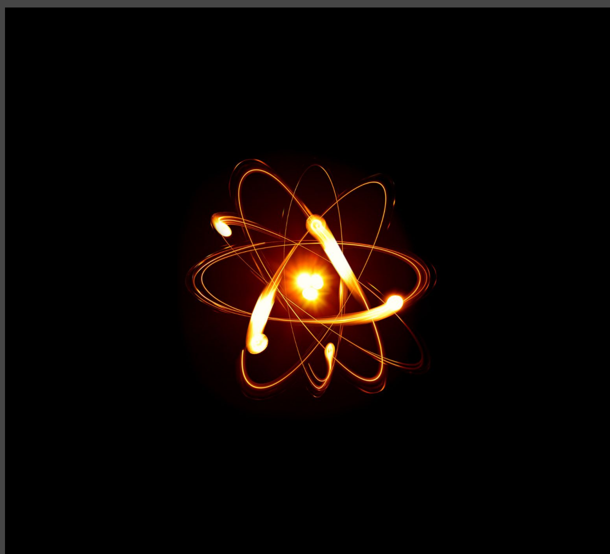
David Peicho

Rendering Theory

Slides available at https://davidpeicho.github.io/teaching/
Found an error? Please contact me at david.peicho@gmail.com

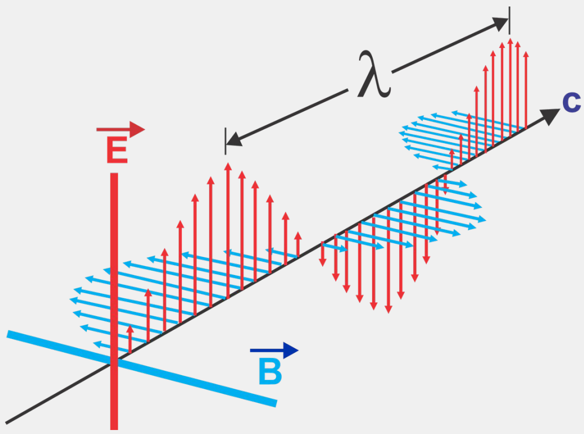


Light - Matter Interactions

Disclaimer

I am not a physicist, and **Quantum Mechanics** is a really complex topic. This section aims at giving you some insight that will help you understand better how our mathematical models are derived.

It's also a rabbit hole, the deeper you question the nature around you, the more questions will start popping in your head.



