T5 - Curs

① $(\mathbb{R}^3, g_0)$ s.v.e., str. canonică

$f: \mathbb{R}^3 \to \mathbb{R}^3$, $f(x) = (x_3, x_2, x_1)$

a) f tr. ort. de sp. 2.

b) reper ortonormat $R = \{e_1, e_2, e_3\}$ ai

$$[f]_{RR} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & \cos\varphi & -\sin\varphi \\ 0 & \sin\varphi & \cos\varphi \end{pmatrix}$$

a) $A = [f]_{R_0R_0} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$.

$AA^T = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = I_3$ $\Big| \Rightarrow f \in O(\mathbb{R}^3)$ de speța 2

$\det A = -1$

b) fr $A = 1 \Rightarrow \varphi = 0$.

$f(x) = -x$.

$\begin{cases} x_3 = -x_1 \\ x_2 = -x_2 \\ x_1 = -x_3 \end{cases} \Leftrightarrow \begin{cases} x_1 + x_3 = 0 \\ x_2 = 0. \end{cases}$

$\Rightarrow (x_1, x_2, x_3) \in <\{(\underbrace{1, 0, -1}_{u})\}>$

e_1	e_2	e_3
1	0	-1
1	0	1

$= -e_2 \quad (2) = (0, -2, 0)$

$<\{u\}>^\perp = \{x \in \mathbb{R}^3 \mid g(x, u) = 0\} = <\{\underbrace{(1, 0, 1)}_{f_2}, \underbrace{(0, 1, 0)}_{f_3}\}>$

$x_1 - x_3 = 0 \Rightarrow x_1 = x_3$

$\{f_2, f_3\}$ reper în $<\{u\}>^\perp$

Aplicăm Gram Schmidt.

$e_2' = f_2 = (1, 0, 1)$ $\qquad e_2 = \frac{1}{\sqrt{2}}(1, 0, 1)$.

$e_3' = f_3 - \frac{g_0(f_3, e_2')}{g_0(e_2', e_2')} e_2' = f_3 = (0, 1, 0)$ $\qquad e_3 = (0, 1, 0)$.

$g_0(f_3, e_2') = 0$

$g_0(e_2', e_2') = 2$

$R_0 = \{e_1^0, e_2^0, e_3^0\} \xrightarrow{C} R = \{e_1, e_2, e_3\}$ sch. de repere ortonormat

$$C = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \end{pmatrix} \qquad \det C = -\frac{1}{2} - \frac{1}{2} = -1$$

(nu sunt la fel orientate)

$$[f]_{RR} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}.$$

② $V = \{A \in M_2(\mathbb{R}) \mid A = A^T\}$, $g : V \times V \to \mathbb{R}$, $g(A, B) = Tr(AB)$

(V, g) sp. vect. eucl.

$A \in V \Rightarrow A = \begin{pmatrix} x_1 & x_2 \\ x_2 & x_3 \end{pmatrix}$, $x_1, x_2, x_3 \in \mathbb{R}$

$B \in V \Rightarrow B = \begin{pmatrix} y_1 & y_2 \\ y_2 & y_3 \end{pmatrix}$, $y_1, y_2, y_3 \in \mathbb{R}$

$$g(A,B) = Tr\begin{pmatrix} x_1y_1 + x_2y_2 & x_1y_2 + x_2y_3 \\ x_2y_1 + x_3y_2 & x_2y_2 + x_3y_3 \end{pmatrix} = x_1y_1 + 2x_2y_2 + x_3y_3$$

$$G = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$G = G^T \Rightarrow g \in L^s(V, V; \mathbb{R})$

$Q : \mathbb{R}^3 \to \mathbb{R}$, $Q(x) = x_1^2 + 2x_2^2 + x_3^2$

signatura $(3, 0) \Rightarrow Q$ poz definită