Programming Tutorial
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385-685
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Section 1.
Variable Assignment and Built in Functions

```
(* Inside of the parentheses and stars are comments. Read these
 before you evaluate the actual code which is in black and blue. *)
```

```
(* Below, we are assigning a value to a variable x. In
 this language variables should be named with lower case
 names.  If we have a variable name with more than one word,
you are encouraged to name it by concatenating the two words and
 capitalizing the second word: secondVariable or firstVariable.  *)
```

```
(* Into the variable x,
we are placing the value 2.  All of the lines in your program
  should end in a semi-colon to supress the output and to separate
  statements. You will get a better feel for their use but for now
  try to put them at the end of every statement. Evaluate the cell by
  clicking inside of the cell and using shift and enter together. Notice
  that before we evaluate the cell, the variable x is blue.
*)
```

```
x = 2;
```

```
(* If you evaluated the cell,
x should turn black. This means that the variable now contains a value.*)
```

```
(* Functions in Mathematica that are a part of the standard
 library of functions will usually be capitalized. It would be
 a bad idea to try to use any of these functions as variable
 names. Evaluate the following code to see the error message.*)
```

```
Print = 2;
```

Set: Symbol Print is Protected.

```
(* The usual syntax for using functions in Mathematica is:
   Function[arg];
arg is short for argument,
which is what programmers call the input of a function. Notice that
 the argument of the function is surrounded by square brackets which
 is different from usual mathematical notation which uses parenthesis.
   Print[x]; returns the value of x to the screen in an output
 cell. Notice that a new cell is born containing 2.  *)
```

```
Print[x];
```

2

```
(* The Print function is smart enough to know not to supress the output even
  if there is a semi-colon. Notice that if we evaluate the following,
we get the same output as Print[x];. Unless the statements are really simple,
it is a good habit to just use the print function
 to print things for now and always use semicolons. *)
```

```
x
```

2

```
(* Just like in math we can make use of Mathematica'
 s functions to use x as a replacement for what it
 contains. Evaluate the next few cells to get a feel for this: *)
```

```
Print[x + 3];
```

5

```
x + 3
```

5

```
Print[x ^ 2]
```

4

```
Print[x^2 + x + 6]
```

12

```
Print[x * x]
```

4

```
(* If we execute the following code we clear the value of the symbol
 x and then we print it. If you look on the right you will notice that
 these two statements are both in the same cell. Clicking inside the
 cell anywhere and pressing shift enter will evaluate all statements
 from top to bottom. First we clear the value of the variable x,
then we print it.  This is our first multi-line cell. In fact most of
  the time you will have cells that have many things inside of them.  *)
```

```
Clear[x];
Print[x];
```

x

```
(* Now what happens if we evaluate the
 previous string of familiar computations on x?  *)
```

```
Print[x + 3];
```

$3 + x$

```
x + 3
```

$3 + x$

```
Print[x ^ 2]
```

x^2

```
Print[x^2 + x + 6]
```

$6 + x + x^2$

```
x * x
```

x^2

```
(* We also have examples of some more exciting functions: Log, Cos,
Sin. We are also given values for familiar mathematical constants like Pi,
E, and I. Notice that they are capitalized. Dont try to use them as
 variables. Use them for what they are, numbers of different sorts. *)
```

```
Cos[1 / 4 * Pi]
```

$$\frac{1}{\sqrt{2}}$$

```
Log[E]
```

1

```
I^2
```

-1

```
(* N is a function that returns the numerical values of the first argument
 to the degree of precision as specified by it's second argument. If a
 function has more than one argument, they will be seperated by commas.  *)
```

```
N[E, 1]
```

```
3.
```

```
N[E, 2]
```

```
2.7
```

```
N[E, 5]
```

```
2.7183
```

```
N[E, 10]
```

```
2.718281828
```

```
N[E, 100]
```

```
2.71828182845904523536028747135266249775724709369995957496696762772407663035354759
 4571382178525166427
```

```
(* You should be able to guess what the function D does based on its
 output and arguments. Here I am creating a second degree polynomial out
 of the unassigned variable x, and putting it into the variable poly1. *)
```

```
poly1 = x^2 + x + 6;
D[poly1, x]
```

```
1 + 2 x
```

```
(* There! We have successfully differentiated a symbolic expression!
 Symbolic manipulation and computation is one of Mathematica'
 s strong suits. (If you play around in the Demonstrations and
    Documentation that we talked about in class see if you can figure
    out how to use the functions that will compute limits for you.)
 You may have guessed just now that D is the differentiation function
 who's second argument is the variable we are differentiating
 with respect to. To quote Ron Swanson: You'd be right.

    Now what happens if we say: *)
```

```
x = 2;
poly1 = x^2 + x + 6;
D[poly1, x]
```

General: 2 is not a valid variable.