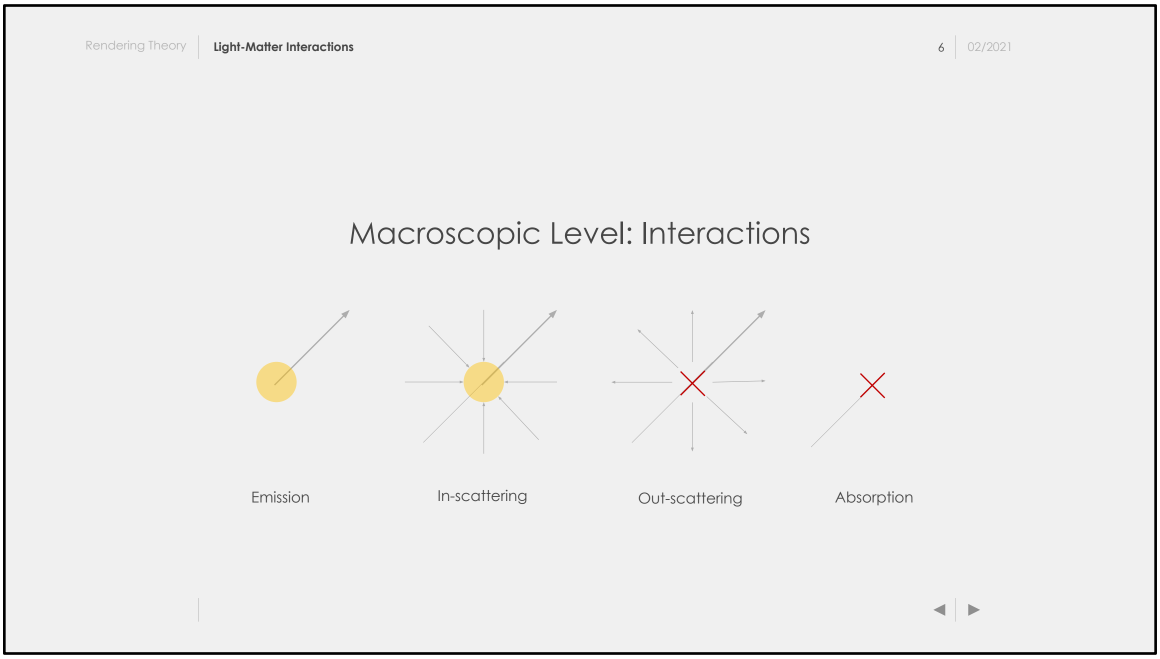
And there be light

Light is an electromagnetic radiation

EM propagation is explained thanks to **Maxwell's equations**

Light is made out of photons

Interacts with matter in several ways



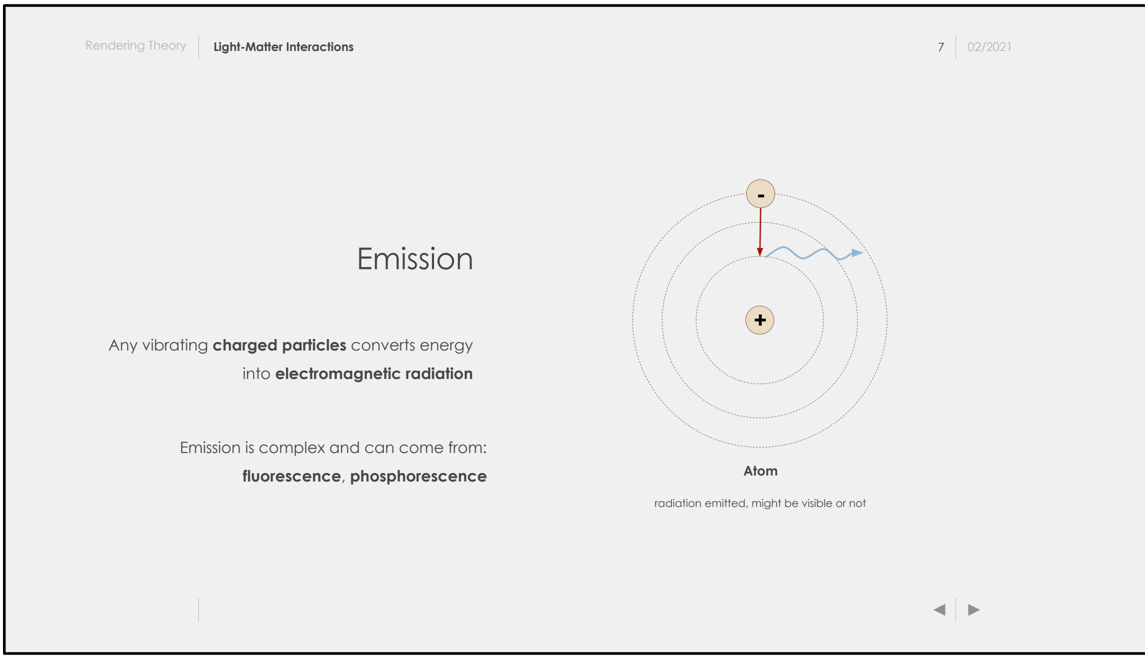
When light interacts with matter, it undergoes interactions that depends on:

- The interface between the two mediums
- The light beam direction
- The wave polarization (out of the scope of this course)

At a macroscopic levels, those interactions are often sorted into three or four categories:

- Emission: light is created (transfer of energy occurs)
- In-Scattering: light arrives at a particle
- Out-Scattering: light leaves a particle
- Absorption: light is absorbed by matter and an energy transfer occurs (e.g: heat)

This is just an overview, and we will go over what happens at the atomic level on the next slides.



Atoms are made out of electrons outside of their nucleus. It's common to depict electrons
as orbiting around their nucleus like here, using the Niels Bohr model.

When an electron is in a higher level of energy, it will try to reach its original state releasing electromagnetic radiation.
The wavelength of the emitted radiation depends on the state the electron was previously in.

