Unit 1.2: Shunt Capacitor Switching

Capacitor switching is a common event on both transmission and distribution circuits. An equivalent circuit for this event type is shown in Fig. 1.4. The calculation of the loop current is found by solving the second order differential equation:

$$L\frac{di}{dt}+\frac{1}{C}\int idt+v_c\left(0\right)=\sqrt{2}V_m\sin\left(\omega t+\theta\right) \tag{1.14}$$

which can be rewritten as:

$$\frac{d^2i}{dt^2}+\frac{1}{LC}i=\frac{\omega\sqrt{2}V_m}{L}\cos\left(\omega t+\theta\right) \tag{1.15}$$

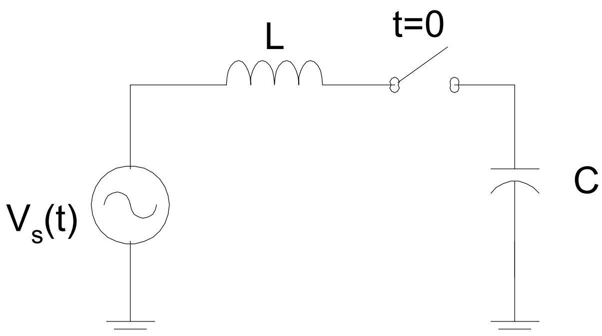

Fig. 1.4 Simple LC Model

There will be two initial conditions for this circuit, which will correspond to the initial inductor current and the initial capacitor voltage. At time t=0, if the system is initially unloaded, then the initial loop current is

$$i\left(0^-\right)=i\left(0^+\right)=0 \tag{1.16}$$

The capacitor will also have an initial charge, depending on how it was last switched out of the circuit. We can use this initial voltage to determine an initial condition for the first derivative of the current at t=0 by realizing that

$$L\frac{di\left(0\right)}{dt}=v_s\left(0\right)-v_c\left(0\right) \tag{1.17}$$

The solution for the loop current will have both a steady-state and a transient component. For a source voltage given by

$$v_s\left(t\right)=\sqrt{2}V_m\sin\left(\omega t+\theta\right) \tag{1.18}$$

then the steady-state current could be determined using phasor analysis as

$$\tilde{I}_{ss}=\frac{V_m\angle\theta}{j\left(\omega L-\frac{1}{\omega C}\right)} \tag{1.19}$$

with

$$i_{ss}(t) = \frac{-\sqrt{2}V_m}{\omega L - \frac{1}{\omega C}} \cos(\omega t + \theta) \tag{1.20}$$

The transient portion solution is obtained by solving:

$$\frac{d^2 i}{dt^2} + \frac{1}{LC} i = 0 \tag{1.21}$$

which has a response dictated by the roots of the characteristic equation

$$s^2 + \frac{1}{LC} = 0 \tag{1.22}$$

where $s_1 = +j\frac{1}{\sqrt{LC}}; s_2 = -j\frac{1}{\sqrt{LC}}$. This results in a transient response of the form

$$i_{tr}(t) = A_1 \cos(\omega_0 t) + A_2 \sin(\omega_0 t) \tag{1.23}$$

with

$$\omega_o = \frac{1}{\sqrt{LC}} \tag{1.24}$$

The complete solution for the current is now given by:

$$i(t) = \frac{-\sqrt{2}V_m}{\omega L - \frac{1}{\omega C}} \cos(\omega t + \theta) + A_1 \cos(\omega_0 t) + A_2 \sin(\omega_0 t) \tag{1.25}$$

To determine the coefficients we apply the initial conditions at t=0:

$$i(0) = 0 = \frac{-\sqrt{2}V_m}{\omega L - \frac{1}{\omega C}} \cos(\theta) + A_1 \tag{1.26}$$

$$\frac{di(0)}{dt} = \frac{v_s(0) - v_c(0)}{L} = \frac{\sqrt{2}V_m \omega}{\omega L - \frac{1}{\omega C}} \sin(\theta) + A_2 \omega_0 \tag{1.27}$$

For which we see that:

$$A_1 = \frac{\sqrt{2}V_m}{\omega L - \frac{1}{\omega C}} \cos(\theta) \tag{1.28}$$

$$A_2 = \frac{v_s(0) - v_c(0)}{\omega_0 L} - \frac{\sqrt{2}V_m (\omega / \omega_0)}{\omega L - \frac{1}{\omega C}} \sin(\theta) \tag{1.29}$$

which when substituted into (1.25) results in

$$i(t) = \frac{-\sqrt{2}V_m}{\omega L - \frac{1}{\omega C}}\cos(\omega t + \theta) + \frac{\sqrt{2}V_m}{\omega L - \frac{1}{\omega C}}\cos(\theta)\cos(\omega_0 t) + \left(\frac{v_s(0) - v_c(0)}{\omega_0 L} - \frac{\sqrt{2}V_m(\omega/\omega_0)}{\omega L - \frac{1}{\omega C}}\sin(\theta)\right)\sin(\omega_0 t) \tag{1.30}$$

What this solution tells us is that switching in a capacitor will result in a current transient which will appear as a high frequency transient superimposed on the power system frequency current. Since we did not include any resistance in our circuit, then this transient does not get damped out.