$\partial_2 12$

Forensics

```
(* We got an error message. You should be asking at this
 point: Why did we get an error? Should we not have just gotten 0 by
    differentiating 12  (the value of that polynomial at 2) with respect to x?

     In this case the first thing we have done is to assign x the value 2.
    Our poly variable then is holding the value of the expression x^2 +
   x + 6 which is equal to 12. Then on the 3rd line we ask
    mathematica to differentiate with respect to x which
    again is just 2. so we are equivaluently saying: *)
D[12, 2]
```

General: 2 is not a valid variable.

$\partial_2 12$

A note on types:

```
(* If something is going wrong for you. See if you can follow the logic
 and values of the variables at each step in the program. What may help
 you the most is to break up the cell into multiple cells and evaluate
 each one at a time to figure out where you have gone wrong exactly. 90
 % of the errors will come from putting something of the wrong data
 type into a function or scenario which calls for a different type. The
 Head function will tell you what the data type of its argument is.*)
Head[
 4]
```

Integer

```
Head[1 / 4]
```

Rational

```
Head[4.0]
```

Real

```
Head[4 + 4 * I]
```

Complex

```
Head[a ^ 3]
```

Power

```
(* You wont need to use Head all the time but it is good to be
 aware that types exist. Mismatched types will be a source of errors
 in your program (errors in your program are also known as bugs).


  In addition to creating variables that hold integers or reals or complex
 numbers, there are other things a variable can hold.  To name a few,
they can hold booleans (named for the logician George Bool),
strings (short for strings of characters), lists
 (containers, essentially tuples, that may hold other things inside of them). You
 could even have lists containing lists. This is how
 (in this language) we will describe matricies.*)

(* Here is an example of a string of characters vs a variable name. They are
 enclosed in quotes. My dear Eliza, We can type plain english as strings!
 (This is an example from another George B.'s book titled Pygmalion)*)

x = 2;

Print[x];

2

Head[x]

Integer

Print["x"];

x

Head["x"]

String

Head["The rain in Spain falls mainly on the planes."]

String
```

Section 2.
Booleans and If statements

```
(* Booleans are truth values. True and False
 are built in values much the same way Pi and E, or 1,
or 2,or 3 are.    There are comparison operators  < and >
 that return True or False. They are, in a sense,
questions. What are some questions we can ask about numbers? *)
```

```
1 < 2
```

```
True
```

```
4 < 3
```

```
False
```

```
(* Slightly more complex, *)

1/Sqrt[n+3] > 1/Sqrt[4*n]
```

$$\frac{1}{\sqrt{3+n}} > \frac{1}{2\sqrt{n}}$$

```
(* Use reduce to see for what n the expression is true:
  (In fact it misses one value, the computers are quite up to us yet I guess. )*)
```

$$\text{Reduce}\left[\frac{1}{\sqrt{3+n}} > \frac{1}{2\sqrt{n}}\right]$$

$n > 1$

```
(* We are putting True into the variable t, and False into the variable f: *)
```

```
t = 1 < 2;
```

```
f = 1 > 2;
```

```
Print[t]
```

```
True
```

```
(* The logical "OR" and "AND" and "NOT" operators are ||, &&,
 and ! respectively. *)
```

```
t || f
```

```
True
```

```
t && f
```

```
False
```

```
(* Not True *)
```

```
!t
```

```
False
```

```
(* Not False *)
```

```
!f
```

```
True
```

```
(* equals equals == is another comparison operator like < or >.
  a == b means "is a equal to b? " and will return True or False *)
```

```
4 == 5
```

False

```
4 == 4
```

True

```
t == !f
```

True

If Statements

```
(* We are now ready for our first control
 structure. So far whenever we have evaluated a cell,
the whole cell was evaluated top to bottom end of story. In the following cell,
we will create a variable i, and then set i equal to itself
  plus 3 . This looks mathematically wrongity wrong wrong. But it
  is in fact a valid statement. i == i + 3 is evaluates to False.  *)
```

```
i = 2;    (* on this line i = 2*)
i = i + 3; (* on this line we are adding one to i*)
Print[i];

(* Finally we print out the desired 5 *)
```

