Algorithm Benchmarking Analysis

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Introduction

- An algorithm's "worst-case" run-time is a useful object of study in computer science.
- Denote $f : \mathbb{N} \to \mathbb{N}$ where for a given input n, $f(n)$ is the steps required for the algorithm to terminate.

Definition
We say a function f has **order** g and write $f(n) \in \mathcal{O}(g(n))$ if there exist $c \in \mathbb{R}$ and $N \in \mathbb{N}$ such that

$$f(n) \leq c \cdot g(n)$$

for all $n \geq N$.

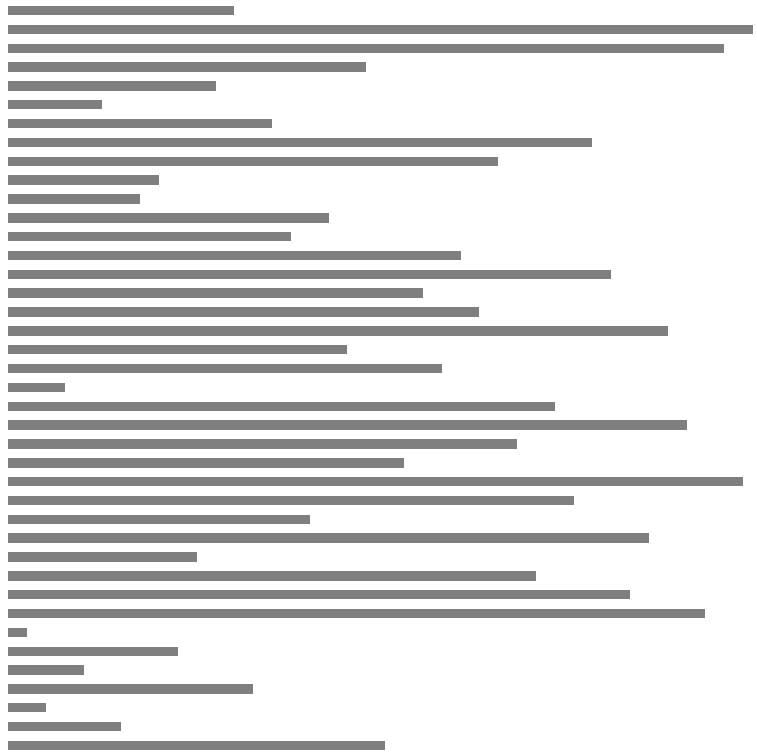
- This is called **Big-Oh** analysis.

Sorting

Problem
Given an array A, rearrange elements of A such that $A[k] \leq A[k+1]$ for all k.

- We consider two algorithms: **insertion sort** and **merge sort**.
- Insertion sort has worst-case complexity $O(n^2)$.
- Merge sort has worst-case complexity $O(n \log_2 n)$.
- In theory, merge sort is **much** faster than insertion sort.

