o Conice fără centru unic

$\Gamma : f(x) = 3x_1^2 - 6x_1x_2 + 3x_2^2 + 2x_1 + 2x_2 - 2 = 0$

$A = \begin{pmatrix} a_{11} & a_{12} \\ a_{12} & a_{22} \end{pmatrix} = \begin{pmatrix} 3 & -3 \\ -3 & 3 \end{pmatrix}$, $\det A = 0 = \delta$ (nu are centru unic).

$\tilde{A} = \begin{pmatrix} A & B^T \\ B & C \end{pmatrix} = \begin{pmatrix} 3 & -3 & 1 \\ -3 & 3 & 1 \\ 1 & 1 & -2 \end{pmatrix}$, $\det \tilde{A} = \Delta = -12 \Rightarrow$ nedegenerată

$\mathcal{R} = \{O; e_1, e_2\} \xrightarrow[\text{rotație}]{\theta} \mathcal{R}\{O; e_1', e_2'\} \xrightarrow[\text{transl.}]{\tau} \mathcal{R}\{P; e_1', e_2'\}$

$Q : \mathbb{R}^2 \to \mathbb{R}$, $Q(x) = 3x_1^2 - 6x_1x_2 + 3x_2^2$

$\det(A - \lambda I_2) = 0 \Rightarrow \lambda^2 - \operatorname{tr} A\, \lambda + \det A = 0 \Rightarrow \lambda^2 - 6\lambda = 0$. $\lambda_1 = 0$, $\lambda_2 = 6$

$Q(x) = 6x_1'^2$

Fie rotația $\theta : x = Rx'$

$A(0 - I_2)x = 0 \Rightarrow x_1 = -x_2 \Rightarrow V_{\lambda_1} = \langle\{\frac{1}{\sqrt{2}}(1, -1)\}\rangle$,

analog $\Rightarrow V_{\lambda_2} = \langle\{\frac{1}{\sqrt{2}}(1, 1)\}\rangle$,

$\Gamma : f(x) = 3x_1^2 - 6x_1x_2 + 3x_2^2 + 2x_1 + 2x_2 - 2 = 0$

$R = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$. $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} x_1' \\ x_2' \end{pmatrix} \Rightarrow \begin{cases} x_1 = \frac{1}{\sqrt{2}}(x_1' + x_2') \\ x_2 = \frac{1}{\sqrt{2}}(-x_1' + x_2') \end{cases}$

$\theta(\Gamma) : 6x_1'^2 + \sqrt{2}(x_1' + x_2') + \sqrt{2}(-x_1' + x_2') - 2 = 0$

$3x_1'^2 + \sqrt{2}x_2' - 1 = \sqrt{2}(x_2' - \frac{1}{\sqrt{2}}) = 0 \Rightarrow 3x_1'^2 + \sqrt{2}(x_2' - \frac{1}{\sqrt{2}}) = 0$.

$\begin{cases} x_1'' = x_1' \\ x_2'' = x_2' - \frac{1}{\sqrt{2}} \end{cases}$

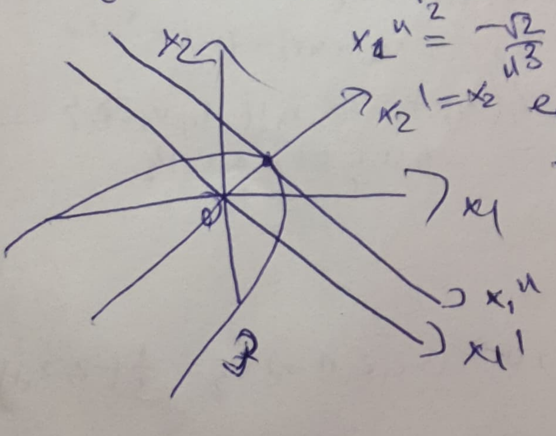
Fie τ translația $\tau : x' = x'' + x_0$, $x_0 = \begin{pmatrix} 0 \\ 1/\sqrt{2} \end{pmatrix}$.

$\left.\begin{aligned} \theta &: x = Rx' \\ \tau &: x' = x'' + x_0 \end{aligned}\right| \Rightarrow x = Rx'' + Rx_0$.

$Rx_0 = \frac{1}{2}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ $P(\frac{1}{2}, \frac{1}{2})$ în rap. cu $\mathcal{R}$.

