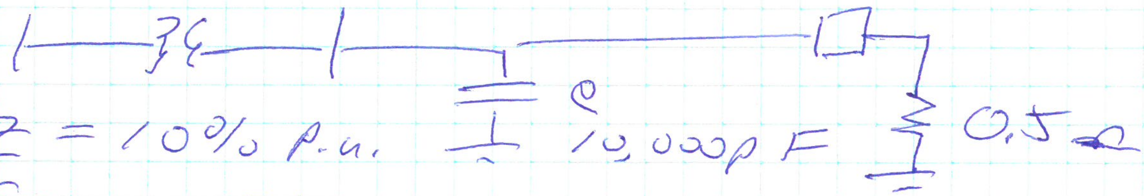
ECE 587 – FALL 2019

TRANSIENT RECOVERY VOLTAGE EXAMPLE

69kV 12.47kV



$Z = 10\%$ p.u.

$S_{BASE} = 25$ MVA

ASSUME CB CLEARS THE FAULT AT CURRENT ZERO. FIND TRV AND PEAK STRESS.

$$Z_{BASE} = \frac{(V_{BASE,LINE})^2}{S_{BASE}} = \frac{(12.47)^2}{25} = 6.22\,\Omega$$

$$\underline{R=0} \Rightarrow L = \frac{X}{\omega} = \frac{0.1 \times 6.22}{377} = 1.65 \times 10^{-3}\,H$$

1.65 mH

$$\tilde{I}_f = \frac{\tilde{V}_s}{R_{fault} + j\omega L} = \frac{\frac{12.47}{\sqrt{3}}\angle\theta}{0.5 + j\,0.622} = \underset{RMS}{9.02\,kA}\angle{-51.2^\circ}$$

$$v_s(t) = \frac{\sqrt{2}\,(12{,}470)}{\sqrt{3}} \cos(377t + \theta)$$

$$i_f(t) = \sqrt{2}\; 9{,}020 \cos(377t + \theta - 51.2^\circ)$$

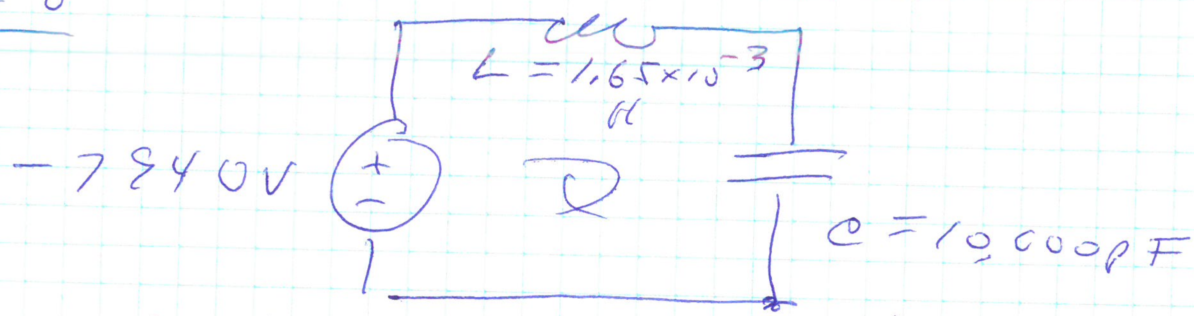
FIND θ, $i_f(t=0) = 0$ $\quad \cancel{377t}^{\,0} + \theta - 51.2^\circ = \frac{\pi}{2}, \frac{3\pi}{2}$

$$\theta = \frac{\pi}{2} + 51.2^\circ$$

$$v_s\left(\theta = \frac{\pi}{2} + 51.2^\circ, t = 0\right) = -7.94 \times 10^3\ V$$

