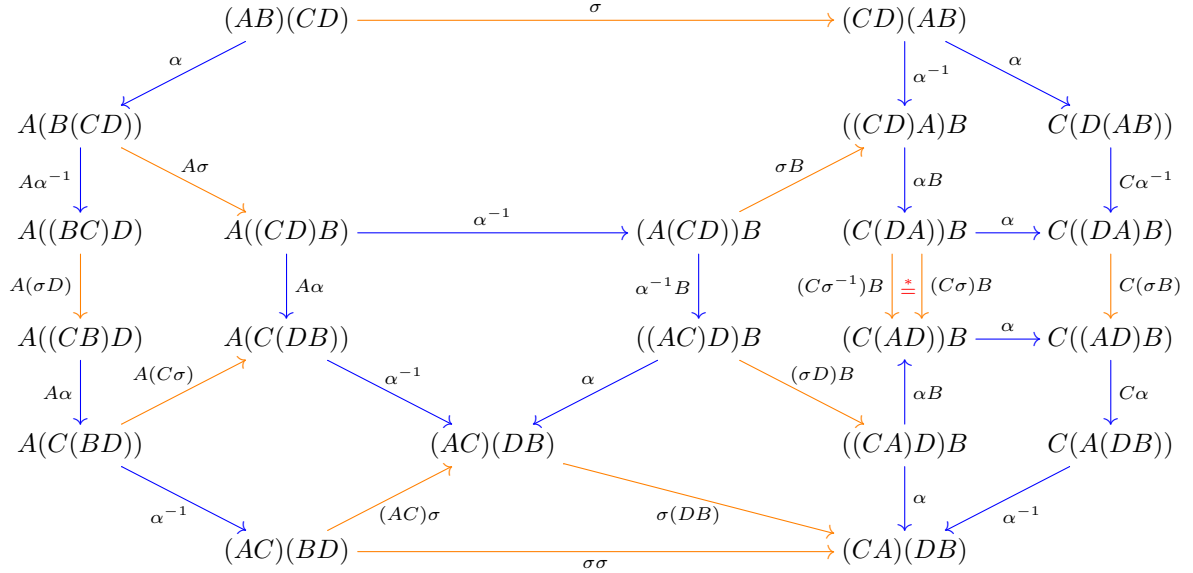


A BRAIDED MONOIDAL CATEGORY IS SYMMETRIC IF AND ONLY IF THE PRODUCT IS BRAIDED

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The following diagram commutes, by virtue of the symmetric monoidal category axioms, and shows in that case $\otimes: \mathcal{C} \times \mathcal{C} \rightarrow \mathcal{C}$ is a *braided* strong monoidal functor. That \otimes is a *strong* monoidal functor is due to Joyal–Street (‘Braided monoidal categories’¹, Proposition 2). The vertical composites down the sides below are the comparison maps for pre- and post- \otimes products given by Joyal–Street.



In the diagram I have written $AB := A \otimes B$ etc, and identity arrows 1_X are written as just X . Isomorphisms arising from associators (α) are in blue, those arising from the braiding (σ) are in orange. Notice that the equality $\stackrel{*}{=}$ is the only place where the symmetry axiom is used.

Conversely, take \mathcal{C} to be merely braided and \otimes a braided functor. If $B = C = I$ the diagram above implies $(I\sigma^{-1})I = (I\sigma)I: (I(DA))I \rightarrow (I(AD))I$. Applying naturality squares for $IX \rightarrow X$ and $XI \rightarrow X$ gives us that $\sigma^{-1} = \sigma$, hence \mathcal{C} is symmetric.

I’m sure this result is rather old, but haven’t found a reference. It follows from high-powered abstract machinery about E_∞ -algebras in **Cat**, but I think this was possible to prove by the time of Mac Lane’s 1963 coherence theorem.

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¹Available at <http://web.science.mq.edu.au/~street/JS1.pdf>