5

```
(* Suppose now that we didn't always want to add one to i whenever we arrive
 at the second line of the program. An If statement will test a condition,
and based on the truth of the condition evaluate some further
 expression. The first argument of If is the test condition. If the
 test condition evaluates to False, the code in the second argument is
 executed (this is called the body of the If statement). If the test
 condition evaluates to False, then the second argument is skipped. This
 is called an If Else Statement. In fact the third argument is
 optional. If you only have the If statement, and no else statement,
you should only have one comma seperating the condition and the body.  *)
```

```
If[1 < 2, Print["Condition is True"], Print["Condition is False"]];
```

Condition is True

```
(* I know that that one was not, but standard practise is to put the condition,
body, and else all on seperate lines. As shown below: *)
```

```
If[1 < 2,
  Print["Condition is True"],
  Print["Condition is False"]
 ];
```

```
Condition is True
```

```
(* Indeed 1 is less than 2 so the body is evaluated and the else is skipped. *)
```

```
(*Note that True and False evaluate to True and False,
so if we leave these as the conditions (or variables containing them),
they will work accordingly. *)
False
```

```
False
```

```
True
```

```
True
```

```
If[True,
  Print["Condition is True"],
  Print["Condition is False"]
 ];
```

```
Condition is True
```

```
If[False,
  Print["Condition is True"],
  Print["Condition is False"]
 ];
```

```
Condition is False
```

```
(* Here I am putting True into t,
then negating t as the condition so it evaluates to False.*)
```

```
t = True;
If[! t,
  Print["Condition is True"],
  Print["Condition is False"]
 ];
```

```
Condition is False
```

```
(* Lets go back to incrementing i. Now instead of just printing a string,
we will do something to a variable depending on the
 truth value of the condition. Here if i is less than 5 we
 will add 5 to i and reset i with this new higher value.*)
```

```
i = 2;
```

```
If[i < 5,
   i = i + 3
 ];
```

```
Print[i];
```

5

```
(* Now I has the value 5. Notice that we have no else body.

  Lets do this again. This time we will
 have more than one statement in the body of the If.*)
```

```
i = 2;
```

```
If[i < 5,
   i = i + 3;
  Print[i];
 ];
```

5

```
(* Notice a few things are different. I put a semi colon at the end of the
  incrementing line where we change i. This is to seperate the increment
  statement from the print statement. Suppose we forgot to put this semi-colon:*)
```

```
i = 2;
```

```
If[i < 5,
   i = i + 3 ×
       Print[i];
 ];
```

2

```
Print[i]
```

2 + 3 Null

Forensics

```
(* Where there is empty space Mathematica is filling in a multiplication. Here
   is our first example of type mismatch. Lets take this piece by piece.

    Mathematica correctly evaluates i < 5 to True.

    then it sees:

    i = i + 3
   Print[i];

    this is identically:

   i = i + 3Print[i];

and it tries to multiply 3 times the print
  statment. This crashes and burns and returns a Null value. Null
  is the empty data type. We are left with 2 + 3Null.
*)


(* In almost every program you write this semester you will
 use if statements. If statements are powerful. Writing programs
 that have more than one line is also powerful. But with great
 power comes great responsibility. Do not forget semicolons.

  In our final example of If statements we will Nest
 them: Suppose that we are making a peanut butter and jelly sandwich. *)
```

In[60]:=
```
peanutButter = False;
jelly = False;
```

```
If[peanutButter == True,
  If[jelly == True,
    Print["Nice job, it looks like you have a good sandwich."],
    Print[
      "Ok, you're a serious fellow I can tell because your sandwich has no jelly
        on it but it does have peanut butter."]
   ];
  , (* below this comma is evaluated in the case peanut butter is false *)
  If[jelly == True,
    Print[
      "Ok, you're a serious fellow I can tell because your sandwich has no peanut
        butter on it but it does have jelly."],
    Print["This is a bread sandwich."]
   ];
 ];
```

```
This is a bread sandwich.
```

```
If[peanutButter == True,
  (* We are now in the body of the if where there is peanutButter on the sandwich
   is there also jelly?*)
  If[jelly == True,
    (* We are now in the body
     of the if where there is peanutButter on the sandwich
     and there is also jelly, in this case: *)
    Print["Nice job, it looks like you have a good sandwich."]
    ,
    (* We are now in the body
     of the if where there is peanutButter on the sandwich
     but no jelly in this case: *)
    Print[
     "Ok, you're a serious fellow I can tell because your sandwich has no jelly
       on it but it does have peanut butter."]
   ];
  ,
  (* We are now past the second comma, cheese is false and we have
   skipped the entire body where we would go if peanutButter we true,
  is there also jelly? *)
  If[jelly == True,
    (* We are now in the body of
     the if where there is no peanutButter on the sandiwich
     but there is  jelly*)
    Print[
     "Ok, you're a serious fellow I can tell because your sandwich has no peanut
       butter on it but it does have jelly."]
    ,
    (* We are now in the place of no peanutButter and no jelly*)
    Print["This is a bread sandwich."]
   ];
 ];
```

