Math 260 Exercises 1.A

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Tuesday August 21th, 2018

Problem 1. *Suppose a and b are real numbers, not both 0. Find real numbers c and d such that $\frac{1}{(a+bi)} = c + di$.*

This problem shows that every complex number has a multiplicative inverse. We can then be sure that our definition of division for complex numbers from class will hold, that is for $\alpha \in \mathbb{C}$ where $\alpha \neq 0$. This is exactly the requirement that for $\alpha = a + bi$, a and b are real numbers which are not both 0.

Problem 2. *Verify properties of $\mathbb{C}$:*

Note from class: Think of λ as a scalar or number like something in $\mathbb{F}$ and think of α, β, and γ as vectors in $\mathbb{F}^n$, (in this case the vectors are actually in $\mathbb{C}^n$) for $n = 1$.

Show that for all α, β, γ λ in $\mathbb{C}$.

1. $\alpha + \beta = \beta + \alpha$ *commutativity of addition*
2. $(\alpha + \beta) + \gamma = \alpha + (\beta + \gamma)$ *associativity of addition*
3. $(\alpha\beta)\gamma = \alpha(\beta\gamma)$ *commutativity of multiplication*
4. *Show that for every α there is a β so that $\alpha + \beta = 0$*
5. *Show that for every α there is a β so that $\alpha\beta = 1$*
6. $\lambda(\alpha + \beta) = \lambda\alpha + \lambda\beta$

Problem 3. *Find $x \in \mathbb{R}^4$ such that $(4, -3, 1, 7) + 2x = (5, 9, -6, 8)$.*

Problem 4. *Explain why there does not exist $\lambda \in \mathbb{C}$ such that $\lambda(2 - 3i, 5 + 4i, -6 + 7i) = (12 - 5i, 7 + 22i, -32 - 9i)$.*

Problem 5. *Verify properties of $\mathbb{F}^n$: Let $x, y, z \in \mathbb{F}^n$ be vectors and $a, b \in \mathbb{F}$ be scalars*

1. *show associativity of addition* $(x + y) + z = x + (y + z)$
2. *show associativity of scalar multiplication* $(ab)x = a(bx)$
3. *show there is a multiplicative identity* $1x = x$
4. *show distributivity of scalar multiplication (vector side)* $a(x+y) = ax+ay$
5. *show distributivity of scalar multiplication (scalar side)* $(a+b)x = ax+bx$