Math 260 Exam 1

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In problems 1 and 2 your wording may be different than that of book. I am looking for correctness in the concepts. Problems $1-3$ are worth 70 points. The test is out of 100.

Problem 1. Give definitions for the following terms:

1. vector space
2. subspace
3. give the 3 conditions for a subspace
4. sum of subspaces
5. direct sum of subspaces
6. linear combination of a list of vectors
7. span of a list of vectors
8. linear independence of a list of vectors
9. basis for a vector space
10. dimension of a vector space

Problem 2. State and prove the Linear Dependence Lemma.

Problem 3. Suppose $p_0, p_1, ..., p_m$ are polynomials in $\mathcal{P}_m(\mathbf{F})$ such that $p_j(2) = 0$ for each j. Prove that $p_0, p_1, ..., p_m$ is not linearly independent in $\mathcal{P}_m(\mathbf{F})$.

Do enough problems to reach the remaining 30 points:

Problem 4. (10 points) Let

$$U = \{(x, y, x+2y, 4x-y, 3y) \in \mathbf{F}^5 \ : \ x, y \in \mathbf{F}\}$$

Find a basis for U. Extend this basis to a basis of $\mathbf{F}^5$ and then use this to find a subspace W such that $W \oplus U = \mathbf{F}^5$.

Problem 5. (5 points) What is the dimension of $\mathcal{P}_m$?

Problem 6. (5 points) Prove $(1, 2), (3, 5)$ is a basis of $\mathbf{F}^2$.

Problem 7. (10 points) Prove $(1, -1, 0), (1, 0, -1)$ is a basis of $\{(x, y, z) \in \mathbf{F}^3 \ : \ x + y + z = 0\}$

Problem 8. (10 points) Suppose U and W are subspaces of V. Then $U + W$ is a direct sum if and only if $U \cap W = \{0\}$.

Problem 9. (5 points) Suppose U_1 and U_2 are subspaces of V. Prove that the intersection $U_1 \cap U_2$ is a subspace of V.

Problem 10. (5 points) Verify both distributive properties in the definition of a vectorspace for $\mathbf{F}^3$.