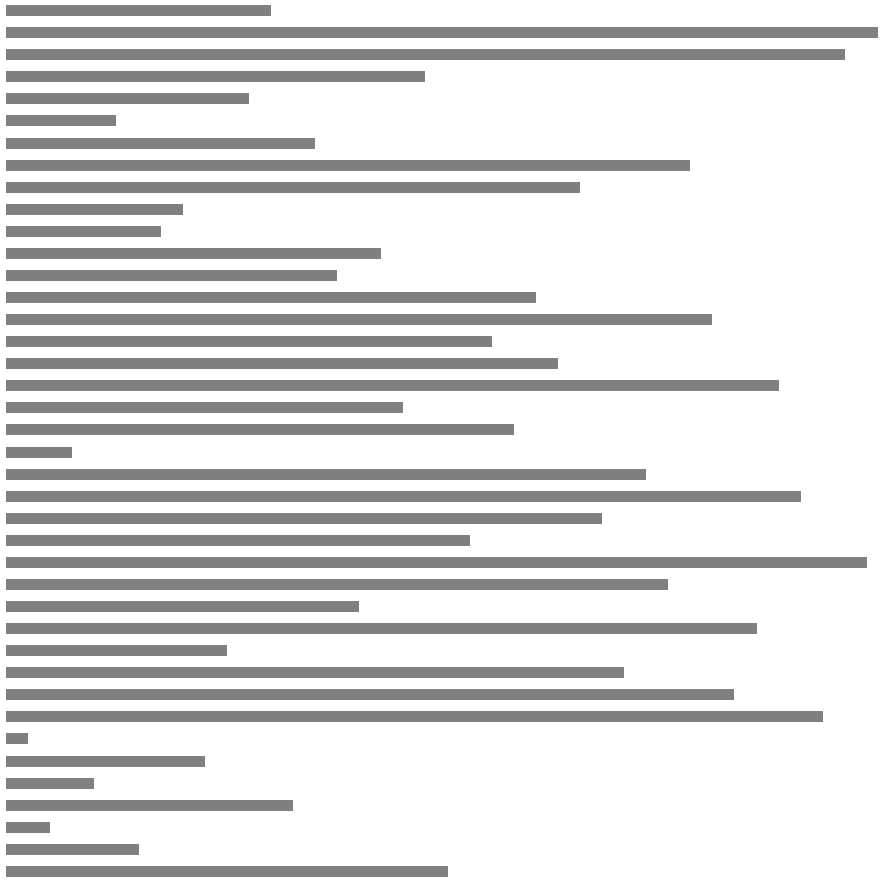
Insertion sort



[1]

[1] Attribution: http://www.sorting-algorithms.com/insertion-sort

Merge sort



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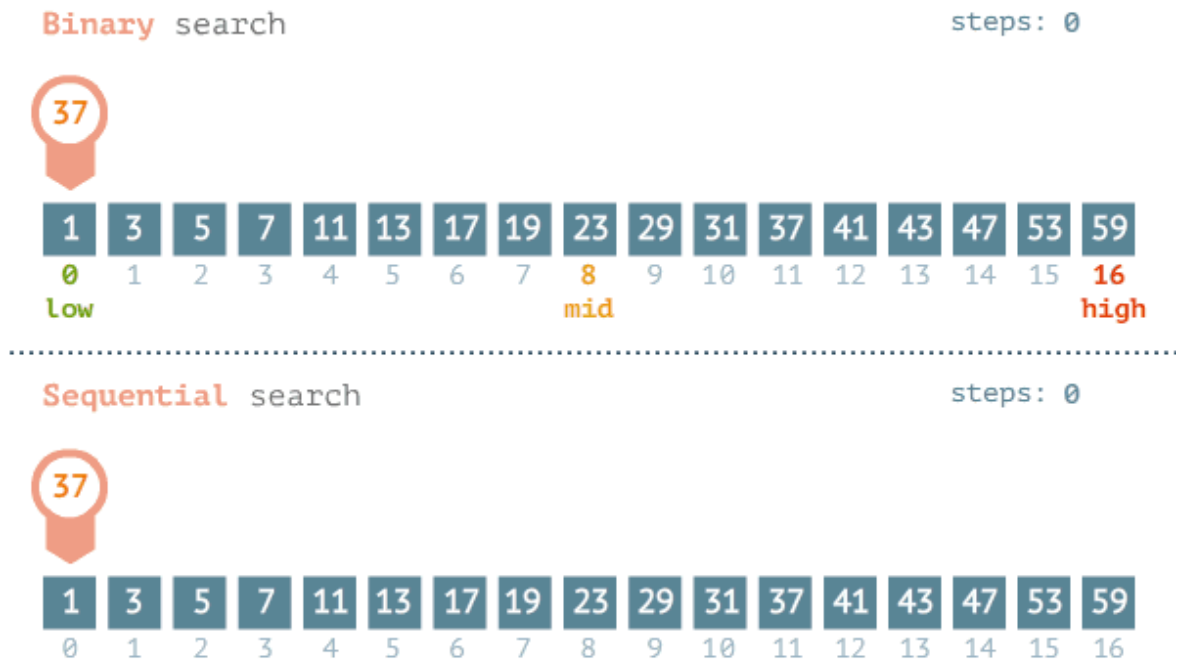
[2] Attribution: http://www.sorting-algorithms.com/merge-sort

Search

Problem
Given a sorted array A and entry v, determine the index k such that $A[k] = v$.

