Three-Phase Distributed-Parameter Lines Modeling

The same approach that was used to develop equations for a single-phase transmission line can also be applied to a three-phase line. To do this we need to work with voltage and current terms that are 3x1 matrices and inductance and capacitance terms that are 3x3 matrices. The differential three-phase line element is shown in Fig. 1. If we have a neutral or shield wire, these need to be reduced out using methods discussed in Unit 1. The capacitance and inductance relationships are now given by:

$$L = \begin{bmatrix} L_{aa} & L_{ab} & L_{ac} \\ L_{ba} & L_{bb} & L_{bc} \\ L_{ca} & L_{cb} & L_{cc} \end{bmatrix} \text{ Henries/meter} \tag{42}$$

$$C = \begin{bmatrix} C_{aa} & C_{ab} & C_{ac} \\ C_{ba} & C_{bb} & C_{bc} \\ C_{ca} & C_{cb} & C_{cc} \end{bmatrix} \text{ Farads/meter} \tag{43}$$

If we now represent the three-phase voltage and currents by:

$$i(x,t) = \begin{bmatrix} i_a(x,t) \\ i_b(x,t) \\ i_c(x,t) \end{bmatrix} \quad ; \quad v(x,t) = \begin{bmatrix} v_a(x,t) \\ v_b(x,t) \\ v_c(x,t) \end{bmatrix} \tag{44}$$

then we can show that the basic first and second-order transmission line relationships used before are still valid in matrix form

$$\frac{\partial v}{\partial x} = -L\frac{\partial i}{\partial t} \tag{45}$$

$$\frac{\partial i}{\partial x} = -C\frac{\partial v}{\partial t} \tag{46}$$

$$\frac{\partial^2 i}{\partial x^2} = LC\frac{\partial^2 i}{\partial t^2} \tag{47}$$

$$\frac{\partial^2 v}{\partial x^2} = LC\frac{\partial^2 v}{\partial t^2} \tag{48}$$

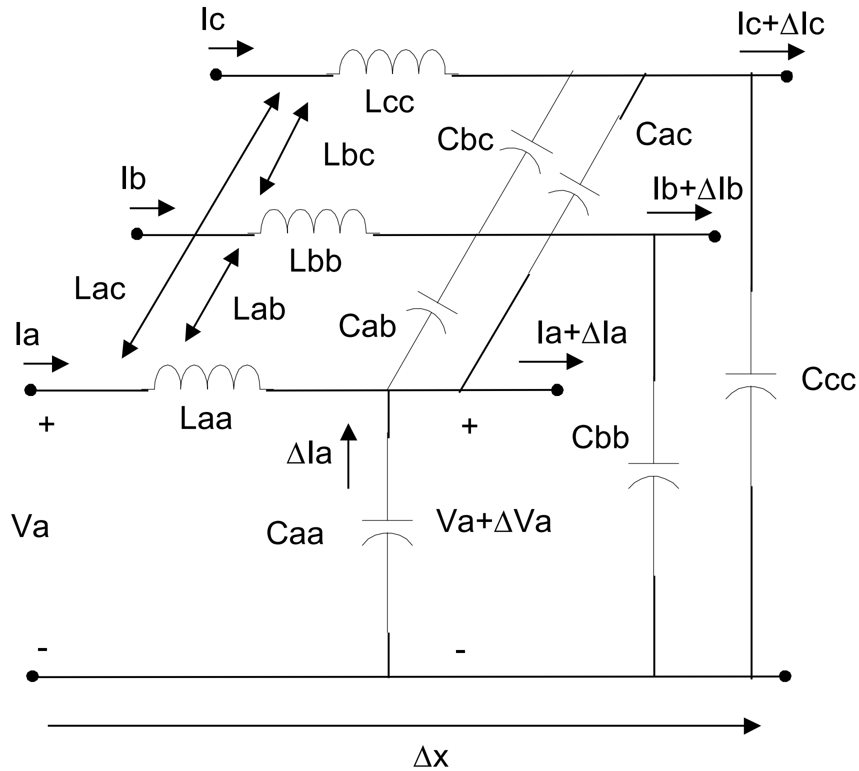

Fig. 17 Model for Three-Phase Differential Element

Equations (45) thru (48) represent sets of three-phase coupled differential equations which would be difficult to solve in their present form. To simplify the analysis we need to decouple these equations using a matrix transformation. The approach taken is similar to that used for symmetrical components. However we need to use a different transformation matrix since the symmetrical component transformation is a steady-state phasor relationship. The transformation matrix used in our case must be able to accommodate instantaneous values. In setting up this transformation it will be simpler to apply if we assume that the diagonal terms in the capacitance and inductance matrices are all equal and that the mutual coupling terms will all be the same as well. This could be based on the assumption that the line is continuously transposed, or could be at least modeled as such. Hence the capacitance and inductance matrices will be assumed to have the form:

