

SheafSystem™ Programmer's Guide

A Guided Tour of the SheafSystem™ Libraries

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1 Introduction

This document shows how to use the basic features of the SheafSystem. It is intended as an introduction for beginners, providing a guided tour of the essential capabilities of the SheafSystem libraries. It does not show how to use every feature of the system; readers are expected to use it as a starting place for further exploration of the reference documentation and class libraries.

The C++ examples in the document are available as source code in the examples subdirectory of the SheafSystemProgrammersGuide module and the reader is encouraged to build and execute the examples along with reading the text. The examples are numbered and the source for example N is in exampleN.cc. There are a few compilation and execution examples in the text, these are given in Linux using the csh shell, Gnu C++, and Gnu make.

2 What you'll need

To take full advantage of this document, you'll need a few things in addition to the document itself, namely:

- an installed copy of the SheafSystemProgrammersGuide module, which includes the examples,
- a C++ compiler,
- a web browser, for viewing the reference documentation, and
- an installed copy of the SheafSystem.

3 The SheafSystem installation

The SheafSystem installer installs all the files of the SheafSystem in a directory tree. We will have to refer to the root of this directory tree repeatedly, so to simplify the notation, we'll let <sheaf_dir> refer to the full path to the root directory of the installation, for instance:

<sheaf_dir> = /usr/local/SheafSystem

Wherever you see <sheaf_dir> in this document, mentally replace it with the full path to your SheafSystem installation.

The installation includes 4 configurations of the libraries: Debug-contracts, Debug-no-contracts, Release-contracts, and Release-no-contracts. The "Debug" configurations are unoptimized and contain symbol information for use by interactive debuggers such as gdb. The "Release" configurations are optimized and contain no debugging information. We'll describe "Contracts" below. Generally speaking, the Release configurations are higher performance than the Debug configurations and the no-contracts configurations are much faster than the contract configurations.

The examples will compile and execute with any configuration, but we will always use the Debug-contracts configuration in the text below.

4 Part I: The sheaf component

4.1 Getting started

4.1.1 PartSpace metaphor

The Part Space document describes the fundamental concepts of the SheafSystem in non-mathematical terms using the common notion of basic and composite parts, tables, and table schema. This document assumes the reader is familiar with the Part Space metaphor.

4.1.2 Sheaf tables

As described in the Part Space document, a SheafSystem database is a collection of tables. Each table is equipped with a covering relation graph describing the lattice order of its rows and another graph describing the lattice order of its columns. Each such object table has an associated table called its schema table and the row graph of the schema table defines the column graph of the object table. A member of the row lattice is represented by a node in the row graph. A member also has a corresponding row in the table if and only if it is a basic part, a join irreducible member (jim) in the row lattice.

There are 3 special tables. the primitive schema table, the primitives table, and the namespace table. The primitives schema table terminates the schema recursion, it is its own schema table. The primitives table describes each primitive type supported by the system.

4.1.3 Namespaces

A namespace table is a special table in each database that serves as a container and table of contents for all the other tables. The SheafSystem includes 3 predefined namespace types: the `sheaves_namespace`, the `fiber_bundles_namespace`, and the `geometry_namespace`. Each of these predefines the sheaf schema for the C++ types

defined in the sheaf, fiber_bundle, and geometry components, respectively. (The fields component doesn't have its own schema).

Creating an instance of a namespace is typically the first thing a client must do to use the SheafSystem, so we start with an example of how to do that using the most basic namespace, sheaves_namespace. This example will also cover the basic mechanics of compiling and linking with the SheafSystem.

4.1.3.1 Example 1: Hello, Sheaf

```
#include "sheaves_namespace.h"
#include "std_iostream.h"

using namespace sheaf;

int main( int argc, char* argv[])
{
    // Create a standard sheaves namespace.

    sheaves_namespace lns("Hello-sheaf");

    // Write its name to cout.

    cout << lns.name();

    return 0;
}
```

This code is in the SheafSystemProgrammersGuide module in examples/sheaf/example1.cc along with a Makefile, the first part of which reads:

```
#
# Path to your C++ compiler
#
CXX =
#
# Path the SheafSystem installation
#
SHEAF_INSTALL_DIR =
SHEAF_LIB_DIR = $(SHEAF_INSTALL_DIR)/Debug-contracts/lib
SHEAF_INC_DIR = $(SHEAF_INSTALL_DIR)/include

example1: example1.cc
    $(CXX) -o example1 -I$(SHEAF_INC_DIR) -L$(SHEAF_LIB_DIR) example1.cc -lsheaves
```

To compile and link the example, you first have to configure the Makefile to your installation by setting the 2 variables CXX and SHEAF_INSTALL_DIR to the actual values for your installation. Then we can compile and link by:

```
>make example1
```

This command will compile example1.cc and link it with the shared library libsheaves.so to create an executable example1 in the current directory. We have to tell the dynamic loader where to find the shared library by setting the environment variable

LD_LIBRARY_PATH to contain the path to the SheafSystem library directory, that is, the same value we set SHEAF_LIB_DIR to in the Makefile, for instance:

```
>setenv LD_LIBRARY_PATH <sheaf_dir>/Debug-contracts/lib
```

To run example32, the LD_LIBRARY_PATH also has to contain the path to the vtk libraries:

```
>setenv LD_LIBRARY_PATH <sheaf_dir>/Debug-contracts/lib:<sheaf_dir>/lib/vtk
```

Now we can execute the example:

```
>./example1  
Hello-sheaf
```

The "handcrafted" Makefile is provided to show the basic mechanics of creating an application with the SheafSystem. The examples module also comes with a CMake build system which doesn't show the details, but may be easier to use, especially on Windows. See the installation how-to document for instructions on using the CMake system.

We've created a sheaves_namespace in this example, but before we can do much with it, we need to learn a few programming patterns that the SheafSystem uses repeatedly.

4.2 Programming patterns

There are a few design features shared by all the classes in the SheafSystem. In this section we will give a quick introduction to the most ubiquitous of these patterns. We'll introduce some more patterns later, as we need them, and also go into some of these initial patterns in more detail.

4.2.1 Design by contract

The sheaf system is implemented using the "design by contract" programming paradigm. We'll cover the essentials of the method and how they are used in the SheafSystem. For a more detailed introduction, see the excellent book Design By Contract, by Example by Richard Mitchell and Jim McKim.

When using design by contract, each class is equipped with an invariant, a set of assertions that must be true at any time control returns the client. (The invariant is not defined when control is within a member function of the class.) Every member function is equipped with preconditions and postconditions. The preconditions are assertions that must be true when control enters the member function; the postconditions must be true when control leaves the member function. The "contract" in "design by contract" is between the client and the member function: if the client guarantees the preconditions are true, the member function ensures the invariant and the postconditions are true.

The invariant, precondition, and postcondition assertions are specified using "invariance", "require", and "ensure" macros, respectively, in the source code. If contracts are enabled when the library is compiled, these clauses will be evaluated as part of the execution of

the member functions. If the conditions specified in the clauses are not true, execution throws an exception with an error message, which usually terminates the program.

The contracts are extremely useful for detecting improper use of the classes and member functions and are thus an important debugging tool. Once client code is correct, the contracts can be disabled to improve efficiency.

The SheafSystem Debug-contracts and release-contracts configurations are compiled with contracts enabled. The Debug-no-contracts and Release-no-contracts are compiled with contracts disabled.

The contracts are also published as an essential part of the reference documentation and are critical to using the sheaf system correctly. Let's look at the reference documentation for the `sheaves_namespace` constructor we used in `example1`. The reference documentation is generated in html, so you can open it with your browser. The main page is `<sheaf_dir>/documentation/C++/index.html`. If you browse to the documentation for class `sheaves_namespace` and click on the constructor `sheaves_namespace(const string& xname)`, you'll find:

`sheaf::sheaves_namespace::sheaves_namespace (const string & xname)`

Creates a sheaves namespace with name `xname`.

Precondition

- `poset_path::is_valid_name(xname)`

Postcondition

- `invariant()`
- `name() == xname`
- `lin_jim_edit_mode()`
- `host() == 0`
- `!index().is_valid()`
- `index().same_scope(member_hub_id_space(false))`
- `has_standard_subposet_ct()`
- `current_namespace() == this`
- `state_is_not_read_accessible()`

So what does this tell us? The precondition:

- `poset_path::is_valid_name(xname)`

tells us exactly what conditions the argument `xname` has to satisfy if we want this call to the constructor to work correctly, namely `is_valid_name(xname)` has to be true. Well, what does that take? If we look up `poset_path::is_valid_name` we find:

`static bool sheaf::poset_path::is_valid_name(const string & xname)`

True if `xname` is not empty and contains only name legal characters.

Postcondition

- `result == (!xname.empty() && (xname.find_first_not_of(name_legal_characters()) == string::npos))`

So `xname` can't be empty and can't contain any characters not in `name_legal_characters()`.

If we click on `name_legal_characters` we find:

static const string & sheaf::poset_path::name_legal_characters()

The characters a name is allowed to contain.

Postcondition

- `result == "ABCDEFGHIJKLMNOPQRSTUVWXYZabcdefghijklmnopqrstuvwxyz0123456789_-,.=+()*?:?"`

So `xname` has to be non-empty and contain only the above characters.

If `xname` satisfies these conditions, which it does in `example1`, then the postcondition gives a great deal of information about what state the `sheaves_namespace` object is in after construction.

The first postcondition is:

- `invariant()`

that is, the invariant has to be satisfied. As we said above, this is an implicit postcondition of every member function, even if we don't explicitly provide it as part of the contract. So what does this mean for `sheaves_namespace`? Well, click on `invariant()` to find:

virtual bool sheaf::namespace_poset::invariant () const

Class invariant.

Invariant

- `poset_state_handle::invariant()`
- `host() == 0`
- `!index().is_valid()`
- `!is_external()`
- `is_attached() ? primitives().is_attached() : true`
- `is_attached() ? (primitives().index() == PRIMITIVES_INDEX) : true`
- `state_is_read_accessible() ? primitives().state_is_read_accessible() : true`
- `is_attached() ? primitives_schema().is_attached() : true`
- `is_attached() ? (primitives_schema().index() == PRIMITIVES_SCHEMA_INDEX) : true`
- `state_is_read_accessible() ? primitives_schema().state_is_read_accessible() : true`

`Sheaves_namespace` inherits `namespace_poset` and doesn't override the invariant, which is a virtual function, so the invariant of `sheaves_namespace` is the invariant of `namespace_poset`. The invariant in a derived class must be at least as strong as the invariant in the base space, so the invariant of `namespace_poset` calls the invariant of its

base class, `poset_state_handle`. Beyond whatever `poset_state_handle::invariant()` ensures, the `namespace_poset` invariant ensures several properties of the data members, `primitives()` and `primitives_schema()` in particular.

As this invariant shows, the conditional expression

- `x ? y : true`

appears frequently in the contracts, so it is worth describing in more detail. As an assertion, this expression can be read "x implies y", that is, x can be either true or false, but if x is true, then y must be true as well. If x is false, there is no condition on y.

The reader is encouraged to examine the `poset_state_handle` invariant to learn what additional invariances `sheaves_namespace` has inherited, but we'll move on to the rest of the postcondition of the constructor. The next postcondition is one you'd expect:

- `name() == xname`

that is, the name of the namespace is the name we gave it.

The remainder of the postconditions ensure various arcane properties of the namespace that we're not very interested in right now. But when your tackling a tough debugging problem, any of these may be just the piece of information you need!

The power of the design by contract method comes from the great amount of detailed information contained in the assertions and two further properties. First, if contracts are turned on, that is if you are using either the Debug-contracts or Release-contracts configuration of the library, the pre- and post-conditions of a function are executed whenever the function is called. Second the contracts exhibited in the documentation are extracted directly from the source code. The combination of the two allows you to reason about the behavior of the code with great confidence while designing, programming, and especially while debugging.

So what happens if the contract for a member function is *not* satisfied? Let's find out by trying to create a `sheaves_namespace` without a name.

4.2.1.1 Example 2: contract for `sheaves_namespace` constructor.

```
#include "sheaves_namespace.h"

using namespace sheaf;

int main( int argc, char* argv[])
{
    cout << "SheafSystemProgrammersGuide Example2:" << endl;

    // Attempt to create a standard sheaves namespace
    // with an empty name. This violates the preconditions
    // of the constructor and will throw an exception and abort.

    sheaves_namespace lns("");
```

```
// Done.

return 0;
}
```

If we compile and run this, assuming we still have `LD_LIBRARY_PATH` set from running example1, we get:

```
>make example2
>./example2
terminate called after throwing an instance of 'std::logic_error'
  what(): 'poset_path::is_valid_name(xname)' in file namespace_poset.cc at line 1941
Abort
```

The error message tells you exactly what assertion failed. If you're debugging, you can walk back up the stack from where the exception was actually thrown to the assertion that failed and inspect local variables, for instance `xname`, to determine what went wrong.

4.2.2 Concurrency control

One of the attractive features of the sheaf data model is that its mathematical formalism provides a natural language for describing concurrency and parallelism. The sheaf system libraries were designed for concurrent programming using an access control mechanism based on the monitor design pattern. Currently, this mechanism is only partially implemented and the SheafSystem libraries are delivered with the access control mechanism disabled. Programmers nevertheless must be aware of certain aspects of the access control mechanism, which we describe in this section. More complete examples are included in Appendix A.

Access to every table is controlled. A client thread can have no access, read access, or read-write access. At any given time, either no client has access, exactly one client has read-write access, or one or more clients have read access. Before reading or writing a table or any of its members, a client must request read access or read-write access, respectively. After accessing the table, the client must release access. If a client requests read access and another client already has write access, or vice versa, the request blocks until the other client releases the conflicting access.

The concurrency control mechanism is "enforced" through precondition clauses in the table member functions. In order to make concurrency control apparent to the client and avoid dead lock, the library routines do not themselves request or release access without the client knowing it. Instead, they "publish" their access requirements as preconditions and let the client control the access.

For instance, in example 1 we invoked `sheaves_namespace::name()`. Consulting the reference documentation, we find for the `name()` member function:

virtual string sheaf::namespace_poset::name() const

The name of this namespace.

Precondition

- `state_is_read_accessible()`

So, if the access control mechanism is enabled, the client must request read access, invoke the name function, and release access:

```
lns.get_read_access();
cout << lns.name() << endl;
lns.release_access();
```

Getting and releasing access can be a tedious programming chore. Furthermore, it is syntactically impossible in some cases, for instance within a pre- or post-condition clause. So many member functions offer an "auto-access" option. These routines will automatically get and release the access they need, if the client allows it by setting an auto-access argument to true. If invoked with the auto-access argument false, the client must get the required access before making the call. These routines also publish their access requirements as preconditions. For instance, the auto-access version of the name function is:

virtual string sheaf::namespace_poset::name(bool xauto_access) const

The name of this namespace.

Precondition

- `state_is_auto_read_accessible(xauto_access)`

Using this version of the name function, the client need only invoke the function with argument "true":

```
cout << lns->name(true) << endl;
```

The function will request read access, get the name, release access, and return the name.

When the access control mechanism is disabled, the client always has read-write access and neither requesting nor releasing access is necessary. Functions with an auto-access argument can be called with either true or false, either will work. However, the access control mechanism doesn't quite disappear from the programmer's view. The auto-access signatures are still present and the access requirements still appear as preconditions in the contracts.

4.2.3 Handles and states

The Sheaf System is object-oriented, so the client interacts with the library by manipulating the various objects presented by the library interface. Lattice members are a prime example. Many of the objects exported by the interface are not however stored as explicit objects internally. Both memory and performance efficiency often require that such objects be implicit - stored as disjoint data items in bulk arrays. The problem of how to present an externally explicit object interface to an internally implicit object is a common software design problem and several similar design patterns - flyweight, proxy,

surrogate, etc., have been developed to address this problem. In the Sheaf System, we call such a surrogate object a handle and the internal data it accesses is called its state.

For the most part, the distinction between handles and states is an implementation detail that the client needs to be only vaguely aware of. The client uses the handle object as if it were stored internally without worrying about the internal details. But there is one aspect the client has to be aware of: the client has to somehow get a handle to the desired object and when finished with it the client may have to explicitly release it.

There are two basic patterns. In the first pattern, some object has a data member which is a handle and it provides an accessor to this data member. For instance, `sheaves_namespace`, like every lattice, has a `top` member. This member is represented by a data member which is a handle and `sheaves_namespace` exports an accessor:

```
sheaves_namespace lns;
namespace_poset_member& ltop = lns.top();
```

The namespace object allocated and owns the handle. The client need not and should not worry about releasing or otherwise deallocating the handle.

The second pattern addresses the more general case in which the number of handles the client needs and what states they should be attached to is not known at compile time. In order to support efficient allocation and deallocation of handles, the system maintains pools of handles which the client can "borrow", use, and return. For instance, we'll see in the next section that index spaces are accessed via handles and the client can get a handle from the appropriate index space family:

```
index_space_handle& lids =
    lns.member_id_spaces(true).get_handle("member_poset_id_space");
```

When accessed in this way, the handle must be released when the client is finished with it:

```
lns.member_id_spaces(true).release_handle(lids, true);
```

How does a client know whether to release a handle or not? Simple, if you got the handle by calling `get_<whatever>`, you need to release it by calling `release_<whatever>`. Release if and only if get!

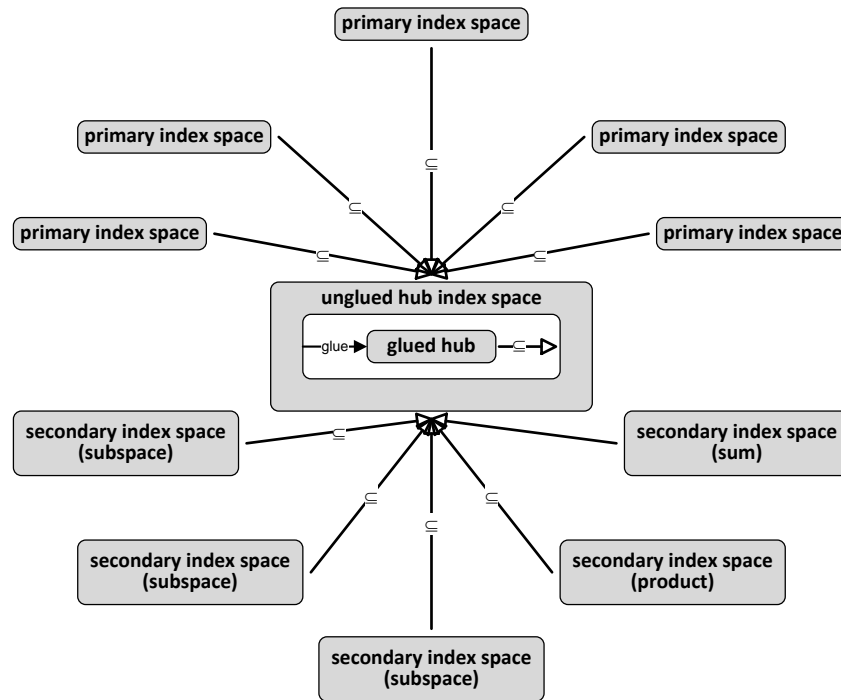


Figure 1: Hub and spoke architecture of an index space family.

4.3 Index spaces and scoped indices, part 1

The members of the row lattice of a table (and hence the members in the column lattice as well) are identified by integer ids. Subsets of the members are very important in the SheafSystem and it is frequently useful to generate a special purpose index scheme for a given subset. Such an index scheme is referred to as an "index space", or "id space" for short. The SheafSystem provides extensive support for defining and using id spaces.

4.3.1 Index spaces and iterators.

More specifically, an index space is a set of integer ids. The system supports the creation and use of a family of index spaces. The fundamental id space of the family is the member id space - the ids automatically generated for the nodes in the row graph. This index space is called the hub id space because the index space family has a hub and spoke architecture as shown in Figure 1. As you can see from the diagram, there are several different kinds of id space and even two hub id spaces, the "unglued" and "glued" versions. For a detailed discussion of this structure see the document "Index Spaces". For the moment, hub id space means unglued hub id space and you should just think of each id space on the rim as a way of indexing some subset of the hub id space, with each spoke representing a map. We'll focus on the basics of how to use id spaces.

As one might expect, the principal use for a member id is to access the features of the member the id refers to. The principal use of a member id space is to iterate over all the members in the subset defined by the id space. Let's look at an example.

4.3.2 Example 3: Iterates over the member hub id space.

```

#include "hub_index_space_handle.h"
#include "index_space_iterator.h"
#include "sheaves_namespace.h"
#include "std_iostream.h"

using namespace sheaf;

int main( int argc, char* argv[])
{
    cout << "SheafSystemProgrammersGuide Example3:" << endl;

    // Create a standard sheaves namespace.

    sheaves_namespace lns("Example3");

    // Get a handle for the member hub id space.

    const index_space_handle& lmb_r_ids = lns.member_hub_id_space(true);

    // Find out how many ids are in the id space.

    cout << lmb_r_ids.name();
    cout << " has " << lmb_r_ids.ct() << " ids.";
    cout << endl;

    // Id spaces are defined as half open intervals, like STL iterators.
    // If the space is "gathered", begin() == 0 and end() = ct().
    // If the space is not gathered, it's "scattered".

    cout << "beginning at " << lmb_r_ids.begin();
    cout << " and ending at " << lmb_r_ids.end();
    cout << " " << (lmb_r_ids.is_gathered() ? "gathered" : "scattered");
    cout << endl;

    // The main thing one does with id spaces is iterate over them.
    // Get an iterator from the iterator pool.

    index_space_iterator& lmb_r_itr = lmb_r_ids.get_iterator();
    cout << endl << "Iterate:" << endl;
    while(!lmb_r_itr.is_done())
    {
        // The current member of the iteration is "pod()".
        // "POD" is an ISO C++ acronym for "plain old data".
        // A pod is an ordinary integer id, in contrast with
        // a "scoped_index" id, to be discussed shortly.

        index_space_iterator::pod_type lpod = lmb_r_itr.pod();

        // Use the id to get the member name.
        // Member name requires a hub id, but since we're using
        // the hub id space, pod and hub pod are the same thing.

        cout << "id: " << lpod;
        cout << " hub id: " << lmb_r_itr.hub_pod();
    }
}

```

```

    cout << " name: " << lns.member_name(lpod, true);
    cout << (lns.is_jim(lpod) ? " is a jim." : " is a jrm.");
    cout << endl;

    // Move on.

    lmbtr_itr.next();
}

// You can reuse an iterator by resetting it.

lmbtr_itr.reset();
cout << endl << "Reiterate:" << endl;
while(!lmbtr_itr.is_done())
{
    index_space_iterator::pod_type lpod = lmbtr_itr.pod();

    cout << "id: " << lpod;
    cout << " hub id: " << lmbtr_itr.hub_pod();
    cout << " name: " << lns.member_name(lpod, true);
    cout << (lns.is_jim(lpod) ? " is a jim." : " is a jrm.");
    cout << endl;

    // Move on.

    lmbtr_itr.next();
}

// If you got an id space or iterator from the pool with get_
// you have to return it to the pool with release_.

lmbtr_ids.release_iterator(lmbtr_itr);

// The id space itself is a data member of the id space family,
// we didn't get it from the pool with get_, so we don't have to
// release it.

// Exit:

return 0;
}

```

If we execute example3 we get:

```

>./example3
SheafSystemProgrammersGuide Example3:
__hub has 6 ids.
beginning at 0 and ending at 6 gathered

```

```

Iterate:
id: 0 hub id: 0 name: bottom is a jrm.
id: 1 hub id: 1 name: top is a jrm.
id: 2 hub id: 2 name: primitives_schema is a jim.
id: 3 hub id: 3 name: namespace_poset_schema is a jim.
id: 4 hub id: 4 name: primitives is a jim.
id: 5 hub id: 5 name: schema definitions is a jrm.

