T6 - Curs

① $\mathcal{D}_1: \frac{x_1-1}{1} = \frac{x_2-3}{1} = \frac{x_3-2}{0}$

$\mathcal{D}_2: \frac{x_1-1}{3} = \frac{x_2}{0} = \frac{x_3-1}{2}$

a) $\mathcal{D}_1, \mathcal{D}_2$ necopl

b) ec. ⊥ comune, $dist(\mathcal{D}_1, \mathcal{D}_2)$

a) $A \in \mathcal{D}_1 \Rightarrow A(1, 3, 2)$

$B \in \mathcal{D}_2 \Rightarrow B(1, 0, 1)$ $\quad \overrightarrow{AB} = (0, -3, -1)$

$$\begin{vmatrix} 1 & 3 & 0 \\ 1 & 0 & -3 \\ 0 & 2 & -1 \end{vmatrix} = 6 + 3 = 9 \neq 0 \Rightarrow \text{necoplanare}$$

b) $P_1(t+1, t+3, 2)$ $\quad u = (1, 1, 0)$

$P_2(3s+1, 0, 2s+1)$ $\quad v = (3, 0, 2)$

$\overrightarrow{P_1P_2}(3s-t, -t-3, 2s-1)$

$$\begin{cases} \langle \overrightarrow{P_1P_2}, u \rangle = 0 \\ \langle \overrightarrow{P_1P_2}, v \rangle = 0 \end{cases} \Rightarrow \begin{cases} 3s - 2t = 3 \\ 11s - 3t = 2 \end{cases} \Rightarrow \begin{cases} s = -5/13 \\ t = -27/13 \end{cases}$$

$P_1\left(\frac{-14}{13}, \frac{12}{13}, 2\right), P_2\left(\frac{-2}{13}, 0, \frac{3}{13}\right)$

$\overrightarrow{P_1P_2}\left(\frac{12}{13}, \frac{-12}{13}, \frac{-23}{13}\right)$

$$\mathcal{D}: \frac{x_1 + \frac{14}{13}}{\frac{12}{13}} = \frac{x_2 - 12/13}{\frac{-12}{13}} = \frac{x_3 - 2}{\frac{-23}{13}}$$

$dist(\mathcal{D}_1, \mathcal{D}_2) = dist(P_1, P_2) = \|\overrightarrow{P_1P_2}\| = \frac{\sqrt{817}}{13}$

② conica $\Gamma: f(x) = 3x_1^2 - 10x_1x_2 + 3x_2^2 + 4x_1 + 4x_2 + 4 = 0$

f. canonica

$$\mathcal{R}(O; e_1, e_2) \xrightarrow[\text{translatie}]{\theta} \mathcal{R}' = \{P_0; e_1, e_2\} \xrightarrow[\text{rotatie}]{G} \mathcal{R}'' = \{P_0; e_1', e_2'\}$$

$A = \begin{pmatrix} 3 & -5 \\ -5 & 3 \end{pmatrix} \Rightarrow \delta = \det A = 9 - 25 = -16 \neq 0 \Rightarrow$ Centru unic.

P_0 centru $\begin{cases} 6x_1 - 10x_2 + 4 = 0 \\ -10x_1 + 6x_2 + 4 = 0 \end{cases} \Rightarrow x_1 = 1, x_2 = 1 \Rightarrow P_0(1,1)$.

$\tilde{A} = \begin{pmatrix} 3 & -5 & 2 \\ -5 & 3 & 2 \\ 2 & 2 & 4 \end{pmatrix} \qquad \Delta = \det \tilde{A} = -128.$

$\theta : x = x' + x_0 \Leftrightarrow \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_1' \\ x_2' \end{pmatrix} + \begin{pmatrix} 1/1 \\ 1/1 \end{pmatrix}$

$\theta(\Gamma) : 3x_1'^2 - 10x_1x_2 + 3x_2'^2 + \frac{\Delta}{\delta} = 0 \quad \left(\frac{\Delta}{\delta} = 8\right)$

$Q(x) = 3x_1'^2 - 10x_1x_2 + 3x_2^2, \quad Q : \mathbb{R}^2 \to \mathbb{R}^2$

$P(\lambda) = \det(A - \lambda I_2) = \begin{vmatrix} 3-\lambda & -5 \\ -5 & 3-\lambda \end{vmatrix} = 9 - 6\lambda + \lambda^2 - 25 =$

