Unit 1.1: Transient Calculation of Fault Current

The fault current calculations introduced in an undergraduate course in power systems typically only considers the sinusoidal steady-state or symmetrical portion of the solution. This sinusoidal current, calculated using phasor analysis, is often referred to as symmetrical since the calculated current has symmetrical positive and negative peak values. However if we revisit the circuit shown in Fig. 1.1 and solve the differential equation which describes this circuit, we see that there is an additional transient DC component in the solution. This DC component will add to the peak current and should be considered when sizing components for short circuit duty. The current calculation which includes this DC transient component is referred to as the asymmetrical current and will be discussed in more detail below.

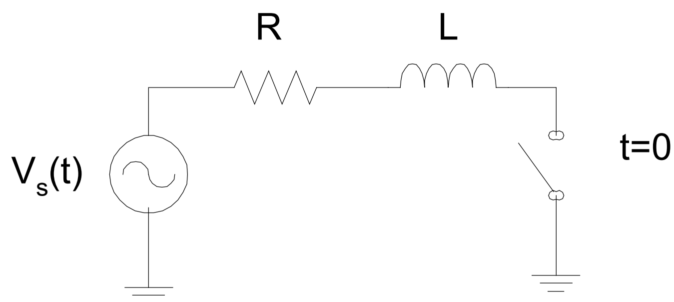

Fig. 1.1 Simple RL Model

Suppose that the equivalent circuit impedance can be described by a resistance R and an inductance L and that the circuit has an equivalent voltage source described by an RMS magnitude V_M and phase shift θ, then the differential equation which describes this circuit is given by:

$$L\frac{di}{dt}+Ri=\sqrt{2}V_M\sin\left(\omega t+\theta\right) \tag{1.1}$$

Since we would like to have the switch representing the fault close at time equal to zero, we represent the voltage at the time of the fault using the phase shift θ. Hence a fault occurring during a peak voltage corresponds to θ=90 degrees.

There are a number a ways to solve the ordinary differential equation. One way to do this would be to employ a classical approach in which one breaks the solution into a steady-state component and a transient component. The steady-state component, sometimes referred to as the particular component, corresponds to what happens in steady-state due to the sinusoidal forcing function. This can be obtained using phasor analysis techniques. The transient component, also referred to as the complementary solution, has a form that can be obtained by looking at the natural response of the circuit without a forcing function.

To obtain the steady-state portion of the solution, simply perform a phasor analysis and convert the result back to the time domain

$$Z=R+j\omega L=\sqrt{R^2+\left(\omega L\right)^2}\angle\phi \tag{1.2}$$

where

$$\phi = a\tan\left(\frac{\omega L}{R}\right) \tag{1.3}$$

Solving for the steady-state current phasor results in

$$\tilde{I}_{ss} = \frac{V_M \angle \theta}{Z} = \frac{V_M}{\sqrt{R^2 + (\omega L)^2}} \angle \theta - \phi \tag{1.4}$$

which in the time domain converts to

$$i_{ss}(t) = \frac{\sqrt{2} V_M}{\sqrt{R^2 + (\omega L)^2}} \sin(\omega t + \theta - \phi) \tag{1.5}$$

To obtain the form of the transient portion of the solution, one needs to look at the natural response of the circuit which is described by

$$L\frac{di}{dt} + Ri = 0 \tag{1.6}$$

The characteristic equation for determining the form of the transient solution is written as

$$\left(s + \frac{R}{L}\right) = 0 \tag{1.7}$$

where $s = -\frac{R}{L}$. Hence the transient form of the solution has the form

$$i_{tr}(t) = Ae^{-(R/L)t} \tag{1.8}$$

in which the constant A must be determined by applying initial circuit conditions.

The total solution for the fault current is the sum total of the steady-state and transient components, so that

$$i(t) = i_{ss} + i_{tr} = \frac{\sqrt{2} V_M}{\sqrt{R^2 + (\omega L)^2}} \sin(\omega t + \theta - \varphi) + Ae^{-(R/L)t} \tag{1.9}$$

To solve for the coefficient A we must apply an initial condition. If there is initially no load on the circuit, then the current flowing through the inductor before the fault occurs will be zero. Since the current flowing through the inductor cannot change instantaneously then

$$i(0^-) = i(0^+) = 0 \tag{1.10}$$

When we apply this initial condition to solve for A, for this initial condition at t=0, then we find that

$$A = -\frac{\sqrt{2} V_M}{\sqrt{R^2 + (\omega L)^2}} \sin(\theta - \varphi) \tag{1.11}$$

And finally

$$i(t)=\frac{\sqrt{2}V_M}{\sqrt{R^2+(\omega L)^2}}\left(\sin\left(\omega t+\theta-\tan^{-1}\left(\frac{\omega L}{R}\right)\right)-\sin\left(\theta-\tan^{-1}\left(\frac{\omega L}{R}\right)\right)e^{-\frac{R}{L}t}\right) \tag{1.12}$$

As one can see, the fault current is dependent on the relative difference between the inductance and resistance in the circuit as well as the point-on-wave at which the fault occurs.

This solution contains two terms, a steady-state component which is sinusoidal and an exponentially decaying DC component. The DC component is a transient which quickly decays. However this DC component can still cause stress on the components through which the fault current flows and should be considered. As an example, consider a 10 kV circuit with a total system impedance of 10 Ohms. For an X/R ratio of 10 and a fault occurring when $\theta = 0$, the fault current has the response shown in Fig. 1.2 below. Note the DC offset in the waveform due to the transient portion of the solution. This transient quickly dies out, resulting in a sinusoidal current which should correspond to an RMS value of V/Z = 1000 Amperes.