$$L = \begin{bmatrix} L_s & L_m & L_m \\ L_m & L_s & L_m \\ L_m & L_m & L_s \end{bmatrix} \quad \text{Henries/meter} \qquad (49)$$

$$C = \begin{bmatrix} C_s & C_m & C_m \\ C_m & C_s & C_m \\ C_m & C_m & C_s \end{bmatrix} \quad \text{Farads/meter} \qquad (50)$$

We can obtain the equivalent self and mutual terms by taking the actual symmetrical values and performing an averaging as follows:

$$\begin{aligned} L_s &= \frac{1}{3}\left(L_{aa}+L_{bb}+L_{cc}\right) \\ L_m &= \frac{1}{3}\left(L_{ab}+L_{bc}+L_{ca}\right) \\ C_s &= \frac{1}{3}\left(C_{aa}+C_{bb}+C_{cc}\right) \\ C_m &= \frac{1}{3}\left(C_{ab}+C_{bc}+C_{ca}\right) \end{aligned} \tag{51}$$

The transformation we will apply here will be what is referred to as the Karrenbauer Transformation. This tranformation will relate phase voltages and currents to modal voltages and currents where

$$v_{p(hase)}(x,t) = \begin{bmatrix} v_a(x,t) \\ v_b(x,t) \\ v_c(x,t) \end{bmatrix} = K \begin{bmatrix} v_g(x,t) \\ v_{l1}(x,t) \\ v_{l2}(x,t) \end{bmatrix} = K v_{m(\text{mod}\,al)}(x,t) \tag{52}$$

$$i_{p(hase)}(x,t) = \begin{bmatrix} i_a(x,t) \\ i_b(x,t) \\ i_c(x,t) \end{bmatrix} = K \begin{bmatrix} i_g(x,t) \\ i_{l1}(x,t) \\ i_{l2}(x,t) \end{bmatrix} = K i_{m(\text{mod}\,al)}(x,t) \tag{53}$$

with

$$K = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -2 & 1 \\ 1 & 1 & -2 \end{bmatrix} \quad ; \quad K^{-1} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 0 \\ 1 & 0 & -1 \end{bmatrix} \tag{54}$$

The modal components g, l1, and l2 are named as follows, for reasons that will be given below:

G - Ground Mode
L1 – Line Mode 1
L2 – Line Mode 2

If we substitute the transformation relationships into equation (45) thru (48) above, we can show that

$$\frac{\partial v_m(x,t)}{\partial x} = -K^{-1}LK\frac{\partial i_m(x,t)}{\partial t} \tag{56}$$

$$\frac{\partial i_m(x,t)}{\partial x} = -K^{-1}CK\frac{\partial v_m(x,t)}{\partial t} \tag{57}$$

$$\frac{\partial^2 i_m(x,t)}{\partial x^2} = K^{-1}LCK\frac{\partial^2 i_m(x,t)}{\partial t^2} \tag{58}$$

$$\frac{\partial^2 v_m(x,t)}{\partial x^2} = K^{-1}LCK\frac{\partial^2 v_m(x,t)}{\partial t^2} \tag{59}$$

We see the decoupling occur when the 3x3 matrices get multiplied out

$$K^{-1}LK = \begin{bmatrix} L_s + 2L_m & 0 & 0 \\ 0 & L_s - L_m & 0 \\ 0 & 0 & L_s - L_m \end{bmatrix} \tag{60}$$

$$K^{-1}CK = \begin{bmatrix} C_s + 2C_m & 0 & 0 \\ 0 & C_s - C_m & 0 \\ 0 & 0 & C_s - C_m \end{bmatrix} \tag{61}$$

$$K^{-1}LCK = \begin{bmatrix} c_g^{-2} & 0 & 0 \\ 0 & c_l^{-2} & 0 \\ 0 & 0 & c_l^{-2} \end{bmatrix} \tag{62}$$

where

$$c_g = \frac{1}{\left[\left(L_s + 2L_m\right)\left(C_s + 2C_m\right)\right]^{1/2}} \tag{63}$$

$$c_l = \frac{1}{\left[\left(L_s - L_m\right)\left(C_s - C_m\right)\right]^{1/2}} \tag{64}$$

