Unit 2.2: EMTP Simulation Approach

Inductor and Capacitor Modeling

The Electromagnetic Transient Program (EMTP) is based on a system simulation approach initially described by Dommel. This approach solves the circuit equations by employing the trapezoidal rule, where

$$x(t) = x(t-\Delta t) + \frac{\Delta t}{2}\left(f(t-\Delta t) + f(t)\right) \tag{2.24}$$

This is referred to as the trapezoidal rule since the integral is approximated by the use of a trapezoidal fit.

To see how this approach works, let us consider how an inductor could be modeled.

The variables associated with the inductor are shown in Fig. 2.6.
Note that instead of writing this in terms of branch voltages, the equations are written in terms of node voltages.
We first start out with the relationship between inductor voltage and current, with

$$v_k(t) - v_m(t) = L\frac{di_{k,m}(t)}{dt} \tag{2.25}$$

Solving this in terms of the inductor current

$$i_{k,m}(t) = i_{k,m}(t-\Delta t) + \frac{1}{L}\int_{t-\Delta t}^{t}\left(v_k(t) - v_m(t)\right)dt \tag{2.26}$$

Applying the trapezoidal rule results in

$$i_{k,m}(t) = i_{k,m}(t-\Delta t) + \frac{\Delta t}{2}\left(\left(\frac{v_k(t-\Delta t) - v_m(t-\Delta t)}{L}\right) + \left(\frac{v_k(t) - v_m(t)}{L}\right)\right) \tag{2.28}$$

Rearranging these terms

$$i_{k,m}(t) = \frac{\Delta t}{2L}\left(v_k(t) - v_m(t)\right) + i_{k,m}(t-\Delta t) + \frac{\Delta t}{2L}\left(v_k(t-\Delta t) - v_m(t-\Delta t)\right) \tag{2.29}$$

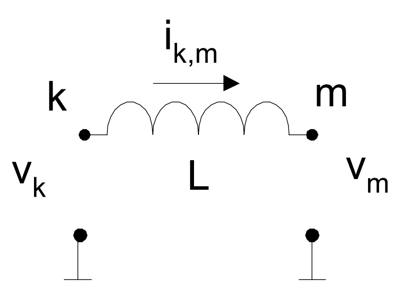

Fig. 2.6 Inductor Example

Equation (2.29) tells us that the inductor current at a given time t is a function of the voltage across the inductor at time t plus the current and voltage at the previous time $t-\Delta t$.

Hence the current state of the inductor is dependent on past states, as if the inductor had a memory property.

To show this more clearly we often break equation (2.29) into two parts, where

$$i_{k,m}(t)=\frac{\Delta t}{2L}\left(v_k(t)-v_m(t)\right)+I_{k,m}(t-\Delta t) \tag{2.30}$$

$$I_{k,m}(t-\Delta t)=i_{k,m}(t-\Delta t)+\frac{\Delta t}{2L}\left(v_k(t-\Delta t)-v_m(t-\Delta t)\right) \tag{2.31}$$

Equation (2.30) and (2.31) can be now represented by the equivalent model shown in Fig. 2.7.
This model shows that an inductor can be represented by an equivalent resistance and current source in parallel.

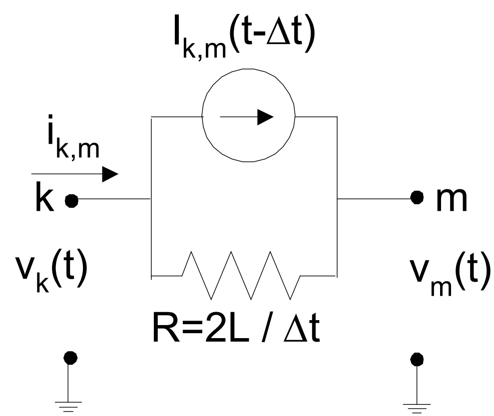

Fig. 2.7 Inductor Equivalent Model

A similar model can now be developed for the capacitor. For the capacitor, voltage and current are related by

$$v_k(t)-v_m(t)=\frac{1}{C}\int_{t-\Delta t}^{t} i_{k,m}(t)dt+v_k(t-\Delta t)-v_m(t-\Delta t) \tag{2.32}$$

where again, the voltages are given in terms of node voltages.

Substituting in for the trapezoidal approximation results in

$$v_k(t)-v_m(t)=\frac{\Delta t}{2C}\left(i_{k,m}(t-\Delta t)+i_{k,m}(t)\right)+v_k(t-\Delta t)-v_m(t-\Delta t) \tag{2.33}$$

Rearranging the terms in order to solve for the current

$$i_{k,m}(t)=\frac{2C}{\Delta t}\left(v_k(t)-v_m(t)\right)-i_{k,m}(t-\Delta t)-\frac{2C}{\Delta t}\left(v_k(t-\Delta t)-v_m(t-\Delta t)\right) \tag{2.34}$$

Again, this can be interpreted using an equivalent circuit model as shown below where

$$i_{k,m}(t) = \frac{2C}{\Delta t}\left(v_k(t) - v_m(t)\right) + I_{k,m}(t - \Delta t) \tag{2.35}$$

$$I_{k,m}(t - \Delta t) = -i_{k,m}(t - \Delta t) - \frac{2C}{\Delta t}\left(v_k(t - \Delta t) - v_m(t - \Delta t)\right) \tag{2.36}$$

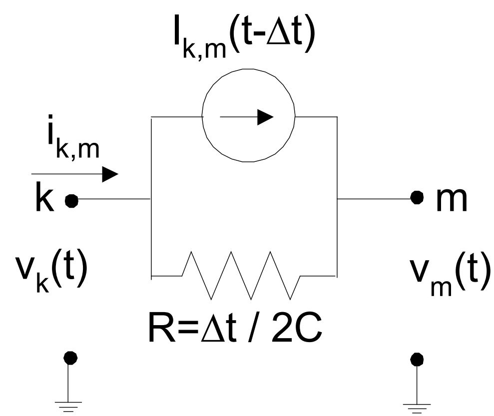

Fig. 2.8 Capacitor Equivalent Model

System Modeling Approach

To see how these types of inductor and capacitor models can be applied, Consider the sample circuit shown below. It consists of both a voltage and current source as well as regular circuit elements. What we want to do in this case is solve for the voltages at nodes 1 and 2.

