1. Project II. Monitoring Whale Populations

The project will have three parts:

- Executive Summary: a one to two page report detailing the conclusions of your modeling written for the client in lay terms.
- Technical Report: a more detailed report fully detailing the model and modeling process for a technical reader.
- Presentation: a 6-8 minute presentation reporting your problem, assumptions, model, analysis, and conclusions to the client.

These three parts are the communication of your work and the grade is based entirely on these three aspects. Each individual team member must also submit a team evaluation form.

Table 1. Teams for Project I

A	B	C	D	E
Harry	Bing	David	Logan	Valerie
Jimin	Evan	Max	Rebelsky	Henry
Rostom	Kaiqian	Andrew	Caleb	Tristan
Royle	Yimin	China		

Background. Ecologist use the following model to represent the growth process of two competing species, x and y:

$$\frac{dx}{dt} = r_1 x(1 - x/K_1) - \alpha_1 xy$$
$$\frac{dy}{dt} = r_2 y(1 - y/K_2) - \alpha_2 xy.$$

The variables x and y represent the number of in each population. The parameters r_i represent the intrinsic growth rates of each species; K_i represents the maximum sustainable population in the absence of competition (known as *carrying capacity*); α_i represents the effect of competition on the population.

The International Whaling Commission (IWC) has studied populations of blue whales and fin whales for several years. These studies have generated the parameter values in Tab. 2 with time t expressed in years.

Table 2. Parameters for the competing species model for blue whales and fin whales.

	Blue	Fin
r	0.05	0.08
K	150,000	400,000
α	10^{-8}	10^{-8}

One objective of the IWC is to set harvesting quotas to properly protect the populations of the whales while enabling the whaling industry to reap an appropriate level of profits. For some time, the value of each whale has been essentially constant with a single blue whale providing $12,000 to the whaler and a fin whale yielding $6,000. For political reasons, the IWC would like to avoid setting quotas and allow the free market to balance the whale populations. However, the IWC has a policy that each whale population must maintain at least half of their environmental carrying capacity. You have been asked to model both of these situations to provide the IWC with the appropriate guidance to determine if they should set quotas for blue and fin whales and if so, how they should set the quota.

Problem. The IWC would like to know the following information.

First, they are interested in the natural populations and would like to know the following in the situation where no commercial whaling was allowed.

- What population levels of blue and fin whales will provide a situation where total whale population grows at the greatest rate each year?
- They have a fair degree of confidence in their research provided in Tab. 2. However, they are interested in knowing how things will change if their research is erroneous.
- They are also confident that the competition parameter α is equal for both species since their primary competition is for krill and both species consume krill at roughly the same pace. If it is true that the competition parameters are equal, will the total growth rate for the whales ever be maximized if one species is driven to extinction?
- A population level is feasible if it is nonnegative and sustainable if the growth rates dx/dt and dy/dt are nonnegative. The IWC would like to know the whale populations that are feasible, sustainable, and what the maximum total population of whales could be.
- Again they are interested in how these values might be affected by their own inaccuracies.
- Assuming still that the competition parameter is equal for both species, how does the maximum total population depend on the strength of competition? Will a particular situation ever lead to the extinction of one species in order to achieve the maximum total whale population?

This previous information will provide the IWC with a basis for making quota decisions. They are interested in balancing the profits of the whaling industry with maintaining a healthy population of whales. The technology in the whaling industry allows them to catch whales at almost any rate they wish. Therefore, the IWC assumes that controlled harvesting can be used to maintain the whale populations at any desired level. In other words, once the populations reach the desired levels, the populations can be held constant by harvesting at a rate equal to the growth rate. Assuming the IWC does not impose any quotas and that harvesting is unregulated, answer the following questions for the IWC.

- What population levels will produce maximum profits for the whaling industry?
- Include a complete sensitivity analysis.
- Will it ever be economically optimal to drive one species of whales to extinction (again assuming $\alpha_1 = \alpha_2$)?
- If Tab. 2 is accurate, what percentage of the carrying capacities for each population will be maintained?

The IWC is committed to its minimum whale population policy. In order to set quotas, they must determine the economic impact of their quotas on the whaling industry.

- What population levels will produce a maximum sustained profit for the whaling industry if the IWC minimum whale population policy is enforced?
- Determine the quotas for each species of whale to enforce the IWC policy.
- Examine the optimal population levels and the sustained profit to the constraints.
- The IWC anticipates that any quota will be met by a petition in international court for a relaxation of the quota. Analyze the potential effects of increased quotas on the yearly revenue for the whaling industry and on the whale populations.

Prepare a one to two page executive summary for the IWC administrators and a technical report for the IWC scientists. Also, you will have approximately seven minutes to present the most important information to the next IWC Congress.

Credit: This is a series of problems from Chapter 2 of *Mathematical Modeling* by M. Meerschaert, First Edition, 1993.