Section 3.
Lists, and Lists of Lists, and Lists of Lists of Lists, and so on...

```
(* Lists in this language are tuples
 seperated by commas surrounded by curly braces:*)
```

```
l1 = {10, 20, 30, 40};
```

```
(* l1 is a variable containing a list of the multiples of 10 from
 10 to 40. Min and Max are functions that can be applied to lists*)

Max[{10, 20, 30, 40}]

40

Min[l1]

10

(* Mean and Total are functions that compute the Mean value of the list,
and the sum total of the elements of the list.*)

Total[l1]

100

Mean[l1]

25

(* In the same way that we could increment a number with the following syntax*)

i = 1;

i = i + 1;

i

2

(* we can add things to lists *)

l2 = {True, 10, 1.5, I};

l2 = Append[l2, "Friday"];

l2

{True, 10, 1.5, i, Friday}

(* Here we have appended to the list l2,
the string "Friday". This example is meant to show that lists
 in general may hold different data types. If we try to check
 what these data types are we might think of trying to apply
 Head which infuriatingly gives us exactly what we asked for: *)

Head[l2]

List

(* However we may use the map function to map head over all of the elements
 of the list. Map takes as its first argument a function and as its
 second a list to which that function will be mapped. Dont use this. When
 we go over loops you will see a better way to do this for now. *)
```

```
Map[Head, l2]
{Symbol, Integer, Real, Complex, String}
```

Wherever possible, i.e. within the limits of your own sanity, do not use Mathematica's fancy built in functions for high level things. Map is an example, do not use Map. AppendTo is fine. Mean can be easily constructed out of total using Length. When we do loops we will see how to build our own Totals and Lengths.

If I ask you to write a program to perform Lagrange Interpolation you will not get credit if you hand in a one line program that says

LagrangeInterpolation[pointSet]

using Mathematica's gigantic standard library. This does not count as you doing the assignment.

```
Total[l1] / Length[l1] == Mean[l1]
True

(* Ok, time for lists of lists.  *)

lofl = {};
AppendTo[lofl, {20, 30, 40}];

lofl
{{20, 30, 40}}

AppendTo[lofl, {20, 30, 40}];

lofl
{{20, 30, 40}, {20, 30, 40}}

AppendTo[lofl, {20, 30, 40}];

lofl
{{20, 30, 40}, {20, 30, 40}, {20, 30, 40}}
```

Good practice note: Notice that I had to type the same identical thing 3 times. If you find yourself typing the same thing over and over and over and over, your program may run, it may give you good output but stop, take a break, come back to it. There is always a simpler way which will let you avoid typing repetitive things, you will think of it. Right now we do not have the machinery to do that in this case but we will shortly.

```
(* Ok now i have a list of lists. Lets see it as a
 Matrix: Note: MatrixForm is just for displaying the data. If I ask you
   to hand in a matrix as homework it should be in matrix form. All
   matricies again are just lists of lists in this language.*)
MatrixForm[
 lofl]
```

$$\begin{pmatrix} 20 & 30 & 40 \\ 20 & 30 & 40 \\ 20 & 30 & 40 \end{pmatrix}$$

```
(* Looking back at the variable lofl*)

lofl

{{20, 30, 40}, {20, 30, 40}, {20, 30, 40}}

(* It is still thankfully just a list of lists. How do we access
  parts of the list? Suppose I wanted to turn the diagonals into
  10* their current selves? Part[A, i] gives me the ith entry of the
  list A. in this case it gives me the first list for i = 1.  *)
i = 1;
Part[lofl, i]

{20, 30, 40}

(* Suppose I just wanted the first entry of the first list I could say: *)
i = 1;
j = 1;
Part[Part[lofl, i], j]


20
```