Since $\omega_o >> \omega$ in most practical power systems, we can make an assumption to simplify the solution. We can assume that the transient is at a high enough frequency that the source voltage looks like a constant, DC value. This is especially true is we were to incorporate damping into the circuit equation. If the source can be modeled as a constant voltage then we only need to solve:

$$L\frac{di}{dt} + \frac{1}{C}\int i dt + v_c(0) = V_m \sin(\theta) \tag{1.31}$$

which can be rewritten as:

$$\frac{d^2 i}{dt^2} + \frac{1}{LC} i = 0 \tag{1.32}$$

The complete solution for the current is now given by:

$$i(t) = A_1 \cos(\omega_0 t) + A_2 \sin(\omega_0 t) \tag{1.33}$$

To determine the coefficients we apply the initial conditions at t=0:

$$i(0) = 0 = A_1 \tag{1.34}$$

$$\frac{di(0)}{dt} = \frac{v_s(0) - v_c(0)}{L} = \omega_0 A_2 \tag{1.35}$$

For which we see that:

$$A_1 = 0 \tag{1.36}$$

$$A_2 = \frac{v_s(0) - v_c(0)}{\omega_0 L} = \frac{v_s(0) - v_c(0)}{\left(\frac{1}{\sqrt{LC}}\right)L} = \frac{v_s(0) - v_c(0)}{\sqrt{\frac{L}{C}}} \tag{1.37}$$

which give us a total current of

$$i(t) = \frac{v_s(0) - v_c(0)}{\sqrt{\frac{L}{C}}} \sin(\omega_o t) \tag{1.38}$$

The term in the denominator has units of impedance and occurs in a variety of power system calculations. It is often called the surge impedance and it is defined by

$$Z_o = \sqrt{\frac{L}{C}} \tag{1.39}$$

One can see that the peak of this transient current is defined by the ratio of the voltage across the switch when the switching action occurs and the surge impedance. Since the time of closing of a mechanical switch is somewhat of a random occurrence, then field measurements will show a variety of transient magnitudes for actual capacitor switchings.

If for some reason we needed to minimize this surge current, there are a number of corrective actions we could take. We could increase the surge impedance or introduce some resistive damping into the circuit. Another possibility is synchronous switching, in which a control is used to close the switch when the voltage across it is minimized.

From the viewpoint of customer power quality, we are often more interested in computing the transient voltage across the capacitor. The customer would be connected in parallel with a power factor correction capacitor being switched on and off as shown in Fig. 1.5. Assuming that the customer load impedance is very high, we can then calculate the capacitor voltage by integrating the current as follows:

$$\begin{aligned}
v_c(t) &= \frac{1}{C}\int_0^t i\,dt + v_c(0) = \frac{1}{C}\int_0^t \frac{v_s(0) - v_c(0)}{Z_o}\sin(\omega_o t)\,dt + v_c(0) \\
&= \frac{1}{C}\left(\frac{v_s(0) - v_c(0)}{Z_o}\right)\left[-\frac{\cos(\omega_o t)}{\omega_o}\bigg|_0^t\right] + v_c(0) = \frac{1}{C}\left(\frac{v_s(0) - v_c(0)}{\sqrt{\frac{L}{C}}}\right)\left[-\frac{\cos(\omega_o t) - 1}{\frac{1}{\sqrt{LC}}}\right] + v_c(0) \\
&= \left(v_s(0) - v_c(0)\right)\left(1 - \cos(\omega_o t)\right) + v_c(0) = v_s(0) - \left(v_s(0) - v_c(0)\right)\cos(\omega_o t)
\end{aligned} \tag{1.40}$$

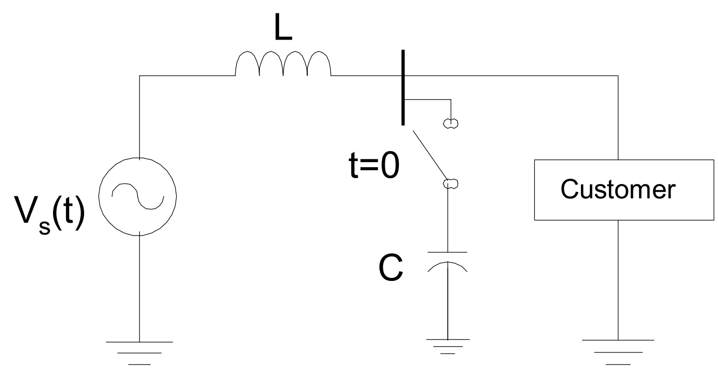

Fig. 1.5 Simple LC Model with Customer

The solution given in (1.40) tells us that the capacitor voltage will oscillate about the source voltage. If the source voltage corresponds to the peak of the voltage source, and the capacitor is charged up to the negative of the source peak so that $v_c(0) = -\sqrt{2}V_m$, this equation tells us that we could theoretically have a voltage across the capacitor which will reach 3x$\sqrt{2}V_M$ when $\omega_o t = \pi$.

