

*All codes must be commented and justified*  
*You get points if your solution is correct*  
*In order to have all the points, you must respect the complexity specifications*

1. Implement a data structure supporting the following operation:

- **bool** `empty()` : returns 1 if there is no element stored in the structure
- **void** `add(int v)` : adds a new element equal to v in the structure
- **int** `next()` : returns and removes from the structure the lower median (i.e., if there are  $n = 2m$  elements stored, exactly  $m-1$  elements should be lower than the output; if there are  $n=2m+1$  elements, exactly  $m$  should be lower than the output).

All the operations should run in  $O(\log(n))$  time /5

2. Consider the following implementation of a binary search tree:

```
typedef struct BinaryTree {  
    int value;  
    struct BinaryTree *tata;  
    struct BinaryTree *stinga;  
    struct BinaryTree *dreapta;  
} BinaryTree;
```

a) Write a function `vector<int> sumOfSubtree(BinaryTree *T)` which computes, for each node of T, the sum of all the values stored in its subtree. The running time should be in  $O(n)$ . /3

b) Write a function `int sumOfInterval(BinaryTree *T, int x, int y, vector<int>& sum)` which outputs the sum of all the values stored in T between x and y. Here, the vector sum is the one computed at the previous question. The running time should be in  $O(\text{height}(T))$ . /2