T6 - Seminar

① $\mathcal{D}: \frac{x_1-1}{4} = \frac{x_2-2}{1} = \frac{x_3}{6}$, $M(1,1,1)$.

a) ec. π, $\mathcal{D} \in \pi$ și M

b) ec. $\mathcal{D}'$ a.î. $M \in \mathcal{D}'$, $\mathcal{D} \parallel \mathcal{D}'$

c) dist $(M, \mathcal{D})$

---

a) $\mathcal{D}: \begin{cases} x_1 = 4t+1 \\ x_2 = t+2 \\ x_3 = 6t \end{cases} \quad t \in \mathbb{R}$

Alegem $A \in \mathcal{D}$, $A(1,2,0)$.

$\langle N, \overrightarrow{AM} \rangle = 0 \Rightarrow$

$\Rightarrow x_1 \cdot 0 + (-1)x_2 + x_3 = 0$

$\pi: -x_2 + x_3 = 0$

b) $\mathcal{D} \parallel \mathcal{D}' \Rightarrow u_{\mathcal{D}'} = u_{\mathcal{D}} = (4,1,6)$

$M \in \mathcal{D}' \Rightarrow \mathcal{D}': \frac{x_1-1}{4} = \frac{x_2-1}{1} = \frac{x_3-1}{6} = t$

$\mathcal{D}': \begin{cases} x_1 = 4t+1 \\ x_2 = t+1 \\ x_3 = 6t+1 \end{cases} \quad t \in \mathbb{R}$

c) $d = \text{dist}(M, \mathcal{D}) = \frac{\|\overrightarrow{MA} \times \overrightarrow{MB}\|}{\|\overrightarrow{AB}\|}$, $A(1,2,0) \in \mathcal{D}$, $B(5,3,6) \in \mathcal{D}$

$\overrightarrow{AB} = (4,1,6)$
$\overrightarrow{MA} = (0,1,-1)$
$\overrightarrow{MB} = (4,2,5)$ $\Rightarrow \overrightarrow{MA} \times \overrightarrow{MB} = \begin{vmatrix} e_1 & e_2 & e_3 \\ 0 & 1 & -1 \\ 4 & 2 & 5 \end{vmatrix} = 7e_1 - 4e_2 - 4e_3 = (7,-4,-4)$

$d = \frac{\sqrt{16+1+36}}{\sqrt{49+2 \cdot 16}} = \frac{\sqrt{53}}{\sqrt{81}} = \frac{\sqrt{53}}{9}$

(2) $\Gamma: f(x) = x_1^2 + x_1 x_2 + x_2^2 - 6x_1 - 16 = 0$

$A = \begin{pmatrix} 1 & 1/2 \\ 1/2 & 1 \end{pmatrix}$ $\delta = \det A = 1 - \frac{1}{4} = \frac{3}{4} > 0 \Rightarrow$ Centru unic

$\begin{cases} 2x_1 + x_2 - 6 = 0 \\ 2x_2 + x_1 = 0 \end{cases} \Rightarrow 3x_2 + 6 = 0 \Rightarrow x_2 = -2 \Rightarrow x_1 = +4$

$P_0(4,2)$ centru unic.

$\tilde{A} = \begin{pmatrix} 1 & 1/2 & -3 \\ 1/2 & 1 & 0 \\ -3 & 0 & -16 \end{pmatrix}$ $\Delta = \det \tilde{A} = -16 - 9 + 4 = -21 \neq 0$

$\theta: x = x' + x_0$

$\theta(\Gamma): x_1'^2 + x_1' x_2' + x_2'^2 + \frac{\Delta}{\delta} = 0$ $\left(\frac{\Delta}{\delta} = -28\right)$

$Q: \mathbb{R}^2 \to \mathbb{R}, Q(x) = x_1'^2 + x_2'^2 + x_1 x_2$

$P(\lambda) = \det(A - \lambda I_2) = \begin{vmatrix} 1-\lambda & 1/2 \\ 1/2 & 1-\lambda \end{vmatrix} = 1 - 2\lambda + \lambda^2 - \frac{1}{4} =$

$= \lambda^2 - 2\lambda + \frac{3}{4} = 0 \Rightarrow \lambda_1 = \frac{3}{2}, \lambda_2 = \frac{1}{2}$.