$\tau \circ \theta(\Gamma) : 3x_1''^2 + \sqrt{2}x_2'' = 0$.

$x_1''^2 = -\frac{\sqrt{2}}{3}x_2''$



$e_1' = \frac{1}{\sqrt{2}}(1, -1)$

$e_2' = \frac{1}{\sqrt{2}}(1, 1)$. $\mathcal{P} : x_1^2 = 2px_2$.

$x_1''^2 = -\frac{\sqrt{2}}{3}x_2'' \Rightarrow$ pe axa x_2 invers

el conice

$\Delta \neq 0$ (nedegenerat) $-\ \delta > 0$ Elipsă, $\emptyset$
$-\ \delta < 0$ Hiperbolă
$-\ \delta = 0$ Parabolă

$\Delta = 0$ (! nedegenerat) $-\ \delta > 0$ Pct. dublu
$-\ \delta < 0$ Drepte concurente
$-\ \delta = 0$ —"— confundate, $\amalg$, $\emptyset$.

Invarianți
- afini: $\frac{\Delta}{\delta}, r', r$
- metrici: $\frac{\Delta}{\delta}, r', r, \Delta, \delta$

Tangente: $M_0(x_1^0, x_2^0) \in \mathcal{E}$. $tg:(M_0)$ $\frac{x_1 x_1^0}{a^2} + \frac{x_2 x_2^0}{b^2} = 1$.
$M_0(x_1^0, x_2^0) \in \mathcal{H}$. $tg:(M_0)$ $\frac{x_1 x_1^0}{a^2} - \frac{x_2 x_2^0}{b^2} = 1$.
$M_0(x_1^0, x_2^0) \in \mathcal{P}$ $tg:(M_0)$ $x_2 x_2^0 = p(x_1 + x_1^0)$.

Elipsa: $c^2 = a^2 - b^2$, $e = \frac{c}{a}$ $PF + PF' = 2a$, $F(c,0)$. $A(a,0)$, $B(0,b)$
$d \cup d': x_1 = \pm a^2/c$

Hiperbola: $c^2 = a^2 + b^2$ $e = \frac{c}{a}$. $|PF - PF'| = 2a$, $F(c,0)$, $A(a,0)$.
$d \cup d' = \pm \frac{a^2}{c}$.

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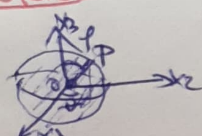
⑪ Cuadrice

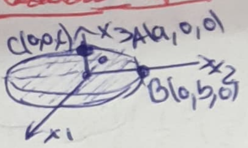
1. Sfera N



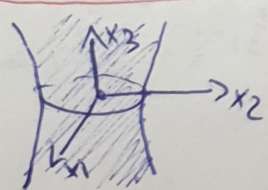
$\Sigma_1(A(a,b,c), R): (x_1-a)^2 + (x_2-b)^2 + (x_3-c)^2 - R^2 = 0$, $a,b,c>0$.
$\mathcal{E}_2: \frac{x_1^2}{a^2} + \frac{x_2^2}{b^2} + \frac{x_3^2}{c^2} = 1$, $a,b,c>0$
$\mathcal{H}_3: \frac{x_1^2}{a^2} + \frac{x_2^2}{b^2} - \frac{x_3^2}{c^2} = 1$, $a,b,c>0$.
$\mathcal{H}_4: -\frac{x_1^2}{a^2} - \frac{x_2^2}{b^2} + \frac{x_3^2}{c^2} = 1$, $a,b,c>0$.
$P_{e_5}: \frac{x_1^2}{a^2} + \frac{x_2^2}{b^2} = 2x_3$, $a>0$, $b>0$.
$P_{h_6}: \frac{x_1^2}{a^2} - \frac{x_2^2}{b^2} = 2x_3$, $a>0, b>0$
$C_{e_7}: \frac{x_1^2}{a^2} + \frac{x_2^2}{b^2} = 1$, $a,b>0$, $x_3 = 0$ $(d \parallel Ox_3)$
$C_{h_8}: \frac{x_1^2}{a^2} - \frac{x_2^2}{b^2} = 1$, $a,b>0$, $x_3 = 0$ $(d \parallel Ox_3)$
$C_{p_9}: x_2^2 = 2px_1$, $p>0$
$d \parallel Ox_3$: $C: \begin{cases} x_2^2 = 2px_1 \\ x_3 = 0 \end{cases}$

