PERFECT NUMBERS

A **perfect number** is one such that all its factors, including one but excluding itself, add up to itself. For example, 6 is a perfect number since 6 has factors $1, 2, 3$ and 6 and $1 + 2 + 3 = 6$. In fact, 6 is the smallest perfect number.

Euclid proved in his **Elements** that a number of the form $2^{p-1}(2^p - 1)$ is a perfect number if $2^p - 1$ is a prime number.

I used this fact to write a programme in Basic to find perfect numbers but first we need a programme on prime numbers for checking if $2^p - 1$ is prime.

```
ready.
10 rem prime numbers
11 rem to calculate prime numbers up to a
20 input a
22 if a=2 then print"2": goto 100
25 print"2 3";
30 for i=2 to a
40 if i=a then 100
50 for d=2 to int(sqr(i)+2)
60 if i/d=int(i/d) then 90
70 next d
80 printi;
90 next i
100 end
```

So now let us see how we can use lines 40-60 to find perfect numbers.

```
10 rem perfect numbers
15 rem to calculate perfect numbers
20 input n
30 if n<6 then print "none":goto 200
35 ir n=6 then print "6 only":goto 200
40 print"6";
45 for i=3 to 26
```

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```
46 rem limit n to 2↑25*(2↑26-1)
47 let y=2↑i-1
50 rem next loop is to check if 2↑i-1 is prime
52 for l=2 to int(sqr(y))
53 if y/l=int(y/l) then 70
54 if y*2↑(i-1)>n then 200
55 next l
57 print",";y*2↑(i-1);
70 next i
200 print
201 print"(this program was written on 26/8/83)"
300 end
```

Unfortunately, line 45 limits us to $2^{25}(2^{26}-1)$, but then the computer has a limited range of numbers: it will never get to $2^{25}(2^{26}-1)$ anyway. I have computed perfect numbers up to 10^{13}.

$$6, 28, 496, 8128, 33\,550\,336, 8.58986906\text{e}+09, 1.37438691\text{e}+11$$

The last two, of course, are only approximations to the actual perfect numbers and are unacceptable in this form.
$8.58986906\text{e}+09 = 8\,589\,869\,060$ when the last two figures are in doubt. In fact it is $8\,589\,869\,056$.

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