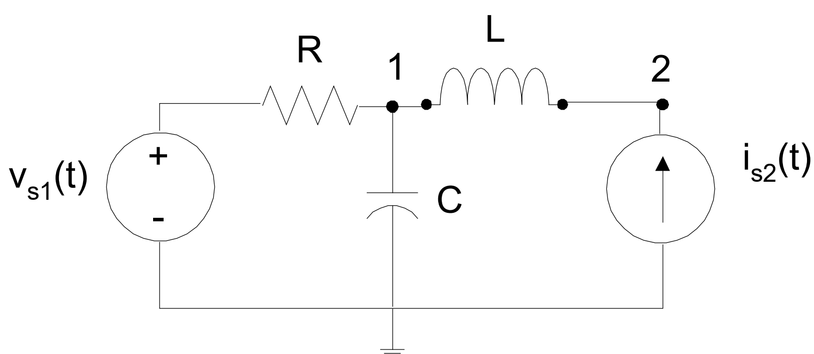

Fig. 2.9 Example Circuit

To apply the trapezoidal formulation developed above, replace the inductor and capacitor in the circuit with the appropriate equivalent models as shown in Fig. 2.10.

Write current law equations at nodes 1 and 2

$$\frac{v_1(t)-v_{s1}(t)}{R}+\frac{v_1(t)}{\Delta t/2C}+I_{1,0}(t-\Delta t)+\frac{v_1(t)-v_2(t)}{2L/\Delta t}+I_{1,2}(t-\Delta t)=0 \tag{2.37}$$

$$\frac{v_2(t)-v_1(t)}{2L/\Delta t}-I_{1,2}(t-\Delta t)-i_{s2}(t)=0 \tag{2.38}$$

with

$$I_{1,2}(t-\Delta t)=i_{1,2}(t-\Delta t)+\frac{\Delta t}{2L}\left(v_1(t-\Delta t)-v_2(t-\Delta t)\right) \tag{2.39}$$

$$I_{1,0}(t-\Delta t)=-i_{1,0}(t-\Delta t)-\frac{2C}{\Delta t}\left(v_1(t-\Delta t)-v_0(t-\Delta t)\right) \tag{2.40}$$

Rearranging these equations results in

$$\left(\frac{1}{R}+\frac{1}{\Delta t/2C}+\frac{1}{2L/\Delta t}\right)v_1(t)+\left(\frac{-1}{2L/\Delta t}\right)v_2(t)=\frac{1}{R}v_{s1}(t)-I_{1,0}(t-\Delta t)-I_{1,2}(t-\Delta t) \tag{2.41}$$

$$\left(\frac{-1}{2L/\Delta t}\right)v_1(t)+\left(\frac{1}{2L/\Delta t}\right)v_2(t)=+i_{s2}(t)+I_{1,2}(t-\Delta t) \tag{2.42}$$

Note that we have a set of equations which are of the form

$$[Y][v]=[i(t)]-[I] \tag{2.43}$$

The admittance matrix $[Y]$ is found similarly to the way the Ybus matrix is formed for steady-state power system analysis. The diagonal entries are simply the sum of the admittance connected to the appropriate node. The off-diagonal entries correspond to the negative of the admittance connecting the appropriate nodes. The right-hand-side of the equality contains current injection terms. The first current term $[i(t)]$ represents the impact of ideal voltage and current sources in the circuit. The second current term $[I]$ represents the system states at the previous time step.

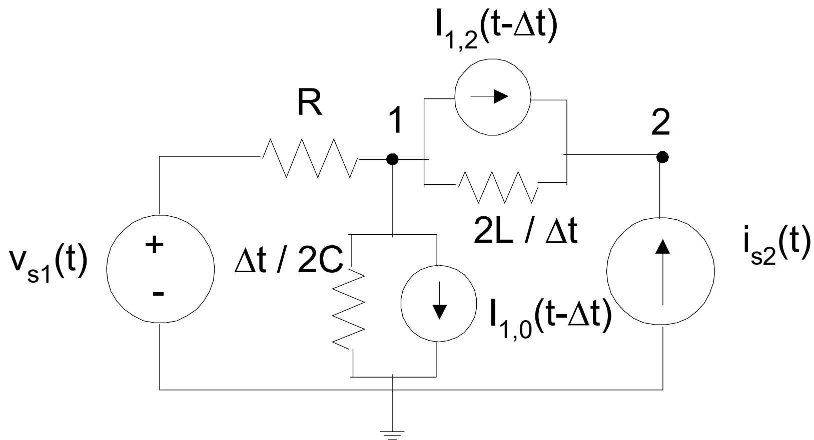

Fig. 2.10 Equivalent Model for Example Circuit

The overall algorithm for simulating they system shown in Fig. 2.9 would occur as follows:

1. Compute the initial conditions for the circuit. We will need to compute both currents and voltages.

2. Select a time step and compute the admittance matrix

3. Increment the time index by the time step

4. Determine the current injection matrices

5. Solve (2.43) to get the node voltages

6. Update the branch currents using the appropriate model

7. If simulation is not finished, go to step 3.

We will see an example of how this algorithm can be applied in the next section.

Extended Example