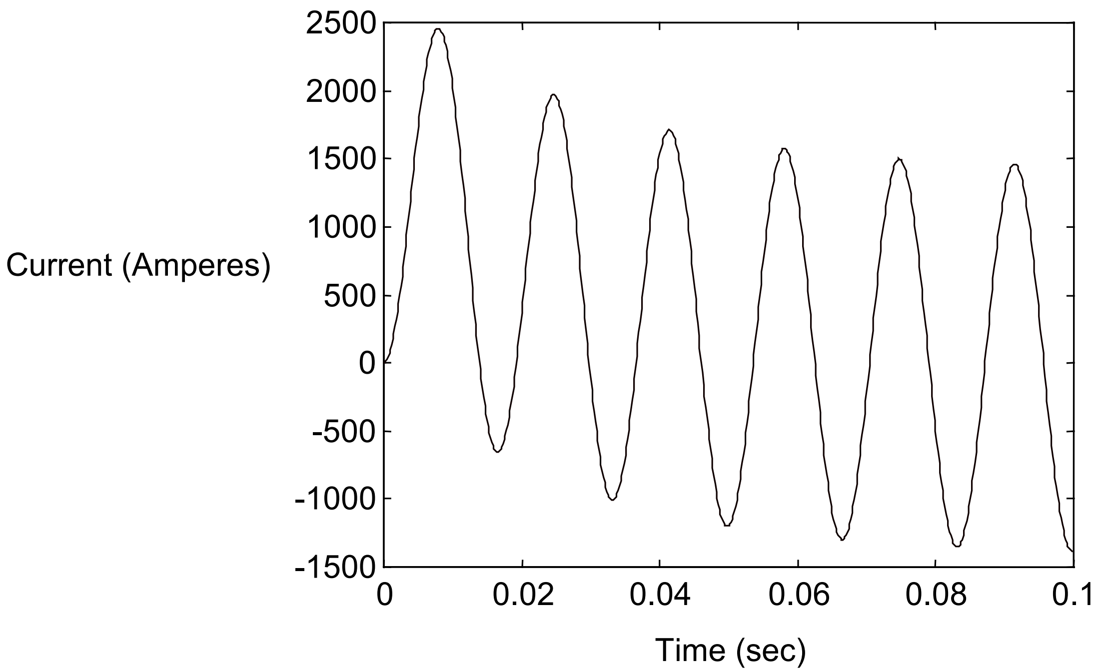

Fig. 1.2 DC Offset in Fault Current

The impact of the DC component depends on the point of the source voltage waveform at which the fault occurs. The variable θ is used to model the point on wave since the fault is assumed to switch on at t=0. In order to calculate the peak current, we can apply the theory of calculus and take the partial of the current equation above with respect to t and then θ. We take these two equations, set them both to zero and solve, in order to get the conditions which correspond to a current peak. What we find from this analysis is that the peak current corresponds to a condition where $\theta = 0$ and a time, t, given by

$$\omega \cos\left(\omega t - \tan^{-1}\left(\frac{X}{R}\right)\right) + \frac{R}{L}\sin\left(\tan^{-1}\left(\frac{X}{R}\right)\right)e^{-\frac{R}{L}t} = 0 \tag{1.13}$$

where X=ωL. This is a transcendental equation which must be solved for t. Note that is a function of the system frequency and the ratio of reactance to resistance.

A useful relationship which can be obtained from this analysis is the ratio between the peak of the asymmetrical current (which include the DC component) and the peak of the symmetrical current. Applying some computer analysis to the transcendental equation above for $\theta = 0$ and $\omega = 377$ radians/sec results in the following ratios for various X/R ratios:

Table 1.1 Offset as Function of X/R Ratio

X/R ratio	10	5	1
I_{asym}/I_{sym}	1.74	1.55	1.07

As the X/R ratio increases, the impact of the DC component becomes more noticeable. As the inductance increases to infinity this ratio would approach 2.0 and as the resistance increases to infinity this ratio would approach 1.0. An X/R ratio of 10 would be typical for high-voltage transmission circuits where the X/R ratio for lower-voltage distribution circuits would drop to 3 or below.

So how can we take the DC component account into our calculations? Well let us suppose that doing phasor analysis, we compute a symmetrical fault current of 1000 A RMS. The instantaneous peak value of this symmetrical current would be a factor of the square root of two larger, or 1414 A. Now let us assume that our circuit has an X/R ratio of 5, then we apply the 'fudge factor' of 1.55 and multiply it by our symmetrical current to get a possible peak asymmetrical current of 2192 A. This current is not the peak that will occur for every fault, but represents a worst case. This peak asymmetrical current represents the peak stress that a component in series with the fault must withstand.

The initial fault current does not always have to be zero. We could have a scenario in which a load exists as shown in Fig. 1.3. Note that the initial current could be computed through a steady-state analysis. The initiation of the fault shorts out the load.

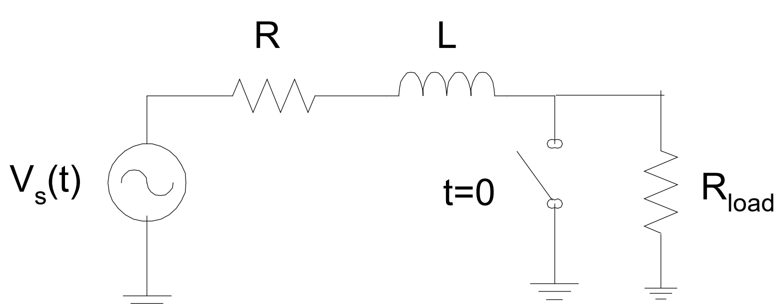

Fig. 1.3 Simple RL Model