Technical Report for Fashion Star Corporation

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Abstract

We provide production forecasts for Fashion Star Corporation's fall line of clothes. We model Fashion Star's production problem as a linear program and solve it using interior point methods. Demand forecasts produced by Fashion Star's market research team are incorporated into the program. Using the baseline model, we offer operational policy recommendations for Fashion Star under a variety of scenarios.

1 Introduction

Fashion Star Corporation is preparing for its fall line of clothes. The firm has already produced designs for a full lineup and has exhibited the new clothing items in fashion shows in New York, Paris, and Milan. Proceeding to the production phase, Fashion Star would like to know what quantities of clothing items will maximize its profit for the upcoming season. The firm has compiled an exhaustive list of the factors for each clothing item (e.g., materials required, labor costs, and machine costs), along with the prices it plans to set for them. It has also provided the maximum available supply for each raw material used in production. Finally, the firm has categorized its designs into a professional line and a casual line. Fashion Star's market research team has provided estimates for the expected demand of a number of items on its lines, but the firm's core business is its professional line. To foster and maintain customer loyalty, Fashion Star wants to guarantee a basic quantity of product in this line for the fall.

In this study we model the firm's production problem as a linear program parameterized by the above information. The linear program is solved by interior point methods, which yields decimal values, not integers. As integer production levels are more useful to interpret, we approximate the solution to the program by finding nearest-integer values to the result. We use the model to predict production levels under a variety of scenarios, such as production when wool input costs are underestimated and whether Fashion Star should sell its clothing in outlet stores. The model provides valuable insight in these scenarios as well as in the basic production problem.

The paper proceeds as follows. Section 2 outlines the baseline model and the accompanying model assumptions that we use to generate production forecasts. Section 3 takes the model from Section 2 and applies it to the parameters and data provided by Fashion Star to obtain forecasts. We then modify the parameter inputs to fit a variety of additional scenarios for the firm and report the results. Finally, Section 4 discusses the usefulness of the linear programming approach, along with its weaknesses in producing viable results. It also uses the data from Section 3 to offer recommendations for Fashion Star's operational policy for its fall line of clothing.

2 Model and assumptions

2.1 Assumptions

We assume that all of the firm's material constraints, material, labor, and machine costs, and demand forecasts are accurate. The estimates for producing clothing items in the Professional and Casual lines are recorded in Table 1 and Table 2 respectively. The cost of the raw materials and the maximum supply of each can be found in Table 3.

Clothing item	Material requirements	Price	Machine and labor costs
Tailored wool slacks	3.0 yards of wool	\$300	\$160
	2.0 yards of acetate for lining		
Cashmere sweater	1.5 yards of cashmere	\$450	\$150
Silk blouse	1.5 yards of silk	\$180	\$100
Silk camisole	0.5 yards of silk	\$120	\$60
Tailored skirt	2.0 yards of rayon	\$270	\$120
	1.5 yards of acetate for lining		
Wool blazer	2.5 yards of wool	\$320	\$140
	1.5 yards of acetate for lining		

Table 1: Professional line.

Clothing item	Material requirements	Price	Machine and labor costs
Velvet pants	3.0 yards of velvet	\$350	\$175
	2.0 yards of acetate for lining		
Cotton sweater	1.5 yards of cotton	\$130	\$60
Cotton miniskirt	0.5 yards of cotton	\$75	\$40
Velvet shirt	1.5 yards of velvet	\$200	\$160
Button-down blouse	1.5 yards of rayon	\$120	\$90

Table 2: Casual line.

In addition to all of these constraints, Fashion Star has also forecasted the demand for many of their items. They believe that no more than 5,500 Velvet pants, 6,000 Velvet shirts, 4,000 Cashmere sweaters, 12,000 Silk blouses, 15,000 Silk camisoles, 5,000 Wool blazers, and 7,000 Tailored wool slacks can be sold. In order to keep their customers happy, they are also committed to producing 2,800 Tailored skirts, 3,000 Wool Blazers, and 4,200 Tailored wool slacks.

Material	Price per yard	Maximum supply
Wool	\$ 9.00	45,000 yards
Acetate	\$ 1.50	28,000 yards
Cashmere	\$60.00	9,000 yards
Silk	\$13.00	18,000 yards
Rayon	\$ 2.25	30,000 yards
Velvet	\$12.00	20,000 yards
Cotton	\$ 2.50	30,000 yards

Table 3: Material Constraints.