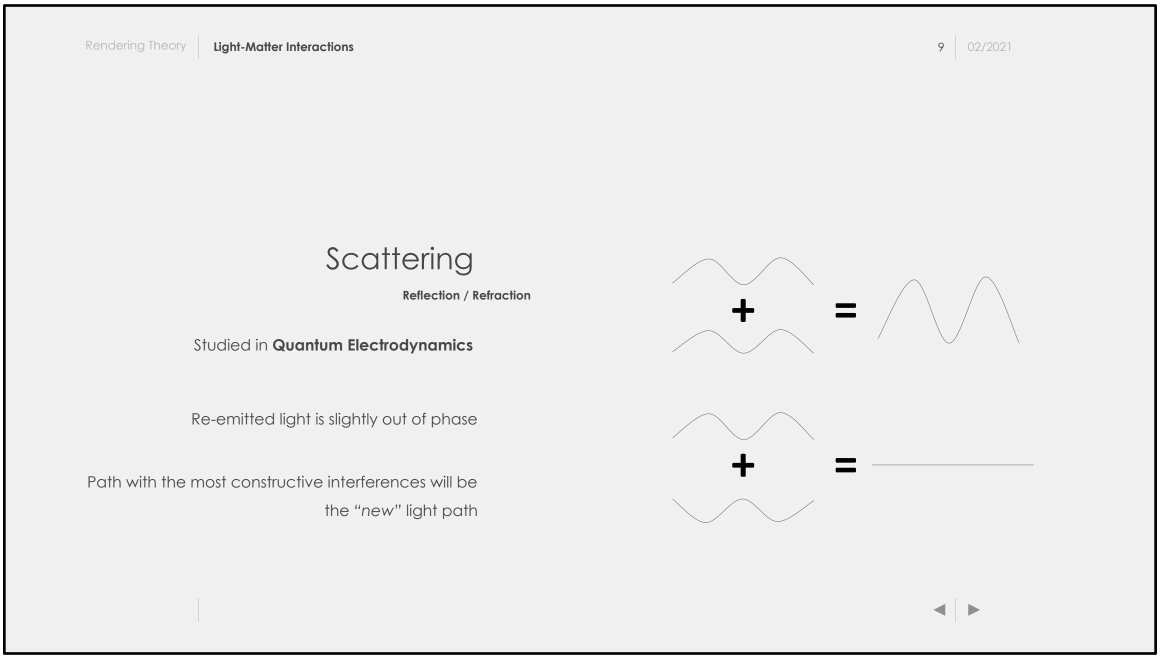
Now the question might be: how do we get an electron to a higher level of energy?
Everything above absolute zero radiates light. There are a lot of conversion that can happen, like
thermal energy into electromagnetic energy.

Just a side note here, the Bohr model isn't fully accurate. Electrons aren't really particles in ring like that.
In reality, electrons occupy **atomic orbitals,** which are region of probabilistic location.

Electromagnetic waves interact with electrons if the incident energy can make the electron
reach a higher state of energy. When going to a higher state, the incident energy is absorbed.

A small amount of time later, the electron goes back to its initial energy state and radiation is
emitted. The wavelength of the emitted radiation depends on the state the electron was previously in.

In some materials, atoms/molecules are close to each other, and absorption is the result of conversion from electromagnetic energy
into themeral energy (vibrations).



QM is inherently probabilistic, and it's not possible to follow the path of a single photon.
However, if we mix QM and the classical view, we can obtain an intuition of what's happening.

Atoms will absorb photons and re-emits them like we saw on the previous slide. However, you can
think of the atom as a punctual point source emitting in every directions. The emitted radiation phase
is dependent on the incoming beam.

When photons interact at the intersection between two mediums, the light reflection/refraction path will be the ones
with the most constructive interferences. At the opposite, the incoming wave will be the path
where most of the destructive interferences occurs.

I want to stress out just one thing: in reality, changing speed is a collective effect and not the result of a single
atom interaction. Photons actually scatter from **all** atom **simultaneously** (crazy,

right?).

At a higher level, Maxwell’s equations help to derive the Fresnel equations that are used
to compute the ratio of reflected light over refracted light.

Final Notes

Any charged particles can interact on electromagnetic radiation (i.e., neutrons, protons)

Quantum Theory and Quantum Electrodynamics can go really far….

I can only advise you to read more about this topic!

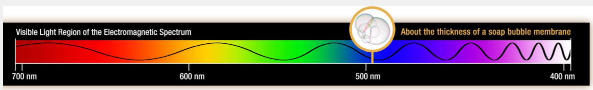
Cool demo showing wave reflection / refraction by Julien Guertault

Don’t forget that the Bohr model we looked at is a more simple representation of what occurs.

Quantum Theory and Quantum Electrodynamics help explain some effects that can’t be explained using classical models.

For instance, it’s not possible to study the path of a single elementary particle. A photon actually doesn’t interact with a single electron at a time, but with all of them **simultaneously**… Crazy, isn’t it?