$Con_{10}: \frac{x_1^2}{a^2} + \frac{x_2^2}{b^2} - \frac{x_3^2}{c^2} = 0$, $a,b,c>0$

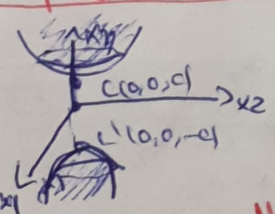
2. ~~Cuadrice~~ Elipsoid N



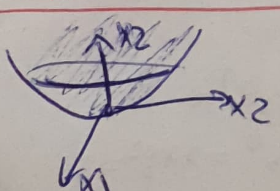
3. Hiperboloidul cu o pânză N



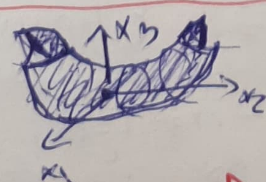
4. Hiperboloidul cu 2 pânze N



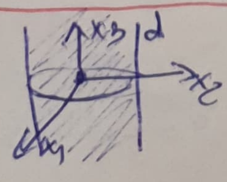
5. Paraboloid eliptic N



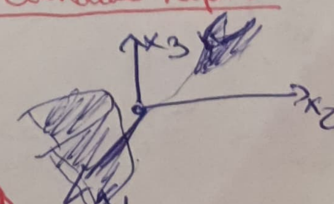
6. Paraboloid hiperbolic N.



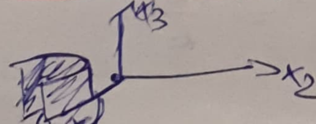
7. Cilindrul eliptic D



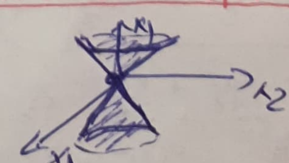
8. Cilindrul hiperbolic D



9. Cilindrul parabolic D



10. ~~Paraboloid~~ Conul pătratic D.



Generatoare H. $G_1: d_\lambda: \begin{cases} \frac{x_1}{a} - \frac{x_3}{c} = \lambda\left(1 - \frac{x_2}{b}\right) \\ \lambda\left(\frac{x_1}{a} - \frac{x_3}{c}\right) = 1 + \frac{x_2}{b} \end{cases}$ $d_\infty: \begin{cases} 1 - \frac{x_2}{b} = 0 \\ \frac{x_1}{a} + \frac{x_3}{b} = 0 \end{cases}$

* C. riglate = cilindru, Pn.

$G_2: \overline{d_\mu}: \begin{cases} \frac{x_1}{a} - \frac{x_3}{c} = \mu\left(1 + \frac{x_2}{b}\right) \\ \mu\left(\frac{x_1}{a} + \frac{x_3}{c}\right) = 1 - \frac{x_2}{c} \end{cases}$ $\overline{d_\infty}: \begin{cases} 1 + \frac{x_2}{b} = 0 \\ \frac{x_1}{a} + \frac{x_3}{c} = 0 \end{cases}$

$\left(a\frac{\lambda\mu+1}{\lambda+\mu}, b\frac{\lambda-\mu}{\lambda+\mu}, c\frac{1-\lambda\mu}{\lambda+\mu}\right) = M.$ $\quad \mu, \lambda \in \mathbb{R}^*$

Generatoare Pn

$G_1: d_\lambda: \begin{cases} \frac{x_1}{a} + \frac{x_2}{b} = \lambda x_3 \\ \lambda\left(\frac{x_1}{a} - \frac{x_2}{b}\right) = 2 \end{cases}$ $d_\infty: \begin{cases} x_3 = 0 \\ \frac{x_1}{a} - \frac{x_2}{b} = 0 \end{cases}$

$G_2: \overline{d_\mu}: \begin{cases} \frac{x_1}{a} - \frac{x_2}{b} = \mu x_3 \\ \mu\left(\frac{x_1}{a} + \frac{x_2}{b}\right) = 2 \end{cases}$ $\overline{d_\infty}: \begin{cases} x_3 = 0 \\ \frac{x_1}{a} + \frac{x_2}{b} = 0 \end{cases}$

