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Rendering Theory

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- 01 Introduction
- 02 Light-Matter Interactions
- 03 Radiometry
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Disclaimer

The rest of the course will assume that...



Light travels in vacuum



We deal only with
opaque surfaces



Interactions occur at
object surface



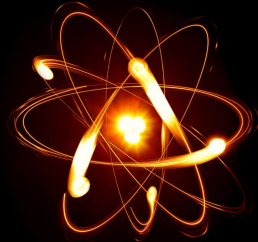
Neglected Effects

- 1 No transmission
- 2 No Subsurface Scattering
- 3 No Phosphorescence
- 4 No Fluorescence

There are ways to simulate some of those effects.
However, it's out of scope for this lesson.

Additional links will be given for readers that want to
go further!





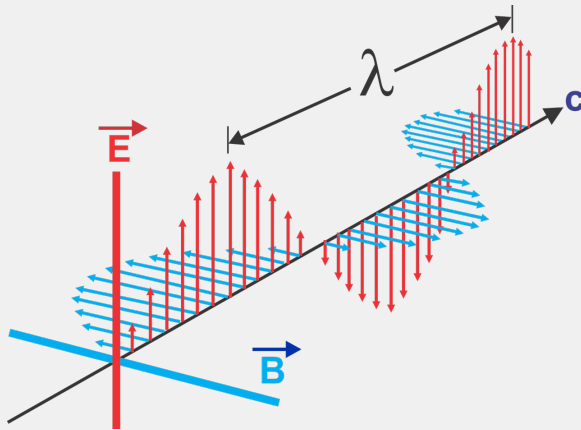
Light - Matter Interactions

Disclaimer

I am not a physicist, and **Quantum Mechanics** is a really complex topic. This section aims at giving you some insight that will help you understand better how our mathematical models are derived.

It's also a rabbit hole, the deeper you question the nature around you, the more questions will start popping in your head.





And there be light

Light is an electromagnetic radiation

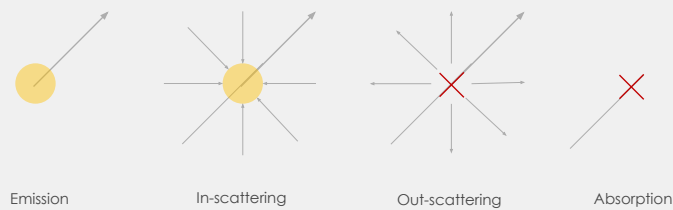
EM propagation is explained thanks to
Maxwell's equations

Light is made out of photons

Interacts with matter in several ways



Macroscopic Level: Interactions



When light interacts with matter, it undergoes interactions that depends on:

- The interface between the two mediums
- The light beam direction
- The wave polarization (out of the scope of this course)

At a macroscopic levels, those interactions are often sorted into three or four categories:

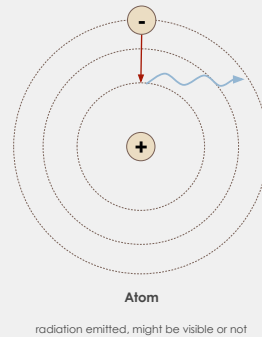
- Emission: light is created (transfer of energy occurs)
- In-Scattering: light arrives at a particle
- Out-Scattering: light leaves a particle
- Absorption: light is absorbed by matter and an energy transfer occurs (e.g: heat)

This is just an overview, and we will go over what happens at the atomic level on the next slides.

Emission

Any vibrating **charged particles** converts energy into **electromagnetic radiation**

Emission is complex and can come from:
fluorescence,



Atoms are made out of electrons outside of their nucleus. It's common to depict electrons as orbiting around their nucleus like here, using the Niels Bohr model.