Radiometry



From now on, we will focus on **classical models**

Radiometry is the study of **electromagnetic radiation**

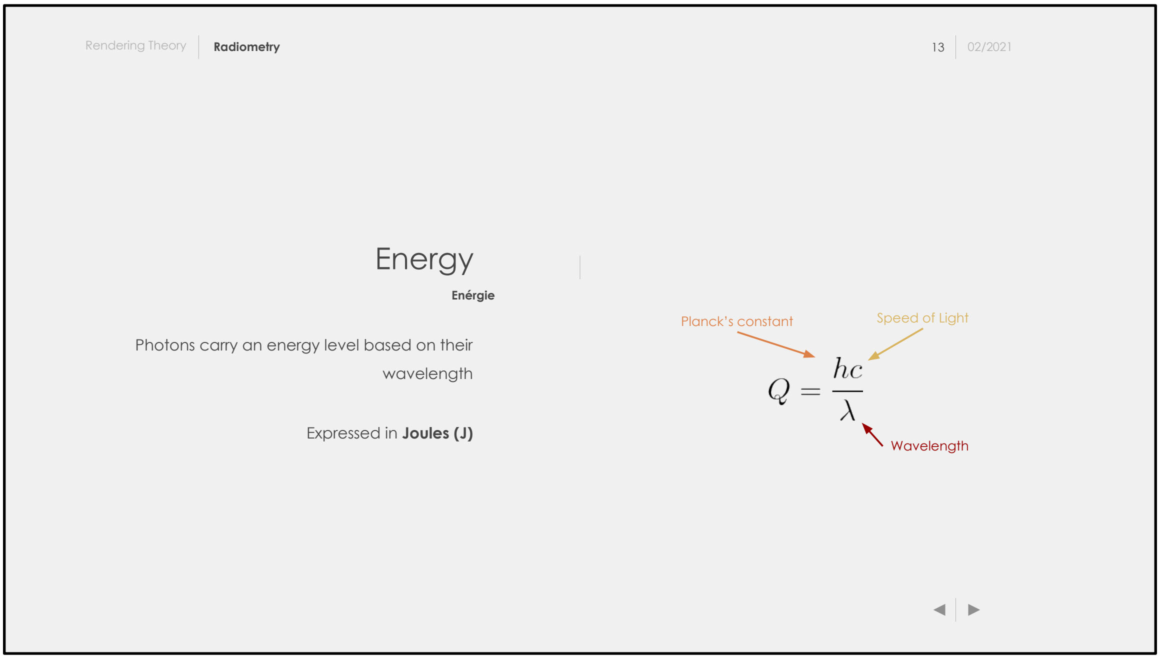
Describes **light propagation** & **reflectance**, i.e., the fundamentals of **rendering**

Let's dive into the quantities involved!

We have had a good look at some of the models that help explain what happens to electromagnetic radiation at the atomic level.

However, for rendering purposes, we will use higher level models.

Radiometry will give us the necessary tools to work our way into the Rendering Equation.
From now on, we will not need to dive into quantum physics and we will keep a much simpler vision:
light is made out of photons that travel on a straight path.



Radiometry is interested into measuring electromagnetic radiation. Electromagnetic radiations are composed of photons which carry energy.

You might remember the equation above from high school, used to compute the total energy of a photon.