```

Reiterate:

id: 0 hub id: 0 name: bottom is a jrm.

id: 1 hub id: 1 name: top is a jrm.

id: 2 hub id: 2 name: primitives_schema is a jim.

id: 3 hub id: 3 name: namespace_poset_schema is a jim.

id: 4 hub id: 4 name: primitives is a jim.

id: 5 hub id: 5 name: schema definitions is a jrm.

4.3.3 Id maps and scoped ids.

As we said above, id spaces are used for indexing subsets. For instance, in a namespace, the member poset id space indexes just the jims, which represent the member posets - the other posets contained in the namespace. There may be several or even many id spaces available in a practical setting. Various member functions may require an index to be in a particular id space, most commonly in the hub id space. The id maps associated with the spokes in the id space family provide the mechanism for translating between id spaces.

Every id space has a map to the (unglued) hub id space. The `index_space_handle` class provides member functions for mapping ids between the id space and the hub id space:

pod_type hub_pod (pod_type xid) const

The pod index in the unglued hub id space equivalent to xid in this id space; synonym for `unglued_hub_pod(pod_type)`.

and

pod_type pod (pod_type xid) const

The pod index in this space equivalent to xid in the hub id space.

Using these functions we can map between id spaces. For instance, if `id1` is an id in `id_space1` and `id_space2` is a different id space, then

```
pod_type id2_eqv_1 = id_space2.pod(id_space1.hub_pod(id1));
```

is the id in `id_space2` that identifies the same member identified by `id1` in `id_space1`, if such an equivalent member exists. The postcondition for the `pod(pod_type)` function is:

- `lis_valid(result) || contains(result)`

So if `id_space2` does not have an equivalent member, `id2_eqv_1` is assigned an invalid value. The value

sheaf::pod_index_type sheaf::invalid_pod_index()

The invalid pod index value.

is reserved as a "null" value for index types. It is currently set to `numeric_limits<pod_index_type>::max()`, but that may change and I only mention that so

you will recognize it if you see it in a print out or in the debugger. It should be used as an opaque value. A pod type can be tested for validity using:

bool sheaf::is_valid(sheaf::pod_index_type xpod_index)

True if and only if xpod_index is valid.

Using the pod and hub_pod functions, the programmer can map ids between id spaces. But it can be tedious. Worse, it may be difficult or impossible for a programmer to track just what id space a given index is in. The scoped_index class provides a convenient mechanism for both managing the connection between an id and the space it belongs to and for automatically mapping between id spaces. We call the id space an id belongs to the scope of the id. A scoped_index is a pair (id, scope). Most member functions that require an id as input are available in two signatures; one signature that takes a pod id and one that takes a scoped id. One can use a scoped id, once it has been initialized, without worrying what scope it is in; any function that accepts a scoped id will translate it to the scope it requires. We'll see more complex examples of mapping between id spaces later, for now let's redo example3 using the member poset id space, the hub id space, id maps and scoped ids.

4.3.4 Example 4: Iterates over the member poset id space.

```
#include "index_space_handle.h"
#include "index_space_iterator.h"
#include "sheaves_namespace.h"
#include "std_iostream.h"

using namespace sheaf;

int main( int argc, char* argv[])
{
    cout << "SheafSystemProgrammersGuide Example4:" << endl;

    sheaves_namespace lns("Example4");

    // Get a handle for the member poset id space;
    // has one member for each poset in the namespace.

    const index_space_handle& lmb_r_ids =
        lns.get_member_poset_id_space(true);

    // Print out the same info we did for the hub id space.

    cout << lmb_r_ids.name();
    cout << " has " << lmb_r_ids.ct() << " ids.";
    cout << endl;

    cout << "beginning at " << lmb_r_ids.begin();
    cout << " and ending at " << lmb_r_ids.end();
    cout << " " << (lmb_r_ids.is_gathered() ? "gathered" : "scattered");
    cout << endl;

    index_space_iterator& lmb_r_itr = lmb_r_ids.get_iterator();
```

```

cout << endl << "Iterate:" << endl;
while(!lmbtr_itr.is_done())
{
    index_space_iterator::pod_type lpod = lmbtr_itr.pod();

    // Use the id to get the member name.
    // Member name requires a hub id which we can get in two ways.
    // The id space will use the map from the id space to the hub
    // to translate any id in the id space to its equivalent in the
    // hub:

    index_space_iterator::pod_type lhup_pod = lmbtr_ids.hub_pod(lpod);

    // The iterator can provide the hub id equivalent for the current
    // id, and it can be faster because for some id space types it can
    // avoid the map lookup.

    lhup_pod = lmbtr_itr.hub_pod();

    cout << "id: " << lpod;
    cout << " hub id: " << lhup_pod;
    cout << " name: " << lns.member_name(lhup_pod, true);
    cout << (lns.is_jim(lhup_pod) ? " is a jim." : " is a jrm.");
    cout << endl;

    // Move on.

    lmbtr_itr.next();
}

// Most member functions are available with two signatures, one
// that takes a pod_index_type and one that takes a scoped_index.
// If you don't want to think about what the scope for an argument
// should be, you can use the scoped_index signature.

// Create a scoped id with scope = member poset id space.

scoped_index lscoped_id(lmbtr_ids);

// The value sheaf::invalid_pod_index() is reserved as a
// "null" value for index types. It is currently set to
// numeric_limits<pod_index_type>::max(), but don't count on it.

cout << endl << "sheaf::invalid_pod_index()= ";
cout << sheaf::invalid_pod_index() << endl;

// When a scoped id is created without a specific pod value,
// it is invalid by default.

cout << "lscoped_id= " << lscoped_id;
cout << " is_valid() ";
cout << boolalpha << lscoped_id.is_valid() << noboolalpha;
cout << endl;

// Reset the iterator and re-iterate using
// the scoped_index signature for member_name.

```



```

    lmbtr_itr.reset();
    cout << endl << "Reiterate:" << endl;
    while(!lmbtr_itr.is_done())
    {
        // Set the scoped id for the current member of the iteration.

        lscoped_id.put_pod(lmbtr_itr.pod());

        // Assignment is overloaded, so you can also say:

        lscoped_id = lmbtr_itr.pod();

        // Use the scoped_index signature to get the member name.

        cout << "scoped_id: " << lscoped_id;
        cout << " name: " << lns.member_name(lscoped_id, true);
        cout << (lns.is_jim(lscoped_id) ? " is a jim." : " is a jrm.");
        cout << endl;

        // Move on.

        lmbtr_itr.next();
    }

    lmbtr_ids.release_iterator(lmbtr_itr);

    // Exit:

    return 0;
}

```

When we run example4 we get:

```

>./example4
SheafSystemProgrammersGuide Example4:
member_poset_id_space has 3 ids.
beginning at 0 and ending at 3 gathered

```

```

Iterate:
id: 0 hub id: 2 name: primitives_schema is a jim.
id: 1 hub id: 3 name: namespace_poset_schema is a jim.
id: 2 hub id: 4 name: primitives is a jim.

```

```

sheaf::invalid_pod_index()= 2147483647
lscoped_id= (2, 2147483647) is_valid() false

```

```

Reiterate:
scoped_id: (2, 0) name: primitives_schema is a jim.
scoped_id: (2, 1) name: namespace_poset_schema is a jim.
scoped_id: (2, 2) name: primitives is a jim.

```

Note the value of the invalid id in the above output; we'll see it again shortly.

4.4 Storage_agent

We've talked about the notion of a SheafSystem database, so there must be some way to make a namespace persistent and indeed there is. Persistent storage is managed by the `storage_agent` class. A `storage_agent` makes it particularly easy to save an entire namespace to disk, as we show in the next example.

4.4.1 Example 5: Write a namespace to a file

```
#include "sheaves_namespace.h"
#include "std_iostream.h"
#include "storage_agent.h"

using namespace sheaf;

int main( int argc, char* argv[])
{
    cout << "SheafSystemProgrammersGuide Example5:" << endl;

    // Create a namespace.

    sheaves_namespace lns("Example5");

    // Write the namespace to a file.

    storage_agent lsa("example5.hdf");
    lsa.write_entire(lns);

    // Exit:

    return 0;
}
```

If we build and run `example5`, it writes the file named in the `storage_agent` constructor, `"example5.hdf"`.

```
>./example5
>ls *.hdf
example5.hdf
```

We'll see shortly how we can view the contents of this file.

4.5 Viewing Namespaces

Once we have a namespace, we'd like to know what it contains. We've already seen how to iterate over the members of the namespace and display their names. Now we'll look at 3 ways that are easier and provide a lot more information.

4.5.1 Stream insertion operator

The base class for namespaces, `namespace_poset`, has a stream insertion operator for writing the contents of the namespace to a stream. The insertion operator is most commonly used for dumping a namespace to `cout` for debugging purposes.

4.5.1.1 Example 6: Write namespace to `cout`

```
#include "sheaves_namespace.h"
#include "std_iostream.h"

using namespace sheaf;

int main( int argc, char* argv[])
{
    cout << "SheafSystemProgrammersGuide Example6:" << endl;

    // Create a namespace.

    sheaves_namespace lns("Example6");

    // Write the namespace to cout.

    cout << lns << endl;

    // Exit:

    return 0;
}
```

When we execute `example6`, it creates quite a lot of output, even for `sheaves_namespace`, which is as close to being empty as a namespace can get. We won't include it here, but the reader should examine the file `example6.cout` in the same directory with `example6.cc`.

For each poset in the namespace, including the namespace itself, the stream insertion operator prints information about the row graph, the subposets of the row graph, and the table. We'll learn more about how to interpret all this information as we go along.

4.5.2 The `dump_shf` utility

The SheafSystem provides the `dump_shf` utility for reading a sheaf file and dumping its contents to `cout` using the stream insertion operator. So if you've written a sheaf file, as we did in example 5, then we can view its contents easily.

4.5.2.1 Example 7: View namespace with `dump_sheaf`

First, make sure you have set your environment using the `set_env_vars` script in the build directory of your Programmer's Guide installation:

```
>source set_env_vars.csh
```

Then run the `dumpsheaf` utility in the `bin` directory of the SheafSystem installation:

```
<sheaf_dir>/Debug_contracts/bin/dumpsheaf example5.hdf
```

4.5.3 The SheafScope interactive file browser

The SheafScope is another SheafSystem utility. It provides an interactive, graphical browser for sheaf files.

4.5.3.1 Example 8: View namespace with SheafScope

Make sure you've set your environment with the `set_env_vars` script as shown above, then run the SheafScope.

```
>java -jar SheafScope.jar example5
```

4.6 Posets

4.6.1 Table or part space or lattice or poset?

We've so far talked about a sheaf database being a collection of sheaf tables. In the Part Space tutorial we talked about sheaf tables as part spaces and in the Analysis and Design tutorial we revealed that a sheaf table could be thought as either a poset-ordered or lattice-ordered relation. So which is it? The answer of course, is (e) all of the above. But when it comes to naming classes, we had to pick one. The one we picked was poset. Most of the classes that implement sheaf tables are posets of some kind or another. The most commonly used type is class poset. The type that is used to represent meshes in the fiber bundles component is `base_space_poset`. The abstract base class for all poset types is `poset_state_handle`.

***Historical note:** `Poset_state_handle`, the abstract base class for all poset types, is as its name says, a handle. But this is a historical artifact. There is no longer any reason for it to be a handle, the state of a poset is an explicit object, and the various handle features are in fact protected so they can't be used.*

4.6.2 Creating posets

A namespace is a factory for posets. Posets are created using the member function template `new_member_poset<T>`, where `T` is the type of poset to create:

```
T& new_member_poset (const string &xname,  
                    const poset_path &xschema_path,  
                    const arg_list &xargs,  
                    bool xauto_access)
```

Creates a new poset with name `xname`, schema specified by `xschema_path`, and table attributes initialized by `xargs`.

But first, we have to have a schema if we want to create a poset. Typically, this means we have to create a schema poset before creating an object poset. We'll see how to create a schema poset shortly, but we can avoid creating a schema poset if the poset we want to create has only a single attribute. Instead, we can use a member of the primitives poset that every namespace has as a schema. We'll do that to get started.

4.6.2.1 Poset_path

The schema for a poset is a member of a schema poset and a member can be identified by a path. A poset path is similar to a file path, but has only two elements, a poset name and a member name. We can create a path by specifying the poset name and member name separately:

```
poset_path lschema_path("primitives", "INT");
```

or by specifying a complete path, like a file path:

```
poset_path lschema_path("primitives_poset/INT");
```

4.6.2.2 Arg_list

Some types of poset cannot be default constructed, so the factory method requires an argument list for the constructor. Each poset type provides a static function `make_args(...)` that sets up the `arg_list`. Every poset type has such a function, even if it can be default constructed. In that case, the `make_args` function itself has no arguments and the `arg_list` it creates is empty. This is in fact the case for the ordinary poset class we want to construct:

```
arg_list largs = poset::make_args();
```

The `arg_list` class has a constructor that takes a string, so if you know the `arg_list` is empty can also just pass an empty string for the `arg_list`, but we'll do it the general way for now.

So now all we need is a name and we can construct the poset. We'll call it "simple_poset":

```
poset& lposet = lns.new_member_poset<poset>("simple_poset",
    lschema_path, largs, true);
```

4.6.3 Accessing posets

Once you've created a poset and have a reference to it, you can access its features. For instance, you can find out what its id is:

```
cout << lposet.index().hub_pod() << endl;
```

All poset types have a stream insertion operator, so once you have access to a poset, you can use the stream insertion operator to print it out:

```
cout << lposet << endl;
```

We'll do more with posets, like creating members, in the next section.

In the meantime, what if a poset already exists, how do you get a reference to it? You can get a reference to a poset by id, which is available in two variants, `pod` and `scoped_index`:

poset_state_handle & member_poset (pod_index_type xindex, bool xauto_access) const

The `poset_state_handle` object referred to by the member with hub id `xindex`.

poset_state_handle & member_poset(const scoped_index &xindex, bool xauto_access) const

The `poset_state_handle` object referred to by the member with index `xindex`.

For instance, assuming that the id of the poset we created above is 5, we can get a reference to it with:

```
poset_state_handle& lpsh1 = lns.member_poset(5, true);
```

You can also access it by path. The member name part of the path can be empty or not, only the poset name will be used.

poset_state_handle & member_poset (const poset_path &xpath, bool xauto_access) const

The `poset_state_handle` object referred to by the member with name `xpath.poset_name()`.

Furthermore, since `poset_path` has a constructor that takes a string literal, you can specify the path as a string literal:

```
poset_state_handle& lpsh2 = lns.member_poset("simple_poset", true);
```

`Poset_state_handle` is the abstract base class for all poset types. If you know the specific type of a poset and want a reference to that type, all three of the above signatures are also available in a templated version:

P& member_poset(pod_index_type xindex, bool xauto_access) const;

The `poset_state_handle` object referred to by the member with hub id `xindex` dynamically cast to type `P&`.

invoked with bracket notation:

```
poset& lposet1 = lns.member_poset<poset>("primitives", true);
```

4.6.4 Deleting posets

Posets live in the namespace. Once you've created one, it stays in the namespace whether you have a reference to it or not. If you want to delete a poset, use the namespace `delete_poset` function, available in the same id and path signatures as `member_poset`, for instance:

void delete_poset (const poset_path &xpath, bool xauto_access)

Delete the poset with name xpath.poset_name().

Example 10 ties the namespace functions we've been discussing into a single example.

4.6.5 Example 9: Creating, accessing, and deleting posets.

```
#include "sheaves_namespace.h"
#include "std_iostream.h"

using namespace sheaf;

int main( int argc, char* argv[])
{
    cout << "SheafSystemProgrammersGuide Example9:" << endl;

    // Create a namespace.

    sheaves_namespace lns("Example9");

    // We use a schema with a single integer attribute.

    poset_path lschema_path("primitives", "INT");

    // The constructor for the ordinary poset class
    // doesn't need any arguments.

    arg_list largs = poset::make_args();

    // Create the poset.

    poset& lposet = lns.new_member_poset<poset>("simple_poset",
        lschema_path, largs, true);

    // Print the poset to cout.

    cout << lposet << endl;

    // Write the namespace to a sheaf file.

    storage_agent lsa("example9.hdf");
    lsa.write_entire(lns);

    // You can get another reference to the poset by id,
    // if you know the id:

    poset_state_handle* lpsh1 = lns.member_poset(6, true);

    // and by path:

    poset_state_handle* lpsh2 = lns.member_poset("simple_poset", true);

    // Delete the poset by path.
    // Invalidates all the above references.
```

```

    lns.delete_poset(lposet.path(), true);

    // Exit:

    return 0;
}

```

When we run example9, the output is once again a bit lengthy to include here, take a look at example9.cout.

4.7 Poset members

We created a poset in the last section, but it was empty. Well, almost empty. As we discussed in the Part Space tutorial, a sheaf table represents a part space. A part space always has a bottom member// corresponding to the empty assembly of basic parts (jims). It also has a top member, corresponding to the assembly of all the basic parts. We automatically create these two composite parts (jrms) when we create the poset. When the poset has no basic parts, the top is equivalent to the bottom as an assembly, but they are still distinct members.

4.7.1 Creating join irreducible members

We can't create any interesting jrms until we have some jims, so we'll start by creating some jims. A poset has a special editing mode, called "jim_edit_mode", for creating jims. Jrms can be created at any time, but to create a jim you have to put the poset into jim_edit_mode. Jim edit mode allows you to directly edit the row graph, creating or deleting members and cover links. Once the jims poset has been defined, you can create composite parts either directly by creating the members and links or algebraically, using the join and meet operations.

Jim edit mode is off by default, so we have enter jim edit mode to create a jim:

void begin_jim_edit_mode (bool xauto_access)

Allow editing of jims and jim covering relation.

(All the functions we will discuss in this section are member functions of class poset or its ancestors unless specifically scoped. Click the "List of all members" item in the extreme upper right of the poset class documentation web page to get a listing of all member functions, both direct and inherited.)

You create a jim with the new_member function:

```

sheaf::pod_index_type new_member (bool xis_jim,
                                   poset_dof_map * xdof_map = 0,
                                   bool   xcopy_dof_map = false )

```

Create a disconnected member with is_jim == xis_jim. If xdof_map != 0, the new member uses it for dof storage, otherwise it creates an instance of array_poset_dof_map. WARNING: this routine leaves a disconnected member in the poset and hence leaves the poset in an

invalid state. The client must properly link the member created by this routine using `new_link` in order to return the poset to a valid state.

Historical note: "Dof" is an acronym for "degree of freedom" but it means "attribute". A "dof_map" is a tuple. Both are historical artifacts, originating in early attempts to interpret fields as relational tuples.

To create a new jim, we call `new_member` with `xis_jim` true and let it create a tuple for the new member by accepting the default values for `xdof_map` and `xcopy_dof_map`. Let's create three new jims, corresponding to the basic parts in the line segment example from the Part Space tutorial, Figure 10, which we reproduce in Figure 2. Well, almost reproduce, Figure 2 has a top member because sheaf tables always have top and bottom members.

The code to create the jims is:

```
lposet.begin_jim_edit_mode(true);
pod_index_type lv0_pod = lposet.new_member(true);
pod_index_type lv1_pod = lposet.new_member(true);
pod_index_type ls0_pod = lposet.new_member(true);
```

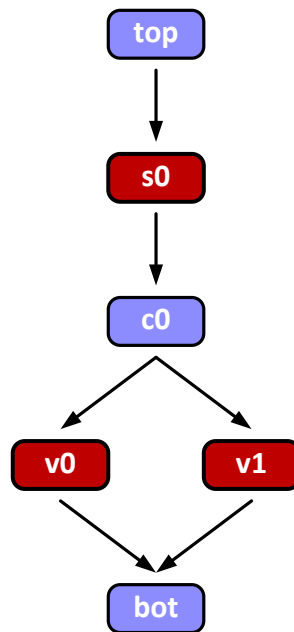


Figure 2: Line segment example from Part Space, Figure 10.

4.7.2 Ordering poset members

You define the ordering relation for the poset by explicitly creating cover links between the jims, using `new_link`:

```
void new_link( pod_index_type xgreater, pod_index_type xless)
```

Insert a cover link from greater to lesser (that is, `xgreater` covers `xlesser`). WARNING: this routine does not ensure that the link is a cover link, that is, it does not remove redundant or conflicting links. Improper use of this routine can produce inconsistent poset states.

Continuing with the line segment example, the segment member should cover the two vertices:

```
lposet.new_link(ls0_pod, lv0_pod);
lposet.new_link(ls0_pod, lv1_pod);
```

Each vertex is an atom, there is no smaller basic part than a vertex, so each vertex should have a cover link to bottom. Similarly, there is no larger part than the segment, so top should cover the segment. We can put these links in explicitly. The id of the top and bottom member are defined in the enumeration `sheaf::standard_member_index` as `TOP_INDEX` and `BOTTOM_INDEX`, respectively. Or, we can get the id from the `top()` or `bottom()` accessors:

```
lposet.new_link(TOP_INDEX, ls0_pod);
lposet.new_link(lv0_pod, BOTTOM_INDEX);
lposet.new_link(lv1_pod, lposet.bottom().index().pod());
```

However, we don't have to explicitly put the links to top and bottom. Remember that the cover relation graph is a directed graph, with links pointing in the "covers" direction. For a given member `p`, the set of lesser members `p` is linked to by outgoing links, that is the set of members `p` covers, is called the lower cover of `p`. The set of larger members that are linked to `p` by the incoming links, the set of members that cover `p`, is called the upper cover of `p`. It is an invariant of a lattice that bottom is the only member with an empty lower cover and top is the only member with an empty upper cover. The `end_jim_edit_mode` function:

```
void end_jim_edit_mode(bool xensure_lattice_invariant = true, bool xauto_access = true)
```

Prevent editing of jims and jim order relation.

will enforce the invariant if we request it by setting the `xensure_lattice_invariant` argument to true. In that case, it automatically links anything with an empty lower cover to bottom and links top to anything with an empty upper cover. This requires a search of the graph for empty covers, so it is more efficient in large graphs to do the linking explicitly.

