Cable

A concentric neutral cable has a phase conductor at the core which is surrounded by neutral strands. The neutral strands are equally spaced. The GMR for the phase conductor and resistance are simply those given in the table. However the GMR and resistance for the cable neutral must take all of the strands into account as follows:

$$GMR_{neutral} = \sqrt[n]{GMR_{strand} \times n \times \left(d_{neutral} / 2\right)^{n-1}} \tag{1}$$

where

GMR_{strand} = GMR of a single concentric neutral strand

n = number of neutral strands

$d_{neutral}/2$ = distance from center of phase conductor to center of concentric ring of neutral strands

The equivalent resistance of the neutral resistance is computed by considering that there are n individual neutral conductors in parallel so that

$$r_{neutral} = r_{strand} / n \tag{2}$$

The spacings needed for the impedance equations must distinguish between the distances associated with just one cable and those relating one cable to an adjacent cable. The phase conductors will be denoted by the subscript 'p' while the equivalent neutral conductors will be represented by the subscript 'n'. Subcripts 'i' and 'j' will be used to refer to cables. For the distance between the phase conductor of cable i and the concentric neutral for cable i

$$D_{ip,in} = d_{neutral} / 2 \quad (3)$$

and for the distance between a concentric neutral i and an adjacent concentric neutral j

$D_{in,jn} = D_{ij}$, the distance between the centers of two adjacent cables (4)

and finally the distance between a concentric neutral i and and an adjacent phase conductor j

$$D_{in,jp} = \sqrt[n]{\left(D_{ip,jp}\right)^n - \left(d_{neutral} / 2\right)^n} \quad (5)$$

Tape Shield Cable
A tape shield cable does not have the concentric neutral wire strands for a neutral, but only a metallic tape which acts as a shield. The AC resistance of the tape shield neutral can be given or computed using the cross-sectional area of the metallic tape. The geometric mean radius of the tape shield neutral, $GMR_{neutral}$, is simply the average radius of the tape shield. The spacings needed for impedance calculations are a bit more straightforward.

Dip,in = radius to center of tape shield

Dip,jp = distance between centers of phase conductors (6)

Dip,jn = distance between centers of phase conductors

Note that for a three-phase set of cables we will be getting a 6x6 matrix for a circuit with no separate neutral and a 7x7 set of equations with a separate neutral. This is because each cable has its own neutral.

Cable Shunt Capacitances
We can also calculate shunt capacitances for cables as well. This was neglected in the section on overhead circuits since the equivalent capacitances were so small. However for cables the charging currents associated with this capacitance can be rather large. For shielded cables, the electric field generated by the phase conductor will be limited to the space between the phase conductor and its neutral. Hence there will be no mutual capacitances, only self capacitances.

For a concentric neutral table, the self shunt admittance is given by

$$Y_{ip,in} = j\omega 2\pi\varepsilon_o\varepsilon_r (1609\ meters / mile) / \left[\ln\left(d_{neutral} / d_{phase}\right) - \left[\ln\left(n * d_{strand} / d_{neutral}\right)\right] / n\right] Siemens / mile \quad (7)$$

For a tape shield cable, the shunt admittance is given by

$$Y_{ip,in} = j\omega 2\pi\varepsilon_o\varepsilon_r(1609\ meters\ /\ mile)\ /\ \ln\left(d_{neutral}\ /\ d_{phase}\right)\ Siemens\ /\ mile \qquad (8)$$

Example Calculation

Precomputed cable impedances are often given in tables in terms of the positive and zero sequence values. Let us work through the calculations for a sample circuit and see how the equations given above can be applied. For our scenario consider a 15 kV, 3-Phase, 175-mil XLP Underground Cable. The insulation is 175-mil cross-linked polyethylene. The cable configuration consists of 3 identical single-phase concentric neutral cables with 1/3 size neutrals. The phase spacing is for a flat configuration of 7.5 inches, 7.5 inches and 15 inches. The earth resistivity is 100 meter-ohms. The conductor temperatures are 90^o C for the phase conductor and 70^o C for the neutral wire. For a 1/0 copper phase conductor with a #14 AWG sized neutral, a power engineering textbook lists the positive and zero sequence values in Ohms per thousand feet as

Z1=0.2182 + j0.0955 Ohms/1000 ft.
Z0=0.5215 + j0.2906 Ohms/1000 ft.

From Fig. 2 we can see that for the 1/0 cable that d_{phase} = 0.0311 feet and that $d_{neutral}$ = 0.0819 feet. The resistances are r_{phase} = 1.1088 Ohms/mile and $r_{neutral}$ = 2.9621 Ohms/mile. The pertinent GMRs are GMR_{phase} = 0.0118 feet and $GMR_{neutral}$ = 0.0318 feet.