By making both Cotton sweaters and Silk blouses, we produce left over scrap material which cannot be refunded. However, this scrap is just enough to use to produce a Cotton miniskirt or Silk camisole, respectively. In order to minimize waste, we assume that every time a Cotton sweater or Silk blouse is made, the scrap is used to produce a miniskirt or camisole as per the material. Fashion Star reports that it has spent \$860,000 in design costs and \$1.2 million in producing shows in New York, Paris, and Milan each. These \$4.46 million in costs are treated as fixed and sunk.

Finally, we assume that all clothing items produced by the firm in its fall line are sold during that season. This reduces the difficulty of estimating expected profit for the fall line, but this also represents the single largest deviation by the model from reality.

2.2 Model

Let $\mathbf{x}$ denote the bundle of production levels for the fall line, and let z denote the profit from the fall line's production (excluding sunk costs). We write

$$\begin{aligned} z &= \mathbf{p}^T\mathbf{x} - \mathbf{c}_L^T\mathbf{x} - \mathbf{c}_M^T\mathbf{x} && (1) \\ &= (\mathbf{p} - \mathbf{c}_L - \mathbf{c}_M)^T\mathbf{x} && (2) \end{aligned}$$

where $\mathbf{p}$ is the price of the bundle (set by Fashion Star), $\mathbf{c}_L$ is the vector of combined labor and machine costs, and $\mathbf{c}_M$ is the vector of material costs associated with each item.

The constraints given in Table 1 and Table 2 are encoded into the matrix and constraint vector below:

$$A = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 & 0 & 2.5 & 0 & 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 & 1.5 & 1.5 & 0 & 0 & 2 & 0 & 0 \\ 0 & 1.5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1.5 & 0.5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 1.5 \\ 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 & 1.5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.5 & 0.5 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \end{pmatrix}, \mathbf{b} = \begin{pmatrix} 5500 \\ 5500 \\ 6000 \\ 4000 \\ 12000 \\ 15000 \\ -2800 \\ -3000 \\ 5000 \\ -4200 \\ 7000 \\ 45000 \\ 28000 \\ 9000 \\ 18000 \\ 30000 \\ 20000 \\ 30000 \\ 0 \\ 0 \end{pmatrix} \tag{3}$$

The demand forecasts are interpreted in A and $\mathbf{b}$ as upper bounds on some of the clothing items, and the customer loyalty policy are likewise interpreted in A and $\mathbf{b}$ as lower bounds on some of the clothing items.

The primal problem is given in its minimization form:

$$\mathbf{x}^* = \arg\min_{\mathbf{x}} -z = -(\mathbf{p} - \mathbf{c}_L - \mathbf{c}_M)^T \mathbf{x} \tag{4}$$

subject to

$$A\mathbf{x} \leq \mathbf{b} \tag{5}$$

$$\mathbf{x} \geq \mathbf{0}. \tag{6}$$

The net profit is therefore given by

$$\pi = (\mathbf{p} - \mathbf{c}_L - \mathbf{c}_M)^T \mathbf{x}^* - c \tag{7}$$

where c is the fixed costs of designing the line and running the three shows.

We solve this problem by using interior point methods. Interior point methods find $\mathbf{x}^*$ by transforming both the primal (maximizing profit) and dual (minimizing slack) problems. We do not diverge into a discussion of the theory here. However, we state the dual problem as follows.

$$\mathbf{y}^* = \arg\max_{\mathbf{y}} \mathbf{b}^T \mathbf{y} \tag{8}$$

subject to

$$A^T \mathbf{y} \geq \mathbf{c}_L + \mathbf{c}_M - \mathbf{p} \tag{9}$$

$$\mathbf{y} \geq \mathbf{0}. \tag{10}$$

We note that the feasible region for the primal problem is convex and bounded, which guarantees the existence of a solution. The dual problem has a useful interpretation in the context of profit maximization. Given the constraint set, the solution to the dual problem gives the shadow prices for each clothing item. They describe the additional profit for the firm if the constraints are relaxed by 1 unit. The solution to the optimization problem is a vector of real numbers, but the production requirements call for integer-valued vectors. Due to the general intractibility of integer linear programming, our approach simply rounds the real solution to fit the integer requirements. In doing so, we introduce the prospect for sub-optimal solutions.

