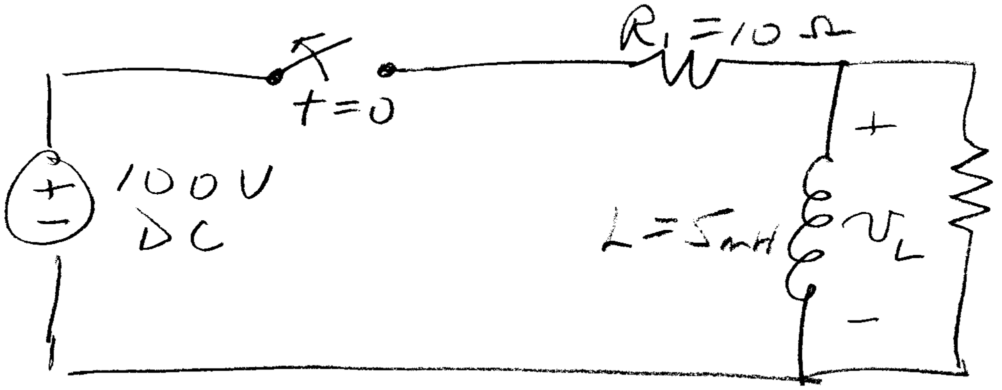
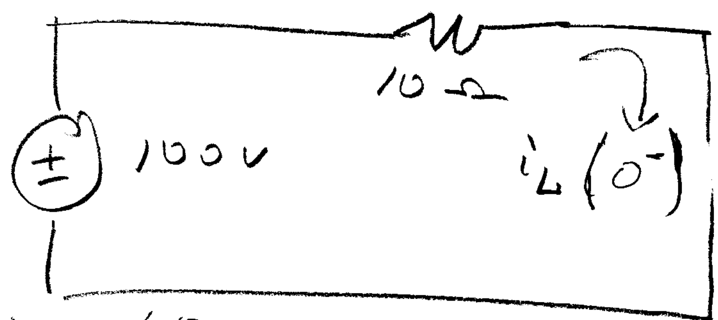
ECE 587 – OLD EXAM

FIRST-ORDER RL TRANSIENT EXAMPLE. FIND $v_L(t)$



BEFORE SWITCHING ACTION, IN STEADY-STATE, INDUCTOR LOOKS LIKE A SHORT CIRCUIT

t < 0

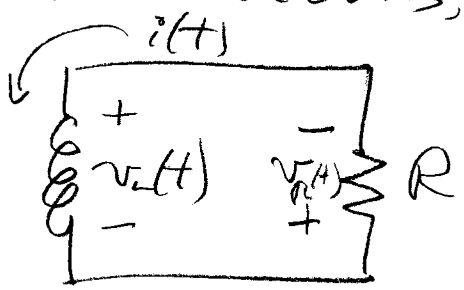


NOTE R_2 SHORTED OUT, HAS NO CURRENT

SO $i_L(0^-) = \frac{100\,V}{10\,\Omega} = 10\,A$

AFTER SWITCHING OCCURS, HAVE THIS CIRCUIT

t ≥ 0



WHERE $i(t=0) = i(0^-) = 10\,A$

WRITING ODE

$$L\frac{di(t)}{dt} + R\,i(t) = 0$$

IN STEADY-STATE $i_{ss}(t) = 0$ SINCE THERE IS NO SOURCE

FOR TRANSIENT PORTION OF SOLUTION, CHARACTERISTIC EQUATION FOUND FROM

$$Ls\,I(s) + R\,I(s) = 0 = (Ls + R)\,I(s) = 0$$

OR $(Ls + R) = 0$ WHERE $i_{TR}(t) = Ae^{st} = Ae^{-\frac{R}{L}t}$

RL FIRST-ORDER (CONT.)

TOTAL SOLUTION $i(t) = i_{ss}(t) + i_{TR}(t) = Ae^{-R/L\,t} \quad t \geq 0$

AT $t=0$ $\; i(0) = Ae^{0} = A = 10\,A$; $\frac{R}{L} = \frac{10}{0.005} = 2000$

$\Rightarrow i(t) = 10\,e^{-2000t}\ A \quad t \geq 0$

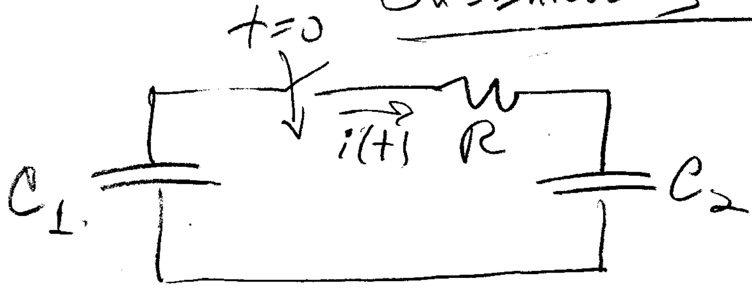
$$v_L(t) = L\,\frac{di_L(t)}{dt} = L\left[10(-2000)e^{-2000t}\right] V$$

$$\boxed{= -100\,e^{-2000t}\ V, \; t \geq 0}$$

SO VOLTAGE DECAYS FROM −100 TO 0.

