Math 260 Exercises 1.B

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Problem 1. *Prove that* $-(-v) = v$ *for every* $v \in V$

Problem 2. *Suppose* $a \in \mathbb{F}$, $v \in V$ *and* $av = 0$. *Prove that* $a = 0$ *or* $v = 0$.

Problem 3. *Suppose* $v, w \in V$. *Explain why there exists a unique* $x \in V$ *such that* $v + 3x = w$.

Problem 4. *The empty set is not a vectorspace. The empty set fails to satisfy only one of the requirements listed in 1.19. Which one?*

Problem 5. *Show that in the definition of a vectorspace (1.19), the additive inverse condition can be replaced with the condition that* $0v = 0$ *for all* $v \in V$. *Here* 0 *on the left side is the number* 0, *and the* 0 *on the right side is the additive identity of* V.

Problem 6. *Let* ∞ *and* $-\infty$ *denote two distinct objects, neither of which is in* $\mathbb{R}$. *Define an addition and scalar multiplication on* $\mathbb{R} \cup \{\infty\} \cup \{-\infty\}$ *as you could guess from the notation. Specifically, the sum and product of two real numbers is as usual, and for* $t \in \mathbb{R}$ *define* $t\infty = -\infty$ *if* $t < 0$, $t\infty = 0$ *if* $t = 0$ *and* $t\infty = \infty$ *for* $t > 0$. *Also* $t + \infty = \infty + t = \infty$, $t + (-\infty) = (-\infty) + t = -\infty$. $\infty + \infty = \infty$, $(-\infty) + (-\infty) = (-\infty)$, $0 = (-\infty) + \infty$.
Is $\mathbb{R} \cup \{\infty\} \cup \{-\infty\}$ *a vectorspace over* $\mathbb{R}$? *Explain.*