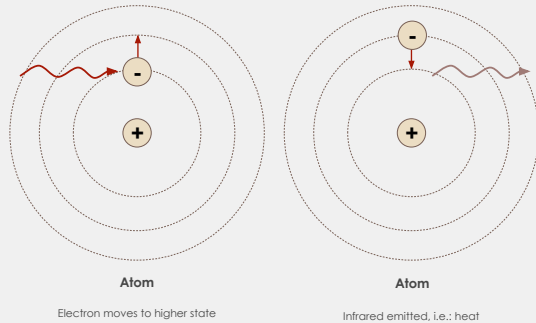
When an electron is in a higher level of energy, it will try to reach its original state releasing electromagnetic radiation. The wavelength of the emitted radiation depends on the state the electron was previously in.

Just a side note here, the Bohr model isn't fully accurate. Electrons aren't really particles in ring like that. In reality, electrons occupy **atomic orbitals**, which are region of probabilistic location.

Absorption

Light may get absorbed when interacting with atoms

Absorption color depends on the atom and the incident light



Electromagnetic waves interact with electrons if the incident energy can make the electron reach a higher state of energy. When going to a higher state, the incident energy is absorbed.

A small amount of time later, the electron goes back to its initial energy state and radiation is emitted. The wavelength of the emitted radiation depends on the state the electron was previously in.

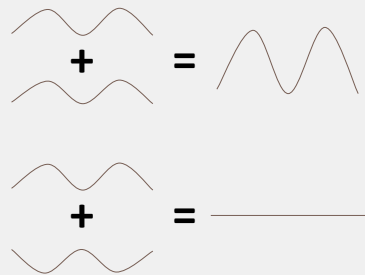
Scattering

Reflection / Refraction

Studied in **Quantum Electrodynamics**

Re-emitted light is slightly out of phase

Path with the most constructive interferences will be the "new" light path



QM is inherently probabilistic, and it's not possible to follow the path of a single photon.

However, if we mix QM and the classical view, we can obtain an intuition of what's happening.

Atoms will absorb photons and re-emits them like we saw on the previous slide.

However, you can

think of the atom as a punctual point source emitting in every directions. The emitted radiation phase

is dependent on the incoming beam.

When photons interact at the intersection between two mediums, the light reflection/refraction path will be the ones

with the most constructive interferences. At the opposite, the incoming wave will be the path

where most of the destructive interferences occurs.

I want to stress out just one thing: in reality, changing speed is a collective effect and not the result of a single

atom interaction. Photons actually scatter from **all** atom **simultaneously** (crazy,

right?).

At a higher level, Maxwell's equations help to derive the Fresnel equations that are used to compute the ratio of reflected light over refracted light.

Final Notes

Any charged particles can interact on electromagnetic radiation (i.e., neutrons, protons)

Quantum Theory and Quantum Electrodynamics can go really far....

I can only advise you to read more about this topic!

[Cool demo](#) showing wave reflection / refraction by [Julien Guertault](#)

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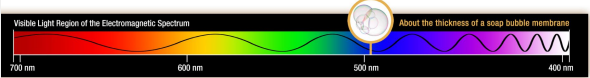
Don't forget that the Bohr model we looked at is a more simple representation of what occurs.

Quantum Theory and Quantum Electrodynamics help explain some effects that can't be explained using classical models.

For instance, it's not possible to study the path of a single elementary particle. A photon actually doesn't interact with a single electron at a time, but with all of them **simultaneously**... Crazy, isn't it?

Radiometry

Rendering Theory |



Visible Light Region of the Electromagnetic Spectrum

700 nm 600 nm 500 nm 400 nm

About the thickness of a soap bubble membrane

From now on, we will focus on **classical models**

Radiometry is the study of **electromagnetic radiation**

Describes **light propagation & reflectance**, i.e., the fundamentals of **rendering**

Let's dive into the quantities involved!

We have had a good look at some of the models that help explain what happens to electromagnetic radiation at the atomic level.

However, for rendering purposes, we will use higher level models.

Radiometry will give us the necessary tools to work our way into the Rendering Equation.

From now on, we will not need to dive into quantum physics and we will keep a much simpler vision:

light is made out of photons that travel on a straight path.

Energy

Énergie

Photons carry an energy level based on their wavelength

Expressed in **Joules (J)**

Planck's constant

Speed of Light

$$Q = \frac{hc}{\lambda}$$

Wavelength



Radiometry is interested into measuring electromagnetic radiation. Electromagnetic radiations are composed of photons which carry energy.