Radiant Flux / Power

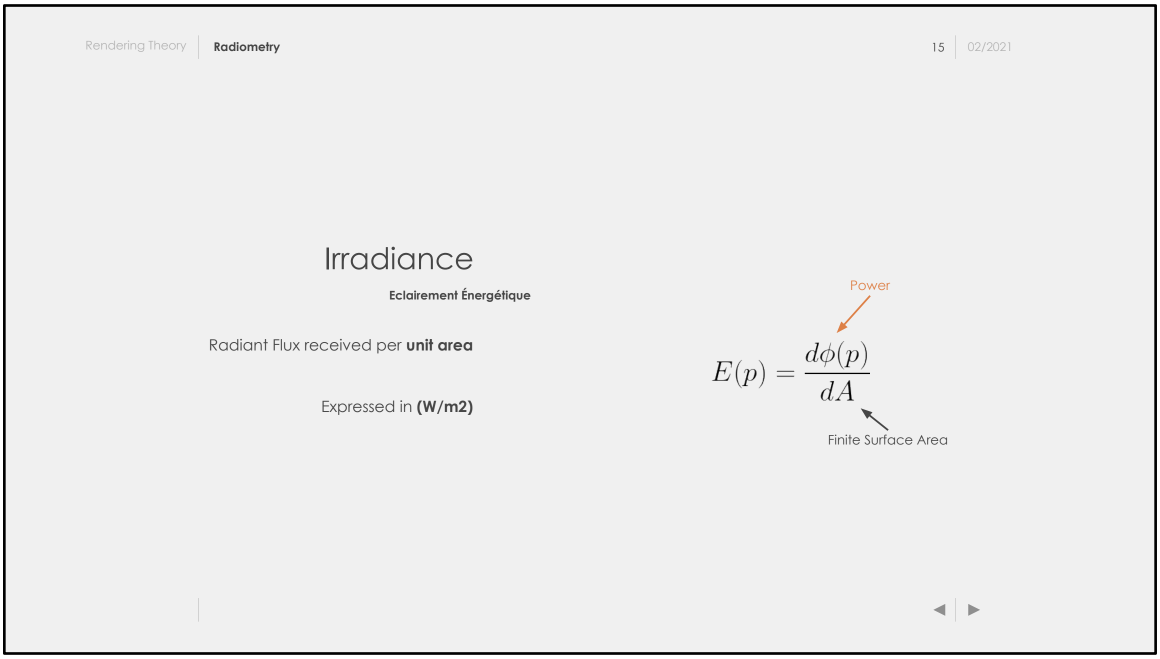
Puissance

Energy going through a surface per unit of time

Expressed in **(J/s)**, i.e., **Watts (W)**

$$\phi = \frac{\overbrace{dQ}^{\text{Energy}}}{\underbrace{dt}_{\text{Differentiable Time}}}$$

Energy per unit of time. In general, rendering will assume a steady state for simplicity.



Irradiance is defined as the radiant flux received per surface of area A.

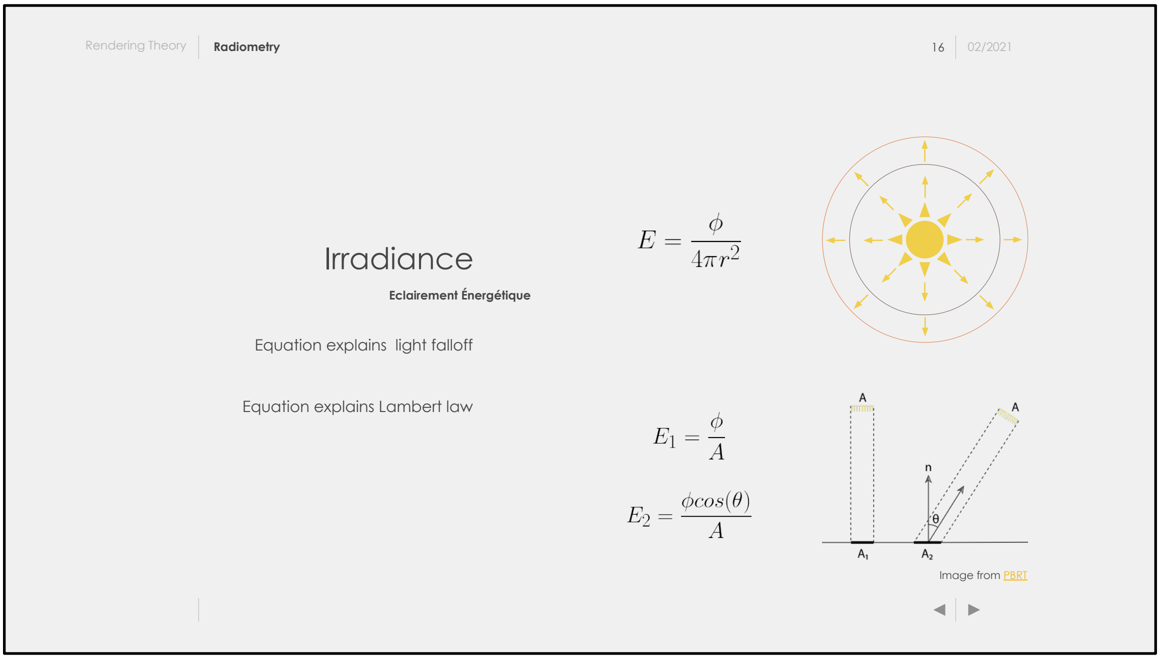
There is a distinction between:

- Incoming radiant flux per unit area is called **Irradiance**
- Outgoing radiant flux per unit area is called **Radiant Exitance**

