Th - Curs

① $f:\mathbb{R}^3 \to \mathbb{R}$ endo, $A = \begin{pmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{pmatrix}$, $A = [f]_{R_0, R_0}$

a) A se poate diagonaliza

b) Reperul pt. diagonaliza

a) $P(\lambda) = \det(A - \lambda I_3) = \begin{vmatrix} 1-\lambda & -3 & 3 \\ 3 & -5-\lambda & 3 \\ 6 & -6 & 4-\lambda \end{vmatrix} = 0$

$\Rightarrow -\lambda^3 + 12\lambda + 16 = 0 \Rightarrow -(\lambda+2)^2(\lambda-4) = 0$

$\lambda_1 = -2, \; m_1 = 2$
$\lambda_2 = 4, \; m_2 = 1$

$V_{\lambda_1} = \{ x \in \mathbb{R}^3 \mid f(x) = \lambda_1 x = -2x \}$

$rg \begin{vmatrix} 3 & -3 & 3 \\ 3 & -3 & 3 \\ 6 & -6 & 6 \end{vmatrix} = 1 \Rightarrow \dim V_{\lambda_1} = 2 = m_1$ ①

$V_{\lambda_2} = \{x \in \mathbb{R}^3 \mid f(x) = \lambda_2 x = 4x\}$

$rg\begin{pmatrix} -3 & -3 & 3 \\ 3 & -9 & 3 \\ 6 & -6 & 0 \end{pmatrix} = 2 \Rightarrow \dim V_{\lambda_2} = 1 = m_2$ (2)

(1), (2) $\Rightarrow$ f diagonalizabilă

b) $3x_1 - 3x_2 + 3x_3 = 0 \Rightarrow x_1 = x_2 - x_3$

$V_{\lambda_1} = <\{(1,1,0), (-1,0,1)\}>$, $R_1 = \{(1,1,0), (-1,0,1)\}$ reper în V_{λ_1}

$\begin{cases} 6x_1 = 6x_2 \\ 3x_1 + 3x_3 = 9x_2 \end{cases} \Leftrightarrow \begin{cases} x_1 = x_2 \\ x_3 = 2x_2 \end{cases}$

$V_{\lambda_2} = <\{(1,1,2)\}>$ $R_2 = \{(1,1,2)\}$ reper în V_{λ_2}

$R = \{(1,1,0), (-1,0,1), (1,1,2)\} = R_1 \cup R_2$

$A' = [f]_{R,R} = \begin{pmatrix} -2 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 4 \end{pmatrix}$

(2) $Q: \mathbb{R}^3 \to \mathbb{R}$, $Q(x) = 5x_1^2 + 6x_2^2 + 4x_3^2 - 4x_1x_2 - 4x_1x_3$

a) f. polară, kerg, nedegenerată

b) Q - f. canonică, poz. definită

a) $g(x,y) = 5x_1y_1 - 2x_1y_2 - 2x_1y_3 - 2x_2y_1 + 6x_2y_2 - 2x_3y_1 + 4x_3y_3$

$\det\begin{pmatrix} 5 & -2 & -2 \\ -2 & 6 & 0 \\ -2 & 0 & 4 \end{pmatrix} = 80 \neq 0 \Rightarrow g$ nedegenerată $\Rightarrow \ker g = \{0_{\mathbb{R}^3}\}$

b) Metoda Jacobi

$\Delta_1 = 5 \neq 0$ $\quad \Rightarrow Q$ pozitiv definită $(\Delta_1, \Delta_2, \Delta_3 > 0)$

$\Delta_2 = 26 \neq 0$

$\Delta_3 = 80 \neq 0$.

$\Rightarrow \exists R'$ ai $Q' = \frac{1}{5}x_1'^2 + \frac{5}{26}x_2'^2 + \frac{13}{40}x_3'^2$

$(3,0)$ - signatura.