We report the solution of the optimization problem in in Table 4. This gives us a net profit of approximately \$2.4 million. It is important to note that in this mathematically optimal solution, no cotton sweaters are made while 60,000 cotton miniskirts are made. Our estimated integer solution, which loses the guarantee of optimality, results in a loss of less than \$1,000 in profit, or less than 0.05% of the optimal solution. The shadow prices for the optimal solution are provided in Table 5.

Clothing item	Quantity
Tailored wool slacks	4,200
Cashmere sweater	4,000
Silk blouse	7,000
Silk camisole	15,000
Tailored skirt	8,066
Wool blazer	5,000
Velvet pants	0
Cotton sweater	0
Cotton miniskirt	60,000
Velvet shirt	6,000
Button-down blouse	9,245
Profit	\$2,402,291.00

Table 4: Initial production line.

3 Model extensions and additional concerns

3.1 Velvet problems

Here we quantify the cost of operational concerns with clothing items in the fall line produced with velvet. Fashion Star disclosed that the velvet shirts required \$500,000 in design and

Clothing Constraints	Shadow price	Material Constraints	Shadow price
Maximum Velvet pants	$0	wool	$0
Maximum Velvet shirts	$22	acetate	$72
Maximum Cashmere sweaters	$210	cashmere	$0
Maximum Silk blouses	$0	silk	$40
Maximum Silk camisols	$33	rayon	$18
Minimum Tailored skirts	$0	velvet	$0
Minimum Wool blazers	$0	cotton	$68
Maximum Wool blazers	$48	silk scrap	$0
Minimum Tailored wool slacks	$34	cotton scrap	$0
Maximum Tailored wool slacks	$0		

Table 5: Shadow Prices for all constraints.

marketing costs, but the model predicts returns of $132,000 for the velvet shirts alone. This means that the firm stands to lose $368,000 by producing velvet shirts. If the $500,000 has already been spent, we recommend continuing to produce velvet shirts since recouping some of the sunk costs is better than recouping none of them. However, if the money has not been spent yet, then velvet shirts are not worth producing.

Next, we consider how to modify the production line if Fashion Star cannot return velvet to its supplier. We estimate the cost for the firm to attempt to renegotiate the amount of velvet that they receive from the supplier. We found that if the same 20,000 yards is received and it can't be returned, net profit decreases by about $27,700 or approximately 1.2%. The changes in the production line are summarized in Table 6.

Clothing item	Old quantity	New quantity
Tailored skirt	8,066	3,177
Velvet pants	0	3,666
Button-down blouse	9,245	15,763
Profit	$2,402,291.00	$2,374,597.12

Table 6: Changes in the production line compared to Table 4 when Velvet cannot be returned.

If Fashion Star attempts and fails to renegotiate with its supplier, the entire relationship may be soured. The risk of failing to renegotiate successfully with the supplier must be weighed against the 1.2% guaranteed hit to profit if the firm chooses not to renegotiate. This problem is beyond the scope of our work. Fashion Star can use this information, however, to determine if such renegotiations are worth their cost.

3.2 Increased input costs

We consider the effect of underestimating costs by $80 for producing a Wool blazer. This presents a serious problem for production, as the baseline model calls for a considerable quantity of Wool blazers. We found that this increase in labor costs decreases net profit by about $350,000 or 14%. The changes in the production line are summarized in Table 7.

Clothing item	Old quantity	New quantity
Tailored skirt	8066	10066
Wool blazer	5000	3000
Button-down blouse	9245	6578
Profit	$2,402,291.00	$2,067,843.75

Table 7: Changes in the production line compared to Table 4 when the cost of producing Wool blazers increases by $80.

The drastic shift in the expected profit suggests that the firm should examine the input costs for Wool blazers and other clothing items with greater scrutiny. The cost of uncertainty in this case is considerable, so it pays to investigate this further.

3.3 Increased acetate

Next, we consider the opportunity of an increased supply of acetate. The shadow price for acetate in Table 5 is $72, the single highest for raw materials. This suggests that an increase by 1 yard of the total supply of acetate increases the expected profit by $72. The shadow price indicates that increasing acetate supply ought to be the highest priority for supply concerns. However, we consider the effects of adding 10,000 yards of acetate, which is not so easily captured by the shadow prices.