- We consider two algorithms: **linear search** and **binary search**.
- Linear search has worst-case complexity $O(n)$.
- Binary search has worst-case complexity $O(\log_2 n)$.
- In theory, binary search is **much** faster than linear search.

Binary and linear search



www.penjee.com

[3] Attribution: http://mrmonline.org/site/binary-search-algorithm/

Iteration and recursion

- Algorithms can usually have both iterative and recursive implementations.
- The dominant view is that iteration is faster than recursion.
- This is due to language-implementation details.

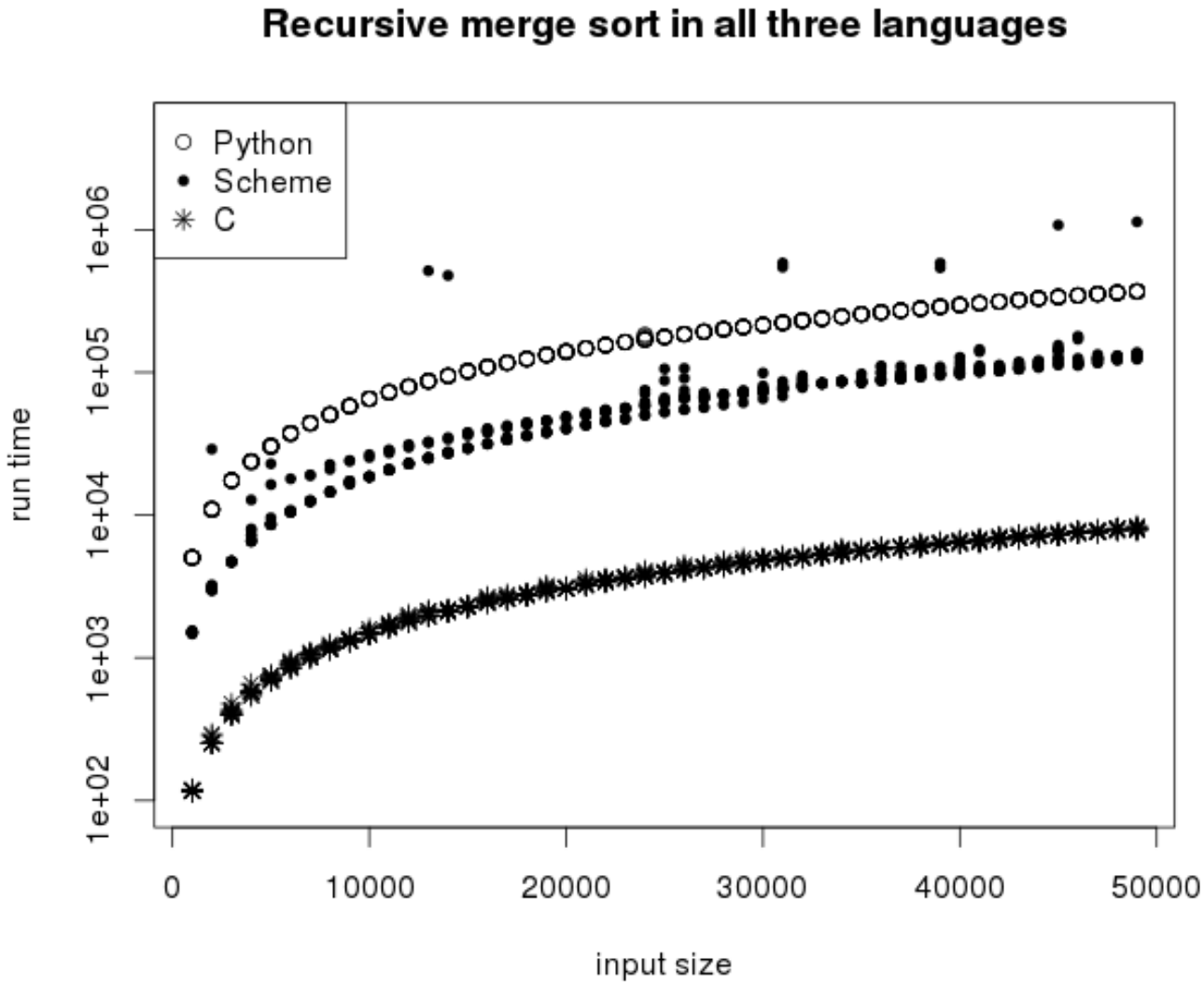
Experiments

- Each algorithm is implemented recursively and iteratively in C, recursively in Scheme, and iteratively in Python.
- It is a common perception that C is faster than both Scheme and Python.
- We generate arrays of length $n = 1000, \ldots 50000$.
- For sorting, the elements are iid $\mathsf{Uniform}(0, n)$
- For searching, the differences between adjacent elements are iid $\mathsf{Uniform}(0, 10)$.
- We measure program execution time, a rough proxy for steps to termination.