Again, If you find yourself typing the same thing over and over and over and over, your program may run, it may give you good output but stop take a break come back to it. There is always a simpler way which will let you avoid typing repetitive things (recall our sandwich shop if statement):

```
(* In this case we can replace the above syntax with: *)
i = 1;
j = 1;
Part[lofl, i, j]

20
```

```
(* In fact there is a better short hand for
 lists: double brackets after the list will give you the
   ith element and then the jth element of the ith element. *)
lofl[[i]]
```

```
{20, 30, 40}
```

```
lofl[[i]][[j]]
```

```
20
```

```
(*Further tools:
  the double semi-colon will give you start and stop indicies to take from *)
```

```
lofl[[i]][[j ;; j + 1]]
```

```
{20, 30}
```

```
(* The following doesnt give us what we would expect: *)
```

```
lofl[[i ;; i + 1]][[j ;; j + 1]]
```

```
{{20, 30, 40}, {20, 30, 40}}
```

```
(*we instead say: *)
```

```
lofl[[i ;; i + 1, j ;; j + 1]]
```

```
{{20, 30}, {20, 30}}
```

```
(*Recall *)
```

```
l2
```

```
{True, 10, 1.5, ⅈ, Friday}
```

```
(* Begining through entry 2*)
```

```
l2[[ ;; 2]]
```

```
{True, 10}
```

```
(* every second entry*)
```

```
l2[[ ;;  ;; 2]]
```

```
{True, 1.5, Friday}
```

```
(* every third entry*)
```

```
l2[[ ;;  ;; 3]]
```

```
{True, ⅈ}
```

Section 4.
For and While

```
(* So far I have made such a big fuss about there being no need to write the same
 thing over and over. Wow I keep saying this. I should take my own advice
 but I can't stress this strongly enough. So here is the real machinery: *)
```

```
For[i = 1, i ≤ 10, i = i + 1, Print[i]];
```

```
1
2
3
4
5
6
7
8
9
10
```

```
(*  For is a looping mechanism. It is a function with four arguments. Its first
 3 arguments specify when and if the fourth argument should be executed
 and how many times.    Lets take a look at the last argument first:
  Print[i]. This is the body of the For loop,
this is where the major statement or collection of statements goes.
  Usually when we write a for loop,
the first 3 lines go on the same line seperated by commas and the body which
 usually has many statements gets multiple lines like an If statement.

  The first argument of the For loop is always executed. It is only executed
 once at the very begining. Then the following things occur. The second,
fourth and then 3rd arguments are executed in that
 order. They are called the test, body, and incrementation.

   In this case the test is the
  statement is i less than or equal to 10?  i ≤ 10
  (this symbol ≤ (analogously ≥)  is generated by typing a < and then a =
     next to each other).

   If the test evaluates to true, the fourth argument, the body,
is executed, in this case just the simple print statement,

Finally the 3rd statement i = i + 1 is executed to increment i.

     Initially we have 1. Then we check 1 ≤ 10, this is true,
we execute the body and print i, which is 1. then we add one to i and update i,
at this point we execute another check,
2 ≤ 10? this is true we execute the body and print i,
which is 2. then we increment i,
at this point we go around and around the
 (maybe its now clear why we call it a loop) loop,
until finally we have printed 10, we then increment to 11,
and 11 is not less than or equal to 10.  11 ≤ 10 evaluates to False,
at which point we exit the loop. For drops us out at the bottom of the loop.

  Note: ++i, and i += 1 are both shorthand for i = i + 1.

                 *)

++i

(* What could we do over and over? *)
```

```
l2
```

```
{True, 10, 1.5, ⅈ, Friday}
```

```
Length[l2]
```

```
5
```

```
(* We are going to iterate over a list using a for
 loop. In the body of the for loop the value of the iterator
 is used to take the part of l2. We then print it out. *)
```

```
For[i = 1, i ≤ Length[l2], ++i,
  Print[
    l2[[i]]
   ];
 ];
```

```
True
10
1.5
ⅈ
Friday
```

```
(* Let's alter a list: *)
```

```
a = {1, 1, 1, 2, 2, 2};
```

```
For[i = 1, i ≤ Length[a], ++i,
  ++a[[i]]
 ];
```

```
a
```

```
{2, 2, 2, 3, 3, 3}
```

```
(* Lets make a 10 by 10 identity matrix: *)
```

```
id = {};
```

```
For[i = 1, i ≤ 10, ++i,
  (*
  We are going to add 10 new rows to
   id. One row will be added each time around the outter loop.
    Inside the inner loop we will populate each row with the correct
   elements. This will involve adding nine zero's and one one.
  *)
  newrow = {};

  For[j = 1, j ≤ 10, ++j,
   If[j == i,
     (*  if i == j we should append to the newrow a 1 otherwise it gets a 0. *)
     AppendTo[newrow, 1],
     AppendTo[newrow, 0]
    ]; (* end If *)
  ]; (* end j loop*)


  (*finally we append to the *)
  AppendTo[id, newrow];

 ]; (*end i loop*)

MatrixForm[id]
```

