

All codes must be justified (either in a separate file or in comments). Your program must be written in C++ (Python is also allowed). Please ONLY send the .h and .cpp files (no executable)!

1) Implement a data structure supporting the following three operations: /5

- a. `void init(const vector<int>& v, int s)`: initialize the data structure content with a fixed n-size vector and an integer s.

Complexity: $O(n \cdot \log(n))$.

- b. `int set(int i)`: modifies the *i*th bit (if it exists) in the binary representation of s.

Complexity: $O(1)$.

- c. `int count()`: returns the number of elements in vector v that are smaller than s.

Complexity: $O(\log(n))$.

2) In what follows, a u-vector is a vector whose all entries are pairwise different.

- a. Write a function `bool isUVector(const vector<int>& v)` that determines whether a vector is a u-vector.

Complexity: $O(n)$. /1

- b. Write a function `bool isInOrder(const vector<int>& v)` that being given a u-vector v, determines whether it is the inorder of some balanced binary search tree.

Complexity: $O(n)$. /1

- c. Write a function `bool isPreOrder(const vector<int>& v)` that being given a u-vector v, determines whether it is the preorder of some binary search tree.

Complexity: $O(n)$. /2

-- Simpler variant: $O(n \cdot \log(n))$. /1

- d. Modify the previous question so that, in the case that v is indeed a preorder, then we also sort this vector.

Complexity: $O(n)$. /1