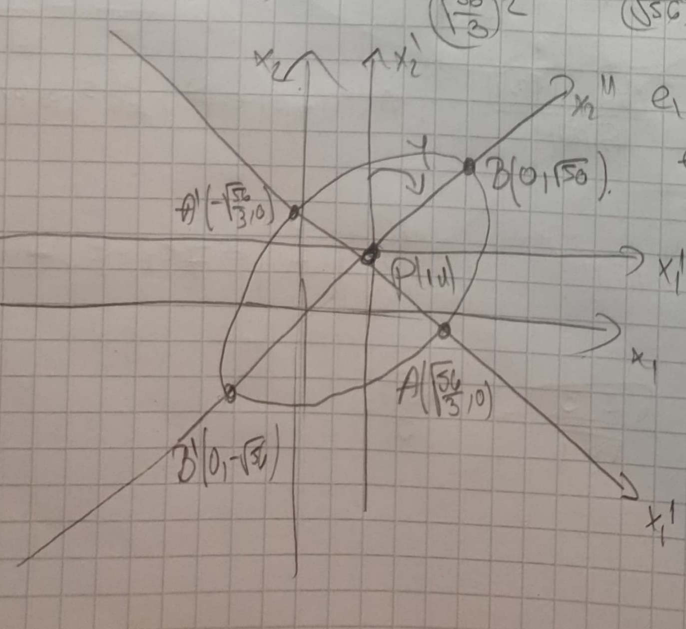
$V_{\lambda_1} = \{x \in \mathbb{R}^2 \mid Ax' = \frac{3}{2}x'\} \Rightarrow A - \frac{3}{2}I_2 = O_3 \Rightarrow V_{\lambda_1} = \langle\{\frac{1}{\sqrt{2}}(1,1)\}\rangle$

$V_{\lambda_2} = \{x \in \mathbb{R}^2 \mid Ax' = \frac{1}{2}x'\} \Rightarrow A - \frac{1}{2}I_2 = O_3 \Rightarrow V_{\lambda_2} = \langle\{\frac{1}{\sqrt{2}}(-1,1)\}\rangle$

Fie τ rotație, $\tau: x' = Rx''$, $R = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \in SO(2)$

$\tau(\theta(\Gamma)): \frac{3}{2}x_1''^2 + \frac{1}{2}x_2''^2 - 28 = 0$.

Avem o Elipsă, $\mathcal{E}: \frac{x_1''^2}{\left(\sqrt{\frac{56}{3}}\right)^2} + \frac{x_2''^2}{(\sqrt{56})^2} = 1$.



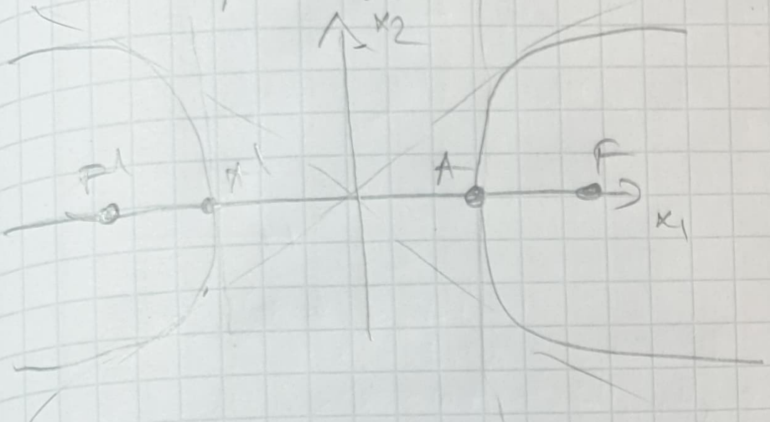
$e_1 = \frac{1}{\sqrt{2}}(1,1)$

$e_2 = \frac{1}{\sqrt{2}}(-1,1)$

(3) $\mathcal{H}: 16x^2 - 25y^2 = 400$

vf, focare, dist foc, ec. as., direct.

$\mathcal{H}: \frac{x^2}{25} - \frac{y^2}{16} = 1$. $a = 5$, $b = 4$. $\Rightarrow c = \sqrt{16+25} = \sqrt{41}$.



(1) Focare: $F(\sqrt{41}, 0)$, $F'(-\sqrt{41}, 0)$

(2) Vârfuri: $A(5, 0)$; $A'(-5, 0)$

(3) Ec. directoare: $d \cup d'$: $x_1 = \pm \frac{25\sqrt{41}}{41}$.

(4) Ec. asimptote: $d_1 \cup d_2$: $x_2 = \pm \frac{4}{5} x_1$

(5) Distanța focară: $FF' = 2c = 2\sqrt{41}$.