When we're finished creating and linking jims, we leave jim edit mode. We'll just take the default arguments, even though we've already linked everything, it won't make any difference for a tiny graph like this one:

```
lposet.end_jim_edit_mode();
```

4.7.3 Accessing poset members

We've created the basic parts and ordered them, but we haven't set any of their attributes, so let's do that now.

4.7.3.1 Member names

We've already seen that every poset member has at least one implicit attribute, it's id, automatically assigned by the system. Every member also has another implicit attribute, supported by the system but not automatically assigned: a name. Any member can be given a name, in fact any member can be given multiple names, but naming is optional. Names are assigned with the `put_member_name` function:

```
void put_member_name(pod_index_type xindex, const string & xname,  
                    bool xunique, bool xauto_access = false )
```

Make `xname` a name for the member with hub id `xindex`; if `xunique`, make `xname` the only name.

We'll give all our basic parts the obvious names:

```
lposet.put_member_name(lv0_pod, "v0", true);  
lposet.put_member_name(lv1_pod, "v1", true);  
lposet.put_member_name(ls0_pod, "s0", true);
```

We can retrieve a name with the `member_name` function:

```
string member_name(pod_index_type xindex, bool xauto_access = false)
```

A name for the member with hub id `xindex`.

For instance:

```
cout << lposet.member_name(lv0_pod);
```

Members can have more than one name. The reader is encouraged to review the other member name functions in the reference documentation for `poset_state_handle`.

4.7.3.2 Schema

The explicit attributes of a member are whatever is defined by the schema for the poset. The schema for a poset is a member of a schema poset. A handle for the schema member is available from the poset:

```
const schema_poset_member& schema () const
```

The schema for this poset (const version).

We'll learn more about member handles shortly. In the meantime, we'll just introduce a few features we need.

Mathematically, each attribute is a component of a tuple and components are traditionally accessed by component id. There is an id space defined for the attributes specified by each schema member. In fact, since the table is partitioned into a row part and a table part for storage efficiency, each schema member defines two id spaces, one for the row

attributes and one for the table attributes. We can get either id space from the schema using the `dof_id_space` accessor function:

```
const index_space_handle&  
schema_poset_member::dof_id_space(bool xis_table_dofs) const
```

The table dof (`xis_table_dof true`) or row dof id space for the schema defined by this.

For instance, the row attribute id space for our example is:

```
const index_space_handle& latt_id_space =  
    poset.schema().dof_id_space(false);
```

Now, because we took a short cut defining the schema, we don't have all the attributes of the original example in the Part Space tutorial. All we have is a single integer attribute called "INT". Since we know that the schema has only a single row attribute, it has to be the first id in the id space, so we can get its id with:

```
pod_index_type latt_pod = lposet.schema().dof_id_space(false).begin();
```

We can also create a scoped id for an attribute:

```
scoped_index latt_id(lposet.schema().dof_id_space(false), latt_pod);
```

4.7.3.3 Member tuple

The attributes are components in the relation tuple of the member, so to access an attribute we need a reference to the tuple or "dof_map", which we can get with:

```
poset_dof_map&  
member_dof_map(pod_index_type xmbr_index, bool xrequire_write_access)
```

The dof map associated with the member identified by `xmbr_index` (mutable version).

We can get the attribute tuple for the first vertex for instance:

```
poset_dof_map& ltuple = lposet.member_dof_map(lv0_pod, true);
```

4.7.3.4 Member attributes

Once we have the tuple, we can get an attribute value with the dof accessor function:

```
primitive_value poset_dof_map::dof(pod_index_type xdof_id) const
```

The dof with name `xname`.

and set an attribute value with the `put_dof` mutator function:

```
void poset_dof_map::put_dof(pod_index_type xdof_id, const primitive_value& xdof)
```

Sets the dof with name `xname` to `xdof`.

The accessor and mutator functions are each available in 3 signatures corresponding to specifying the attribute by pod id, scoped id, or name.

4.7.3.5 Primitive_value

The attribute value returned by the accessor functions and accepted by the mutator functions is of type `primitive_value`, which is essentially a wrapper for any primitive type. It allows a single signature for each function to support any attribute type. `Primitive_value` is essentially a union that can store any primitive plus some type information. It also has conversion operators to and from every primitive type that make it easy to put a primitive value in to a `primitive_value` and get it back again.

You can put a value into a `primitive_value` with either a constructor or an assignment:

```
primitive_value lpval(int(0));
lpval = float(1);
```

and get it back again with an assignment:

```
float lf = lpval;
```

You can find out what type of value a `primitive_value` is currently holding with the `id` function by testing it against the `id()` member of the `primitive_traits` template for the appropriate type:

```
if(lpval.id() == primitive_traits<float>.id())
{
    float lf = lpval;
}
```

4.7.3.6 Setting an attribute

Setting the attribute for the vertex `v0` takes three steps: get the tuple, put the attribute in a `primitive_value` wrapper, put the attribute in the tuple.

```
poset_dof_map ltuple = lposet.member_dof_map(lv0_pod, true);
primitive_value lpv(int(0));
ltuple.put_dof(latt_pod, lpv);
```

These three steps have to be done, but they don't have to be done explicitly. Implicit conversions actually make it much simpler:

```
lposet.member_dof_map(lv1_pod, true).put_dof(latt_pod, int(0));
lposet.member_dof_map(ls0_pod, true).put_dof(latt_pod, int(1));
```

We can also set an attribute value using either the scoped id signature:

```
lposet.member_dof_map(lv0_pod, true).put_dof(latt_id, int(0));
```

or the name signature:

```
lposet.member_dof_map(lv0_pod, true).put_dof("INT", int(0));
```

In practice, you use whichever signature is most convenient.

4.7.3.7 Getting an attribute

The implicit conversions work when getting an attribute value as well:

```
int ldim = lposet.member_dof_map(lv0_pod, false).dof(latt_pod);
```

As with the mutator, the accessor works with scoped id or name:

```
int ldim = lposet.member_dof_map(lv0_pod, false).dof(latt_id);
int ldim = lposet.member_dof_map(lv0_pod, //false).dof("INT");
```

4.7.4 Creating join reducible members

We can create jrms and link them up at any time, whether we're in `jim_edit_mode` or not. To create a jrm, just set the `xis_jim` argument to false in `new_member`.

```
poset_index_type lc0_pod = lposet.new_member(false);
lposet.put_member_name(lc0_pod, "c0", true);
```

Then link it up:

```
lposet.new_link(ls0_pod, lc0_pod);
lposet.new_link(lc0_pod, lv0_pod);
lposet.new_link(lc0_pod, lv1_pod);
```

But we can't stop here. We have to make sure the cover relation is indeed a cover relation. The segment no longer covers the two vertices, so we have to remove those links using `delete_link`:

void delete_link (pod_index_type xgreater, pod_index_type xlesser)

Delete the cover link between `xgreater` and `xlesser`.

which is pretty straight forward:

```
lposet.delete_link(ls0_pod, lv0_pod);
lposet.delete_link(ls0_pod, lv1_pod);
```

Remember, when you're editing the graph, it's your job to get it right! In particular, it's your job to make sure there are no transitive links (links equivalent to a path) like the two we just removed. An invalid graph can produce subtle errors that are difficult to track down. We'll see shortly that in many cases, you can create and link up a jrm in a single step using the join operation. In that case, the system does all the graph editing and makes sure the graph is valid.

4.7.5 Creating join equivalent members

What if we create another jrm, let's call it `c1`, and link it between `s0` and `c0`:

```
pod_index_type lc1_pod = lposet.new_member(false);
lposet.delete_link(ls0_pod, lc0_pod);
lposet.new_link(ls0_pod, lc1_pod);
lposet.new_link(lc1_pod, lc0_pod);
```

The result is shown in Figure 3, but what does it mean? Is it even legal, since `c1` has only a single member in its lower cover, doesn't it have to be a jim?

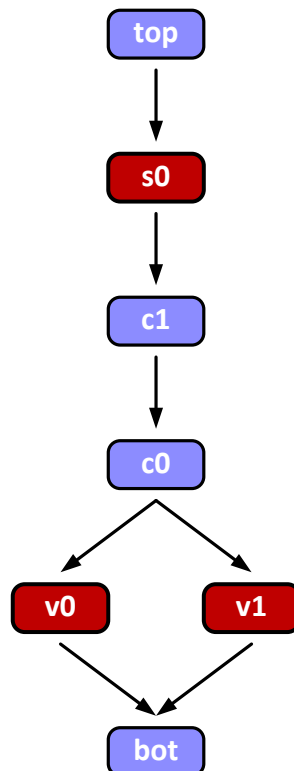


Figure 3: A join equivalent member.

Well, to answer the first question, let's think about part space. A jrm is a composite part in the part space metaphor and a composite part is an assembly of basic parts. More specifically, it is the assembly of the basic parts in its down set, the set of all parts below it in the graph. Pretty clearly, the set of basic parts in the down set of our new jrm is exactly the same as the set of basic parts in the down set of `c0`. So our new jrm is a distinct member, but as an assembly it is identical to `c0`. In other words, the new jrm is a copy of `c0`. We say that `c1` is join equivalent to `c0` and call it a join equivalent member or "jem", pronounced like "gem".

Now for the second question: is it legal? The short answer is yes, but why this is true takes a little bit of explanation. The interested reader can find the full answer in Appendix B. For those not interested in the mathematical details, feel free to copy members

whenever you please, as many times as you please. In fact, in section 11 we'll see some functions that make it easy to make copies.

4.7.6 Deleting poset members

Deleting a member is the inverse of creating it. You have to unlink it, delete it, and relink the remaining members appropriately. We've already seen how to delete links. To delete a member, use the `delete_member` function:

virtual void delete_member (pod_index_type xindex)

Delete the member with index `xindex`. Warning: this routine does not delete links; it will leave any links to this member dangling.

So let's delete the `jem` we just created:

```
lposet.delete_link(lc1_pod, lc0_pod);
lposet.delete_link(ls0_pod, lc1_pod);
lposet.new_link(ls0_pod, lc0_pod);
lposet.delete_member(lc1_pod);
```

4.7.7 Example 10: Reading a sheaf file; manipulating poset members with the poset interface

We've covered the basics of creating, linking, accessing, and deleting poset members using the poset interface. Let's collect everything we've covered together into a single example. We'll learn to read a poset from a file as well.

```
#include "sheaves_namespace.h"
#include "poset.h"
#include "poset_dof_map.h"
#include "std_iostream.h"
#include "storage_agent.h"

using namespace sheaf;

int main( int argc, char* argv[])
{
    cout << "SheafSystemProgrammersGuide Example10:" << endl;

    // Create a namespace.

    sheaves_namespace lns("Example10");

    // Populate the namespace from the file we wrote in example9.
    // Retrieves the simple_poset example.

    storage_agent lsa("example9.hdf", sheaf_file::READ_ONLY);
    lsa.read_entire(lns);

    // Get a reference to the poset "simple_poset".

    poset_path lpath("simple_poset");
    poset& lposet = lns.member_poset<poset>(lpath, true);
```



```

// Allow creation of jims.

lposet.begin_jim_edit_mode(true);

// Create jims for the two vertices and the segment.

pod_index_type lv0_pod = lposet.new_member(true);
pod_index_type lv1_pod = lposet.new_member(true);
pod_index_type ls0_pod = lposet.new_member(true);

// Make the segment cover the vertices.

lposet.new_link(ls0_pod, lv0_pod);
lposet.new_link(ls0_pod, lv1_pod);

// Top covers the segment.

lposet.new_link(TOP_INDEX, ls0_pod);

// The vertices cover bottom.

lposet.new_link(lv0_pod, BOTTOM_INDEX);
lposet.new_link(lv1_pod, lposet.bottom().index().pod());

// We're finished creating and linking jims.

lposet.end_jim_edit_mode();

// Give each jim a name..

lposet.put_member_name(lv0_pod, "v0", true);
lposet.put_member_name(lv1_pod, "v1", true);
lposet.put_member_name(ls0_pod, "s0", true);

// Print the names to cout.

cout << lposet.member_name(lv0_pod) << endl;
cout << lposet.member_name(lv1_pod) << endl;
cout << lposet.member_name(ls0_pod) << endl;

// Get the row attribute id space and pod and
// scoped ids for the only attribute.

const index_space_handle& latt_id_space =
    lposet.schema().dof_id_space(false);
pod_index_type latt_pod =
    lposet.schema().dof_id_space(false).begin();
scoped_index latt_id(lposet.schema().dof_id_space(false), latt_pod);

// Get the attribute tuple for vertex 0.

poset_dof_map& ltuple = lposet.member_dof_map(lv0_pod, true);

// Set the only attribute of v0 to its dimension, 0.
// Do the first one explicitly, without any automatic conversion.

primitive_value lpv(int(0));

```

```
ltuple.put_dof(latt_pod, lpv);

// Set attributes for v1 and s0 relying on conversions.

lposet.member_dof_map(lv1_pod, true).put_dof(latt_pod, int(0));
lposet.member_dof_map(ls0_pod, true).put_dof(latt_pod, int(1));

// Get attributes back and write them to cout.

int lv0_dim = lposet.member_dof_map(lv0_pod, false).dof(latt_pod);
int lv1_dim = lposet.member_dof_map(lv1_pod, false).dof(latt_pod);
int ls0_dim = lposet.member_dof_map(ls0_pod, false).dof(latt_pod);

cout << "v0 dim= " << lv0_dim;
cout << " v1 dim= " << lv1_dim;
cout << " s0 dim= " << ls0_dim;
cout << endl;

// Create a jrm named c0.

pod_index_type lc0_pod = lposet.new_member(false);
lposet.put_member_name(lc0_pod, "c0", true);

// Link it up:

lposet.new_link(ls0_pod, lc0_pod);
lposet.new_link(lc0_pod, lv0_pod);
lposet.new_link(lc0_pod, lv1_pod);

// Delete the now obsolete links from s0 to the vertices.

lposet.delete_link(ls0_pod, lv0_pod);
lposet.delete_link(ls0_pod, lv1_pod);

// Create a jem; a copy of c0, call c1.

pod_index_type lc1_pod = lposet.new_member(false);
lposet.put_member_name(lc1_pod, "c1", true);
lposet.delete_link(ls0_pod, lc0_pod);
lposet.new_link(ls0_pod, lc1_pod);
lposet.new_link(lc1_pod, lc0_pod);

// Output the finished poset to cout:

cout << lposet << endl;

// Delete c1.

lposet.delete_link(lc1_pod, lc0_pod);
lposet.delete_link(ls0_pod, lc1_pod);
lposet.new_link(ls0_pod, lc0_pod);
lposet.delete_member(lc1_pod);

// Exit:

return 0;
}
```

Make sure to run example9 before example10 and in the same directory so that example10 can find the file example9.hdf. The output from example10 is in the file example10.cout

4.8 Poset member handles

The poset member class hierarchy provides an alternate interface for manipulating poset members. `Abstract_poset_member` is the abstract base class for the hierarchy, with immediate descendants `partial_poset_member` and `total_poset_member`. The partial/total adjective refers to whether the interface supports restriction of a member to only part of its schema; `partial_poset_member` does and `total_poset_member` doesn't. `Partial_poset_member` is the base class for the various types of sections in the `section_spaces` cluster of the `fiber_bundles` component, for which restriction is an important operation. `Total_poset_member` is the type of member for ordinary posets and is the base class for the various algebraic type in the `fiber_spaces` cluster in `fiber_bundles`, for which restriction doesn't really make much sense.

The most prominent feature of `abstract_poset_member` and its descendants is that they are handles. We've already seen index space handles. Poset member handles are similar in concept but are used somewhat differently. First, poset member handles are not stored in pools managed by the system; the client directly creates and destroys them. The default constructor creates an unattached handle (all the following functions are direct or inherited members of `total_poset_member`):

total_poset_member()

Default constructor; creates a new, unattached `total_poset_member` handle.

Once created, you can "attach" a handle to a state:

void attach_to_state(const poset_state_handle * xhost, pod_index_type xindex)

Attach this handle to the state with index `xindex` in the current version of host `xhost`.

The `attach_to_state` function is available in several signatures for specifying the member to attach to by pod id, scoped id, name and some other variations, see the reference documentation.

You can combine the construction and attachment into a single step, also available in several signatures, for instance:

total_poset_member(const poset_state_handle * xhost, pod_index_type xindex)

Creates a new `total_poset_member` handle attached to the member state with index `xindex` in `xhost`.

You detach a handle, so it is unattached again.

void poset_component::detach_from_state()

Detach this handle from its state, if any.

You can create a handle and a new jim state in a single step:

**total_poset_member(poset_state_handle* xhost, poset_dof_map* xdof_map=0, bool
xcopy_dof_map=false, bool xauto_access=true)**

Creates a new jim (join-irreducible member) attached to a new member state in xhost.

You can use an existing handle to create a new state, as usual with several signatures, for instance:

**void new_jim_state(poset_state_handle* xhost, poset_dof_map* xdof_map = 0,
bool xcopy_dof_map = false, bool xauto_access = true)**

Creates a new jim (join-irreducible member) state in xhost and attaches this to it.

The poset member classes provides most of the operations we've already seen in the poset interface. For instance:

void create_cover_link(abstract_poset_member* xlesser)

Insert a link from this to lesser; make lesser <= this.

and

void delete_cover_link(abstract_poset_member* lesser)

Delete the link from this to lesser; make lesser incomparable to this.

In addition, the poset member classes provide several operations not available (yet) in the poset interface. In particular, it provides the lattice algebra operations, join and meet:

**total_poset_member* total_poset_member::l_join(abstract_poset_member* other,
bool xnew_jem = true)**

Lattice join of this with other, auto-allocated version. The lattice join is the least upper bound in the lattice generated by the jims in the poset.

**total_poset_member* sheaf::total_poset_member::l_meet(abstract_poset_member* other,
bool xnew_jem = true)**

Lattice meet of this with other, auto-allocated version. The lattice meet is the greatest lower bound in the lattice generated by the jims in the poset.

4.8.1 Example 11: Manipulating poset member with the poset_member interface.

We provide examples of the poset member handle interface by redoing Example 10 using the poset member interface instead of the poset interface.

```
#include "sheaves_namespace.h"
#include "poset.h"
#include "poset_dof_map.h"
#include "std_iostream.h"
#include "storage_agent.h"
#include "total_poset_member.h"

using namespace sheaf;

int main( int argc, char* argv[])
{
    cout << "SheafSystemProgrammersGuide Example11:" << endl;

    // Create a namespace.

    sheaves_namespace lns("Example11");

    // Populate the namespace from the file we wrote in example9.
    // Retrieves the simple_poset example.

    storage_agent lsa("example9.hdf", sheaf_file::READ_ONLY);
    lsa.read_entire(lns);

    // Get a reference to the poset "simple_poset".

    poset_path lpath("simple_poset");
    poset& lposet = lns.member_poset<poset>(lpath, true);

    // Create an unattached handle.

    total_poset_member lmbr;
    cout << "lmbr is_attached() = " << boolalpha << lmbr.is_attached();
    cout << endl;

    // Attach it to the top member of our poset.

    lmbr.attach_to_state(&lposet, TOP_INDEX);
    cout << "lmbr attached to " << lmbr.name() << endl;

    // Reattach it to the bottom member.

    lmbr.attach_to_state(&lposet, BOTTOM_INDEX);
    cout << "lmbr attached to " << lmbr.name() << endl;

    // Unattach it.

    lmbr.detach_from_state();
    cout << "lmbr is_attached() = " << lmbr.is_attached() << endl;

    // Allow creation of jims.

    lposet.begin_jim_edit_mode(true);

    // Create jims for the two vertices and the segment.

    total_poset_member lv0(&lposet);
```

```

total_poset_member lv1(&lposet);
total_poset_member ls0(&lposet);

// Make the segment cover the vertices.

ls0.create_cover_link(&lv0);
ls0.create_cover_link(&lv1);

// Top covers the segment.

lposet.top().create_cover_link(&ls0);

// The vertices cover bottom.

lv0.create_cover_link(&lposet.bottom());
lv1.create_cover_link(&lposet.bottom());

// We're finished creating and linking jims.

lposet.end_jim_edit_mode();

// Give each jim a name..

lv0.put_name("v0", true, true);
lv1.put_name("v1", true, true);
ls0.put_name("s0", true, true);

// Print the names to cout.

cout << lv0.name() << endl;
cout << lv1.name() << endl;
cout << ls0.name() << endl;

// Get the row attribute id space and pod and scoped ids
// for the only attribute.

const index_space_handle& latt_id_space =
    lposet.schema().dof_id_space(false);
pod_index_type latt_pod =
    lposet.schema().dof_id_space(false).begin();
scoped_index latt_id(lposet.schema().dof_id_space(false), latt_pod);

// Set attributes for v0, v1, and s0 relying on conversions.

lv0.dof_map(true).put_dof(latt_pod, int(0));
lv1.dof_map(true).put_dof(latt_pod, int(0));
ls0.dof_map(true).put_dof(latt_pod, int(1));

// Get attributes back and write them to cout.

int lv0_dim = lv0.dof_map(false).dof(latt_pod);
int lv1_dim = lv1.dof_map(false).dof(latt_pod);
int ls0_dim = ls0.dof_map(false).dof(latt_pod);

cout << "v0 dim= " << lv0_dim;
cout << " v1 dim= " << lv1_dim;
cout << " s0 dim= " << ls0_dim;

```

```

    cout << endl;

    // Create a jrm named c0.
    // C0 is the join of v0 and v1, so we can create it
    // and link it up in a single step using the join operator.

    total_poset_member* lc0 = lv0.l_join(&lv1, false);
    lc0->put_name("c0", true, true);

    // Create a jem; a copy of c0, Call it c1.

    total_poset_member lc1(*lc0, true);
    lc1.put_name("c1", true, true);

    // Output the finished poset to cout:

    cout << lposet << endl;

    // Delete c1.

    lc1.delete_state(true);

    cout << "c1 is_attached() = " << boolalpha << lc1.is_attached();
    cout << endl;

    // Exit:

    return 0;
}

```

4.9 Subposets

A subposet is, as the name suggests, a subset of a poset. We've already seen that subsets can be represented by id spaces, so why subposets? The answer is mostly historical, the id space concept emerged as a generalization of subposets and id maps. The id maps no longer exist as independent entities, they have been subsumed by the id spaces, but subposet lives on, at least for a while.

A subposet is essentially a bit vector with a bit for each member of the poset. The bit corresponding to a member is set (true) if the member of the poset is a member of the subposet. The bit vector representation is efficient in both time and memory for subsets that contain more than a few percent of the total poset, which is one reason the subposet class is still used.