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

```
(* Lets make a while loop. This is simpler but more dangerous
 than a For loop. There are no Iterators. While takes two arguments,
a test condition and a body. It evaluates this pair
 infinitely as long as the test condition is true. *)
```

```
In[301]:= i = 1;
While[i ≤ 5,
   Print[i];
  ++i;
 ];
```

1

2

3

4

5

```
(*Note the difference in output if we switch the increment and the print. *)
```

In[303]:=
```
i = 1;
While[i ≤ 5,
  ++i;
   Print[i];
 ];
```

2

3

4

5

6

```
(* When would you use a while loop?

  How about when you dont know exactly how many iterations your
 loop will take to get its answer. In this case you would want to
 use a while loop so that you dont run for too long or too short. *)

(* Suppose we had a loop that kept going based on two conditions. Then
 the test expression could be a little more complicated. Here
 our test condition says keep going when: a≤ 5 or b ≤ 10. *)

a = 1;
b = 1;
While[a <= 5 || b <= 10,
  Print["New step"];
  Print["a == ", a];
  Print["b == ", b];
  Print["a * b == ", a * b];
  a = a + 1;
  b = 2 * b;
 ];
```

```
New step
a == 1
b == 1
a * b == 1
New step
a == 2
b == 2
a * b == 4
New step
a == 3
b == 4
a * b == 12
New step
a == 4
b == 8
a * b == 32
New step
a == 5
b == 16
a * b == 80
```

```
(* Suppose we wanted to know how many times we went
 around the while loop? We could initialize a variable with 1,
then add one to the variable every time we go around the
 while loop. In this way after the loop ends we could ask,
how many times we went around by just looking inside this variable.*)

counter = 0;
a = 1;
b = 1;
While[a <= 5 || b <= 10,
  counter = counter + 1;
  Print["a * b == ", a * b];
  a = a + 1;
  b = 2 * b;
 ];
Print["We went around the while loop exactly ", counter, " times"]
```

```
a * b == 1
a * b == 4
a * b == 12
a * b == 32
a * b == 80
We went around the while loop exactly 5 times
```

```
(* A while loop can run
 forever: This can happen when the test condition always evaluates to true. *)
```

WARNING: While loops can be dangerous

IF YOU ARE STUCK IN AN INFINITE LOOP:

Go to the menu tab and select

Evaluation -> Quit Kernel -> Local

This will stop the program in it’s tracks and clear all variables of their values. From black to blue all the symbols go.

After you have finished coding and before you hand something in to me:

Follow the above instructions to quit the kernel. Then reevaluate your whole program to make sure that your program runs smoothly.

Section 5. User defined functions

```
(* So far we have seen many functions that were created as part of the language,
built-ins so they call them. How could we make our own? In this language

   f(x) = x^2

  is written as:

*)
```

```
In[24]:= f[x_] := x^2;
```

In[25]:=
```
f[2]
```

Out[25]= 4

```
(* Don't forget the underscore when
 declaring what arguments the function will take.

  Halfway through the program if you are going to change
 the function then define the function without the semi colon *)
```

In[26]:=
```
f[x_] = x^2;
```

In[27]:=
```
f[2]
```

Out[27]= 4

```
(* Please also try to use the colon when we define
  functions based on other functions we will not use the := . *)
```

```
(* Here we define a function g of several variables,
and then we define a function h in terms of g. *)
```