For a six conductor system, we have 15 distances that need to be defined:

$D_{ap,bp} = D_{bp,cp} = D_{an,bn} = D_{bn,cn} =$ 0.625 feet
$D_{ap,cp} = D_{an,cn} =$ 1.25 feet
$D_{ap,an} = D_{bp,bn} = D_{cp,cn} =$ 0.0819/2 = 0.041 feet
$D_{ap,bn} = D_{bp,cn} = D_{bp,an} = D_{cp,bn} =$

$$\sqrt[n]{\left(D_{ip,jp}\right)^n - \left(d_{neutral}\ /\ 2\right)^n} = \sqrt[6]{\left(0.625\right)^6 - \left(0.0819\ /\ 2\right)^6} = 0.625$$

$$D_{ap,cn} = D_{cp,an} = \sqrt[n]{\left(D_{ip,jp}\right)^n - \left(d_{neutral}\ /\ 2\right)^n} = \sqrt[6]{\left(1.250\right)^6 - \left(0.0819\ /\ 2\right)^6} = 1.25$$

If the ground has a soil resistivity of 100 Ohm-meters and the calculations are for 60 Hz, then

$$D_e = 2160\sqrt{\frac{100}{60}} = 2788.5\ ft.$$

Applying the formulas for calculating the six-wire impedance matrix

$$Z'_{ap,ap} = R_{phase} + R_e + j0.1213 \ln \frac{D_e}{GMR_{phase}} = 1.1088 + 0.0953 + j0.1213 \ln \frac{2788.5}{.0118}$$
$$= 1.2041 + j1.5008 \ (Ohms\ /\ mile)$$

and

$$Z'_{ap,bp} = R_e + j0.1213 \ln \frac{D_e}{D_{ap,bp}} = 0.0953 + j0.1213 \ln \frac{2788.5}{0.625}$$
$$= 0.0953 + j1.0193 \ (Ohms\ /\ mile)$$

which when applied to the other terms leads to

$$\left[Z_{abcn}'\right] = \begin{bmatrix} 1.2041 + j1.5008 & 0.0953 + j1.0193 & 0.0953 + j0.9352 & 0.0953 + j1.3498 & 0.0953 + j1.0193 & 0.0953 + j0.9352 \\ & 1.2041 + j1.5008 & 0.0953 + j1.0193 & 0.0953 + 1.0193 & 0.0953 + j1.3498 & 0.0953 + j1.0193 \\ & & 1.2041 + j1.5008 & 0.0953 + j0.9352 & 0.0953 + j1.0193 & 0.0953 + j1.3498 \\ & & & 3.0574 + j1.3806 & 0.0953 + j1.0193 & 0.0953 + j0.9352 \\ & & & & 3.0574 + j1.3806 & 0.0953 + j1.0193 \\ & & & & & 3.0574 + j1.3806 \end{bmatrix}$$

Kron reducing out the effect of the three neutrals results in the 3 x 3 impedance matrix

$$\left[Z_{abc}\right] = \begin{bmatrix} 1.6766 + j0.8476 & 0.5379 + j0.3552 & 0.5111 + j0.2905 \\ & 1.6911 + j0.8160 & 0.5379 + j0.3552 \\ & & 1.6766 + j0.8476 \end{bmatrix}$$

(Ohms/mile)

Equivalent self and mutual inductances needed for sequence values are obtained by using averages

$$Z_s = \frac{\left(1.6766 + j0.8476\right) + \left(1.6911 + j0.8160\right) + \left(1.6766 + j0.8476\right)}{3} = 1.6814 + \mathrm{j}0.8370$$

$$Z_m = \frac{\left(0.5379 + j0.3552\right) + \left(0.5111 + j0.2905\right) + \left(0.5379 + j0.3552\right)}{3} = 0.5289 + \mathrm{j}0.3336$$

and using these terms to calculate

$$Z_1 = Z_s - Z_m = 1.1525 + j0.5034 \ \text{Ohms/mile}$$
$$Z_0 = Z_s + 2Z_m = 2.7393 + j1.5043 \ \text{Ohms/mile}$$

which in terms of Ohms per 1000 ft. turn out to be

$$Z_1 = Z_s - Z_m = 0.2183 + j0.0953 \text{ Ohms/1000 feet}$$
$$Z_0 = Z_s + 2Z_m = 0.5188 + j0.2849 \text{ Ohms/1000 feet}$$

These results are suprisingly good considering that the cable used in the reference book may not have had exactly the same parameters as the one assumed in this problem.