The other reasons subposet is still around is that it is used for three critical roles in the system. The first critical role is the "whole" subposet. The whole subposet defines which ids are actually members of the poset; when a member is deleted the corresponding id continues to exist, but the member is no longer in the whole subposet. The functionality of whole subposet has been largely replaced by the member_hub_id_space.

The second critical role for subposet is the jims subposet. A member is a jim if and only if it is in the jims subposet. Finally, as we will learn shortly, subposets are used as filters in depth first searches of the graph.

Whenever possible, clients should use id spaces instead of subposets.

The following example shows the basic features of subposets.

4.9.1 Example 12: Subposets

```
#include "sheaves_namespace.h"
#include "poset.h"
#include "poset_dof_map.h"
#include "std_iostream.h"
#include "storage_agent.h"
#include "subposet.h"

using namespace sheaf;

int main( int argc, char* argv[])
{
    cout << "SheafSystemProgrammersGuide Example12:" << endl;

    // Create a namespace.

    sheaves_namespace lns("Example12");

    // Populate the namespace from the file we wrote in example9.
    // Retrieves the simple_poset example.

    storage_agent lsa_read("example10.hdf", sheaf_file::READ_ONLY);
    lsa_read.read_entire(lns);

    // Get a reference to the poset "simple_poset".

    poset_path lpath("simple_poset");
    poset& lposet = lns.member_poset<poset>(lpath, true);

    // Create a subposet called "jrms".

    subposet ljrms(&lposet);
    ljrms.put_name("jrms", true, false);

    // Test to see if it is empty (constructor ensures that it is).

    cout << "subposet " << ljrms.name();
    cout << " is empty()= " << boolalpha << ljrms.is_empty();
    cout << endl;

    // Put top, c1, c0, and bottom into the subposet.
    // Use all the different signatures for insert_member.

    scoped_index ltop_id(lposet.top().index());
    ljrms.insert_member(ltop_id);
```



```

    pod_index_type lc1_pod = lposet.member_id("c1", false);
    ljrms.insert_member(lc1_pod);

    pod_index_type lc0_pod = lposet.member_id("c0", false);
    ljrms.insert_member(lc0_pod);

    abstract_poset_member* lbot = &lposet.bottom();
    ljrms.insert_member(lbot);

    // Check if it contains the member we just put in.

    cout << "contains top: " << ljrms.contains_member(ltop_id) << endl;
    cout << "contains c1: " << ljrms.contains_member(lc1_pod) << endl;
    cout << "contains c0: " << ljrms.contains_member(lc0_pod) << endl;
    cout << "contains bottom: " << ljrms.contains_member(lbot) << endl;

    // Get an iterator for the members and print out their names.

    cout << "Subposet jrms contains:";
    index_iterator litr = ljrms.indexed_member_iterator();
    while(!litr.is_done())
    {
        // Print out the member name.

        cout << " " << lposet.member_name(litr.index(), false);
        litr.next();
    }
    cout << endl;

    // Remove top and bottom.

    ljrms.remove_member(lposet.top().index());
    ljrms.remove_member(&lposet.bottom());

    // Print out the member names again.

    cout << "Subposet jrms contains:";
    litr.reset();
    while(!litr.is_done())
    {
        // Print out the member name.

        cout << " " << lposet.member_name(litr.index(), false);
        litr.next();
    }
    cout << endl;

    // Exit:

    return 0;
}

```

4.10 Traversing the graph

Once you've constructed a poset or retrieved one from storage, you typically want to move around in it, explore it. This requires traversing (searching) the covering relation

graph and the SheafSystem provides iterators for that purpose. There are two kinds cover iterators and depth-first iterators.

4.10.1 Cover id spaces and iterators

Local searches, in the immediate neighborhood of a given poset member are accomplished with cover iterators. Remember that every member except bottom has a set of outgoing links to the lower cover of the member and every member except top has a set of incoming links to its upper cover. The lower and upper cover of each member are represented as id spaces:

index_space_handle& poset_state_handle::

get_cover_id_space(bool xlower, pod_index_type xmbr_hub_id) const

Allocates a handle for the lower (xlower true) or upper (xlower false) cover id space of the member with hub id xmbr_hub_id from the pool of id spaces.

As always, if you get_ it you have to release_ it when you're finished with it.

void release_cover_id_space (index_space_handle &xcover_id_space) const

Returns xcover_id_space to the pool of id spaces.

We can get the id space from the poset and get/release an iterator from the id space or get/release the iterator directly from the poset:

index_space_iterator& poset_state_handle::

get_cover_id_space_iterator(bool xlower, pod_index_type xmbr_hub_id) const

Allocates an iterator for the lower (xlower true) or upper (xlower false) cover id space of the member with hub id xmbr_hub_id from the pool of id space iterators.

void poset_state_handle::

release_cover_id_space_iterator (index_space_iterator &xcover_itr) const

Returns xcover_itr to the pool of id space iterators.

The cover_id_space and cover_id_space_iterator functions are also available from poset member handles.

4.10.1.1 Example 13: Cover id spaces and iterators

```
#include "index_space_handle.h"
#include "index_space_iterator.h"
#include "poset.h"
#include "sheaves_namespace.h"
#include "std_iostream.h"
#include "storage_agent.h"
#include "total_poset_member.h"
```

```
using namespace sheaf;
```

```
int main( int argc, char* argv[])
```

```

{
    cout << "SheafSystemProgrammersGuide Example12:" << endl;

    // Create a namespace.

    sheaves_namespace lns("Example12");

    // Populate the namespace from the file we wrote in example10.
    // Retrieves the simple_poset example.

    storage_agent lsa_read("example10.hdf", sheaf_file::READ_ONLY);
    lsa_read.read_entire(lns);

    // Get a reference to the poset "simple_poset".

    poset_path lpath("simple_poset");
    poset& lposet = lns.member_poset<poset>(lpath, true);

    // Get the hub id for member "c0".

    pod_index_type lc0_pod = lposet.member_id("c0", false);

    // Get an iterator for the lower cover to member "c0".

    index_space_handle& lc0_lc1 = lposet.get_cover_id_space(true,
lc0_pod);
    index_space_iterator& lc0_lc_itr1 = lc0_lc1.get_iterator();

    // Iterate over the members of the lower cover.

    cout << "Lower cover of c0 is:";
    while(!lc0_lc_itr1.is_done())
    {
        cout << "  " << lposet.member_name(lc0_lc_itr1.hub_pod());
        lc0_lc_itr1.next();
    }
    cout << endl;

    // We're finished with the iterator, release it.

    lc0_lc1.release_iterator(lc0_lc_itr1);

    // Get an iterator for the upper cover of c0.
    // The enums LOWER and UPPER are defined in the sheaf namespace
    // true and false, respectively.

    index_space_iterator& lc0_uc_itr1 =
        lposet.get_cover_id_space_iterator(UPPER, lc0_pod);

    // Iterate over the members of the upper cover.

    cout << "Upper cover of c0 is:";
    while(!lc0_uc_itr1.is_done())
    {
        cout << "  " << lposet.member_name(lc0_uc_itr1.hub_pod());
        lc0_uc_itr1.next();
    }
}

```

```
cout << endl;

// We're finished with the iterator, release it.

lposet.release_cover_id_space_iterator(lc0_uc_itr1);

// Repeat all the above using the poset member handle interface.

// Get a handle for member c0.

total_poset_member lc0_mbr(&lposet, "c0");

// Get an iterator for the lower cover to member "c0".

index_space_handle& lc0_lc2 = lc0_mbr.get_cover_id_space(true);
index_space_iterator& lc0_lc_itr2 = lc0_lc2.get_iterator();

// Iterate over the members of the lower cover.

cout << "Lower cover of c0 is:";
while(!lc0_lc_itr2.is_done())
{
    cout << "  " << lposet.member_name(lc0_lc_itr2.hub_pod());
    lc0_lc_itr2.next();
}
cout << endl;

// We're finished with the iterator, release it.

lc0_lc2.release_iterator(lc0_lc_itr2);

// Get an iterator for the upper cover of c0.
// The enums LOWER and UPPER are defined in the sheaf namespace
// true and false, respectively.

index_space_iterator& lc0_uc_itr2 =
    lc0_mbr.get_cover_id_space_iterator(UPPER);

// Iterate over the members of the upper cover.

total_poset_member luc_mbr;
cout << "Upper cover of c0 is:";
while(!lc0_uc_itr2.is_done())
{
    luc_mbr.attach_to_state(&lposet, lc0_uc_itr2.hub_pod());
    cout << "  " << luc_mbr.name();
    lc0_uc_itr2.next();
}
cout << endl;

// We're finished with the iterator, release it.

luc_mbr.release_cover_id_space_iterator(lc0_uc_itr2);

// Exit:

return 0;
```

}

4.10.2 Depth first traversal

Global searches, over the larger portions of a graph than the immediate neighborhood of a single member are accomplished using the `depth_first_itr` family of iterators. As implied by the name, these iterators execute a depth first search of the graph. Readers unfamiliar with depth first search should consult a basic computer science text such as Aho and Ulman "Foundations of Computer Science".

A depth first iterator starts from a given member called the anchor for the iteration and transitively follows all the links in a given direction, either down or up. A depth first search always visits all the children of a node before it visits any of the siblings of the node. There are three locations relative to a given member *p* where some action may be performed in a depth first search, as shown in Figure 4. The previsit action occurs immediately before visiting the first child, so it is the first opportunity for acting on *p*. The postvisit action occurs after all the children have been visited, so it is the last action opportunity for *p* with *p* as the current member of the iteration. The link action for a given child occurs "on the link", immediately after visiting the child. The link action for a child, with the parent as current member, occurs immediately after the postvisit action for the child, with the child as current member. So a link action can still affect a node after the postvisit action on that node has occurred.

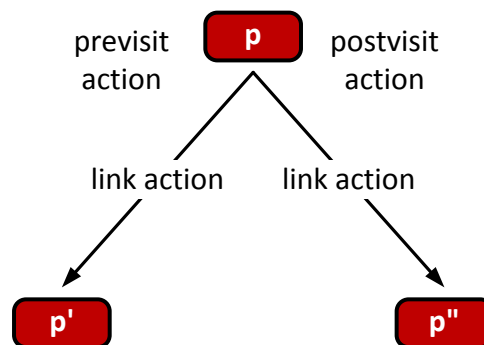


Figure 4: Actions in depth first search.

The members of the `depth_first_itr` hierarchy differentiate on which of the action opportunities are exposed to the client. Class `preorder_itr` returns control to the user only in the previsit position, immediately before iterating over the links. Class `postorder_itr` returns control to the client only in the postvisit position, after visiting all the children. Class `biorder_itr` returns control in both the previsit and postvisit positions. Class `linkorder_itr` exposes the link action opportunities. Class `trioorder_itr` exposes all the action opportunities.

The `postorder_itr` is the most frequently used iterator. Postorder has the extremely useful property that by the time a given node is visited, all the nodes less than it in the order relation have been visited.

The `preorder_itr` is the next most frequently used. It is useful because the depth first search can be truncated from the preorder position. Instead of visiting all the children, transitively, iteration can be forced to jump over the children to the next sibling. This is useful for finding maximal and minimal members of some subset, as we'll see in the example.

`Biorder_itr`, `linkorder_itr`, and `triororder_itr` are useful for advanced algorithms, especially iterations that change the graph.

All five of these classes are actually class templates, for instance:

template<typename T> class sheaf::postorder_itr< T >

Specialization of the filtered depth-first iterator which exposes the `POSTVISIT_ACTION` to the client.

The parameter specifies the type of data structure used to track which nodes in the graph have already been visited. Three specializations are provided: `zn_to_bool` (a bit vector type), `set`, and `hash set`. The specializations are provided because they have different performance characteristics. Let N be the number of nodes in the graph and M be the number visited in a given traversal. `Zn_to_bool` uses $O(1)$ time to check if a node has been visited already, but uses one bit per node and hence uses $O(N)$ memory, independent of how much of the graph is being searched. `Zn_to_bool` is most efficient for traversal of the entire graph, but the $O(N)$ memory means it takes $O(N)$ time to initialize the marker structure, a serious problem for repeated small searches. `Set` uses $O(\log(M))$ time to check if a node has been visited and $O(M)$ memory, so it is better for small searches. `Hash_set` uses $O(1)$ time and $O(M)$ memory, so it is very good for small searches, but uses more memory than `zn_to_bool` when applied to the whole graph. Typedefs are provided for the three specializations, for instance:

typedef postorder_itr<zn_to_bool> sheaf::zn_to_bool_postorder_itr

Postorder_itr<T> using `zn_to_bool` for `_has_visited` markers.

typedef postorder_itr< set<pod_index_type> > sheaf::set_postorder_itr

Postorder_itr<T> using `set` for `_has_visited` markers.

typedef postorder_itr< hash_set<pod_index_type> > sheaf::hash_set_postorder_itr

Postorder_itr<T> using `hash_set` for `_has_visited` markers.

When we create an iterator, we specify the anchor, a filter, the search direction (down or up), and strictness (whether the anchor itself is visited):

postorder_itr(const abstract_poset_member & xanchor, pod_index_type xfilter_index, bool xdown, bool xstrict)

Creates an iterator anchored at `xanchor`, filtered by `xfilter_index`. If `xdown`, iterate in the down direction, otherwise iterate up. If `xstrict`, iterate over strict up/down set only.

The filter is a subposet; the iterator only returns control to the client if the node is a member of the subposet.

4.10.2.1 Example 14: Depth first iterators

```
#include "poset.h"
#include "postorder_itr.h"
#include "preorder_itr.h"
#include "sheaves_namespace.h"
#include "std_iostream.h"
#include "storage_agent.h"
#include "total_poset_member.h"

using namespace sheaf;

int main( int argc, char* argv[])
{
    cout << "SheafSystemProgrammersGuide Example14:" << endl;

    // Create a namespace.

    sheaves_namespace lns("Example14");

    // Populate the namespace from the file we wrote in example10.
    // Retrieves the simple_poset example.

    storage_agent lsa_read("example10.hdf", sheaf_file::READ_ONLY);
    lsa_read.read_entire(lns);

    // Get a reference to the poset "simple_poset".

    poset_path lpath("simple_poset");
    poset& lposet = lns.member_poset<poset>(lpath, true);

    // Create a postorder iterator for the down set of top.
    // Filter with the jims subposet.
    // DOWN and UP are enums for true and false, respectively.
    // Include the anchor itself in the search.

    zn_to_bool_postorder_itr
    lpost_itr(lposet.top(), JIMS_INDEX, DOWN, NOT_STRICT);

    // Find all the jims.

    while(!lpost_itr.is_done())
    {
        cout << lposet.member_name(lpost_itr.index(), false);
        cout << " is a jim";
        cout << endl;
        lpost_itr.next();
    }

    // Find just the minimal jims (atoms).
    // Iterate up from the bottom and truncate when we find something.

    zn_to_bool_preorder_itr
```

```

    lpre_itr(lposet.bottom(), JIMS_INDEX, UP, NOT_STRICT);
    while(!lpre_itr.is_done())
    {
        cout << lposet.member_name(lpre_itr.index(), false);
        cout << " is an atom";
        cout << endl;
        lpre_itr.truncate();
    }

    // Exit:

    return 0;
}

```

4.11 Schema posets

We took a short cut in section 9.2 "Creating posets" to avoid having to create a schema poset before we knew how to create an ordinary poset. Now that we've seen the basics of creating posets and poset members, we can return to the task of creating a schema poset.

Actually, there is not much special about a schema poset. It is just an ordinary poset, except the schema of a schema poset has to be the "schema schema", the top member of the `primitives_schema` poset, and it has to have a couple of specific subposets created in it. But the programmer doesn't have to worry about these details because the `namespace_poset` has a factory method for making schema posets:

poset & new_schema_poset (const string &xname, bool xauto_access)

Creates a new schema poset with name xname.

So let's create the cell schema that was used for the line segment example in the Part Space tutorial. The cell schema is shown in Figure 5, which reproduces Figure 14 from the Part Space tutorial.

Using the factory method, we can create the cell schema poset in a single line:

```
poset& lcell_schema = lns.new_schema_poset("cell_schema", true);
```

Once we've got the schema poset, it's time to create some members. The class `schema_poset_member` is a specialized poset member handle for creating and manipulating schema members. Typically the most convenient way to construct a schema member is with:

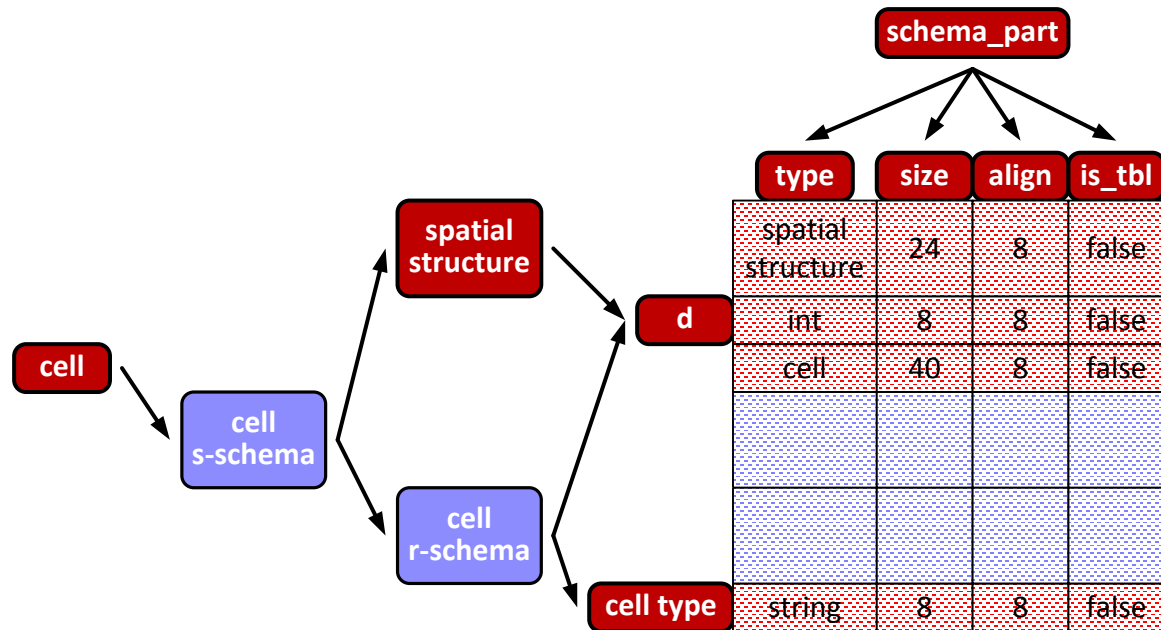


Figure 5: The cell schema table copied from the Part Space tutorial, Figure 14.

```
schema_poset_member(const namespace_poset &xns, const string &xname,
                    const poset_path &xparent_path,
                    const wsv_block<schema_descriptor> &xdof_specs,
                    bool xschematize,
                    bool xauto_access=true)
```

Creates a new jim which conforms to the schema with path `xparent_path` and has additional dofs with names, types, and roles specified by `xdof_specs`. `Xschematize` is obsolete and ignored.

This constructor supports single inheritance. The `xparent_path` argument specifies the path to the base class for the type being defined. The `xdof_space` argument specifies any additional data members. Class `schema_descriptor` is essentially a struct with members for the name, type, and `is_tbl` properties of an attribute. The "wsv" in `wsv_block` stands for "whitespace separated value" and the `wsv_block` template is essentially an array with features for easily entering elements as strings. Both define appropriate constructors and implicit conversions to make it easy to define the schema for individual attributes. The easiest way to explain how they work is with an example. Recall the cell class hierarchy from the Part Space tutorial:

```
class cell: public spatial_structure
{
    string cell_type;    // The type of spatial cell
}

class spatial_structure
{
    int d;               // The spatial dimension of the structure.
```

```
}
```

Class `cell` inherits class `spatial_structure`, so first we create the `spatial_structure` schema. It doesn't inherit anything, so we specify `bottom` as the parent. `Poset_path` has a constructor that takes a string literal, so we can specify the path using a string literal and rely on implicit conversion to `poset_path`. `Spatial_structure` has one data member, name `"d"`, type `int`, not a table attribute. The `wsv_block<schema_descriptor>` combination lets you just string the attribute specifications together, separated by white space. The set of possible primitive types is specified in the enum `primitive_type` in file `primitive_types.h` in the include directory of the SheafSystem installation. The enumerator for type `int` is `"INT"`. So the spec for the only data member of `spatial structure` is `"d INT false"`. For more attributes, just string them together, separated by whitespace. So the call to create `spatial_structure` is:

```
schema_poset_member lspatial(lns, "spatial_structure",
"cell_schema_poset/bottom", "d INT false", false, true);
```

`Cell` inherits `spatial_structure` and adds one data member, name `cell_type`, type `string`, not a table attribute. The primitive type corresponding to a C++ string object is a C string, with enumerator `C_STRING`, so we create the `cell` schema with:

```
schema_poset_member lcell(lns, "cell", lspatial.path(),
"cell_type C_STRING false", false, true);
```

So now we've created the schema for the `spatial_structure` and `cell` types, with `cell` inheriting `spatial_structure`. Let's verify that `cell` inherits `spatial_structure`. We do that with:

```
bool sheaf::schema_poset_member::  
conforms_to(const schema_poset_member & xother) const
```

True if the dofs defined by this agree in type and in order with the dofs defined by `xother`.
(This schema may contain additional dofs as well.)

So we can test whether `lcell` inherits `lspatial` with:

```
cout << "cell conforms to spatial_structure= ";  
cout << boolalpha << lcell.conforms_to(lspatial);  
cout << endl;
```

Each schema member defines an id space for its row attributes and an id space for its table attributes. We can use ids from the attribute id space to access the properties of the attributes using, for instance:

```
sheaf::size_type sheaf::schema_poset_member::  
size(pod_index_type xdof_id, bool xis_table_dof) const
```

The number of bytes in the table dof (`xis_table_dof true`) or row dof referred to by `xdof_id` in the schema defined by this.

There are similar accessors for name, type, alignment, and offset.

We demonstrate the use of all of these features and more in Example 15.