ASSUME $\omega_0 \gg \omega = 377$ R/S

$t = 0$



AT $t = 0$ $\quad i_L(0) = 0 \; ; \; v_c(0) = 0$

KVL $\quad L\frac{di}{dt} + v_c = V_s \; ; \; i = C\frac{dv_c}{dt}$

$$\frac{1}{LC}\left(LC\frac{d^2 v_c}{dt^2} + v_c = V_s\right)$$

$$\frac{d^2 v_c}{dt^2} + \frac{1}{LC} v_c = \frac{1}{LC} V_s$$

$$v^c_{ss}(t) = V_s$$

$$\frac{d^2 v_c}{dt^2} + \frac{1}{LC} v_c = 0 \Longrightarrow \text{CHAR EQN}$$

$$S^2 + \frac{1}{LC} = 0$$

$$v^c_{TR}(t) = A_1 \cos \omega_0 t + A_2 \sin \omega_0 t \qquad S_1, S_2 = \pm j\frac{1}{\sqrt{LC}}$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$v_c(t) = V_s + A_1 \cos \omega_0 t + A_2 \sin \omega_0 t$$

$$v_c(0) = 0 = V_s + A_1 \Longrightarrow A_1 = -V_s$$

$$\frac{dv_c(0)}{dt} = \frac{i_c(0)}{C} = 0 = 0 - A_1 \omega_0 \sin(0) + A_2 \omega_0 \cos(0)$$

$$A_2 = 0$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = 2.46\times10^{5} \text{ RAD/SEC}$$

$(\sim 40 \text{ KHz})$

$$v_c(t) = -7940\left[1-\cos(2.46\times10^{5}t)\right] \quad t\ge 0$$

$$\left|V_{\substack{PEAK\\STRESS}}\right| = 2\times7940 \simeq \underline{15.9 \text{ kV}}$$

$$\frac{X}{R_s} = 6 \qquad Z_{BASE} = 6.22\,\Omega$$

$$|Z| = \sqrt{R^2 + X^2} = 0.1\times6.22 = 0.622\,\Omega$$

$$X = 6R \qquad 0.622 = \sqrt{R^2+(6R)^2} = \sqrt{37}\,R_s$$

$$R_s = \frac{0.622}{\sqrt{37}} = 0.102\,\Omega \Rightarrow X = 6R = 0.613\,\Omega$$

$$\hat{I}_f = \frac{\hat{V}_s}{(R_f+R_s)+j\omega L} = \frac{12.47/\sqrt{3}}{(0.5+0.102)+j0.613} = 8.37\angle\theta-45.5^\circ \text{ KA}$$

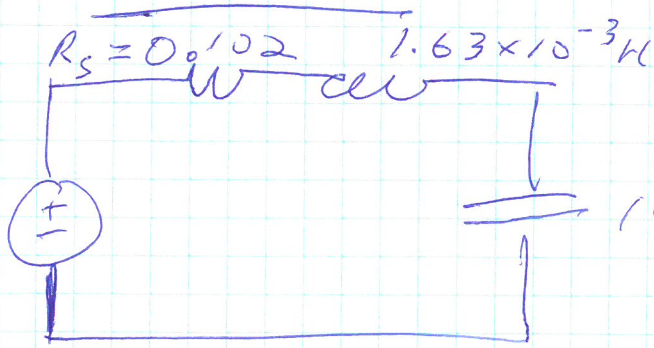
$$v_s(t) = \frac{\sqrt{2}\,12{,}470}{\sqrt{3}}\cos(377t+\theta)$$

$$i_f(t) = \sqrt{2}\,8{,}374\cos(377t+\theta-45.5^\circ)$$

For $i_f(t=0)=0 \Rightarrow \theta-45.5^\circ = \frac{\pi}{2}$; $\theta = \frac{\pi}{2}+45.5^\circ$

