Classical Autonomous Systems – Autumn 2021
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Exercise Session 4

Figure 1: Longitudinal dynamics of a helicopter

1. Consider the longitudinal dynamic model of a helicopter, given by the linear state-space representation

$$\frac{d}{dt}\begin{bmatrix} q \\ \theta \\ u \end{bmatrix} = \begin{bmatrix} -0.4 & 0 & -0.01 \\ 1 & 0 & 0 \\ -1.4 & 9.8 & -0.02 \end{bmatrix}\begin{bmatrix} q \\ \theta \\ u \end{bmatrix} + \begin{bmatrix} 6.3 \\ 0 \\ 9.8 \end{bmatrix}\delta \qquad (1)$$

where $u(t)$ is the longitudinal velocity of the helicopter expressed in the earth-fixed frame, $\theta(t)$ is the pitch angle and $q(t)$ the pitch angular velocity, while $\delta(t)$ is the control input representing the angle of the rotor thrust with respect to the helicopter (see Figure 1). Open a new Simulink file, and create a subsystem where your plant will be.

2. In a Matlab file, define the matrices corresponding to the linear state-space representation (1). Check whether this system is stable.

3. In the plant/helicopter subsystem, use a state-space block to implement your system (sampling time for your simulation: $T_s = 0.01s$).

4. Implement a state-feedback controller that will stabilize the helicopter around the origin. Use first the command `place` to tune your controller. Check that it works for different initial conditions.

5. Change your controller into a Linear Quadratic Regulator and tune it using the command `lqr`.

6. Add a feedforward gain so that the helicopter stabilizes around the desired velocity of 10 m/s.

7. Re-implement the previous controller digitally: first discretize the plant using the `c2d` command with sampling time $10T_s$. Second, obtain the discrete-time controller gain using again the `lqr` command.

8. Implement your discrete-time controller in Simulink. In your implementation, include zero-order hold blocks to represent the effect of the discretizing/sampling continuous-time signals.