Measuring polygonal “niceness”

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[1]AMS 545: Computational Geometry
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May 1, 2018

Preliminaries

- $\lambda(A)$ is the area of a set $A \subseteq \mathbb{R}^2$.
- Let $P \subseteq \mathbb{R}^2$ denote a simple bounded closed polygon.
- ∂P is the boundary of P.
- $[x, y]$ is the closed line segment bounded by $x, y \in \mathbb{R}^2$.

Measures of “niceness”

Definition (α-fatness)

The **$\boldsymbol{\alpha}$-fatness score** is given by

$$\alpha(P) = \inf\{\frac{\lambda(B(x,\rho)\cap P)}{\lambda(B(x,\rho))} : x \in \partial P, \rho > 0, P \not\subseteq B(x,\rho)\}$$

- For every point $x \in \partial P$, sweep through all balls of radius ρ, centered at x, that don't contain P.
- Compute the proportion of area covered by both P and $B(x, \rho)$.
- Find the smallest such proportion. This is $\alpha(P)$.
- It's much easier when the ball is a square!

Measures of “niceness”

Definition (Chord-area)

The **chord-area score** is given by

$$s_\lambda(P) = \inf_{x,y\in\partial P} \max(\lambda(P'), \lambda(P'')),$$

where the chord $[x, y]$ partitions $P = P' \cup P''$.

- This is a “minimax”-esque definition. We want the least bad resulting split.

Implementation

- The measurements were implemented in C++ using CGAL with exact arithmetic kernel.
- We used a δ-boundary discretizing scheme: the length of $[x_k, x_{k+1}]$ is at most $\delta > 0$ for consecutive boundary vertices.
- Generating useful test polygons was tricky.
- We ran our measurements on [THESE CLASSES OF POLYGONS].

Results