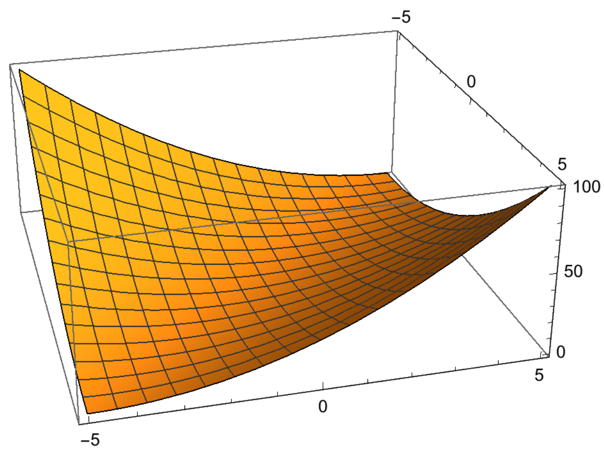
In[24]:=
```
g[x_, y_] := x^2 + 2*x*y + y^2;
```

In[25]:=
```
h[x_] := g[x, 2];
```

```
(* Use Plot and Plot3D respectively for functions of one or two variables.*)
```

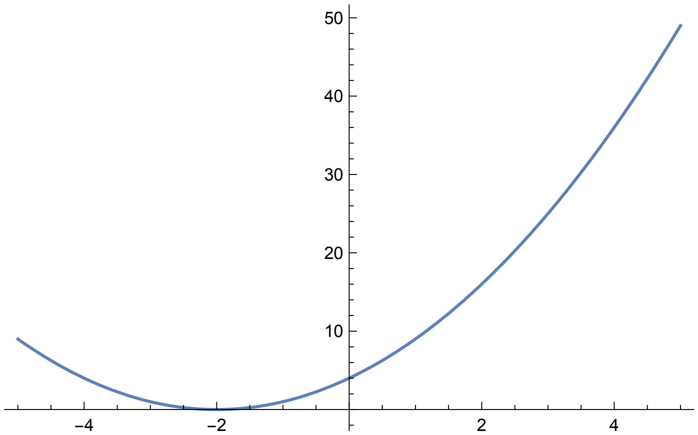
In[26]:=
```
Plot3D[g[x, y], {x, -5, 5}, {y, -5, 5}]
```

Out[26]=



In[27]:=
```
Plot[h[x], {x, -5, 5}]
```

Out[27]=




```
(* We can also apply functions to lists: *)
```

In[28]:=
```
l = {};
```

In[29]:=
```
For[i = -5, i ≤ 5, ++i,
 AppendTo[l, i];
]
```

In[30]:=
```
l
```

Out[30]= {-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5}

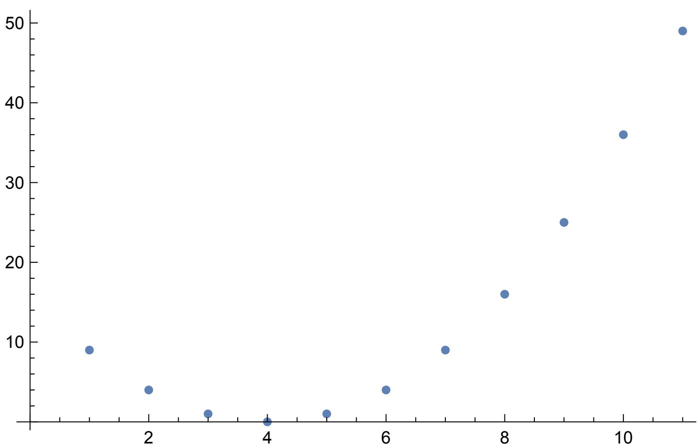
```
(* We will now apply h to the list: *)
```

In[31]:=
```
h[l]
```

Out[31]= {9, 4, 1, 0, 1, 4, 9, 16, 25, 36, 49}

In[32]:=
```
ListPlot[h[l]]
```

Out[32]=

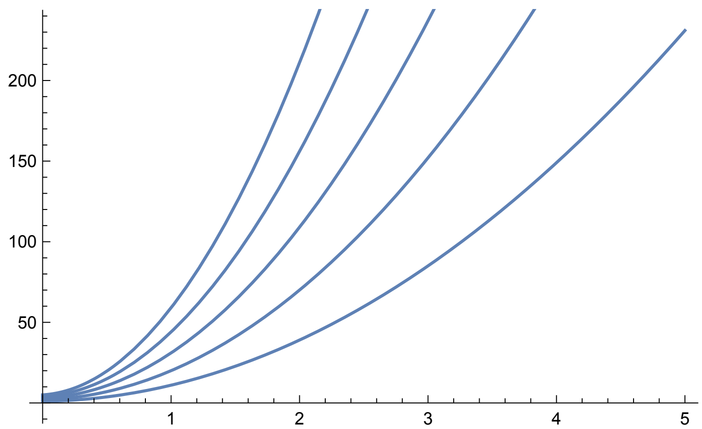


In[33]:=
```
(* Breifly We will Define a Family of Functions:
  This will be useful when we program methods of interpolating functions.*)

plotlist = {};
For[i = 1, i ≤ 5, ++i,
  fᵢ[x_] := h[i] * x^2 + i * x + i;
  AppendTo[plotlist, Plot[fᵢ[x], {x, 0, 5}]];
 ];
```