$\Rightarrow \lambda_1 = 8, \lambda_2 = -2. \quad \lambda^2 - 16.$

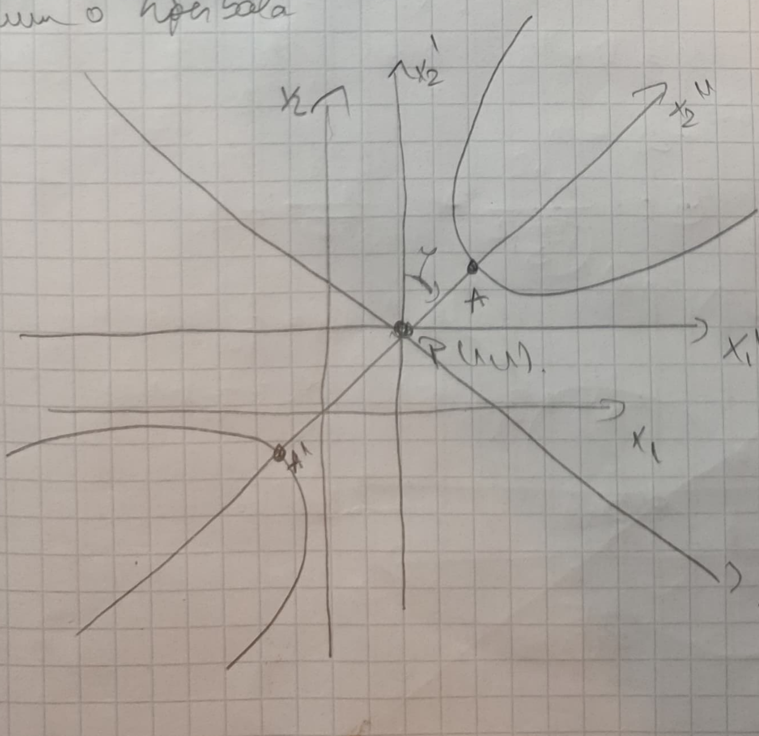
$V_{\lambda_1} = \{x \in \mathbb{R}^2 \mid Ax' = 8x'\} \to A - 8I_2 = O_2 \Rightarrow V_{\lambda_1} = \langle\{\frac{1}{\sqrt{2}}(1,-1)\}\rangle.$

$V_{\lambda_2} = \{x \in \mathbb{R}^2 \mid Ax' = -2x'\} \Rightarrow A + 2I_2 = O_2 \Rightarrow V_{\lambda_2} = \langle\{\frac{1}{\sqrt{2}}(1,1)\}\rangle$

Fie τ rotație, $\tau : x' = Rx''$, $R = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \in SO(2)$

$\tau(\theta(\Gamma)) : 8x_1''^2 - 2x_2''^2 + 8 = 0 \Rightarrow -x_1''^2 + \frac{x_2''^2}{4} = 1.$

Avem o hiperbola



$e_1 = \frac{1}{\sqrt{2}}(1,-1)$

$e_2 = \frac{1}{\sqrt{2}}(1,1)$

$\mathcal{H} : \frac{-x_1''^2}{1^2} + \frac{x_2''^2}{2^2} = 1.$

$A(0,2), A'(0,-2).$

(3) $A(1,0)$, $d_1 \cup d_2 = 2x_1 \pm x_2 = 0$

ec. $\mathcal{H}$, e, ec. dir

$A(1,0) \Rightarrow A$ vf. hip $\Rightarrow a = 1$

$d_1 \cup d_2 : x_2 = \pm \frac{b}{a} x_1 \Rightarrow x_2 = \pm b x_1 \Rightarrow b = 2$

$\mathcal{H} = \frac{x_1^2}{1^2} - \frac{x_2^2}{2^2} = 1$

(1) Ec. directoare:

$c = \sqrt{a^2 + b^2} = \sqrt{5}$

$d \cup d' : x_1 = \pm \frac{\sqrt{5}}{5}$

(2) Excentricitatea.

$e = \frac{c}{a} = \sqrt{5}$