4.11.1 Example 15: Schema poset

```
#include "index_space_iterator.h"
#include "schema_poset_member.h"
#include "sheaves_namespace.h"
#include "std_iostream.h"
#include "storage_agent.h"
#include "wsv_block.h"

using namespace sheaf;

int main( int argc, char* argv[])
{
    cout << "SheafSystemProgrammersGuide Example15:" << endl;

    // Create a namespace.

    sheaves_namespace lns("Example15");

    // Create the cell schema poset.

    poset& lposet = lns.new_schema_poset("cell_schema_poset", true);

    // Create the schema for spatial_structure.
    // It doesn't inherit anything, so specify bottom as the parent.
    // It has one data member, name "d", type "INT", not a table
    // attribute

    schema_poset_member lspatial(lns, "spatial_structure",
        "cell_schema_poset/bottom", "d INT false", false, true);

    // Cell inherits spatial_structure and adds one data member,
    // name cell_type, type C_STRING, not a table attribute.

    schema_poset_member lcell(lns, "cell", lspatial.path(),
        "cell_type C_STRING false", false, true);

    // Verify that cell inherits spatial_structure.

    cout << endl;
    cout << "cell conforms to spatial_structure= ";
    cout << boolalpha << lcell.conforms_to(lspatial);
    cout << endl;
    cout << endl;

    // Get an iterator for the row attribute id space of the cell schema.

    index_space_iterator& litr =
        lcell.dof_id_space(false).get_iterator();
    while(!litr.is_done())
    {
        // Print attribute info.
```

```

    pod_index_type latt_pod = litr.pod();

    cout << "name: " << lcell.name(latt_pod, false);
    cout << " size: " << lcell.size(latt_pod, false);
    cout << " alignment: " << lcell.alignment(latt_pod, false);
    cout << " type: " << lcell.type(latt_pod, false);
    cout << " offset: " << lcell.offset(latt_pod, false);
    cout << endl;

    litr.next();
}
lcell.dof_id_space(false).release_iterator(litr);

// Print out the schema.

cout << lposet << endl;

// Test the schema by creating a poset using it.

poset& ltest =
    lns.new_member_poset<poset>("test", lcell.path(), "", true);

ltest.begin_jim_edit_mode(true);
pod_index_type lmbr0 = ltest.new_member(true);
pod_index_type lmbr1 = ltest.new_member(true);
pod_index_type lmbr2 = ltest.new_member(true);
ltest.end_jim_edit_mode(true, true);

ltest.put_member_name(lmbr0, "v0", true, false);
ltest.member_dof_map(lmbr0, true).put_dof("d", int(0));
ltest.member_dof_map(lmbr0, true).put_dof("cell_type", "vertex");

ltest.put_member_name(lmbr1, "v1", true, false);
ltest.member_dof_map(lmbr1, true).put_dof("d", int(0));
ltest.member_dof_map(lmbr1, true).put_dof("cell_type", "vertex");

ltest.put_member_name(lmbr2, "s0", true, false);
ltest.member_dof_map(lmbr2, true).put_dof("d", int(1));
ltest.member_dof_map(lmbr2, true).put_dof("cell_type", "segment");

cout << ltest << endl;

// Exit:

return 0;
}

```

5 Part II: The fiber bundle component

The sheaf component provides the fundamental mechanism for defining arbitrary persistent data types. The fiber bundle component uses this capability to define the types of the fiber bundle data model: base spaces, fiber spaces, and section spaces. We'll describe the basic functionality provided by the SheafSystem for each of these roles.

5.1 Fiber_bundles_namespace

The `fiber_bundles_namespace` class provides the sheaf schema for the specialized types defined by the fiber bundle component. As with the sheaf component, the first step in using the fiber bundle component is to create a namespace object, but for the fiber bundle component, it must be a `fiber_bundles_namespace`. You create a `fiber_bundles_namespace` object in the same way you create a `sheaves_namespace`, just give the constructor a name:

`fiber_bundles_namespace(const string &xname)`

Creates a fiber bundles namespace with name `xname`.

For example:

```
fiber_bundles_namespace lns("example16");
```

For the remainder of Part II, any namespace we mention is a `fiber_bundles_namespace`.

5.2 Base spaces

As described in the Part Spaces For Scientific Computing tutorial and the Sheaf System Analysis and Design Tutorial, the base space role in a property association is typically represented by a mesh. A mesh is a decomposition of some domain into a collection of smaller parts for the purpose of representing the dependence of one or more properties on position within the domain. In the sheaf system, a mesh is represented as a poset, with a jim for each of the basic parts in the decomposition. As with all tables in the SheafSystem, a base space table needs a schema.

5.2.1 Base space schema

Our discussion of the sheaf component delayed discussion of schema posets to the end of the discussion because we had to know how about posets in general before discussing schema posets. But you need a schema poset before you can create an object poset, so from here on we discuss the schema up front.

The `fiber_bundle_bundles` namespace defines a single universal schema for any base spaces. The base space schema poset is accessible from the namespace:

`poset& fiber_bundles_namespace::base_space_schema_poset()`

The poset defining the schemas for `base_space_member` and descendants.

However, the programmer rarely needs to even be aware of this poset, much less use it, because the system takes care of it automatically.

5.2.2 Base space posets

Meshes are represented using the class `base_space_poset`. A `base_space_poset` inherits `poset`, so it is a poset, but the class has several member functions useful for manipulating meshes. The namespace provides a factory method for creating base space posets:

```
template <typename B>  
base_space_poset& new_base_space(const poset_path& xbase_space_path,  
                                const arg_list& xargs = "",  
                                const poset_path& xschema_path = "",  
                                int xmax_db = -1,  
                                bool xauto_access = true);
```

Find or create a new base space for meshes (blocks) of type B.

The function is templated on the kind of members the mesh will contain. The type `base_space_member_prototype` can be used to create a poset that will accept any other type of member, we discuss those shortly.

The only argument that must always be provided to `new_base_space` is the name, in the form of a poset path. If the other arguments are not provided, the `new_base_space` function will apply defaults appropriate for the type of member the poset is being created for.

An exception to depending on default values is that if the maximum cell dimension `B::DB` specified by the member type B is -1, then `max_db`, must be provided either through `xargs` or directly with the `xmax_db` argument. The value for DB is specified in the reference documentation for each member type.

The reason for the additional arguments is that the system is designed to be extensible. You can program derived classes that inherit class `base_space_poset`. and such specializations may require arguments for the constructor, hence the `xargs` argument. Similarly, you can define specialized schema, even for class `base_space_poset`, hence the `xschema_path` argument. But these are advanced topics we won't cover in this introduction.

An example will show how this works. Let's return to the line segment example we used in the discussion of the sheaf component and re-do it as a base space poset. To create a base space with schema general enough for any type of member, specify `base_space_member_prototype` as the member type. Consulting the reference documentation for `base_space_member_prototype`, we find `DB = -1`. So we have to provide `max_db`; we'll use `xargs` argument. Like every poset, `base_space_poset` provides the `make_args` function:

```
static sheaf::arg_list fiber_bundle::base_space_poset::make_args(int xmax_db)
```

Makes a constructor `arg_list` for an instance with maximum intrinsic dimension `xmax_db`. Intended for use with `fiber_bundles_namespace::new_base_space`.

For our line segment example the maximum dimension is 1, so:

```
arg_list largs = base_space_poset::make_args(1);
base_space_poset& lbsp=
    lns.new_base_space<base_space_member_prototype>("mesh", largs);
```

5.2.3 Example 21: Creating a base_space_poset.

```
#include "base_space_member_prototype.h"
#include "base_space_poset.h"
#include "fiber_bundles_namespace.h"
#include "std_iostream.h"
#include "storage_agent.h"

using namespace sheaf;
using namespace fiber_bundle;

int main( int argc, char* argv[])
{
    cout << "SheafSystemProgrammersGuide Example21:" << endl;

    // Create a namespace.

    fiber_bundles_namespace lns("Example21");

    // The constructor for base_space_poset needs
    // max_db = 1 for line_segment mesh.

    arg_list largs = base_space_poset::make_args(1);

    // Create the poset.

    base_space_poset& lposet =
        lns.new_base_space<base_space_member_prototype>("mesh", largs);

    // Print the poset to cout.

    cout << lposet << endl;

    // Write the namespace to a sheaf file.

    storage_agent lsa("example21.hdf");
    lsa.write_entire(lns);

    // Exit:

    return 0;
}
```

5.2.4 Base space members

Base space members can be created in two ways. They can be created individually, like we did previously in our `simple_poset` example. However, this is tedious for anything larger than the tiniest meshes and, worse, the result makes inefficient use of memory. So the system provides "block" members. A block is a collection of cells, all of the same

type. We'll describe creating individual members in this section. Blocks can only be created with the handle interface, so we'll discuss them in section 5.2.5.

A `base_space_poset` is a poset so we can create individual members just like we did previously for the "simple_poset" example. The only difference is that `base_space_poset` provides special support for creating and initializing the attribute tuples for standard cell types.

```
const sheaf::scoped_index&  
fiber_bundle::base_space_poset::  
new_row_dof_map(const string & xprototype_name)
```

Creates a new dof map with dofs equal to those of the prototype with name `xprototype_name`.

The names of the prototypes for individual members defined by the `base_space_member_prototypes` poset in the `fiber_bundles_namespace` are shown in Table 1.

Table 1: Standard prototypes for individual cells

0-cells	1-cells	2-cells	3-cells
point	segment	triangle	tetra
		quad	hex
		general_polygon	general_polyhedron

Any of these can be used with `new_row_dof_map` to create a pre-initialized attribute tuple for the named type of cell. As an example, we recreate the line segment mesh using `base_space_poset`.

5.2.4.1 Example 22: Base space members using the `base_space_poset` interface.

```
#include "base_space_poset.h"  
#include "fiber_bundles_namespace.h"  
#include "poset_dof_map.h"  
#include "std_iostream.h"  
#include "storage_agent.h"  
  
using namespace sheaf;  
using namespace fiber_bundle;  
  
int main( int argc, char* argv[] )  
{  
    cout << "SheafSystemProgrammersGuide Example22:" << endl;  
  
    // Create a namespace.  
  
    fiber_bundles_namespace lns("Example22");  
  
    // Populate the namespace from the file we wrote in example21.  
    // Retrieves the simple_poset example.
```



```

storage_agent lsa_read("example21.hdf", sheaf_file::READ_ONLY);
lsa_read.read_entire(lns);

// Get a reference to the poset "mesh".

poset_path lpath("mesh");
base_space_poset& lposet =
    lns.member_poset<base_space_poset>(lpath, true);

// Create attribute tuples for member prototypes
// "vertex" and "segment". These will be automatically
// initialized with all the proper attribute values.

scoped_index lv_tuple_id = lposet.new_row_dof_map("point");
scoped_index ls_tuple_id = lposet.new_row_dof_map("segment");

// Allow creation of jims.

lposet.begin_jim_edit_mode(true);

// Create the first vertex and assign its attributes at the same time
// by providing the attribute tuple to use.

pod_index_type lv0_pod =
    lposet.new_member(true, lv_tuple_id.hub_pod());
lposet.put_member_name(lv0_pod, "v0", true);

// Create the second vertex with the same attribute tuple.
// v0 and v1 share the tuple; essential memory savings
// for large meshes.

pod_index_type lv1_pod =
    lposet.new_member(true, lv_tuple_id.hub_pod());
lposet.put_member_name(lv1_pod, "v1", true);

// Create the segment with the segment tuple.

pod_index_type ls0_pod =
    lposet.new_member(true, ls_tuple_id.hub_pod());
lposet.put_member_name(ls0_pod, "s0", true);

// Print the attribute tuples for the members.

cout << "v0 tuple: " << lposet.member_dof_map(lv0_pod, false);
cout << endl;
cout << "v1 tuple: " << lposet.member_dof_map(lv1_pod, false);
cout << endl;
cout << "s0 tuple: " << lposet.member_dof_map(ls0_pod, false);
cout << endl;

// Make the segment cover the vertices.

lposet.new_link(ls0_pod, lv0_pod);
lposet.new_link(ls0_pod, lv1_pod);

// Top covers the segment.

```

```

lposet.new_link(TOP_INDEX, ls0_pod);

// The vertices cover bottom.

lposet.new_link(lv0_pod, BOTTOM_INDEX);
lposet.new_link(lv1_pod, lposet.bottom().index().pod());

// We're finished creating and linking jims.

lposet.end_jim_edit_mode();

// Print the finished poset.

cout << lposet << endl;

// Write it to a file for later use.

storage_agent lsa_write("example22.hdf", sheaf_file::READ_WRITE);
lsa_write.write_entire(lns);

// Exit:

return 0;
}

```

5.2.5 Base space member handles

There are a number of handle classes for members of base space posets: `base_space_member` and the several block handle classes.

5.2.5.1 Base_space_member

`Base_space_member` is the general interface to members of `base_space_posets`. It is a `total_poset_member` that knows about the attributes that all base space members have in common:

int db () const

The base space dimension.

const pod_index_type& type_id () const

The cell type id of this. The id of the prototype of this in the "cell_types" id space of the prototypes poset.

const char* type_name() const

The cell type name.

int refinement_depth() const

The refinement depth.

5.2.5.2 Blocks

As we said before, a block is a collection of cells of the same type. In other words, a block is pretty much what we think of as a mesh, or at least a simple mesh. Some meshes require multiple blocks, perhaps of different kinds of cells.

More precisely, a collection of cells and the maximal dimension cells, which we refer to as "zones" or "elements", are all the same. Each maximal dimension cell may contain other kinds of cells in its downset. For instance, zones of any dimension typically contain vertices. The SheafSystem supports several different types of blocks, differentiated by the type of zones and how they are connected to each other:

Point blocks have zones are that isolated, disconnected 0-dimensional points. For class `point_block_1d`, the points can be indexed by a single index., class `point_block_2d` by a pair of indices (i, j) and class `point_block_3d` by a triple of indices (i, j, k)

Structured blocks are traditional i, (i,j) or (i,j,k) grids. The zones are 1, 2, or 3 dimensional boxes (line segments, quadrangles, or hexahedra, respectively) arranged in a regular 1, 2, or 3 index array and connected by their boundaries. Classes `structured_block_1d`, `structured_block_2d`, and `structured_block_3d`, respectively.

Zone-nodes blocks are finite element style meshes. The zones are line segments, triangles, quadrangles, tetrahedra, or hexahedra with explicit client-specified nodal connectivity. Class `zone_nodes_block`.

Unstructured blocks are more general finite element style meshes. The zones are copies of a client-specified zone template, connected with explicit client-specified nodal connectivity. The zone templates can, for instance, contain faces and or edges. The zone templates defined in the base space member prototypes poset are shown in Table 2.

Table 2: Standard prototypes for zone templates

0-cells	1-cells	2-cells	3-cells
	<code>segment_complex</code>	<code>triangle_nodes</code>	<code>tetra_nodes</code>
		<code>triangle_complex</code>	<code>tetra_complex</code>
		<code>quad_nodes</code>	<code>hex_nodes</code>
		<code>quad_complex</code>	<code>hex_faces_nodes</code>
			<code>hex_complex</code>

More general meshes, in fact any decomposition of a domain, can be represented using the sheaf system, but we will not discuss such meshes in this document.

The implementation of each block type is optimized for the type. Unstructured blocks can represent any of the other block types, but will not be as efficient.

5.2.5.2.1 Creating blocks

Blocks can only be created using the appropriate handle class. For instance, to create our line segment mesh as a `structured_block_1d` use:

```
fiber_bundle::structured_block_1d::  
structured_block_1d(poset * xhost, const size_type & xi_size, bool xauto_access )
```

Creates a new handle attached to a new state in `xhost` with `i_size() == xi_size`.

The attribute `i_size()` is the size of the mesh, that is, the number of zones:

```
sheaf::size_type fiber_bundle::structured_block_1d::i_size() const
```

The number of local cells (zones) in the `i` direction. The number of vertices in the `i` direction is `i_size() + 1`.

So we can create the entire line segment mesh with just:

```
structured_block_1d lblock(&lposet, 1, true);
```

5.2.5.2.2 Block id spaces

Every block provides 3 block specific id spaces:

- the local id space for all the members of the block,
- the zone id space for the zones in the block, and
- the vertex id space for the vertices in the block.

The local id space is a data member of the block handle (`homogeneous_block` is a base class for all blocks):

```
const sheaf::index_space_handle&  
fiber_bundle::  
homogeneous_block::local_id_space() const
```

The id space for the members of this block, including the block itself.

The zone id space and the vertex id space are accessed with the usual get/release pattern. for instance:

```
sheaf::index_space_handle&  
fiber_bundle::homogeneous_block::  
get_zone_id_space(bool xauto_access) const
```

Allocates a handle from the pool of handles for the id space of zones in this block.

and

```
void  
fiber_bundle::homogeneous_block::
```

release_zone_id_space(index_space_handle& xid_space, bool xauto_access) const

Returns the zone id space xid_space to the pool of handles.

Iterators for these id spaces are also available directly from the block handle, for instance:

sheaf::index_space_iterator&
fiber_bundle::homogeneous_block::
get_zone_id_space_iterator(bool xauto_access) const

Allocates a zone id space iterator from the pool of iterators.

and

void
fiber_bundle::homogeneous_block::
release_zone_id_space_iterator(index_space_iterator& xitr, bool xauto_access) const

Returns the zone id space iterator xitr to the pool of iterators.

Getting the iterators directly is more efficient than first getting the id space and then getting the iterator from the id space. This can be important in deeply nested loops.

As an example of the use of blocks, we recreate the line segment mesh yet again, this time using structured_block_1d.

5.2.5.2.3 Example 23: Blocks

```
#include "base_space_poset.h"
#include "index_space_iterator.h"
#include "fiber_bundles_namespace.h"
#include "poset_dof_map.h"
#include "std_iostream.h"
#include "std_sstream.h"
#include "storage_agent.h"
#include "structured_block_1d.h"

using namespace sheaf;
using namespace fiber_bundle;

int main( int argc, char* argv[])
{
    cout << "SheafSystemProgrammersGuide Example23:" << endl;

    // Create a namespace.

    fiber_bundles_namespace lns("Example23");

    // Populate the namespace from the file we wrote in example21.
    // Retrieves the simple_poset example.

    storage_agent lsa_read("example21.hdf", sheaf_file::READ_ONLY);
    lsa_read.read_entire(lns);
```

```

// Get a reference to the poset "mesh".

poset_path lpath("mesh");
base_space_poset& lposet =
    lns.member_poset<base_space_poset>(lpath, true);

// Create the line segment mesh as a structured block.
// Set xauto_access to true so it will invoke begin_jim_edit_mode
// and end_jim_edit_mode as needed, so we don't have to do it
// ourselves. See the precondition for details.

structured_block_1d lblock(&lposet, 1, true);
lblock.put_name("block", true, true);

// Iterate over the zones and give them names. (There's only 1).

index_space_iterator& lzitr =
    lblock.get_zone_id_space_iterator(true);
while(!lzitr.is_done())
{
    stringstream lstr;
    lstr << "s" << lzitr.pod();
    lposet.put_member_name(lzitr.hub_pod(), lstr.str(), true, false);
    lzitr.next();
}
lblock.release_zone_id_space_iterator(lzitr, true);

// Iterate over the vertices and give them names.

index_space_iterator& lvitr =
    lblock.get_vertex_id_space_iterator(true);
while(!lvitr.is_done())
{
    stringstream lstr;
    lstr << "v" << lvitr.pod();
    lposet.put_member_name(lvitr.hub_pod(), lstr.str(), true, false);
    lvitr.next();
}
lblock.release_vertex_id_space_iterator(lvitr, true);

// Print the finished poset.

cout << lposet << endl;

// Write it to a file for later use.

storage_agent lsa_write("example23.hdf", sheaf_file::READ_WRITE);
lsa_write.write_entire(lns);

// Exit:

return 0;
}

```

5.2.6 Subposets

Class `base_space_poset` maintains a subposet for the cells in the poset:

```
const sheaf::subposet&  
fiber_bundle::base_space_poset::  
cells() const
```

The subposet containing all the cells, of any dimension.

It also maintains a separate subposet for cells of each dimension:

```
const sheaf::subposet&  
fiber_bundle::base_space_poset::  
d_cells(int xd) const
```

The subposet containing the cells of dimension `xd` (const version).

The d-cells subposets are also available as id spaces:

```
const sheaf::mutable_index_space_handle&  
fiber_bundle::base_space_poset::  
d_cells_id_space(int xd) const
```

The id space for the subposet containing the cells of dimension `xd` (const version).

It's better to use the id space instead of the subposet because, as we mentioned in section 4.9, id spaces will eventually completely replace subposets.

5.2.7 Traversing the graph

The cover id space iterators and depth first iterators can be used to traverse the row graph of a `base_space_poset`, just like any ordinary poset. In addition, `base_space_poset` provides two additional id spaces and iterators, connectivity and adjacency.

5.2.7.1 Connectivity

The connectivity id space indexes the vertices in the downset of a zone. The connectivity id space is available either from `base_space_poset` or from a block handle. For instance:

```
sheaf::index_space_handle&  
fiber_bundle::base_space_poset::  
get_connectivity_id_space(pod_index_type    xzone_id, bool xauto_access) const
```

Allocates an id space handle from the connectivity handle pool attached to the connectivity id space state for zone with id `xzone_id`.

and

```
void  
fiber_bundle::base_space_poset::
```

release_connectivity_id_space_iterator(index_space_iterator& xid_space, bool xauto_access)

Returns the id space iterator xid_space to the connectivity iterator pool.

As usual, there is a similar interface for get/release of connectivity iterators.

5.2.7.2 Adjacency

The adjacency id space indexes the zones in the upset of a zone. The adjacency id space is available either from base_space_poset or from a block handle. For instance:

sheaf::index_space_handle&

fiber_bundle::base_space_poset::

get_adjacency_id_space(pod_index_type xvertex_id, bool xauto_access) const

Allocates an id space handle from the adjacency handle pool attached to the adjacency id space state for zone with id xvertex_id.

void

fiber_bundle::base_space_poset::

release_adjacency_id_space(index_space_handle& xid_space, bool xauto_access) const

Returns the id space handle xid_space to the adjacency handle pool.

There is a similar interface for get/release of adjacency iterators.

5.3 Fiber spaces

The classes in the fiber spaces cluster provide the various algebraic types used in theoretical physics to describe the properties of particles and systems. There are currently more than 40 such types, organized into an inheritance hierarchy based on their mathematical definitions. The core of the hierarchy is shown in Figure 6.

The central types of this hierarchy are:

- Vd: abstract vector of dimension d
- Tp: general tensor of degree p
- ATp: antisymmetric tensor of degree p
- Stp: symmetric tensor of degree p

As shown in the figure, the hierarchy differentiates on symmetry and degree. In addition, since a tensor is fundamentally a map on vectors and a tensor space is defined with respect to the specific kind of vector it maps, the hierarchy differentiates on vector type as well. The figure shows only a portion of the complete hierarchy supported by the Sheaf System, the portion corresponding to degrees 0 through 2 and Euclidean vector spaces E2 and E3.

Although the mathematical structure of this hierarchy is critical to the design and implementation of the Sheaf System class libraries, applications typically only use the leaf classes in the core of the hierarchy, as shown in the figure.