You might remember the equation above from high school, used to compute the total energy of a photon.

Radiant Flux / Power

Puissance

Energy going through a surface per unit of
time

Expressed in (J/s), i.e., **Watts (W)**

$$\phi = \frac{dQ}{dt}$$

Energy

Differentiable Time



Energy per unit of time. In general, rendering will assume a steady state for simplicity.

Irradiance

Eclairement Énergétique

Radiant Flux received per **unit area**

Expressed in **(W/m²)**

$$E(p) = \frac{d\phi(p)}{dA}$$

Power
Finite Surface Area

Irradiance is defined as the radiant flux received per surface of area A.

There is a distinction between:

- Incoming radiant flux per unit area is called **Irradiance**
- Outgoing radiant flux per unit area is called **Radiant Exitance**

As stated in PBRT book, the irradiance equation makes us realize two things:

- For a perfect point light, the energy falls off with the squared distance from the light origin
- Helps us understand the Lambert Cosine Law

Shining a light with an angle at a surface will create a larger projected area (**A₂** on the drawing). A larger area implies a smaller Irradiance. It also makes sense intuitively, the flux is then dispatched on a larger surface and each “*point*” receives less power and so less energy.

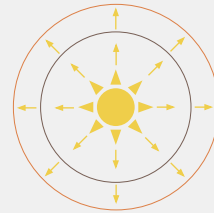
Irradiance

Eclairement Énergétique

Equation explains light falloff

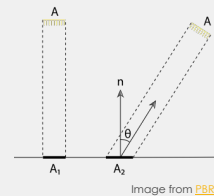
Equation explains Lambert law

$$E = \frac{\phi}{4\pi r^2}$$



$$E_1 = \frac{\phi}{A}$$

$$E_2 = \frac{\phi \cos(\theta)}{A}$$



As stated in PBRT book, the irradiance equation makes us realize two things:

- For a perfect point light, the energy falls off with the squared distance from the light origin
- Helps us understand the Lambert Cosine Law

Shining a light with an angle at a surface will create a larger projected area (**A2** on the drawing). A larger area implies a smaller Irradiance. It also makes sense intuitively, the flux is then dispatched on a larger surface and each “*point*” receives less power and so less energy.

Solid Angle

Angle solide

Area of a projected shape onto the **Unit Sphere**

Expressed in **steradians (sr)**

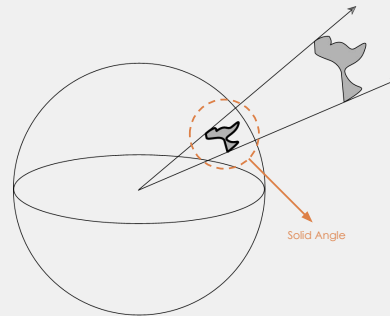


Image from [PBR1](#)

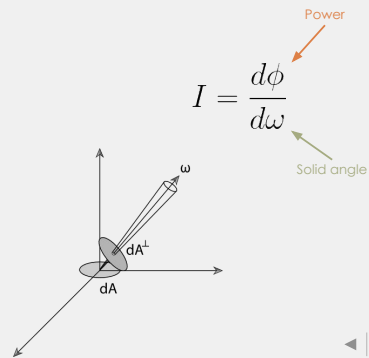
We are often interested into bringing directionality when talking about radiometry. Solid angles describes the projected area of an object onto the unity sphere. You can imagine for instance being an observer at the center of the sphere, looking at the projected silhouette of the object on the edges of the sphere.

Radiant Intensity

Intensité Énergétique

Power corresponds to the energy per unit of time

Expressed in (W/sr)



In physics, it's defined as the radiant flux (the power) per unit area.

In the case of Radiometry however, the intensity is defined as the **power per unit solid angle**.

Be careful here, intensity isn't exactly synonymous to 'strength' or 'magnitude'. It can be seen as a strength over a given direction and area.

With intensity, it's possible to derive the strength and direction of light in a cone.