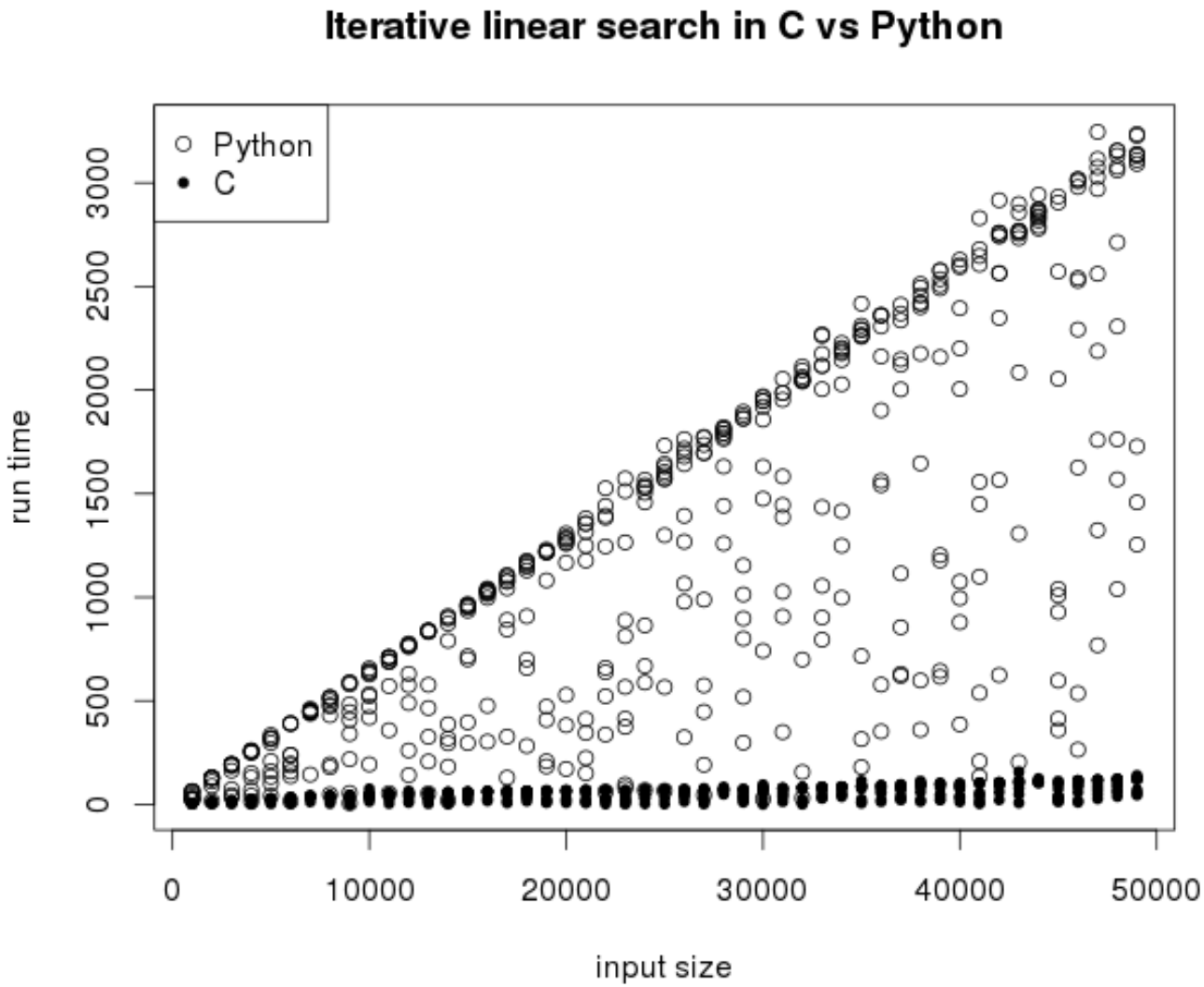
Research Questions

- Does the theorized relationship between input and run-time manifest in practice?
- Is merge sort empirically faster than insertion sort?
- Is binary search empirically faster than linear search?
- Is iteration faster than recursion?
- Is C faster than Scheme or Python?
- What is the relationship between Scheme and Python?