As stated in PBRT book, the irradiance equation makes us realize two things:

- For a perfect point light, the energy falls off with the squared distance from the light origin
- Helps us understand the Lambert Cosine Law

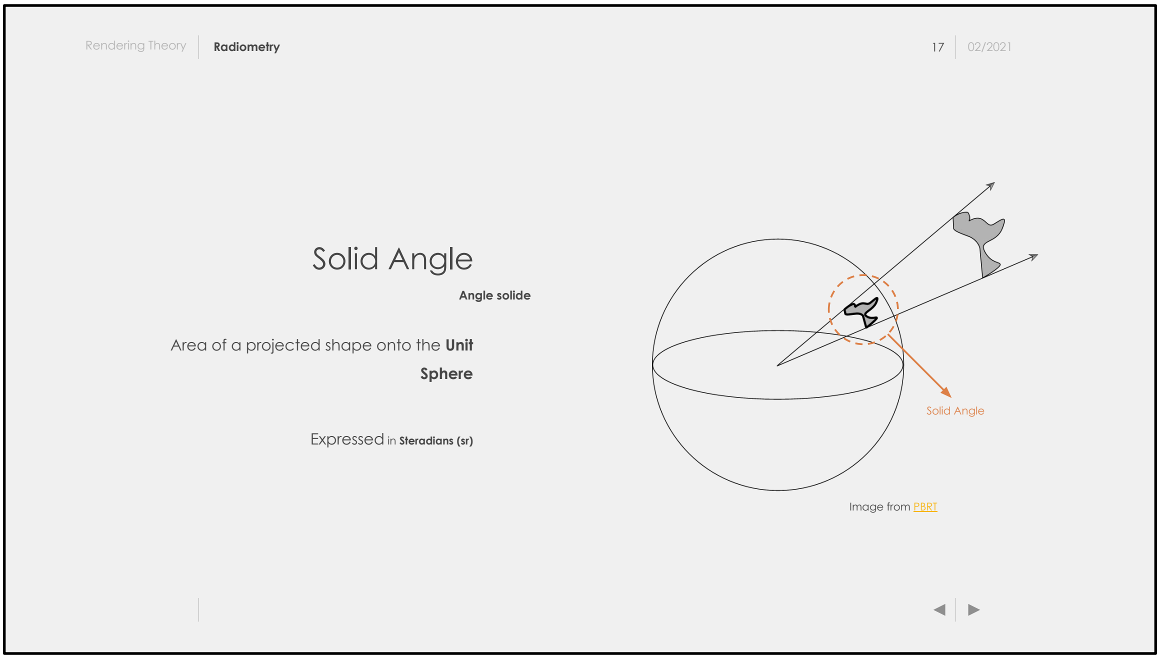
Shining a light with an angle at a surface will create a larger projected area (**A2** on the drawing). A larger
area implies a smaller Irradiance. It also makes sense intuitively, the flux is then dispatched on a larger
surface and each *"point"* receives less power and so less energy.



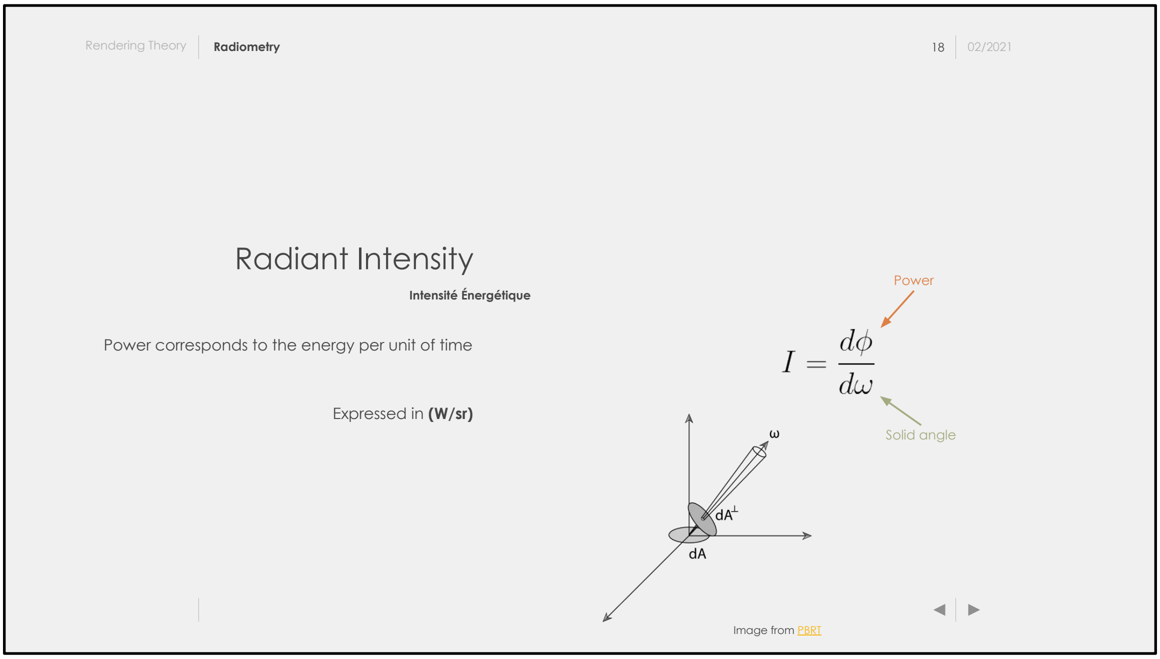
As stated in PBRT book, the irradiance equation makes us realize two things:

- For a perfect point light, the energy falls off with the squared distance from the light origin
- Helps us understand the Lambert Cosine Law

Shining a light with an angle at a surface will create a larger projected area (**A2** on the drawing). A larger
area implies a smaller Irradiance. It also makes sense intuitively, the flux is then dispatched on a larger
surface and each *"point"* receives less power and so less energy.



We are often interested into bringing directionality when talking about radiometry. Solid angles describes the projected area of an object onto the unity sphere. You can imagine for instance being an observer at the center of the sphere, looking at the projected
silhouette of the object on the edges of the sphere.



In physics, it’s defined as the radiant flux (the power) per unit area.
In the case of Radiometry however, the intensity is defined as the **power per unit solid angle**.

Be careful here, intensity isn’t exactly synonymous to ‘strength’ or ‘magnitude’. It’s the strength over a given direction and area.

When taking the intensity emitted by a light source, the solid angle corresponds to the surface into which the flux is emitted.
When computing the received intensity at a point, the solid angle corresponds to the area subtended by the source **as seen** from the point.

With intensity, it’s possible to derive the strength and direction of light.
Because we care mostly about direction, we will in general the limit of a differential cone, i.e., infinitesimally small solid angle.

Radiance

Luminance Energétique

Radiant Intensity leaving an area **A**, over solid angle omega

Expressed in **(W.sr^{-1}.m^{-2})**

$$L(p,\omega) = \frac{dE_\omega(p)}{d\omega} = \frac{d\phi(p)}{d_\omega dA^\perp}$$

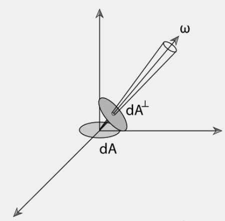
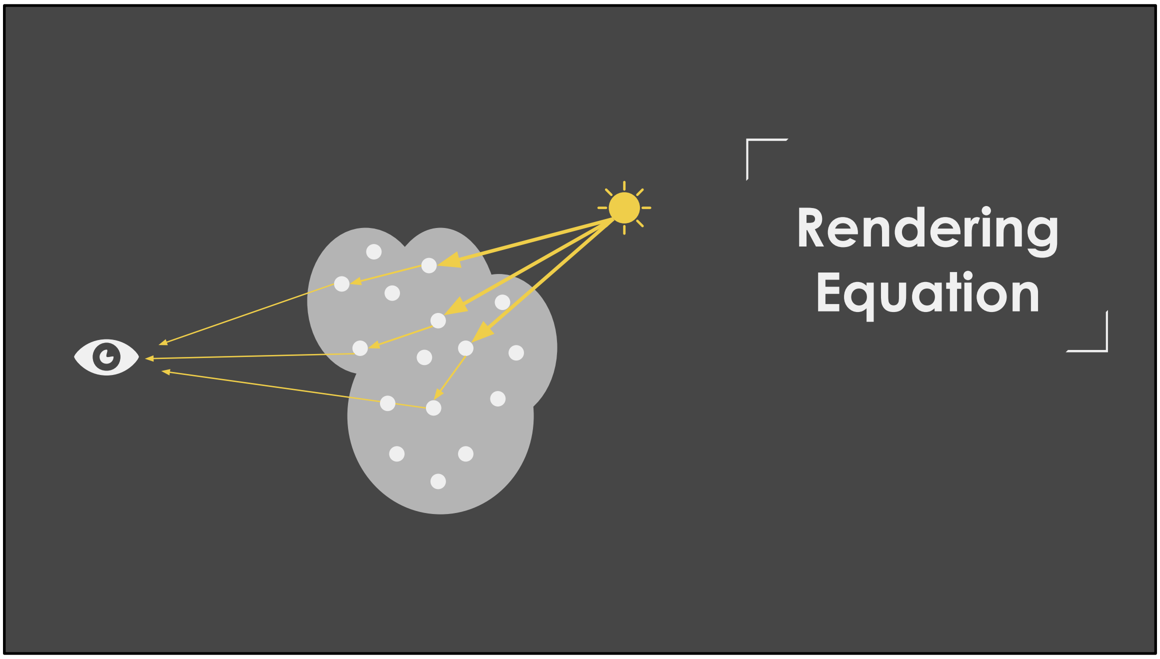