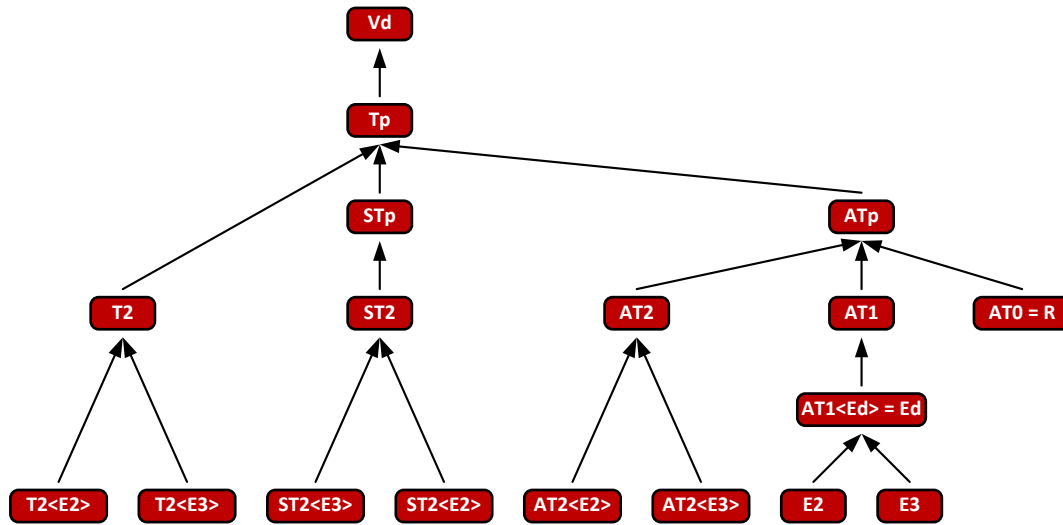


Figure 6: The core of the physical property inheritance hierarchy.

5.3.1 Fiber space schema

The sheaf schema for these types is defined in the "fiber_space_schema" table in the fiber_bundle_namespace. Example 24 prints a useful listing of all the types defined in the fiber_space_schema poset, and provides an interesting example of how to use the biorder_itr. The output is in file example24.cout

5.3.1.1 Example 24: Fiber schema subobject hierarchy.

```

#include "biorder_itr.h"
#include "fiber_bundles_namespace.h"
#include "poset.h"
#include "std_iostream.h"

using namespace sheaf;
using namespace fiber_bundle;

int main( int argc, char* argv[])
{
    cout << "SheafSystemProgrammersGuide Example24:" << endl;

    // Create a namespace.

    fiber_bundles_namespace lns("Example24");

    // Get a handle to the fiber space schema poset.

    poset& lfiber_schema =
    lns.member_poset<poset>(lns.standard_fiber_space_schema_poset_name(),
    true);

    // Use a biorder_itr to print out the subobject hierarchy.

    cout << endl;
    cout << "Subobject hierarchy in " << lfiber_schema.name(true);

```

```

cout << endl << endl;

schema_poset_member lschema(lfiber_schema.top());

int ltab_ct = 0;
set_biorder_itr litr(lfiber_schema.top(), true, true);
while(!litr.is_done())
{
    pod_index_type lhup_pod = litr.index().hub_pod();
    switch(litr.action())
    {
        case set_biorder_itr::PREVISIT_ACTION:
            if(lhup_pod != BOTTOM_INDEX)
            {
                if(lfiber_schema.is_jim(lhup_pod))
                {
                    for(int i=0; i<ltab_ct; i++) cout << "    ";
                    cout << lfiber_schema.member_name(lhup_pod, true);
                    if(lfiber_schema.is_atom(lhup_pod))
                    {
                        lschema.attach_to_state(lhup_pod);
                        cout << ": " << lschema.type();
                        cout << (lschema.is_table_dof() ? "    table" : "    row");
                    }
                    cout << endl;
                    ltab_ct++;
                }
            }
            break;
        case set_biorder_itr::POSTVISIT_ACTION:
            if(lhup_pod != BOTTOM_INDEX)
            {
                if(lfiber_schema.is_jim(lhup_pod))
                {
                    ltab_ct--;
                    if(ltab_ct == 0)
                    {
                        cout << endl;
                    }
                    litr.put_has_visited(lhup_pod, false);
                }
            }
            break;
        default:
            break;
    }
    litr.next();
}

lschema.detach_from_state();

// Exit:

return 0;
}

```

5.3.2 Fiber space posets

There are a number of specialized poset types for fiber space, corresponding the central types of the hierarchy. Specifically:

- `tuple_space`: poset for abstract tuples
- `vd_space.h`: poset for abstract vectors of dimension `d`
- `tp_space`: poset for general tensors of degree `p`
- `stp_space`: poset for symmetric tensors of degree `p`
- `atp_space`: poset for antisymmetric tensors of degree `p`
- `at1_space`: poset for antisymmetric tensors of degree 1 (ordinary vectors)
- `at0_space`: poset for antisymmetric tensors of degree 0 (scalars)
- `gln_space`: poset for general linear transformations of a vector space of dimension `n`
- `jcb_space`: poset for Jacobians

The Jacobian and linear transformation types are not shown in Figure 6.

These spaces are created using the `fiber_bundles_namespace` factory method:

```
template<typename F>
typename F::host_type&
fiber_bundle::fiber_bundles_namespace::
new_fiber_space(const poset_path& xfiber_space_path = "",
                const arg_list& xfiber_space_args = "",
                const poset_path& xfiber_space_schema_path = "",
                bool xauto_access = true)
```

Find or create a new fiber space for fibers of type `F`. Simplified interface uses default values for prerequisite vector and scalar spaces, if needed. To explicitly specify prerequisite spaces use the factory method for the category associated with `F`, i.e. `new_vector_space`, `new_tensor_space`, etc.

To show how this works, let's consider making a poset for ordinary 3D Euclidean vectors, represented by class `e3`. Each fiber type specifies or inherits a typedef for the type of poset it needs. For instance, consulting the reference documentation for `e3` we find:

```
typedef at1_space fiber_bundle::at1::host_type
```

The type of host poset.

That is, the typedef `e3::host_type` is actually inherited from class `at1`, where it is defined as class `at1_space`. So we want to create an `at1_space`.

The `new_fiber_space` function will apply a series of defaults based `F`, so we can create our space using all the defaults with:

```
at1_space& le3_space = lns.new_fiber_space<e3>();
```

The `new_fiber_space` function will examine the arguments, apply defaults and try to find the resulting spaces. If it can find them, it will use them, otherwise it will create them. This applies in particular to the space we think we are creating; if it already exists, `new_fiber_space` will just return a reference to it.

In simple cases, these defaults work fine. But if you need multiple spaces and need to make they're not interacting by using the same defaults (more on this problem shortly) then you can precisely specify the arguments as needed. The first argument, just specifies the name you want the space to have, in the form of a `poset_path` object. The second argument is an `arg_list` object for the constructor. We've seen these before, with the `sheaves_namespace::new_member_poset` factory method. As with the posets we've already seen, all the fiber space posets support the `make_args` function for creating an `arg_list`. However, in the fiber space cluster these functions are all named "`make_arg_list`" rather than `make_args`. For instance:

```
static sheaf::arg_list
fiber_bundle::at1_space::
make_arg_list(const poset_path& xscalar_space_path)
```

Creates an arg list which conforms to the schema of this.

Every vector space is defined with respect to some space of scalars and the `xscalar_path` argument specifies the scalar space for the `at1_space` we are creating. We can also create and use a custom schema for our `at1_space` using the `xfiber_space_schema_path` argument, but we won't explore that option.

The `scalar_space_path` is an example where using the defaults can cause trouble in complex cases. You can wind up with two different vector spaces using the same scalar space. There's nothing wrong with this in principle; it all depends on what you intended.

Example 25 shows how to create a fiber space both by default and with explicit arguments.

5.3.2.1 Example 25: Creating a fiber space.

```
#include "at1_space.h"
#include "e3.h"
#include "fiber_bundles_namespace.h"
#include "poset.h"
#include "std_iostream.h"

using namespace sheaf;
using namespace fiber_bundle;

int main( int argc, char* argv[])
{
    cout << "SheafSystemProgrammersGuide Example25:" << endl;

    // Create a namespace.

    fiber_bundles_namespace lns("Example25");
```

```

// Create a space for e3 objects; use all the defaults.

atl_space& le3_space = lns.new_fiber_space<e3>();

cout << "e3 space name: " << le3_space.name() << endl;
cout << "e3 space schema name: " << le3_space.schema().name();
cout << endl;
cout << "e3 space scalar space path: ";
cout << le3_space.scalar_space_path();
cout << endl;

// Create another space for e3 objects,
// specifying the arguments explicitly.

arg_list largs = atl_space::make_arg_list("e3_at0_space");

atl_space& lanother_e3_space =
    lns.new_fiber_space<e3>("another_e3_space", largs);

cout << endl;
cout << "another e3 space name: " << lanother_e3_space.name();
cout << endl;
cout << "another e3 space schema name: ";
cout << lanother_e3_space.schema().name();
cout << endl;
cout << "another e3 space scalar space path: ";
cout << lanother_e3_space.scalar_space_path();
cout << endl;

// Exit:

return 0;
}

```

5.3.3 Fiber space members

The specialized fiber space posets all inherit poset and the fiber space members can be manipulated with the member interface on the poset classes the same way we've seen in previous examples. The fiber space poset classes do add a few member functions associated with the algebraic types they represent, especially the tensor classes, but those are advanced features beyond the scope of this introduction.

5.3.4 Fiber space member handles

The fiber space member handles are the heart of the fiber space subcomponent. As described above, there are a number of them. But for practical purposes, the leaf classes in Figure 6 are the important ones: the Euclidean vectors spaces in dimension 3 and 3 and the several tensor types defined over these vector spaces.

We've already seen how to create a poset for e3, now let's create some members in that space. We've already seen in section 4.8 how use total_poset_member handles to create

new members and fiber space member are no different. Every e3 vector we want to create is a jim, so we can use the constructor:

```
fiber_bundle::e3::  
e3(poset_state_handle* xhost, bool xauto_access = true)
```

Creates a new e3 handle attached to a new jim state in xhost.

Class e3, like most of the fiber types, is a descendant of the abstract vector space class vd and hence has components. We can set and get component values in a variety of ways. The simplest are:

```
void  
fiber_bundle::vd::  
put_component(int xindex, value_type xvalue)
```

Sets the value of the xindex-th component to xvalue.

and

```
vd::value_type  
fiber_bundle::vd::  
component(int xindex) const
```

The value of the xi-th component.

5.3.4.1 Algebraic operations and namespaces

Each fiber type is associated with an algebra. For instance, the algebra associated with class e3 is Euclidean vector algebra. We refer to all these specific algebra operations collectively as the fiber algebra. The algebraic operations are defined as namespace scope functions rather than member functions and there is a subnamespace of fiber_bundle for each algebra. The subnamespace is named after the associated class, for instance the general vector algebra is in namespace fiber_bundles::vd_algebra and the 3D Euclidean vector algebra is in namespace fiber_bundle::e3_algebra. Appendix C summarizes the operations available in each algebra, see the namespace reference documents under Namespaces/Namespace list/fiber_bundle for a complete listing.

The fiber algebra is still under development. Currently, there is only partial support for the fiber algebra on the persistent fiber types, but more complete support for volatile fiber types.

5.3.4.2 Persistent, volatile, and POD types

The fiber space types are most often used for short calculations, typically inside a loop where the fiber variable is just a temporary. For such purposes, the schema, access control, and persistence properties of the sheaf tables are unnecessary, inconvenient, and frequently inefficient. For this reason, every persistent fiber type F has an associated volatile type, F_lite. For instance e3 has e3_lite. The lite types have all the same data members and algebraic operations as the full fiber types, but don't "live" in a table and

hence aren't persistent. The volatile types don't have an explicit schema and aren't access-controlled.

Lite types are intended to be easy and efficient to use as temporary variables. In particular, they can be created without a table. For instance:

```
fiber_bundle::e3_lite::  
e3_lite(const value_type& x, const value_type& y, const value_type& z)
```

Creates a new instance with components x, y, z.

Both the persistent fiber types and the lite fiber types are C++ classes with virtual functions. As a result, the byte by byte layout of instances of these types is compiler defined and usually contains data in addition to the data members defined by the SheafSystem. This makes it difficult to pass to any untyped, generic context, which includes the sheaf i/o subsystem as well as external C or FORTRAN functions. For this reason, each fiber type has a third associated type, the "plain old data" or "POD" type. (This nomenclature may seem informal to the point of being flippant, but it is in fact the official ISO C++ standard terminology!). For instance `e3::row_dofs_type` and `e3_lite::row_dofs_type` both are typedefed to the pod type `e3_row_dofs_type`.

The pod types satisfy the requirements of C++ POD types. They have no user-defined constructors or assignment operators and only public data members and member functions. They support data member access and conversion to and from full or lite types, but not algebraic operations.

The pod type provides the mechanism for easy conversion between persistent and volatile types. Both support construction from, conversion to, and assignment from the pod type. For instance:

```
fiber_bundle::e3_lite::  
e3_lite(const row_dofs_type & xrow_dofs)
```

Creates a new instance with row dofs xrow_dofs.

```
fiber_bundle::e3_lite::  
operator e3_lite::row_dofs_type& ()
```

Conversion (cast) operator to convert to the associated row dofs type (non const version).

```
fiber_bundle::e3_lite&  
fiber_bundle::e3_lite::  
operator= (const row_dofs_type & xrow_dofs)
```

Row_dofs_type assignment operator.

Implicit conversion to the pod type and construction or assignment from the pod type allows seemingly direct conversion and assignment between the persistent and volatile types. Example 26 shows how persistent and types work, and how they work together.

5.3.4.3 Example 26: Persistent and volatile types.

```

#include "atl_space.h"
#include "e3.h"
#include "fiber_bundles_namespace.h"
#include "poset.h"
#include "std_iostream.h"

using namespace sheaf;
using namespace fiber_bundle;
using namespace fiber_bundle::vd_algebra;
using namespace fiber_bundle::ed_algebra;
using namespace fiber_bundle::e3_algebra;

int main( int argc, char* argv[])
{
    cout << "SheafSystemProgrammersGuide Example26:" << endl;

    // Create a namespace.

    fiber_bundles_namespace lns("Example26");

    // Create a space for e3 objects; use all the defaults.

    atl_space& le3_space = lns.new_fiber_space<e3>();

    // Create some persistent vectors.

    e3 i_hat(&le3_space, true);
    e3 j_hat(&le3_space, true);

    // Set the components of i_hat individually.

    i_hat.put_component(0, 1.0, false);
    i_hat.put_component(1, 0.0, false);
    i_hat.put_component(2, 0.0, false);

    // Set the components of j_hat all at once.

    j_hat.put_components(0.0, 1.0, 0.0);

    // Create and initialize some volatile vectors

    e3_lite i_hat_v(i_hat);
    e3_lite j_hat_v;
    j_hat_v = j_hat;

    cout << "i_hat_v:" << i_hat_v << endl;
    cout << "j_hat_v:" << j_hat_v << endl;

    // "Auto-allocated" add creates result on heap.

    e3_lite* v1_v = add(i_hat_v, j_hat_v);

    cout << "v1_v:\t" << *v1_v << endl;

```



```

// Pre-allocated add requires result as argument

e3_lite v2_v;
add(i_hat_v, j_hat_v, v2_v);

cout << "v2_v:\t" << v2_v << endl;

// "Self-allocated" add uses first arg as result.

e3_lite v3_v(i_hat_v);
add_equal(v3_v, j_hat_v);

cout << "v3_v:\t" << v3_v << endl;

// Operations are also available as overloaded operators,
// whenever it makes sens.

e3_lite* v4_v = i_hat_v + j_hat_v;

cout << "v4_v:\t" << *v4_v << endl;

e3_lite v5_v(i_hat_v);
v5_v += j_hat_v;

cout << "v5_v:\t" << v5_v << endl;

// All the usual operations of Euclidean vector algebra are
// available as functions, and also as operators, when it
// makes sense. For instance:

// Multiplication by scalar

e3_lite v6_v;
multiply(i_hat_v, 2.0, v6_v);

cout << "v6_v:\t" << v6_v << endl;

e3_lite v7_v(i_hat_v);
v7_v *= 2.0;

cout << "v7_v:\t" << v7_v << endl;

// Dot product.

e3::value_type ls0 = dot(i_hat_v, v5_v);
e3::value_type ls1 = i_hat_v*v5_v;

size_type lprec = cout.precision(18);
cout << "ls0:\t" << scientific << setw(27) << ls0 << endl;
cout << "ls1:\t" << scientific << setw(27) << ls1 << endl;
cout.precision(lprec);

// Cross product

e3_lite k_hat_v;
cross(i_hat_v, j_hat_v, k_hat_v);

```

```

    cout << "k_hat_v:" << k_hat_v << endl;

    k_hat_v = v6_v;
    k_hat_v ^= v5_v;

    cout << "k_hat_v:" << k_hat_v << endl;

    // Normalization.

    normalize(k_hat_v);

    cout << "k_hat_v:" << k_hat_v << endl;

    // Convert volatile to persistent;
    // uses implicit coversion to row_dofs_type.

    e3 k_hat(le3_space, k_hat_v, true);

    // Clean up.
    // Workaround for soon to be fixed bug.

    i_hat.detach_from_state();
    j_hat.detach_from_state();
    k_hat.detach_from_state();

    // Exit:

    return 0;
}

```

5.3.5 Subposets

There are no additional subposet features associated with fiber spaces.

5.3.6 Traversing the graph

There are no additional graph traversal methods associated with the fiber spaces.

5.4 Section spaces

We saw in Part Spaces For Scientific Computing and The Sheaf System Analysis and Design Tutorial that a section is a member of a section space poset and in practical cases the schema for a section space poset is the tensor product of the base space lattice and the fiber schema lattice. The set of jims of the tensor product is the Cartesian product of the jims of the base space and the jims of the fiber space schema. The members of the tensor product are pairs (base space member, fiber schema member) and joins of such members with no components in common. A pair (base space member, fiber schema member) corresponds to a "homogenous" representation, the fiber schema is the same everywhere on the given base space member. A join of pairs corresponds to an "inhomogeneous" representation, a collection of homogeneous schema for different parts of the base space. What the fiber schema is depends on which base space member you are talking about.

A direct representation of this poset, using the same method as the other posets we've described, would require instantiating the set of jims and any jrms we are interested in. Such a naive implementation is however infeasible - in practical cases the Cartesian product of the base space jims and fiber schema jims is just too large to store.

So some method other than just storing the product needs to be used. The approach currently implemented in the SheafSystem provides only a subspace of the entire tensor space, specifically the Cartesian subspace consisting of only the pairs (base space member, fiber schema member), and hence the current implementation can support only homogeneous representations.

This approximation, which we will refer to as the product subspace approximation, synthesizes the product on the fly, using special iterators, whenever it is needed. As a result, a section space schema poset is actually *empty*, it contains only the two standard members top and bottom. We can attach `section_space_schema_member` handles to members as if they actually existed, but there are no member objects stored in the poset. All this is transparent to the client, as long as one accesses the poset through the special member handles, iterators, and id spaces provided for the purpose.

5.4.1 Section space schema

Section spaces differ from the base spaces and fiber spaces we've seen so far in an important way. Both the base space and the fiber spaces could use a predefined schema. The schema of a section space depends on the base space, so the schema cannot be predefined. It has to be created once we know what the base space should be.

In order to create a section space schema we need to know three pieces of information:

1. the base space, the mesh the section space is based on;
2. the fiber space schema, the type of the dependent variable; and
3. the representation type ("rep type"), the method used to discretize a section.

The base space must always be specified by the client, but a given section class may specify the fiber space and possibly the rep type. So each section class provides a static function `make_schema` that takes the specific arguments needed to create a schema. For instance, the section class `sec_e2` corresponds to a 2D vector section. The fiber space (e2) is fixed for this section type, so the client need only specify the base space and representation.

5.4.1.1 Representation type

A representation type is identified by three further pieces of information:

4. the discretization subposet, the subset of cells in the mesh with which the section data (attributes) are associated;

5. the evaluation subposet, the subset of cells with which the interpolation functions are associated; and
6. the evaluation method, the type of interpolation function.

A typical rep type would specify the vertices as the discretization subposet, the zones as the evaluation subposet, and linear interpolation as the evaluation method. We can visualize a representation type by tagging the members of the discretization and evaluation subposets in the graph of the base space, as shown for our line segment example in Figure 7.

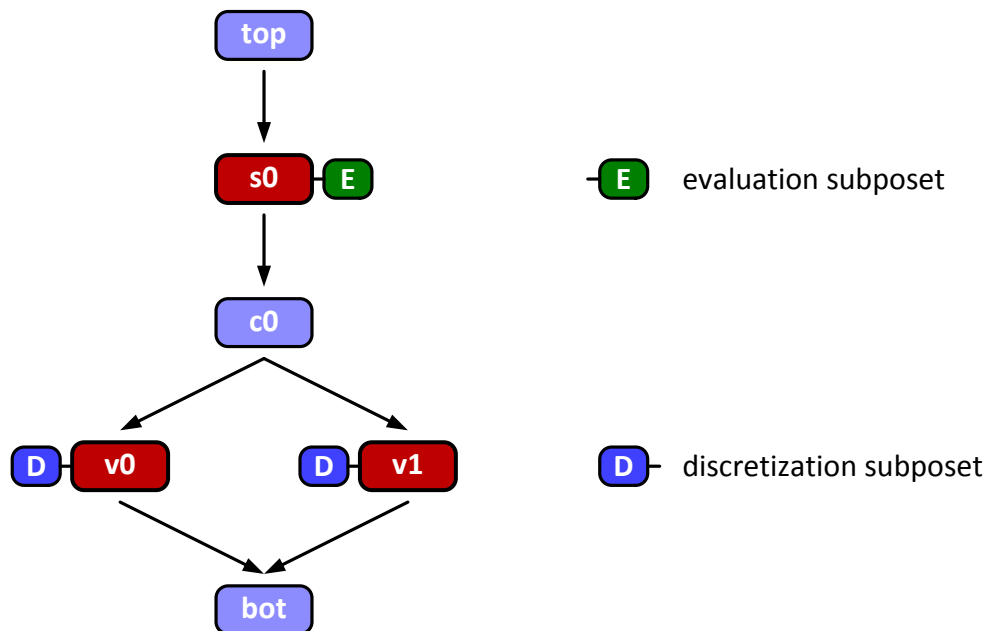


Figure 7: Rep type for the line segment mesh.

5.4.1.2 Sec_rep_descriptors

The poset "sec_rep_descriptors" in the fiber bundles namespace provides a number of predefined rep types, with names derived from these three quantities. For instance, "vertex_element_dlinear" is a representation with the data at the vertices, interpolated over the zones (elements) using linear, bilinear, or trilinear interpolation, depending on the dimension of the mesh and the type of zone. Another example is "element_element_constant", for which the data is at the centers of the zones and is "interpolated" over the zones as a constant value. There are a number of other predefined rep types that address common cases, or the client can define specialized rep types. The standard types predefined in the sec_rep_descriptors poset of the fiber_bundles_namespace are shown in Table 3.