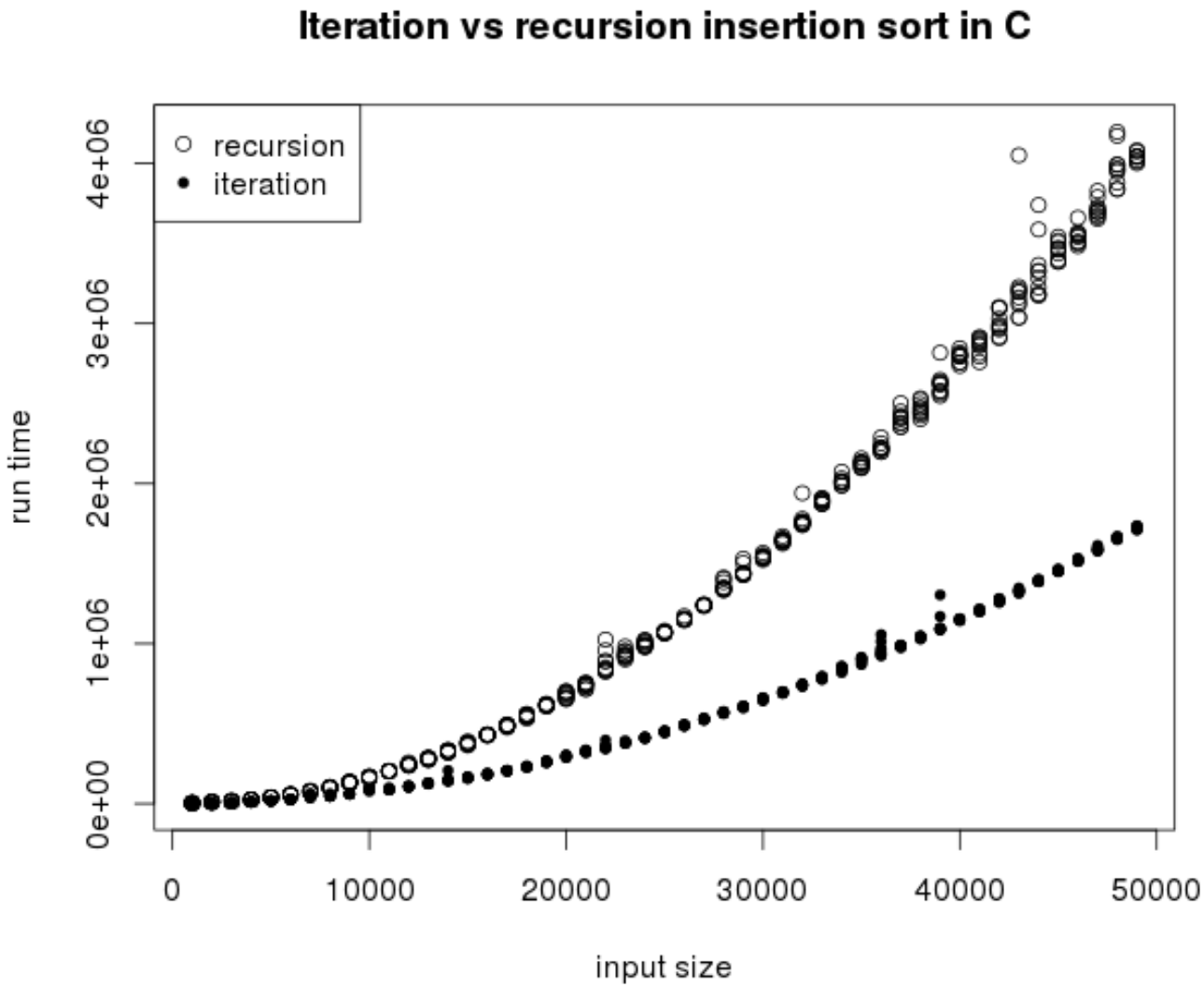
Comparing languages



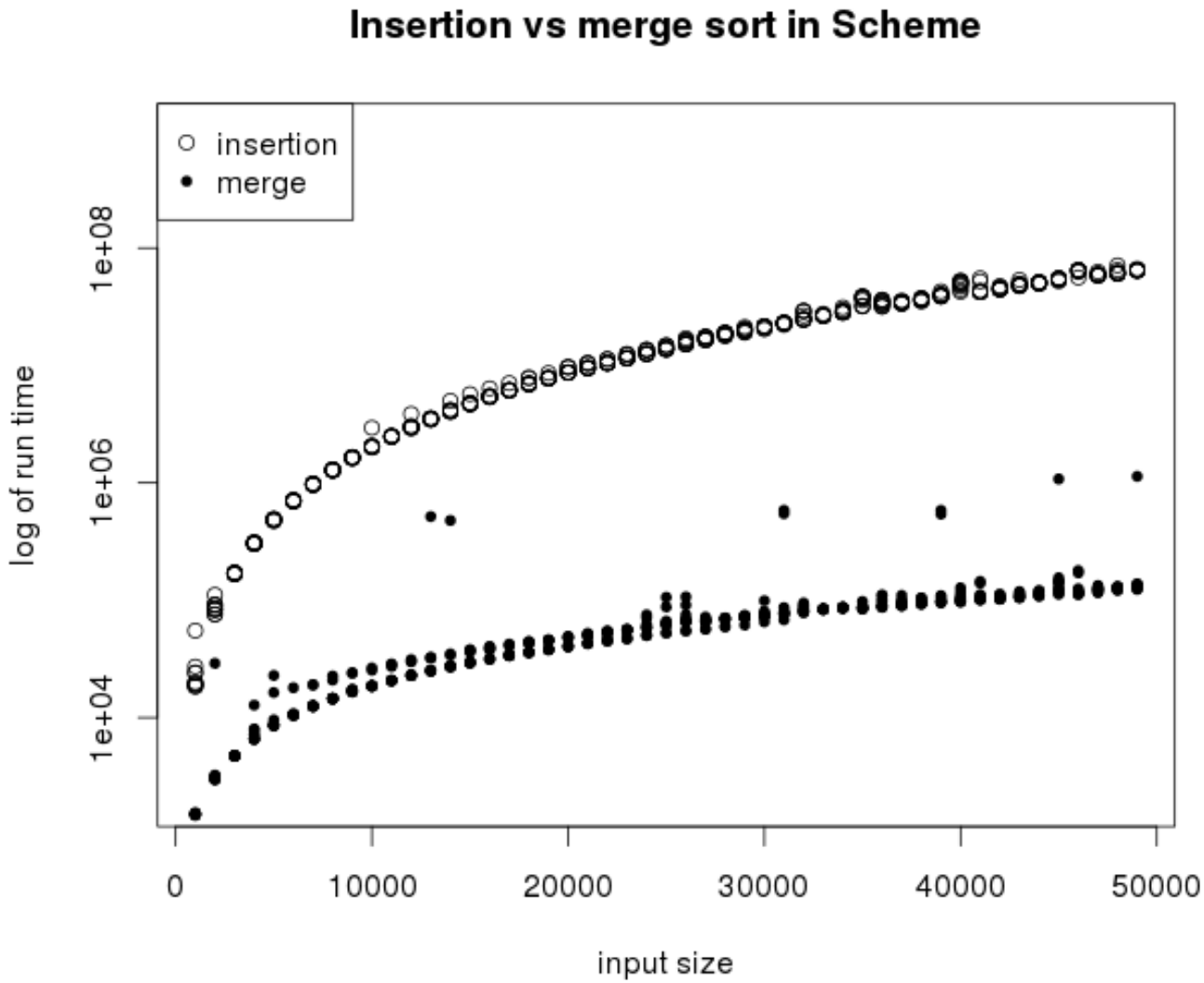
Comparing languages



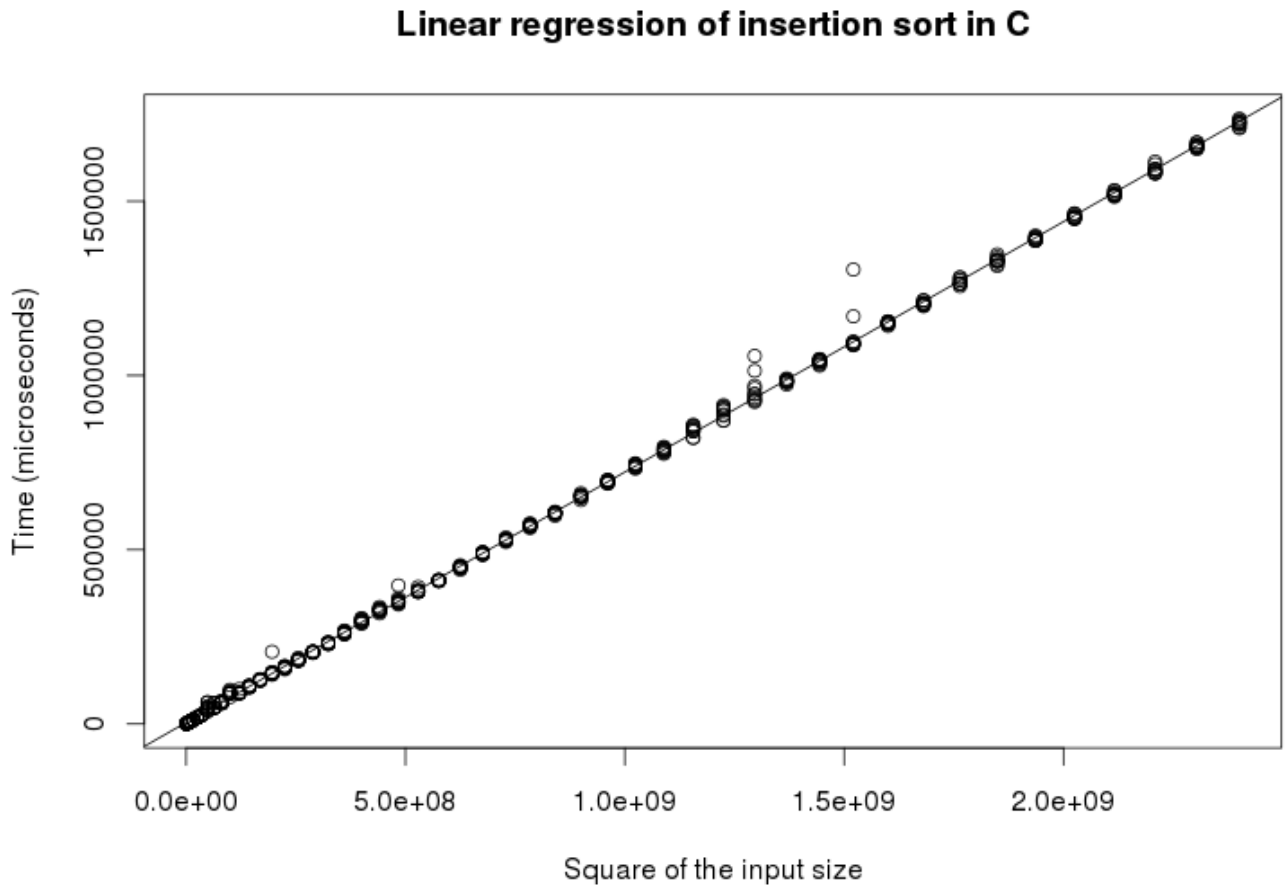
Comparing iteration and recursion



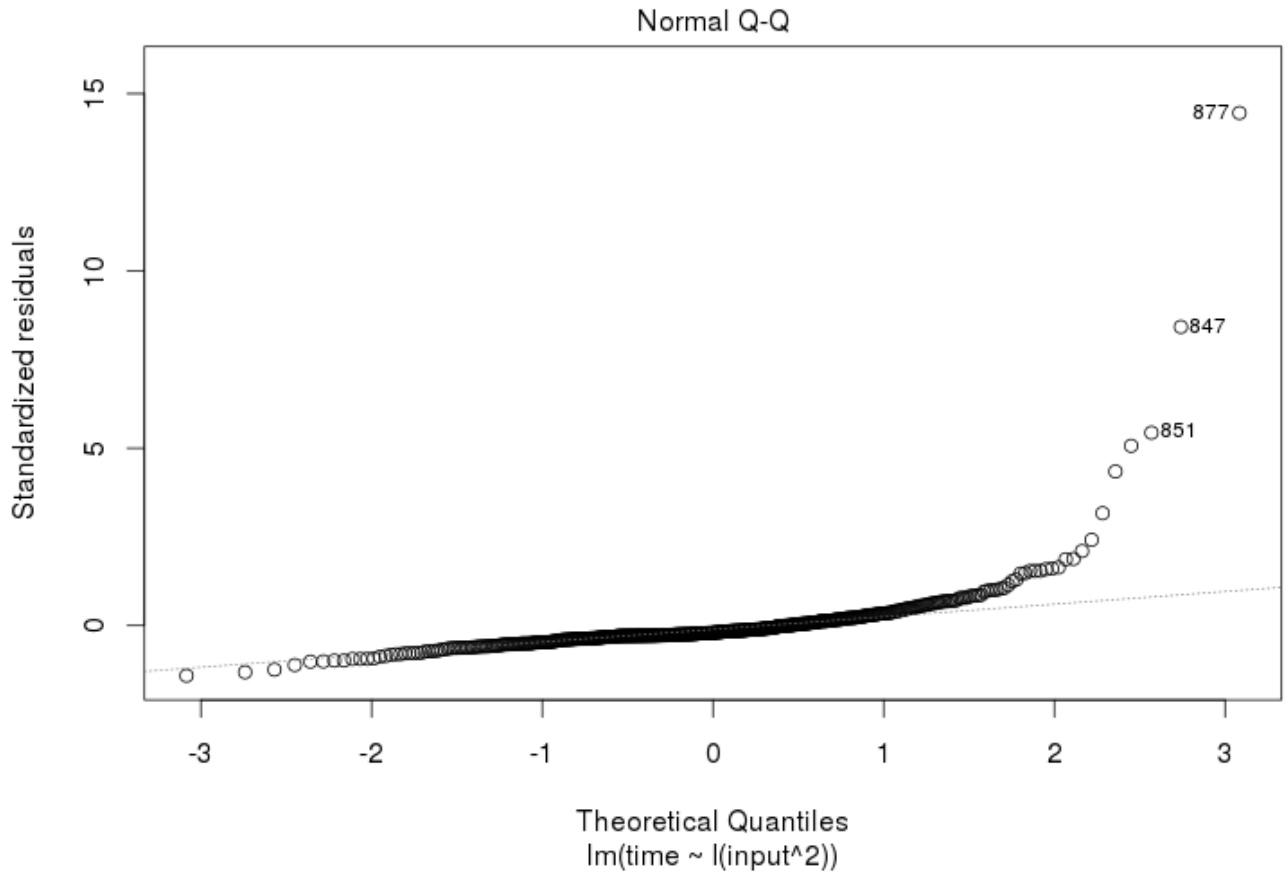
Comparing sorting algorithms



Fitted theoretical model



Fitted theoretical model



Methodology and Results

- We used paired t-tests to identify significant speed differences between
  - recursion vs iteration
  - C, Scheme, and Python,
  - linear vs binary search; insertion vs merge sort
- We found significant ($p < 0.001$) speed differences in almost every comparison.
- We found no significant speed difference between recursive and iterative binary search in C ($p = 0.24$).

Methodology and Results

- We used transformed linear models to determine if empirical data supports theoretical upper bounds of performance
- We found that the theoretical models better fit the data in almost every case.
- We did not find this for binary search in C.

Conclusion

- The overwhelming evidence is that C is the fastest language, iterative is the fastest paradigm, and merge sort and binary search are the fastest algorithms
- We found no significant results for binary search in C because its run time is sub-microsecond.
- Should every program be written in C?
- Should every algorithm be implemented iteratively?