Radiance

Luminance Énergétique

Radiant Intensity leaving an area **A**, over solid angle ω

Expressed in (**W.sr⁻¹.m⁻²**)

$$L(p, \omega) = \frac{dE_{\omega}(p)}{d\omega} = \frac{d\phi(p)}{d_{\omega}dA^{\perp}}$$

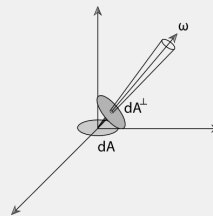
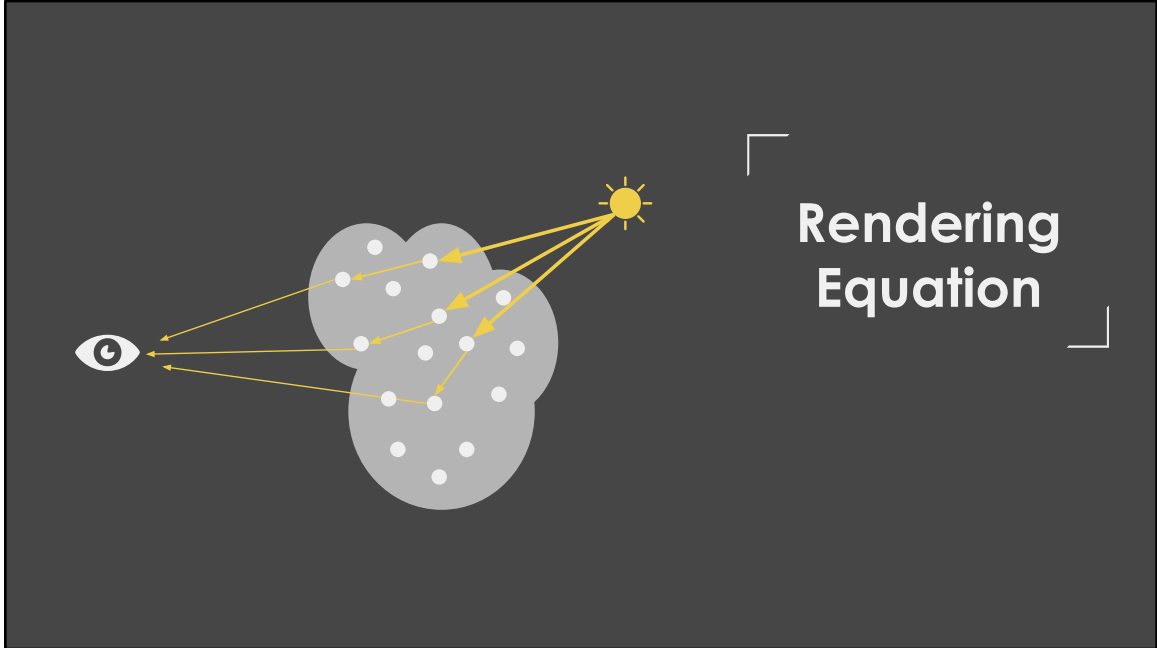


Image from [PBRT](#)

Radiance allows to express irradiance or radiant exitance in terms of solid angle, bringing directionality into our equations!

Remember that our final goal is to perform rendering, which basically consists in computing radiance coming from viewing direction onto a sensor (e.g., eye, camera).

Radiance is also a combination of all the previous quantity we have seen, and can be used to get back to other quantity by using integration.



We have talked about light-matter interactions as well as Radiometry. Now we need to come up with an equation that can represent those interactions and gives us information about the spectral distribution of light as it undergoes reflection / refraction.

Fortunately for us, a lot of smart people spent time figuring it out, and came up with an equation.

Definition


Rendering Equation

We are interested in computing the radiance reaching the viewer

This equation will allow us to compute it!

Recursive,

$$L_o(p, \omega_o) = \int_{\Omega} f_r(p, \omega_o, \omega_i) L_i(p, \omega_i) n \cdot \omega_i d\omega_i$$



BRDF

The above equation gives the radiance leaving in the direction ω_o , due to illumination at point p from all directions in the hemisphere oriented in the hemisphere aligned to the surface normal.