Table 3: Standard sec_rep_descriptors

representation type	discretization subposet	evaluation subposet	evaluation method
vertex_element_dlinear	__vertices	__elements	dlinear
vertex_cells_dlinear	__vertices	__cells	dlinear
element_element_constant	__elements	__elements	constant
vertex_block_dlinear	__block_vertices	__blocks	dlinear
vertex_block_uniform	__block_vertices	__blocks	uniform
vertex_vertex_constant	__vertices	__vertices	constant

5.4.1.3 Creating section space schema

Section space schema are created with a factory method. The base space and rep_type must already exist. If the fiber space also already exists, we can use the simplified factory method:

```

sheaf::poset_path
fiber_bundle::fiber_bundles_namespace::
new_section_space_schema(const poset_path& xsection_space_schema_path,
                           const poset_path& xrep_path,
                           const poset_path& xbase_space_path,
                           const poset_path& xfiber_space_path,
                           bool xauto_access)

```

Creates a new section space schema and member in namespace xns with name xname and representation, base space and fiber space specified by xrep_path, xbase_space_path, and xfiber_space_path, respectively. Note that the objects referred to by xrep_path, xbase_space_path, and xfiber_space_path must already exist and that xfiber_space_path specifies the fiber space, not the fiber schema.

Note that this factory method does not create just a section space schema poset. It creates a section space schema, which is a member in a section space schema poset. This requires that the xsection_space_schema_path argument be "full", that is, specify both a poset name and a member name. Of course, given the special representation of the section space schema poset, the member doesn't really exist, but it has a name anyway!

Example 27 creates the schema for an e2 section space on our line segment mesh.

5.4.1.4 Example 27: Creating a section space schema.

```

#include "at1_space.h"
#include "base_space_poset.h"
#include "e2.h"
#include "fiber_bundles_namespace.h"
#include "index_space_iterator.h"
#include "poset_dof_map.h"

```

```

#include "binary_section_space_schema_poset.h"
#include "std_iostream.h"
#include "std_sstream.h"
#include "storage_agent.h"
#include "structured_block_1d.h"

using namespace sheaf;
using namespace fiber_bundle;

int main( int argc, char* argv[])
{
    cout << "SheafSystemProgrammersGuide Example27:" << endl;

    // Create a namespace.

    fiber_bundles_namespace lns("Example27");

    // Populate the namespace from the file we wrote in example23.
    // Retrieves the mesh poset example.

    storage_agent lsa_read("example23.hdf", sheaf_file::READ_ONLY);
    lsa_read.read_entire(lns);

    // Create a space for e2 objects; use all the defaults.

    atl_space& le2_space = lns.new_fiber_space<e2>();

    // Create a section space schema
    // named "e2_on_mesh/e2_on_block"
    // using
    // "vertex_cells_dlinear" sec_rep_descriptor,
    // the "block" member in the "mesh" poset as the base space,
    // and the fiber space we just created.
    // Specify xauto_access = true to allow begin/end jim edit mode.

    poset_path lsssp_path =
        lns.new_section_space_schema("e2_on_mesh/e2_on_block",
"sec_rep_descriptors/vertex_cells_dlinear",
                                "mesh/block",
                                le2_space.path(),
                                true);

    // Get a handle to the poset

    binary_section_space_schema_poset& lsssp =
        lns.member_poset<binary_section_space_schema_poset>(lsssp_path,
false);

    // Print the finished poset.

    cout << lsssp << endl;

    // Write it to a file for later use.

    storage_agent lsa_write("example27.hdf", sheaf_file::READ_WRITE);
    lsa_write.write_entire(lns);

```

```
// Exit:
return 0;
}
```

5.4.2 Section space posets

There are a number of specialized poset types for section spaces, one for each specialized type of fiber space. Specifically:

- `sec_tuple_space`: poset for sections with fiber abstract tuple
- `sec_vd_space.h`: poset for sections with fiber abstract vectors of dimension `d`
- `sec_tp_space`: poset for sections with fiber general tensors of degree `p`
- `sec_stp_space`: poset for sections with fiber symmetric tensors of degree `p`
- `sec_atp_space`: poset for sections with fiber antisymmetric tensors of degree `p`
- `sec_at1_space`: poset for sections with fiber antisymmetric tensors of degree 1 (ordinary vectors)
- `sec_at0_space`: poset for sections with fiber antisymmetric tensors of degree 0 (scalars)
- `sec_jcb_space`: poset for sections with fiber Jacobians

Like the fiber spaces, these posets are created with a `fiber_bundles_namespace` factory method:

```
template<typename S>
typename S::host_type&
fiber_bundle::fiber_bundles_namespace::
new_section_space(const poset_path& xsection_space_path,
                  const poset_path& xbase_space_path,
                  const poset_path& xrep_path = "",
                  bool xauto_access = true )
```

Finds or creates the section space and section space_schema specified by the arguments for sections of type `S`. Simplified interface uses default values for prerequisite vector and scalar spaces, if needed. To explicitly specify prerequisite spaces use the factory method for the category associated with `S`, i.e. `new_vector_section_space`, etc.

Also like, the fiber types, each section type provides a typedef "host_type" for the type of poset it requires. For instance, for a section of type `sec_e2`, that is a section with fiber `e2`, we have:

```
typedef sec_at1_space fiber_bundle::sec_at1::host_type
```

That is, the typedef `sec_e2::host_type` is inherited from class `sec_at1`, where it is defined as class `sec_at1_space`.

Note that this factory method does not require a specification of the schema for the section space. Although in the last section we saw how to explicitly create a section space

schema, in most practical cases we do not need to, we can let this factory method apply reasonable defaults and find or create a schema. As with the fiber space, there are additional factory method signatures that give the programmer complete control over the create of the section space schema and section space posets, but these are only required for advanced techniques that are beyond the scope of this tutorial.

Example 28 creates a section space for `sec_e2` on the line segment mesh.

5.4.2.1 Example 28: Creating a section space.

```
#include "fiber_bundles_namespace.h"
#include "sec_atl_space.h"
#include "sec_e2.h"
#include "std_iostream.h"
#include "storage_agent.h"

using namespace sheaf;
using namespace fiber_bundle;

int main( int argc, char* argv[])
{
    cout << "SheafSystemProgrammersGuide Example28:" << endl;

    // Create a namespace.

    fiber_bundles_namespace lns("Example28");

    // Populate the namespace from the file we wrote in example27.
    // Retrieves the base space, fiber space and section space schema.

    storage_agent lsa_read("example27.hdf", sheaf_file::READ_ONLY);
    lsa_read.read_entire(lns);

    // Create a section space for sec_e2 sections
    // on the line segment mesh
    // using default values for fiber space, rep, and schema.

    poset_path lbase_path("mesh/block");
    poset_path lssp_path("e2_on_block");

    sec_e2::host_type& lhost =
        lns.new_section_space<sec_e2>(lssp_path, lbase_path);

    // Print the finished poset.

    cout << lhost << endl;

    // Write it to a file for later use.

    storage_agent lsa_write("example28.hdf", sheaf_file::READ_WRITE);
    lsa_write.write_entire(lns);

    // Exit:

    return 0;
}
```



```
}
```

5.4.3 Section space members

The specialized section space posets all inherit poset and the section space members can be manipulated with the member interface on the poset classes the same way we've seen in previous examples. As with the fiber space poset classes, the section space class add a few member functions associated with the algebraic types they represent, especially the tensor classes, but those are advanced features beyond the scope of this introduction.

5.4.4 Section space member handles

There is a section space member handle type for each fiber type and, as with the fiber types, it is the leaf types in Figure 6 that are important in practice.

5.4.4.1 Creating and accessing sections.

We create sections in much the same way we create fiber objects. We'll use `sec_e2` as an example:

```
fiber_bundle::sec_e2::  
sec_e2 (sec_rep_space* xhost,  
        section_dof_map* xdof_map = 0,  
        bool xauto_access = true )
```

Creates a new `sec_e2` handle attached to a new jim state in `xhost`.

We usually create let the constructor create the attribute tuple by accepting the default arguments.

5.4.4.1.1 Section id spaces

In the product subspace approximation, an instance of the fiber is stored for each member of the discretization subposet in the base space and the `fiber_space_schema` defines the data that represents each instance of the fiber.

This means that there are 3 id spaces associated with the section attributes: the fiber attribute id space, the discretization id space, and the section attribute id space. The fiber attribute id space is the row attribute id space of the fiber schema. The discretization id space is the id space of the discretization subposet. The section attributes id space is the Cartesian product of the discretization id space and the fiber dof id space.

The interface for accessing the section values is based on these 3 id spaces, with an accessor function, a mutator function and an iterator for each id space.

5.4.4.1.2 Discretization id access

Typically the most convenient access is via discretization id. One can iterate over all the discretization "points" of a section and, since an instance of the fiber type is associated with each discretization id, one can get or put a fiber value:

The discretization id space is available from the schema:

```
const index_space_handle&  
fiber_bundle::section_space_schema_member::  
discretization_id_space() const
```

The id space for the discretization members in the down set of the base space of this (const version).

The value at each discretization id is a copy of the fiber. The accessor, `get_fiber`, and the mutator, `put_fiber`, are defined in the base class `sec_vd`:

```
void  
fiber_bundle::sec_vd::  
get_fiber(pod_index_type xdisc_id, vd_lite& xfiber) const
```

Sets `xfiber` to the fiber referred to by discretization id `xdisc_id`.

```
void  
fiber_bundle::sec_vd::  
put_fiber(pod_index_type xdisc_id, const vd_lite& xfiber)
```

Sets the fiber referred to by discretization id `xdisc_id` to `xfiber`.

5.4.4.1.3 Section attribute id access

Occasionally it is more convenient to access the individual attributes. In that case, one can iterate directly over the attribute ids of a section and get or put an individual attribute value. The row attribute id space is available from a section schema just like it is from an ordinary schema member, in fact the section schema inherits the `dof_id_space` accessor from `schema_poset_member`:

```
const sheaf::index_space_handle&  
sheaf::schema_poset_member::  
dof_id_space(bool xis_table_dofs) const
```

The table dof (`xis_table_dof true`) or row dof id space for the schema defined by this.

For convenience and clarity, the accessor `row_dof_id_space()` is defined as a synonym for `dof_id_space(false)`.

The individual attributes can be accessed in the same way as those of ordinary poset members, as described in section 4.7.3.4. In addition, since every attribute corresponds to a pair (discretization id, fiber attribute id), there are accessor and mutation signatures for this combination of ids, inherited from base class `sec_rep_space_member`:

```

void
fiber_bundle::sec_rep_space_member::
get_dof(pod_index_type xdisc_id,
        pod_index_type xfiber_dof_id,
        void* xdof,
        size_type xdof_size ) const

```

Copies the dof referred to by xdisc_id, xfiber_dof_id into xdof.

```

void
fiber_bundle::sec_rep_space_member::
put_dof(pod_index_type xdisc_id,
        pod_index_type xfiber_dof_id,
        const void* xdof,
        size_type xdof_size )

```

Sets the dof referred to by xdof_id to the value at xdof.

5.4.4.1.4 Fiber attribute id access

One can also access the section values by fiber attribute id. This access method is included mostly for mathematical completeness. It is rarely used in modern applications but can be useful in the context of legacy code oriented towards vector processor architectures.

Since the fiber attributes are usually the components of the fiber, the collection of section attributes with a given fiber attribute id usually constitute a component of the section. Hence, this access method is called component access.

```

void
fiber_bundle::sec_rep_space_member::
get_component(pod_index_type xfiber_dof_id,
              void* xcomponent,
              size_type xcomponent_size) const

```

Sets xcomponent to the component referred to by fiber id xfiber_dof_id.

```

void
fiber_bundle::sec_rep_space_member::
put_component(pod_index_type xfiber_dof_id,
              const void* component,
              size_type xcomponent_size)

```

Sets the xcomponent referred to by fiber id xfiber_dof_id to xcomponent.

5.4.4.1.5 Example 29: Creating and accessing sections.

```

#include "e2.h"
#include "fiber_bundles_namespace.h"

```

```

#include "index_space_handle.h"
#include "index_space_iterator.h"
#include "sec_at1_space.h"
#include "sec_e2.h"
#include "std_iostream.h"
#include "storage_agent.h"

using namespace sheaf;
using namespace fiber_bundle;
using namespace fiber_bundle::vd_algebra;

int main( int argc, char* argv[])
{
    cout << "SheafSystemProgrammersGuide Example29:" << endl;

    // Create a namespace.

    fiber_bundles_namespace lns("Example29");

    // Populate the namespace from the file we wrote in example28.
    // Retrieves the section space.

    storage_agent lsa_read("example28.hdf", sheaf_file::READ_ONLY);
    lsa_read.read_entire(lns);

    // Get a handle for the section space.

    poset_path lssp_path("e2_on_block");

    sec_e2::host_type& lhost =
        lns.member_poset<sec_e2::host_type>(lssp_path, true);

    // Create a section.

    sec_e2 lsec1(&lhost);

    // Initialize the first section using the disc id interface.

    // First, get an instance of the lite fiber type
    // (e2_lite in this case).

    sec_e2::fiber_type::volatile_type lfiber(0.0, 1.0);

    // Iterate over the discretization setting the fibers.

    index_space_handle& ldisc_id_space =
        lsec1.schema().discretization_id_space();

    index_space_iterator& ldisc_itr = ldisc_id_space.get_iterator();
    while(!ldisc_itr.is_done())
    {
        // Note that we use the pod id, not the hub pod id,
        // the id has to be in the disc id space.

        lsec1.put_fiber(ldisc_itr.pod(), lfiber);
        ldisc_itr.next();
    }
}

```

```

// Create a second section and initialize it
// from the first section using disc id interface.

sec_e2 lsec2(&lhost);

sec_e2::value_type lval = 1.0;
ldisc_itr.reset();
while(!ldisc_itr.is_done())
{
    lsec1.get_fiber(ldisc_itr.pod(), lfiber);

    lfiber *= lval;
    lval *= 2;

    lsec2.put_fiber(ldisc_itr.pod(), lfiber);
    ldisc_itr.next();
}

// Create a third section and initialize it
// from the first section using section attribute id interface.

sec_e2 lsec3(&lhost);

const index_space_handle& lsec_id_space =
    lsec1.schema().row_dof_id_space();
index_space_iterator& lsec_id_itr = lsec_id_space.get_iterator();
while(!lsec_id_itr.is_done())
{
    lval = lsec1.dof(lsec_id_itr.pod());
    lval *= 2.0;
    lsec3.put_dof(lsec_id_itr.pod(), lval);
    lsec_id_itr.next();
}
lsec_id_space.release_iterator(lsec_id_itr);

// Create a fourth section and initialize it
// from the first section using disc id, fiber id interface.

sec_e2 lsec4(&lhost);

const index_space_handle& lfiber_id_space =
    lsec1.schema().fiber_schema().row_dof_id_space();
index_space_iterator& lfiber_id_itr = lfiber_id_space.get_iterator();

ldisc_itr.reset();
while(!ldisc_itr.is_done())
{
    while(!lfiber_id_itr.is_done())
    {
        lsec1.get_dof(ldisc_itr.pod(), lfiber_id_itr.pod(),
                     &lval, sizeof(lval));

        lval *= 3;

        lsec4.put_dof(ldisc_itr.pod(), lfiber_id_itr.pod(),
                     &lval, sizeof(lval));
    }
}

```

```

        lfiber_id_itr.next();
    }
    lfiber_id_itr.reset();
    ldisc_itr.next();
}

// Create a fifth section

sec_e2 lsec5(&lhost);

// Allocate a buffer to store a section component
// All the components are the same size for a sec_e2.

size_type lcomp_size = lsec1.schema().component_size(0);

sec_e2::dof_type* lcomp = new sec_e2::dof_type[lcomp_size];

// Initialize the 5th section using component access.

lfiber_id_itr.reset();
while(!lfiber_id_itr.is_done())
{
    lsec1.get_component(lfiber_id_itr.pod(), lcomp, lcomp_size);
    lsec5.put_component(lfiber_id_itr.pod(), lcomp, lcomp_size);
    lfiber_id_itr.next();
}

lfiber_id_space.release_iterator(lfiber_id_itr);
ldisc_id_space.release_iterator(ldisc_itr);

// Print the finished poset.

cout << lhost << endl;

// Write it to a file for later use.

storage_agent lsa_write("example29.hdf", sheaf_file::READ_WRITE);
lsa_write.write_entire(lns);

// Exit:

return 0;
}

```

5.4.4.2 Algebraic operations and namespaces

The section type hierarchy repeats the fiber type hierarchy: for every type F in the fiber hierarchy there is a section type S with fiber of type F . Every fiber operation has a corresponding section operation. See Appendix D for additional discussion.

5.4.4.3 Coordinate sections

The positions of the cells in a mesh are not an intrinsic feature of the mesh in the sheaf system. Positions are assigned to the cells by defining a coordinates section on the mesh.

A mesh can have a single coordinates section or many coordinates sections. For instance, if the positions of the vertices change with time, then the coordinates at each time step can be stored as separate sections.

Any section which defines a one-to-one mapping of points in the base space to position can be used as a coordinates section, but the sheaf system provides two types of sections specifically intended for coordinates: uniform coordinates and general coordinates.

5.4.4.3.1 Creating uniform coordinates on structured blocks

Uniform coordinates can be used only on structured blocks. Uniform coordinates are defined by positions at the corners of the mesh and are interpolated uniformly (actually, d-linearly, where d is the dimension of the mesh) on the interior of the mesh. Assuming the positions are chosen to be a d-dimensional box, uniform coordinates make the mesh a "regular" grid - the vertices are evenly spaced and the zones are all the same size.

5.4.4.3.2 Creating general coordinates

General coordinates can be created on any type of block using the `sec_e1`, `sec_e2`, or `sec_e3` classes. The client has to explicitly set the attributes of the coordinates after creating the section. In the case of complex mesh shapes, as is usually the case with zone-node and unstructured blocks, the block vertex id space is just a simple id space. The positions of the vertices were presumably determined by the mesher, at the same time the nodal connectivity was defined, and this position information can be used.

5.4.4.3.3 Example 30: Creating coordinate sections

```
#include "e2.h"
#include "fiber_bundles_namespace.h"
#include "index_space_handle.h"
#include "index_space_iterator.h"
#include "sec_atl_space.h"
#include "sec_e1.h"
#include "sec_e1_uniform.h"
#include "std_iostream.h"
#include "storage_agent.h"
#include "structured_block_ld.h"

using namespace sheaf;
using namespace fiber_bundle;
using namespace fiber_bundle::vd_algebra;

int main( int argc, char* argv[])
{
    cout << "SheafSystemProgrammersGuide Example30:" << endl;

    // Create a namespace.

    fiber_bundles_namespace lns("Example30");
```

```

// Create a new base space with structured_block_1d with 2 segments.

arg_list largs = base_space_poset::make_args(1);
base_space_poset& lbase_host =
    lns.new_base_space<structured_block_1d>("mesh2", largs);
structured_block_1d lblock(&lbase_host, 2, true);
lblock.put_name("block", true, true);

// Create a section space for uniform coordinates

poset_path lelu_path("el_uniform_on_block");
poset_path lbase_path("mesh2/block");
poset_path lelu_rep_path("sec_rep_descriptors/vertex_block_uniform");

sec_el_uniform::host_type& lelu_host =
    lns.new_section_space<sec_el_uniform>(lelu_path, lbase_path,
                                          lelu_rep_path, true);

// Create the coordinates section.
// Uniform coordinates are initialized by the constructor.

sec_el_uniform lelu_coords(&lelu_host, -1.0, 1.0, true);
lelu_coords.put_name("uniform_coordinates", true, true);

// Create a section space for general coordinates;
// use the default vertex_element_dlinear rep.

poset_path lel_path("el_on_block");

sec_el::host_type& lel_host =
    lns.new_section_space<sec_el>(lel_path, lbase_path);

// Create a general coordinates section.

sec_el lel_coords(&lel_host);

// Have explicitly initialize general coordinates.
// In a more realistic case, the coordinate values
// would come from some external source such as a mesher.
// But this mesh only has 3 vertices, so we'll just make
// something up that's definitely not uniform..

sec_el::dof_type lcoord_values[3] = { 0.0, 1.0, 10.0};

sec_el::fiber_type::volatile_type lfiber;
index_space_handle& ldisc_id_space =
    lel_coords.schema().discretization_id_space();
index_space_iterator& ldisc_itr = ldisc_id_space.get_iterator();
while(!ldisc_itr.is_done())
{
    lfiber = lcoord_values[ldisc_itr.pod()];
    lel_coords.put_fiber(ldisc_itr.pod(), lfiber);
    ldisc_itr.next();
}
ldisc_id_space.release_iterator(ldisc_itr);

// Print the finished posets.

```



```

    cout << lelu_host << endl;
    cout << lel_host << endl;

    // Write it to a file for later use.

    storage_agent lsa_write("example30.hdf", sheaf_file::READ_WRITE);
    lsa_write.write_entire(lns);

    // Exit:

    return 0;
}

```

5.4.4.4 Multi-valued sections

Sometimes we need to represent a property that is multivalued. Consider a domain which consists of two parts ("patches") made of different materials. At the boundary between the two patches, density may have some value in the one material and a different value in the other material, making it effectively double-valued on the boundary. As another example, some objects, such as the surface of the Earth, cannot be completely covered by a single coordinate system. In such cases the standard practice is to decompose the object into overlapping patches with a single-valued coordinate system on each patch.

In both these cases, the result is a set of ordinary single-valued sections, one on each patch. It is very useful to treat such a collection as a single section which is multi-valued where the patches overlap. We call such a multi-valued section a multisection and each single-valued piece is referred to as a branch of the multisection. Since a multisection is a collection of single-valued sections, each of which is a jim in section space, a multisection is represented by a jrm in section space.

5.4.4.4.1 Creating a multisection

Creating a multisection just requires specifying the patches on which the section is single-valued, for instance:

```

fiber_bundle::sec_e2::
sec_e2(sec_rep_space* xhost, const subposet& xbase_parts, bool xauto_access)

```

Creates a new handle attached to a new jrm state which is a multi-section with a branch for each member of the base space contained in xbase_parts.

5.4.4.4.2 Accessing the attributes of a multisection

A multisection is a jrm and hence does not itself have attributes, only the branches have attributes. Each multisection defines an id space for its branches. The base class sec_rep_space_member supplies the usual get/release protocol, inherited by all section types:

```

sheaf::mutable_index_space_handle&
fiber_bundle::sec_rep_space_member::

```

get_branch_id_space(bool xauto_access)

Allocate an id space handle from the handle pool attached to the branch id space for this member.

void

fiber_bundle::sec_rep_space_member::

release_branch_id_space(index_space_handle& xid_space, bool xauto_access) const

Returns the id space handle xid_space to the handle pool.