$$v_s\left(\frac{\pi}{2}+45.5^\circ\right) = \underline{-7262\text{ V}}$$

For $t=0$



$V_s = -7262$

$i_L(0) = i_c(0) = 0$

$v_c(0) = 0$

$\longrightarrow$

KVL $L\frac{di}{dt} + Ri + v_c = V_s$; $i = C\frac{dv_c}{dt}$

$$LC\frac{d^2v_c}{dt^2} + RC\frac{dv_c}{dt} + v_c = V_s$$

$$\boxed{\frac{d^2v_c}{dt^2} + \frac{R}{L}\frac{dv_c}{dt} + \frac{1}{LC}v_c = \frac{V_s}{LC}}$$

$$v_{SS}(t) = K \Rightarrow \cancel{\frac{d^2K}{dt^2}}^{0} + \frac{R}{L}\cancel{\frac{dK}{dt}}^{0} + \frac{1}{LC}K = \frac{V_s}{LC}$$

$$K = V_s \Rightarrow v_{SS}(t) = V_s = -7262$$

CHAR. EQN $s^2 + \frac{R}{L}s + \frac{1}{LC} = 0 \Rightarrow b = \frac{R}{L},\ c = \frac{1}{LC}$

$$s_1, s_2 = \frac{-b}{2a} \pm \frac{\sqrt{b^2-4ac}}{2a} = -\underbrace{31.4}_{\alpha} \pm j\underbrace{2.48\times10^5}_{\omega_1}$$

$$v_{c_{TR}}(t) = e^{-31.4t}\left[A_1\cos(2.48\times10^5t) + A_2\sin(2.48\times10^5t)\right]$$

$$\boxed{v_c(t) = -7262 + e^{-31.4t}\left[A_1\cos(2.48\times10^5t) + A_2\sin(2.48\times10^5t)\right]}$$

APPLY BOUNDARY CONDITIONS

$$v_c(0) = 0 = -7262 + \cancel{e^{(0)}}^{1}\left[A_1\cancel{\cos(0)}^{1} + A_2\cancel{\sin(0)}^{0}\right]$$

$$\boxed{A_1 = 7262}$$

APPLY CHAIN RULE $\frac{d}{dt}\left[f(t)g(t)\right] = \frac{df(t)}{dt}g(t) + f(t)\frac{dg(t)}{dt}$

$$\frac{dv_c(0)}{dt} = \frac{i_L(0)}{C} = 0 = 0 - 31.4\cancel{e^{(0)}}^{1}\left[A_1\cancel{\cos(0)}^{1} + A_2\cancel{\sin(0)}^{0}\right]$$
$$+ e^{(0)}\left[-A_1\omega_1\cancel{\sin(0)}^{0} + A_2\omega_1\cancel{\cos(0)}^{1}\right]$$

$$\Rightarrow A_2 = \frac{31.4A_1}{\omega_1} = \frac{31.4(7262)}{2.48\times10^5} = \underbrace{0.9204}_{\text{SMALL!}}$$

$$v_c(t) = -7262 + e^{-31.4t}\left[7262\cos(2.48\times10^{5}t) + \underbrace{\cancel{0.9204\sin(2.48\times10^{5}t)}}_{\simeq 0}\right]$$

LOOK AT $2.48\times10^{5}t = \pi$

$$t = 1.2674\times10^{-5}$$

$$v_{c\,PEAK} \simeq -7262 + \underbrace{e^{-31.4(1.2674\times10^{-5})}}_{=0.9996}\left[7262(-1)\right]$$

NOT MUCH ATTENUATION!

$$\simeq 2(-7262)$$

$$\simeq \underline{-14.52}$$

SOMEWHAT CLOSE TO WHAT WE SEE IN PSCAD SIMULATION

SO ADDING RESISTANCE DID DROP PEAK STRESS FROM 15.9 → 14.5 kV, BUT NOT A HUGE REDUCTION.

MOSTLY THIS WAS DUE TO SOURCE VOLTAGE AT FAULT CLEARING TIME BEING LOWER IN MAGNITUDE AS OPPOSED TO EXPONENTIAL DAMPING IN SOLUTION.