Clothing item	Old quantity	New quantity
Tailored skirt	8066	14733
Button-down blouse	9245	356
Profit	$2,402,291.00	$3,121,230.75

Table 8: Changes in the production line compared to Table 4 when 10000 more yards of acetate can be obtained.

The changes in the production line are summarized in Table 8. We found that if the supplier can provide an additional 10,000 yards of acetate, profit increases by almost $720,000 or 30%. However, this increase is largely driven by a massive increase in the amount of Tailored skirts produced. As with the Cotton miniskirts, since we do not have a forecasted demand for Tailored skirts, the model allows their production to increase without bound. We do not know how feasible it is to sell almost 15,000 Tailored skirts.

3.4 Outlet stores

Finally, we consider whether or not Fashion Star should continue utilizing their outlet stores. The firm has hired a branding consultant who recommends eliminating outlet stores to "bolster the Fashion Star brand as a high-end clothing line". The firm would consider moving any unsold items during the fall to outlet stores and sell them for 60% of the cost. To this end, we consider three possible discounting regimes:

Under the assumption that the discount is from the basic unit costs of production, selling at 60% of the cost is not profitable, since each discounted item sold would lead to a loss of 40% the input costs.

Under the assumption that the discount is from the price tag of each clothing item, Fashion Star would be selling the items for 60% of the total revenue. In this case, we find that the only item worth selling additional units in outlet stores is the cashmere sweaters. In this case, profit increases by $60,500 or 2.4%. The changes in the production line are summarized in Table 9.

Clothing item	Old quantity	New quantity
Cashmere sweater	4000	6000
Profit	$2,402,291.00	$2,462,852.62

Table 9: Changes in the production line compared to Table 4 if we sell items at the outlet store for 60% of the revenue.

Under the assumption that the discount is the price that yields 60% of the profit for full price items, the total profit increases by about $600,000 or 25%. The changes in the production line are summarized in Table 10.

Clothing item	Old quantity	New quantity
Cashmere sweater	4000	6000
Silk blouse	7000	0
Silk camisole	15000	36000
Velvet shirt	6000	13333
Profit	$2,402,291.00	$3,002,333.33

Table 10: Changes in the production line compared to Table 4 if we sell items at the outlet store for 60% of the profit.

Of these three possible cases, the second interpretation best matches the likely policy implementation Fashion Star would follow. This means that the most likely use of the outlet stores is to sell extra cashmere sweaters.

4 Discussion and recommendations

The expected profit from solving the baseline model is $2.4 million; however, in every scenario that we looked at, the expected profit exceeds $2 million. We found that increasing the acetate supply dramatically increases the expected profit to $3 million, but this relies on selling a large amount of Tailored skirts which may not be feasible. The minimum profit of

$2 million occurred when production costs were significantly underestimated for individual items. In particular, underestimating the labor costs of of wool blazers cuts roughly $350,000 off of overall profit. This is a substantial revision. As such, we highly recommend a more thorough estimation of the true costs. We would also recommend that better forecasting be done for both Cotton miniskirts and Tailored skirts, since both of these are very profitable per yard of material used.

The quantity of Tailored skirts that our solution calls for producing may be infeasible, but this calls into question a greater weakness of the model presented in this work. The model presumes that "supply creates its own demand," meaning that every item produced for the fall line is sold during the season. For those clothing items with demand forecasts provided by Fashion Star's market research team, the model treats the forecast as an upper bound on production (i.e., customers will buy up to the quantities forecasted). For clothing items without such demand forecasts, the model tacitly assumes that customers will purchase a potentially unlimited supply of product. The deviation from reality here is substantial, and one of the consequences may be that the results rely too heavily on Tailored and Cotton skirts.

The model does indicate that there is large potential for growth if the firm can negotiate higher available levels of certain supply materials. Using the shadow price results as a guide, raising the acetate, cotton, and silk levels could drastically increase profit, as could increasing demand for Cashmere sweaters. These are useful directions to pursue if at all possible before the fall season begins, though if the demand critique raised above poses a substantial problem for overestimating profit, these directions would need reconsideration.

Based on the expected increased profit of $60,500 or 2.4% for using outlet stores, it may be worth listening to the branding consultant. As far as velvet goes, although negotiations with the supplier are not yet finalized, good relationships throughout the supply chain are important in the fashion industry. The potential loss of $27,700 may be worth sustaining in order to keep a good relationship with the velvet supplier.