In[35]:=
```
Show[plotlist]
```

Out[35]=



Section 6. Error

```
(*  When Pandora opened the box of all of
 the horrible computational errors in the universe,
many types of error sprang out. To name a few: Type 1, Type 2,
systemic error, roundoff error, compound error. Lets talk about some of these.

  When we program Gauss Jordan
 elimination: you will see that a small error in the top left hand
   corner of the matrix will propagate through the entirety of our
   comptuations compounding itself in the bottom right. The larger the
   matrix the larger the error.  When you type your less than the wrong
   way in the test condition of a while loop and you spiral off into
   an infinite oblivion of Print statements this is an example of an
   error in logic or a systemic error. When we approximate a function,
or its integral, or its derivative we are going to have
 some error by virtue of the fact that we
 are looking for an approximation. However,
approximations wouldnt be of very much use to us unless
 we had some method for bounding the error. *)
```

```
(* suppose we know the function f(1) = 1,
and f(1.1) = 1.21; We could then use Newton's quotient
 to estimate the value of the derivative at the point 1. *)
```

```
In[36]:= a = 1;
b = 1.1;
fa = 1;
fb = 1.21;

In[40]:= estd = (fb - fa) / (b - a);

In[41]:= Print[estd]

2.1
```

```
(* this is our estimate for the derivative of the function at 1.*)

(* In fact, our function is *)
```

```
In[42]:= f[x_] := x^2;

In[43]:= g[y_] = D[f[y], y];

In[44]:= g[1]

Out[44]= 2
```

```
(* to do this correctly we omit the second colon in the definition of g *)
```

```
(* See the definition of error, absolute error,
relative error ,relative absolute error. from class notes*)
```

```
In[46]:= e = g[1] - estd

Out[46]= -0.1

In[48]:= absoluteError = Abs[e]

Out[48]= 0.1

In[49]:= relativeError = e / g[1]

Out[49]= -0.05

In[50]:= relativeAbsError = Abs[relativeError]

Out[50]= 0.05
```

Homework Due Wednesday August 30th:

For these exercises you will need to define functions f and g.

If you can: Use a For loop to compute the error for each value of x. You will have an expression like 10^(-i) somewhere in the body of the For loop. Append the error to a list, then print the list and plot the list using ListPlot.

Exercises:

1. Consider the function $f(x,y) = ((x+y)^2 - 2xy - y^2)/x^2$. We expect that if $x \neq 0$, then $f(x,y) = 1$. Set $y = 10^3$ and compute f for $x = 10.0^{-1}$, 10.0^{-2}, 10.0^{-3}, 10.0^{-4}, 10.0^{-5}, 10.0^{-6}, 10.0^{-7}, 10.0^{-8}. For each value of x compute the absolute error.

2. Repeat Problem 1 with $g(x,y) = (x+y)^2/x^2 - 2xy/x^2 - y^2/x^2$. Why are the results different? Is $g[x, 10^3]$ the same function as $f[x, 10^3]$?

3. Repeat Problem 1 using $x = 10^{-1}$, 10^{-2}, 10^{-3}, $10^{(-4}$, 10^{-5}, 10^{-6}, 10^{-7}, 10^{-8}. Why are the results different?

Additionally:

Write a program that plots the iteration counts for each number from 1 to a few thousand (of your choice). Some number high enough that it doesn't take forever to run but so that you get something interesting.

Begin by writing a while loop that plays the game for a single number. Once this code works, use a For loop to make your program run this while loop a number of times.

You can find the rules here:
https://en.wikipedia.org/wiki/Collatz_conjecture

The final structure of the program should look something like this:

list = {};

For[i = 1, i <= 1000, ++i,
n = i;
count = 0;

Play the Collatz game starting with n, counting the number of steps it takes to finish, use a while loop

```
append count to list
];

ListPlot[list];
```