Get_branch_id_space_iterator and release_branch_id_space_iterator are also provided for direct access to the iterators.

The attributes of each branch may be accessed as described above for an ordinary section.

Example 31 shows how to create and access multisections.

5.4.4.4.3 Example 31: Multisections

```
#include "e2.h"
#include "fiber_bundles_namespace.h"
#include "index_space_handle.h"
#include "index_space_iterator.h"
#include "sec_atl_space.h"
#include "sec_e2.h"
#include "std_iostream.h"
#include "std_sstream.h"
#include "storage_agent.h"

using namespace sheaf;
using namespace fiber_bundle;
using namespace fiber_bundle::vd_algebra;

int main( int argc, char* argv[])
{
    cout << "SheafSystemProgrammersGuide Example31:" << endl;

    // Create a namespace.

    fiber_bundles_namespace lns("Example31");

    // Populate the namespace from the file we wrote in example30.

    storage_agent lsa_read("example30.hdf", sheaf_file::READ_ONLY);
    lsa_read.read_entire(lns);

    // Create a section space for sec_e2;
    // use the default vertex_element_dlinear rep.

    poset_path le2_path("e2_on_block");
    poset_path lbase_path("mesh2/block");

    sec_e2::host_type& le2_host =
        lns.new_section_space<sec_e2>(le2_path, lbase_path);
```

```

// Create a multisection.
// We'll use the evaluation subsubset for the xbase_parts arg,
// create one branch on each segment.

sec_e2 lmulti(&le2_host, le2_host.schema().evaluation(), true);

// Set the attributes of the branches.
// Because lfiber is incremented every time
// and middle vertex is in both branches,
// multisection will be discontinuous at middle vertex.

e2_lite lfiber(0.0, 0.0);
e2_lite lfiber_inc(1.0, 1.0);

// Get a branch iterator for the multisection.

index_space_iterator& lbranch_itr =
    lmulti.get_branch_id_space_iterator(false);
while(!lbranch_itr.is_done())
{
    // Attach a handle to the current branch.
    // Note that arg has to be in the hub id space.

    sec_e2 lbranch(&le2_host, lbranch_itr.hub_pod());

    // Give the branch a name.

    stringstream lstr;
    lstr << "branch" << lbranch_itr.pod();
    lbranch.put_name(lstr.str(), true, false);

    // Get discretization iterator for the current branch
    // and iterate over fibers, setting the section.

    index_space_handle& ldisc_id_space =
        lbranch.schema().discretization_id_space();
    index_space_iterator& ldisc_itr = ldisc_id_space.get_iterator();
    while(!ldisc_itr.is_done())
    {
        lbranch.put_fiber(ldisc_itr.pod(), lfiber);
        lfiber += lfiber_inc;
        ldisc_itr.next();
    }
    ldisc_id_space.release_iterator(ldisc_itr);
    lbranch_itr.next();
    lbranch.detach_from_state();
}
lmulti.release_branch_id_space_iterator(lbranch_itr, false);

// Print the finished section space.

cout << le2_host << endl;

// Exit:

return 0;

```

}

5.4.5 Subposets

we've already discussed the discretization and evaluation subposets, and the use of subposets for definition of branches in a multisection. All of these subposets are actually in the base space. Section spaces and section space schema define no additional client-visible subposets/

5.4.6 Traversing the graph

All the section space posets are ordinary posets as far as graph traversal is concerned, so the cover iterators and `depth_first_itr`s we discussed above can be used for section spaces. The only additional feature introduced in section spaces is the branch id space and iterators for multisections, which we discussed in the preceding section.

Section space schema posets are a different story. As we've already mentioned, section space schema posets are virtual. The membership of these is not actually instantiated as ordinary members, so ordinary traversal via either cover iterators or `depth_first_itr`s will not produce the expected results. Section space schema posets are traversed with special iterators that know about the product subspace approximation used in these posets. However, although not protected or private in any way, these iterators are essentially internal to the system. Clients need rely only on the attribute id spaces and iterators we've discussed above, at least for the introductory level use which is the subject of this tutorial.

6 Part III: The fields component

The classes in the fields component represent the usual physics notion of field, that is, some property as a function of global coordinates. The field abstraction is the natural context for a number of useful functions, most notably calculus. The fields component follows the pattern already established by the section space cluster: there is a field type `field_F` for each fiber type `F` in the fiber spaces cluster. The fields component is still under construction at this time, so the functionality discussed here will only discuss a few of the features already implemented.

6.1 Creating fields

A fields is just a pair of sections, a coordinates section and a property section. It is C++ object, but not a sheaf object. It does not have a schema, does not live in a table, and is not persistent. We create a field from the two sections, a coordinates section and a property section. A scalar field, for instance:

```
fields::field_at0::
```

```
field_at0 (const sec_ed & xcoordinates, const sec_at0& xproperty, bool xauto_access)
```

Create an instance with coordinates `xcoordinates` and property `xproperty`.

6.2 Creating a property as a function of the coordinates

It's quite common to want to set the property to some given function of the coordinates. If the coordinates and the property sections both use the same discretization, the task is simple - we just iterate over the discretization id space, get the coordinate fiber, compute the given function and put the property fiber. But if the property and coordinates are not on the same discretization, the task is substantially more complicated. For each property discretization point, we have to find the coordinate evaluation member that contains the point, gather the coordinate attributes for that evaluation member, evaluate the coordinate section at the property discretization point, then compute the property attributes from the coordinate attributes. The `put_property_dofs` function takes care of all this for us. All the client has to provide is a simple function that does the last step - calculates the property given the coordinates:

```
void
fields::field_vd::
put_property_dofs(property_dof_function_type xproperty_dofs_fcn, bool      xauto_access)
```

Sets the dofs of `property()` to values computed by `xproperty_dofs_fcn`.

The typedef for the function signature is:

```
typedef void(*fields::field_vd::property_dof_function_type)(block<sec_vd_value_type>&
      xglobal_coords, block<sec_vd_dof_type>& xproperty_dofs)
```

The type of the function for computing the property dofs at given global coordinates.

This is one of those C/C++ typedefs that is virtually impossible to decipher, so the library provides an example, largely so you can understand the signature:

```
void
fields::field_vd::
property_dof_function_example(block< sec_vd_value_type >& xglobal_coords,
      block< sec_vd_dof_type >& xproperty_dofs) static
```

Example property dof function; property value is $x*1000000 + y*1000 + z$, intended to be easy to check field dofs for correctness.

6.3 Evaluating the property at given coordinates

The defining function of the field object is to evaluate the property at given global coordinates. This is more complicated than it seems at first. The property is a section, a function of patch and local coordinates. To evaluate it, we need to know what patch and local coordinates the global coordinates value corresponds to, in other words, we have to invert the global coordinate section. The task is even more complicated if the coordinates and the property aren't defined on the same discretization and evaluation subposets, for example if the coordinate representation is uniform and the property representation is `vortex_element_dlinear`. The `property_at_coordinates` method takes care of all this complexity:

```

void
fields::field_vd::
property_at_coordinates(sec_vd_value_type xcoord_base[],
                        int xcoord_ct,
                        sec_vd_value_type xprop_base[],
                        int xprop_ub) const

```

The value of the property at the given global coordinates.

6.4 Pushing a property from one mesh to another

One of the most useful features of the field abstraction is that it provides enough context to support moving a property from one mesh to another. This process, often called "remapping" in the scientific computing literature, is called "pushing" in the sheaf system because that is what it is called in the fiber_bundle literature. As with put_property_dofs and property_at_coordinates, this task is considerably more complicated than it appears at first. The push function takes care of the complexity and makes it easy:

```

void
fields::
push(const field_vd& xsrc, field_vd& xdst, bool xauto_access)

```

Pushes xsrc.property() to xdst.property().

6.5 Algebraic operations and namespaces

The field type hierarchy repeats the fiber type hierarchy: for every type F in the fiber hierarchy there is a field type field_F with property section of type sec_F. Every fiber algebra has a corresponding field algebra in namespace fields::field_F_algebra. See the reference documentation for a complete listing of the namespaces and operations.

Example 32 illustrates the features of fields.

6.6 Example 32: Fields

```

#include "at0.h"
#include "e1.h"
#include "fiber_bundles_namespace.h"
#include "field_at0.h"
#include "index_space_handle.h"
#include "index_space_iterator.h"
#include "sec_at0.h"
#include "sec_at0_space.h"
#include "sec_at1_space.h"
#include "sec_e1.h"
#include "sec_e1_uniform.h"
#include "std_iostream.h"
#include "std_sstream.h"
#include "storage_agent.h"
#include "structured_block_1d.h"

```

```
using namespace sheaf;
```

```

using namespace fiber_bundle;
using namespace fields;

namespace
{
    // Function to compute the property dofs as a function of the
    // coordinates. As an example, assume the property is scalar and
    // set it to the distance from the coordinate origin.
    // For the 1d case we will use, this will just give us the
    // coordinate back again, so it will be easy to see if we get
    // the right answer.

    void
    property_dof_function(block<sec_vd_value_type>& xglobal_coords,
                        block<sec_vd_dof_type>& xproperty_dofs)
    {
        sec_vd_value_type dist = 0.0;
        for(int i= 0; i<xglobal_coords.ct(); ++i)
        {
            dist += xglobal_coords[i]* xglobal_coords[i];
        }

        dist = sqrt(dist);
        xproperty_dofs[0] = dist;
        return;
    }
} // end unnamed namespace.

int main( int argc, char* argv[])
{
    cout << "SheafSystemProgrammersGuide Example32:" << endl;

    // Create a namespace.

    fiber_bundles_namespace lns("Example32");

    // Populate the namespace from the file we wrote in example30.

    storage_agent lsa_read("example30.hdf", sheaf_file::READ_ONLY);
    lsa_read.read_entire(lns);

    // Get the coordinates space.

    poset_path lel_path("el_on_block");
    sec_el::host_type& lel_host =
        lns.member_poset<sec_el::host_type>(lel_path, true);

    // Get a handle for the coordinates section

    sec_el lel_coords(&lel_host, "general_coordinates");

    // Create a scalar section space.

```

```

poset_path lat0_path1("at0_on_block");
poset_path lbase_path1("mesh2/block");

sec_at0::host_type& lat0_spacel =
    lns.new_section_space<sec_at0>(lat0_path1, lbase_path1);

// Create a scalar section for the property

sec_at0 lprop1(&lat0_spacel);

// Create a scalar field

field_at0 lscalar_field1(lcl_coords, lprop1, true);

// Set the property to the value defined by
// the put_property_dofs_example function above.

lscalar_field1.put_property_dofs(property_dof_function, true);

// Get the property value at coordinate = 5.0.

sec_el::value_type lx = 5.0;
sec_at0::value_type lp;

lscalar_field1.property_at_coordinates(&lx, 1, &lp, 1);

cout << endl << "property at x= 5.0 is " << lp << endl;

// Print the finished section space.

cout << lat0_spacel << endl;

// Create a different base space with 3 segments

arg_list largs = base_space_poset::make_args(1);
base_space_poset& lbase_host =
    lns.new_base_space<structured_block_ld>("mesh3", largs);
structured_block_ld lblock3(&lbase_host, 3, true);
lblock3.put_name("block3", true, true);

// Create a section space for uniform coordinates
// on the new base space.

poset_path lelu_path("el_uniform_on_block3");
poset_path lbase_path2("mesh3/block3");
poset_path lelu_rep_path("sec_rep_descriptors/vertex_block_uniform");

sec_el_uniform::host_type& lelu_host =
    lns.new_section_space<sec_el_uniform>(lelu_path, lbase_path2,
                                         lelu_rep_path, true);

// Create the coordinates section for the new base space.
// Uniform coordinates are initialized by the constructor.

sec_el_uniform lelu_coords(&lelu_host, 0.0, 3.0, true);
lelu_coords.put_name("uniform_coordinates", true, true);

```



```
// Create a scalar section space on the new base space.

poset_path lat0_path2("at0_on_block3");

sec_at0::host_type& lat0_space2 =
    lns.new_section_space<sec_at0>(lat0_path2, lbase_path2);

// Create a scalar section for the property

sec_at0 lprop2(&lat0_space2);

// Create a scalar field on the new base space.

field_at0 lscalar_field2(lelu_coords, lprop2, true);

// Push the property from the first base space to the second.

push(lscalar_field1, lscalar_field2, true);

// Print out the property section space on the new base space.

cout << lat0_space2 << endl;

// Exit:

return 0;
}
```

Appendix A Concurrency control examples

The access control mechanism is a work in progress. The control mechanism itself is complete and is implemented both for multiple threads using pthreads and for single threads. When the library is compiled with threads enabled and a client requests read access and another client already has write access, or vice versa, the request blocks until the other client releases the conflicting access. When the library is compiled with threads disabled, requests do not block, they return immediately. The library is currently delivered with threads disabled because the use of threads and concurrency in the library is only partially implemented and not tested. The access control mechanism is disabled by default but can be enabled by the programmer. These examples demonstrate use of the manual and auto-access mechanisms.

A.1 Example A1: manual access control

```
#include "sheaves_namespace.h"
#include "std_iostream.h"

using namespace sheaf;

int main( int argc, char* argv[])
{
    cout << "SheafSystemProgrammersGuide ExampleA1:" << endl;

    // Enable concurrency control; must be called
    // before any other library call.

    read_write_monitor::enable_access_control();

    // Create a standard sheaves namespace.

    sheaves_namespace* lns = new sheaves_namespace("ExampleA1");

    // Write its name to cout.
    // Requires read access to the namespace.

    // Be polite, request access.
    // If threads are enabled and another thread has
    // read-write access, execution will block until it
    // releases access. Otherwise, the request will succeed
    // immediately.

    // You can nest requests as deep as you want, or at least
    // until the integer depth counter overflows.

    cout << "request depth " << lns->access_request_depth() << endl;
    lns->get_read_access();
    cout << "request depth " << lns->access_request_depth() << endl;
    lns->get_read_access();
    cout << "request depth " << lns->access_request_depth() << endl;

    // Invoke the operation.
```

```

    cout << lns->name() << endl;

    // Be proper, release access so this thread
    // or another can get write access.
    // Have to match every request with a release.

    cout << "request depth " << lns->access_request_depth() << endl;
    lns->release_access();
    cout << "request depth " << lns->access_request_depth() << endl;
    lns->release_access();
    cout << "request depth " << lns->access_request_depth() << endl;

    // Delete the namespace, requires read-write access.
    // Be polite, request access. If threads are enabled
    // and another thread has either read or read-write
    // access, execution will block until it releases access.
    // Otherwise, the request will succeed immediately.
    // This client must not already have read-only access,
    // see precondition for details.

    lns->get_read_write_access(false);

    // Invoke the operation.

    delete lns;

    // Deletion is the only case where the client
    // cannot be proper and release access.

    // Create another namespace.

    lns = new sheaves_namespace("Example3B");

    // Invoking a function that requires access
    // without first getting access violates the
    // precondition of the function.
    // The following will throw an exception and abort.

    cout << lns->name() << endl;

    return 0;
}

```

If you compile and run example A1, the output is:

SheafSystemProgrammersGuide ExampleA1:

```

request depth 0
request depth 1
request depth 2
Example3A
request depth 2
request depth 1
request depth 0
terminate called after throwing an instance of 'std::logic_error'
  what(): 'is_external() ? name_space()->state_is_read_accessible() : state_is_read_accessible()'
in file poset_state_handle.cc at line 1178

```

Abort

A.2 Example A2: auto-access control

```
#include "sheaves_namespace.h"
#include "std_iostream.h"

using namespace sheaf;

int main( int argc, char* argv[])
{
    cout << "SheafSystemProgrammersGuide ExampleA2:" << endl;

    // Enable concurrency control; must be called
    // before any other library call.

    read_write_monitor::enable_access_control();

    // Create a standard sheaves namespace.

    sheaves_namespace* lns = new sheaves_namespace("ExampleA2");

    // Write its name to cout.
    // Requires read access to the namespace.
    // Invoke the auto-access version of the operation with
    // auto-access set to true.
    // Operation will request and release access as needed.

    cout << lns->name(true) << endl;

    return 0;
}
```

If you compile and run example A2, the output is:

```
SheafSystemProgrammersGuide ExampleA2:
ExampleA2
```

Appendix B Join equivalent members and lexicographic ordering

Is it legal to have a jrm with a single member in its lower cover? The answer is no, in an ordinary finite distributive lattice as defined in the text books, but yes, in a SheafSystem lattice. In the mathematical view, the lattice is fully instantiated, the order relation fully enumerated, and a member is a jim if and only if it has a single member in its lower cover. So `c1` couldn't be a jrm. In this view, the join operator is a query that finds the member which is the least upper bound of the joinands. The Birkhoff representation theorem is a consequence of the order relation and it, in turn, implies two additional facts. First, for lattice members p and p' , the set of jims in the downset of p is included in the set of jims of p' if and only if $p \leq p'$. Second, the jims in the downset of a join is the union of the jims of the joinands.

But on the computer, we can't afford the memory to fully instantiate all the members of the lattice and enumerate the order relation. So the SheafSystem reverses the

mathematical argument. We initially instantiate only the poset of join irreducible members and the covering relationships between them. That is, a member is a jim because we specify that it is when we create it, not because of some property of the cover or order relation. The join operator isn't a query, it's a constructor that creates a jrm and places it in the cover relation so that the set of jims in its down set is the union of the jims of the joinands. The Birkhoff representation theorem is satisfied by construction and the order relation is derived from it: $p \leq p'$ if and only if the set of jims in the down set of p is included in the jims of p' .

But what if we join the same set of joinands twice? We can interpret the second operation in two ways. Either the join operator finds the existing join and returns it as the result, or it constructs a second member, with the same jims in its downset as the first join. The SheafSystem supports both approaches, but in the latter case, where should the second join be placed in the order relation? The set of jims of the second join is equal to the set of jims of the first, so the order relation says they are equal, but they are not the same object. The SheafSystem breaks the tie by extending its definition of the order relation to what is called a lexicographic order. A lexicographic order is a generalization of ordinary dictionary order. To place words in order, first we sort on the first letter. If two words have the same first letter, we sort on the second letter, and so on. The SheafSystem uses a lexicographic order in which the first letter is the set of jims in the down set and the second letter is the order of creation. So when we construct the second join, it has the same set of jims but it was constructed after the first join, so it is greater than the first join. The second join is thus linked immediately above the first join. A third copy would be linked above the second copy, and so on.

Appendix C Fiber Algebra

As described above, the classes in the fiber spaces cluster provide the various algebraic types used in theoretical physics to describe the properties of particles and systems. Each of these algebraic types has an associated set of operations, an algebra. In this appendix we summarize the operations associated with each type. See the reference documentation for a complete description of the operations and the numerous signatures supported for each operation.

C.1 Abstract vectors (vector algebra)

Class `vd` and its descendants support the operations of vector algebra (linear algebra). The main operations are:

<code>add</code>	add one vector to another.
<code>subtract</code>	subtract one vector from another.
<code>multiply</code>	multiply a vector by a scalar.
<code>divide</code>	divide a vector by a scalar.
<code>min</code>	find the minimum component of a vector.
<code>max</code>	find the maximum component of a vector.

contract contract a vector with a covector.

C.2 Euclidean vectors (Euclidean vector algebra)

dot the Euclidean scalar product of two vectors.
length the length of a vector.
put_length sets the length of a vector.
normalize sets the length of a vector to 1.
cross the vector ("cross") product of an e3 vector with another.

C.3 General tensors

Class tp and its descendants support general tensor algebra. The main operations are:

tensor the tensor product of one tensor with another.
alt the antisymmetric ("alternating") part of a tensor.
sym the symmetric part of a tensor.
contract contract a tensor on one contravariant and one covariant index.

C.4 Antisymmetric tensors (exterior algebra)

Class atp and its descendants support exterior algebra. the main operations are:

wedge the exterior ("wedge") product of two antisymmetric tensors.
hook the interior ("hook") product of an antisymmetric tensor and a vector.
star the Hodge star ("dual") of an antisymmetric tensor.

C.5 Symmetric tensors (symmetric algebra)

symmetric_product the symmetric product of one symmetric tensor with another.
trace the trace of a degree 2 symmetric tensor.
determinant the determinant of a degree 2 symmetric tensor.
to_principal_axes diagonalizes a degree 2 symmetric tensor

C.6 Metric tensors

raise make a given index of a tensor contravariant.
lower make a given index of a tensor covariant

C.7 Jacobians

push push a vector from the domain to the range of a Jacobian
pull pull a covector from the range to the domain of a Jacobian

C.8 Transformation groups

inverse the inverse of a linear transformation
transform_basis_by transform the basis and components of a tensor

C.9 Multiple signatures

Each of the above operations is represented by a family of functions implementing multiple signatures for the operation. Typically each operation is provided for both regular fiber types and the associated lite fiber types. In addition, auto-allocated, pre-allocated and self-allocated variants are provided. Finally, whenever it makes sense, operator variants are provided.

The full fiber algebra consists of all the various signatures for all the various operations and thus contains approximately 100 functions.

Appendix D Section algebra

The section type hierarchy repeats the fiber type hierarchy: for every type F in the fiber hierarchy there is a section type S with fiber of type F . Every fiber operation has a corresponding section operation. Sections also support additional operations, not defined for fibers. The most important of these are the real-valued functions of scalar sections. A scalar section is essentially a map from the base space to reals and we can compose this with any map from reals to reals to form another map from the base space to reals, i.e. another scalar section. Namespace `fiber_bundle::sec_at0_algebra` defines such an operation for each math library function defined by the C++ standard. We summarize these in the following:

<code>fabs</code>	absolute value of a scalar section.
<code>ceil</code>	ceiling of a scalar section.
<code>floor</code>	floor of a scalar section.
<code>sqrt</code>	square root of a scalar section.
<code>pow</code>	power of a scalar section.
<code>cos</code>	cosine of a scalar section.
<code>sin</code>	sine of a scalar section.
<code>tan</code>	tangent of a scalar section.
<code>acos</code>	arc cosine of a scalar section.
<code>asin</code>	arc sine of a scalar section.
<code>atan</code>	arc tangent of a scalar section.
<code>atan2</code>	arc tangent of a scalar section.
<code>cosh</code>	hyperbolic cosine of a scalar section.
<code>sinh</code>	hyperbolic sine of a scalar section.

<code>tanh</code>	hyperbolic tangent of a scalar section.
<code>exp</code>	exponential of a scalar section.
<code>log</code>	log base e of a scalar section.
<code>log10</code>	log base 10 of a scalar section.
<code>modf</code>	integer and fractional parts of a scalar section.
<code>frexp</code>	fractional and exponent parts of a scalar section.
<code>fmod</code>	remainder of scalar section.
<code>ldexp</code>	scalar section times a power of 2.