NOTE BEFORE SWITCHING ACTION $v_L(0^-) = 0\,V$.
AT $t=0$, $v_L(0) = -100\,V$. BUT THIS IS OK SINCE VOLTAGE ACROSS INDUCTOR CAN CHANGE INSTANTANEOUSLY.

GREENWOOD 2.3



$R = 5\,\Omega$
$C_1 = 60\,\mu F$
$C_2 = 40\,\mu F$
$Q_{C_1}(0) = 1\,C\,;\; Q_{C_2}(0) = 0$

$$Q = CV \Rightarrow V_{C_1}(0) = \frac{1}{60\,\mu F} = \underline{16{,}667\,V}$$

$V_{C_2}(0) = 0\,V$

WRITING KVL $\quad +\frac{1}{C_1}\int i\,dt + v_{C_1}(0) + Ri + \frac{1}{C_2}\int i\,dt + v_{C_2}(0) = 0$

TAKING DERIVATIVE OF BOTH SIDES $\quad \frac{d}{dt}\left(+\frac{1}{C_1}\int i\,dt + \cancel{v_{C_1}(0)}^{0} + Ri + \frac{1}{C_2}\int i\,dt + \cancel{v_{C_2}(0)}^{0} = 0\right)$

$$+\frac{1}{C_1}i + R\frac{di}{dt} + \frac{1}{C_2}i = 0$$

$$\frac{di}{dt} + \frac{1}{RC_{eq}}i = 0 \qquad \text{WHERE } C_{eq} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2}}$$

CHAR EQN. $\quad s + \frac{1}{RC_{eq}} \Rightarrow s = -\frac{1}{RC_{eq}}\,;\quad C_{eq} = 24\,\mu F \quad \frac{1}{RC_{eq}} = 8333$

IN STEADY-STATE $\quad i_{ss}(t) = 0$ SO

$$i(t) = i_{TR}(t) = Ae^{-\frac{1}{RC_{eq}}t} \qquad t \geq 0$$

AT $t=0$, SINCE CAPACITOR VOLTAGE CANNOT CHANGE INSTANTANEOUSLY

$$i(0) = \frac{16{,}667 - 0}{5} = 3{,}333\,A$$

SO APPLYING INITIAL CONDITION

$$i(0) = 3{,}333 = Ae^{-\frac{1}{RC_{eq}}(0)} = A \Rightarrow A = 3{,}333$$

$$\boxed{i(t) = 3333\,e^{-8333t}\,A \qquad t \geq 0}$$

(a) PEAK CURRENT $= \underline{3{,}333\,A}$

(b) $i(200\,\mu sec) = \underline{629.6\,A}$

GREENWOOD 2.3

(c.) $v_{c2}(t) = \frac{1}{C_2}\int_0^t i\,dt + v_{c2}(0)$

$= \frac{1}{C_2} 3333 \frac{1}{-8333} e^{-8333t}\Big|_0^t + 0$

$= -10{,}000 e^{-8333t} + 10{,}000 + 0$

As $t \to \infty \Rightarrow v_{c2}(\infty) = \underline{10{,}000V}$

$E = \frac{1}{2} C_2 v^2 = \frac{1}{2}(40\times10^{-6})(10{,}000)^2 = \boxed{2000 \text{ Joules}}$

(d.) $v_{c1}(t) = \frac{1}{C_1}\int_0^t (-i)\,dt + v_{c1}(0)$

$= \frac{1}{C_1}(-3333)\left(\frac{1}{-8333}\right) e^{-8333t}\Big|_0^t + 16{,}667$

$= 6{,}666 e^{-8333t} - 6{,}666 + 16{,}667$

As $t \to \infty$ $v_{c1}(0) = \underline{10{,}000V}$

NOTE IN EQUILIBRIUM, CAPACITORS HAVE SAME VOLTAGE.

INITIAL ENERGY IN C_1 IS $\frac{1}{2}(60\times10^{-6})(16{,}667)^2$
$= 8333$ JOULES

FINAL ENERGY IN C_1 IS $\frac{1}{2}(60\times10^{-6})(10{,}000)^2$
$= 3000$ JOULES

RESISTOR DISSIPATES $8333 - (2000 + 3000) = \underline{3{,}333 \text{ J.}}$