References

[Ber97] Claude Berge. *Topological Spaces: including a treatment of multi-valued functions, vector spaces, and convexity.* Courier Corporation, 1997.

This is a very important book. Consider a function $u : X \times Y \to R$.

[DG18] François Dufour and Alexandre Genadot. On the expected total reward with unbounded returns for markov decision processes. *Applied Mathematics & Optimization*, pages 1–18, 2018.

Dufour and Genadot recall the definition of a strongly coercive function from a paper of E. J. Blder ("Existence without explicit compactness in stochastic dynamic programming"). I will translate this here so that I can compare it directly with the definition of a $\mathbb{K}$-inf-compact function. Recall that u is $\mathbb{K}$-inf-compact if

1. u is lower semi-continuous;
2. whenever $\{(x_n, y_n)\}_{n=1}^{\infty} \in \mathrm{Gr}(\Phi)$ such that $x_n \to x$ and $\sup_{n\in\mathbb{N}} u(x_n, y_n) < \infty$, then there exists a subsequence $(y_{n_k})_{k=1}^{\infty} \subseteq (y_n)_{n=1}^{\infty}$ such that $y_{n_k} \to y \in \Phi(x)$.

According to Dufour and Genadot, u is strongly coercive if whenever $\{(x_n, y_n)\}_{n=1}^{\infty} \subseteq \mathrm{Gr}(\Phi)$ such that $x_n \to x$ and $\liminf_{n\to\infty} u(x_n, y_n) = \ell < \infty$, there exists a subsequence $(y_{n_k})_{k=1}^{\infty} \subseteq (y_n)_{n=1}^{\infty}$ such that $y_{n_k} \to y \in \Phi(x)$ and $u(x_{n_k}, y_{n_k}) \leq \ell$. We claim that these two definitions are equivalent.