Of course a real power system would have some damping associated with it that would keep peaks from approaching this magnitude. A more realistic scenario which includes some line resistance is shown in Fig. 1.6.

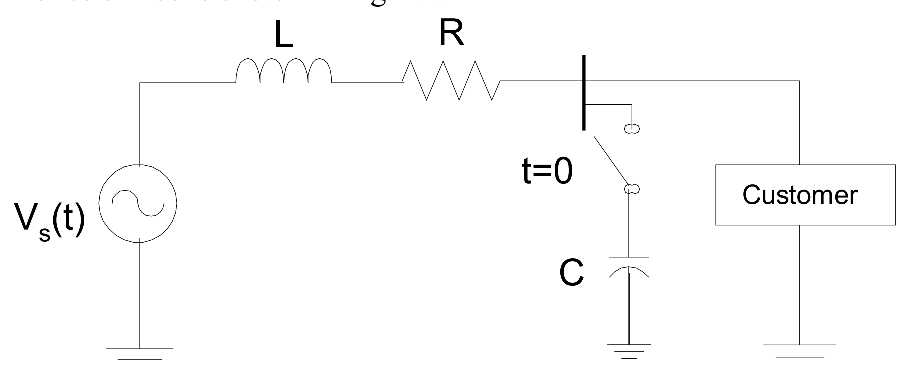

Fig. 1.6 Capacitor Switching in Circuit with Damping

Again, rather than trying to come up with the full-blown solution for the current, let us assume that the source voltage can be assumed to be constant. So to get the current we must solve:

$$L\frac{di}{dt} + Ri + \frac{1}{C}\int idt + v_c(0) = \sqrt{2}V_m \sin(\theta) \tag{1.41}$$

which can be rewritten as:

$$\frac{d^2i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{1}{LC}i = 0 \tag{1.42}$$

which has a response dictated by the roots of the characteristic equation

$$s^2 + \frac{R}{L}s + \frac{1}{LC} = 0 \tag{1.43}$$

where assuming that the circuit is underdamped

$$s_1 = -\frac{R}{2L} + j\frac{\sqrt{4/LC - (R/L)^2}}{2}$$

$$s_2 = -\frac{R}{2L} - j\frac{\sqrt{4/LC - (R/L)^2}}{2} \tag{1.44}$$

This results in a response of the form:

$$i(t) = e^{-\left(\frac{R}{2L}\right)t}\left(A_1\cos(\omega_1 t) + A_2\sin(\omega_1 t)\right) \tag{1.45}$$

where

$$\omega_1 = \frac{\sqrt{4/LC - (R/L)^2}}{2} = \sqrt{\omega_o^2 - (R/2L)^2} \tag{1.46}$$

To determine the coefficients we apply the initial conditions at t=0:

$$i(0) = 0 = A_1 \tag{1.47}$$

$$\frac{di(0)}{dt} = \frac{v_s(0) - v_c(0)}{L} = \omega_1 A_2 \tag{1.48}$$

which give us a total current of

$$i(t) = \frac{v_s(0) - v_c(0)}{\omega_1 L}e^{-\left(\frac{R}{2L}\right)t}\sin(\omega_1 t) \tag{1.49}$$

Since the source is actually sinusoidal, we should not assume that (1.49) means that the capacitor current goes to zero, since there will be a steady-state sinusoidal component. What this is telling us is that the high-frequency transient component is damped and will be driven to zero.

The characteristic equation given in (1.43) has several other solutions. If there is a large amount of resistance in the circuit, the solution would be overdamped. When $\left(\frac{R}{L}\right)^2 > 4\omega_o^2$, then both of the roots of the characteristic equation are real with

$$s_1 = -\frac{R}{2L} + \frac{\sqrt{(R/L)^2 - 4/LC}}{2}$$

$$s_2 = -\frac{R}{2L} - \frac{\sqrt{\left(R/L\right)^2 - 4/LC}}{2} \tag{1.50}$$

This will give us a response that consists of two decaying exponential terms. This could occur if the resistance represented a large damping resistor. It is also possible to have a situation where the roots are exactly equal to each other, and this case is referred to as critically damped.