The transformation converts (56) thru (59) into three decoupled equations each. For example for (56) we get

$$\frac{\partial v_g(x,t)}{\partial x} = -\left(L_s + 2L_m\right)\frac{\partial i_g(x,t)}{\partial t} \tag{65}$$

$$\frac{\partial v_{l1}(x,t)}{\partial x} = -\left(L_s - L_m\right)\frac{\partial i_{l1}(x,t)}{\partial t} \tag{66}$$

$$\frac{\partial v_{l2}(x,t)}{\partial x} = -\left(L_s - L_m\right)\frac{\partial i_{l2}(x,t)}{\partial t} \tag{67}$$

and likewise for (59) this expands to

$$\frac{\partial^2 v_g(x.t)}{\partial x^2} = \frac{1}{c_g^2}\frac{\partial^2 v_g(x,t)}{\partial t^2} \tag{68}$$

$$\frac{\partial^2 v_{l1}(x.t)}{\partial x^2} = \frac{1}{c_l^2}\frac{\partial^2 v_{l1}(x,t)}{\partial t^2} \tag{69}$$

$$\frac{\partial^2 v_{l2}(x.t)}{\partial x^2} = \frac{1}{c_l^2}\frac{\partial^2 v_{l2}(x,t)}{\partial t^2} \tag{70}$$

These three sets of decoupled equations, for the ground, line 1, and line 2 modes respectively, tell us that a three-phase line can be represented by three decoupled single-phase lines. Each of the three single-phase circuits would be modeled the same way as we had modeled the single-phase line earlier. The first of these three circuits represents what we call the "ground mode", and it has an inductance given by Ls+2Lm, a capacitance given by Cs+2Cm, with a speed of propagation given by C_g. The second and third circuits represent what we call "line" modes. The inductance is given by Ls-Lm, the capacitance by Cs-Cm, and the speed of propagation is given by C_l.

To see what the physical meaning of the ground vs. line mode represents, suppose that we had a three-phase circuit in which we had a traveling waveform with the following modal magnitudes:

$$i_g = 0 \quad , \quad i_{l1} \neq 0 \quad , \quad i_{l2} = 0 \tag{71}$$

When we apply the K transformation and convert these values back to the phase domain we get

$$i_a = i_{l1} \quad , \quad i_b = -2i_{l1} \quad , \quad i_c = i_{l1} \tag{72}$$

Adding these quantities up gives us the earth/ground return current which is equal to zero. We could say in this case that the line mode is the component without an earth/ground return. Similarly if we look at the ground mode by itself, with

$$i_g \neq 0 \quad , \quad i_{l1} = 0 \quad , \quad i_{l2} = 0 \tag{73}$$

Again, applying the K transformation and converting these values back to the phase domain:

$$i_a = i_g \quad , \quad i_b = i_g \quad , \quad i_c = i_g \tag{74}$$

which sums up to an earth/neutral return current of $-3i_g$.

When we want to simulate a three-phase line transient the procedure is as follows:

1. Decompose the three-phase voltage or current into modal components using K^{-1}
2. Determine the equivalent line parameters for the ground and line modes. For the ground mode we have

$$c_g = \frac{1}{\sqrt{(L_s + 2L_m)(C_s + 2C_m)}} \qquad \tau_g = \frac{length}{c_g} \qquad Z_g = \sqrt{\frac{L_s + 2L_m}{C_s + 2C_m}}$$

and for the line modes

$$c_l = \frac{1}{\sqrt{(L_s - L_m)(C_s - C_m)}} \qquad \tau_l = \frac{length}{c_l} \qquad Z_l = \sqrt{\frac{L_s - L_m}{C_s - C_m}}$$

3. Next, simulate the transient on the ground, line 1, and line 2 mode single-phase equivalents, just like solving for a single-phase circuit.
4. Convert the results back to the phase domain using the K transformation.