$P\left(a\frac{\lambda+\mu}{\lambda\mu}, b\frac{\lambda-\mu}{\lambda\mu}, \frac{2}{\lambda\mu}\right), d_\lambda \cap \overline{d_\mu} \Rightarrow x_3 = \frac{2}{\lambda\mu}.$

(Ex1) $\Gamma: 5x_1^2 + 6x_2^2 + 7x_3^2 - 4x_1x_2 + 4x_2x_3 - 10x_1 - 8x_2 + 14x_3 - 6 = 0$

$A = \begin{pmatrix} 5 & -2 & 0 \\ -2 & 6 & 2 \\ 0 & 2 & 7 \end{pmatrix}$ $\det A = \delta = 2\cdot 81 \neq 0 \Rightarrow$ C. unic. $\tilde{A} = \begin{pmatrix} 5 & -2 & 0 & -5 \\ -2 & 6 & 2 & -4 \\ 0 & 2 & 7 & 7 \\ -5 & -4 & 7 & -6 \end{pmatrix}$ $\det \tilde{A} = \Delta = -36\cdot 81$ $\Rightarrow$ C. Ndg.

$\begin{cases} 10x_1 - 4x_2 - 10 = 0 \\ 12x_2 - 4x_1 + 4x_3 - 8 = 0 \\ 14x_3 + 4x_2 + 14 = 0 \end{cases} \Rightarrow P_0(1, 0, -1).$

Fie $\theta: X = X' + X_0, \quad X_0 = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$

$\theta(\Gamma) = 5x_1'^2 + 6x_2'^2 + 7x_3'^2 - 4x_1'x_2' + 4x_2'x_3' + \frac{\Delta}{\delta} = 0.$ $\quad \frac{\Delta}{\delta} = -18$

$P(\lambda) = \det(A - \lambda I_3) \Rightarrow \lambda_1 = 3 \Rightarrow V_{\lambda_1} = \langle\{(\frac{2}{3}, \frac{2}{3}, -\frac{1}{3})\}\rangle$

$\lambda_2 = 6 \Rightarrow V_{\lambda_2} = \langle\{(\frac{2}{3}, -\frac{1}{3}, \frac{2}{3})\}\rangle$

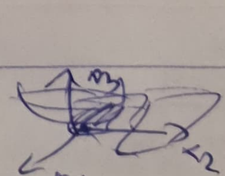
$\lambda_3 = 9 \Rightarrow V_{\lambda_3} = \langle\{(\frac{1}{3}, -\frac{2}{3}, -\frac{2}{3})\}\rangle$

$R = \begin{bmatrix} \frac{2}{3} & \frac{2}{3} & \frac{1}{3} \\ \frac{2}{3} & -\frac{1}{3} & -\frac{2}{3} \\ -\frac{1}{3} & \frac{2}{3} & -\frac{2}{3} \end{bmatrix}.$

$\tau(\theta(\Gamma)): 3x_1''^2 + 6x_2''^2 + 9x_3''^2 - 18 = 0.$

$\frac{x_1''^2}{6} + \frac{x_2''^2}{3} + \frac{x_3''^2}{2} = 1 \Rightarrow$ Elipsoid.

(Ex2) $\mathcal{P}_e: \frac{x_1^2}{12} + \frac{x_2^2}{4} = x_3, \quad \pi: x_1 - x_2 - 2x_3 - 1 = 0.$



$\pi': \frac{x_1 x_1^0}{6} + \frac{x_2 \cdot x_2^0}{2} = x_3 + x_3^0 \quad (\mathcal{P}_e: \frac{x_1^2}{6} + \frac{x_2^2}{2} = 2x_3)$

$N_\pi = (1, -1, -2)$

$N_{\pi'} = \left(\frac{x_1^0}{6}, \frac{x_2^0}{2}, -1\right)$

$\frac{\frac{x_1^0}{6}}{1} = \frac{\frac{x_2^0}{2}}{-1} = \frac{-1}{-2} = \frac{1}{2} \Rightarrow x_1^0 = 3, x_2^0 = -1. \rightarrow P_0(3, -1, x_3^0) \in \mathcal{P}_e$

$\frac{9}{12} + \frac{1}{4} = x_3^0 \Rightarrow x_3^0 = 1.$

$\text{dist}(P_0, \pi) = \frac{|3 + 1 - 2 - 1|}{\sqrt{1 + 1 + 4}} = \frac{\sqrt{6}}{6}.$