Classical Autonomous Systems – Autumn 2021
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Exercise Session 2

1. **Lorenz system.** Consider the Lorenz system described by

$$\begin{cases} \dot{x} = \sigma(y - x) \\ \dot{y} = x(\rho - z) - y \\ \dot{z} = xy - \beta z \end{cases} \tag{1}$$

with $\rho = 28$, $\sigma = 10$, $\beta = 8/3$.

1.1. Implement this system in Simulink using a "MATLAB Function" block and a single integrator (use non-zero initial conditions). Use block "XY Graph" to exhibit the chaotic behavior of the system.

1.2. Using the block "To Workspace" and the command "plot3" in Matlab, represent/plot the so-called Lorenz attractor.

1.3. Discretize this system and implement the discrete version in Simulink using a single unit delay block.

2. **ODE to state-space conversion.** Consider the following differential equation

$$\dddot{y} - 4\ddot{y} + 7\dot{y} - 2y = 3u \tag{2}$$

2.1. Give a state-space representation of (2).

2.2. Calculate the transfer function corresponding to (2) (recall that $d^k y(t)/dt^k$ in the time domain gives $s^k Y(s)$ in the Laplace one).

2.3. In Matlab, use the function "tf2ss" to obtain a state-space representation of (2). Comment?

3. **Linearization of a controlled pendulum**. Consider the actuated pendulum (robot with one degree of freedom)

$$ml^2\ddot{\theta} + d\dot{\theta} + mgl\sin\theta = u \tag{3}$$

where $m = 50$, $l = 1$, $g = 9.8$ and $d = 0.1$.

3.1. Implement this nonlinear system in Matlab/Simulink.

3.2. Compute the value of u^* to maintain the system in the equilibrium $\mathbf{x}^* = \begin{bmatrix} \pi/4 \\ 0 \end{bmatrix}$. Apply this input to the model you programmed in the previous question.

3.3. Compute a linear approximation of the actuated pendulum around $\mathbf{x}^*$ and implement it in your file next to the original nonlinear system.

3.4. compare the response of both linear and nonlinear systems for different initial conditions (hint: the comparison should be fair...).