As you can see, this equation is recursive and depends on incoming radiance from all over the hemisphere oriented around the surface normal. This is why rendering photorealistic images is a computational heavy problem: you need to compute incoming light from many directions **recursively** to determine the final color of a pixel.

We will first break down together this equation, and see how it can be retrieved. There exist several mathematical tools to describe how light is reflected and transmitted. However, as mentioned at the beginning of this course, we will only focus on opaque surfaces with interactions occurring at the object surfaces.

Bidirectional Reflectance Distribution Function (BRDF)

$$f_r(p, \omega_i, \omega_o)$$

Returns the ratio of **light scattered** at **p** from ω_i to ω_o

A physical BRDF matches the following conditions:

reciprocal. i.e.,

energy conservation. i.e.,

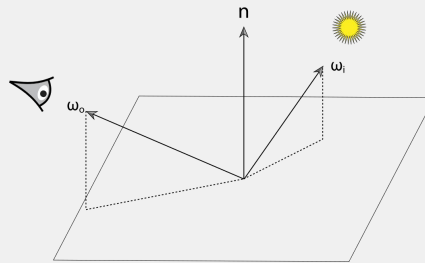


Image from [PBRt](#)

The BRDF is used to weight incoming radiance over the hemisphere. It describes how much light from a direction ω_i gets reflected in direction ω_o . Stated differently, the BRDF helps you know how much of the light coming from a particular direction will contribute to the result direction (ω_o).

The function is driven by the material properties and helps define its appearance.

It's really important to remember the two conditions listed above. A BRDF must follow those rules in order to be physical. The energy conservation rule is especially important because energy is always transferred between systems.

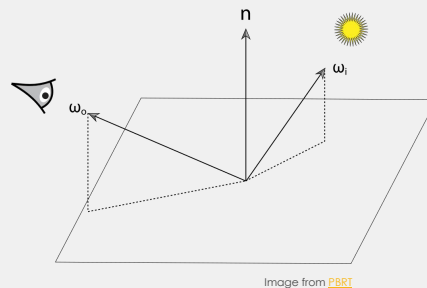
You may also wish to have a non-physical BRDF to perform some non-realistic rendering, but that's another story.

Bidirectional Reflectance Distribution Function (BRDF)

BRDF is used for opaque surface and are only applicable for surface interactions

More advanced distribution functions exists, such as the **BTDF** and the **BSSRDF**

BRDFs are derived from models, or captured using a **Gonioreflectometer**



In reality, light penetrates an object and interacts with particles. Lights will scatters inside the material and gets absorbed or eventually leave the material.

“Remaining” wavelengths are what gives objects their colors.

This is why there are other formalism, such as the:

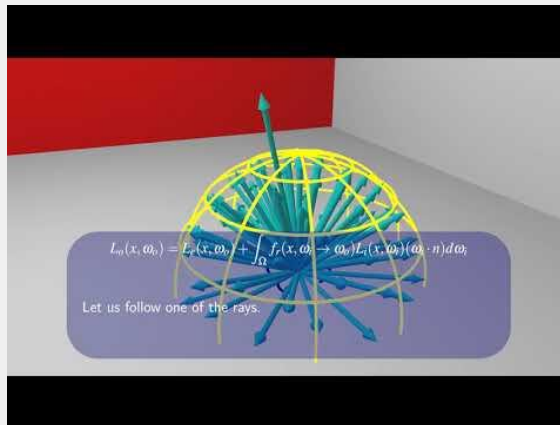
- **BTDF**: Describes the behavior of transmission
- **BSSRDF**: Describes the subsurface behavior of light. Light enters at point A but might leave at point B

My goal here is just to present other mathematical tools that can help us describe more complex

behaviors that exist in nature. We mentioned already that we only care about a single point of interaction at the object

surface. So for this entire course, we will **always** focus on **BRDFs** and nothing else.

Rendering Equation Video



Final Notes

Rendering equation uses all quantities we have seen

The rendering equation is what we solve when generating 3D images

A good explanation of how to derive the equation is available in [PBRT](#)



References

References

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Thanks

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