Image from PBRT

Radiance allows to express irradiance or radiant exitance in terms of solid angle, bringing directionality into our equations!

Remember that our final goal is to perform rendering, which basically consists in computing radiance coming from viewing direction onto a sensor (e.g., eye, camera).

Radiance is also a combination of all the previous quantity we have seen, and can be used to get back to other quantity by using integration.



We have talked about light-matter interactions as well as Radiometry. Now we need to come up with
an equation that can represents those interactions and gives us information about the spectral
distribution of light as it undergoes reflection / refraction.

Fortunately for us, a lot of smart people spent time figuring it out, and came up with an equation.

Disclaimer

The rest of the course will assume that...

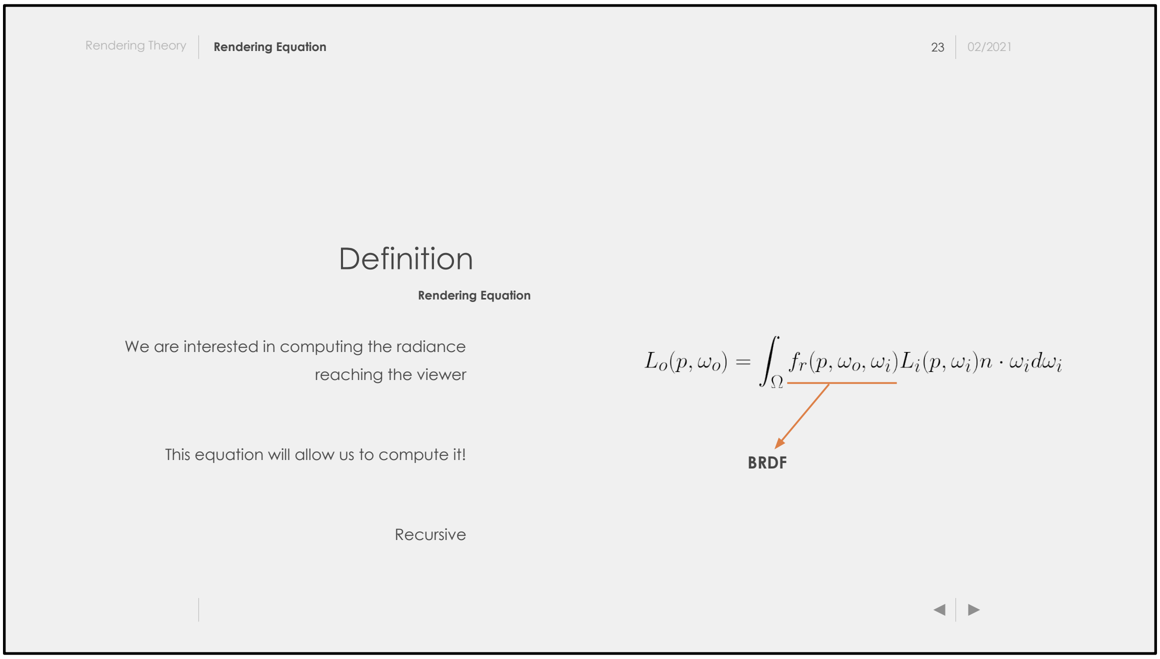
- Light travels in vacuum
- We deal only with opaque surfaces
- Interactions occur at **object surface**

Neglected Effects

1. No transmission
2. No Subsurface Scattering
3. No Phosphorescence
4. No Fluorescence

There are ways to simulate some of those effects. However, it's out of scope for this lesson.

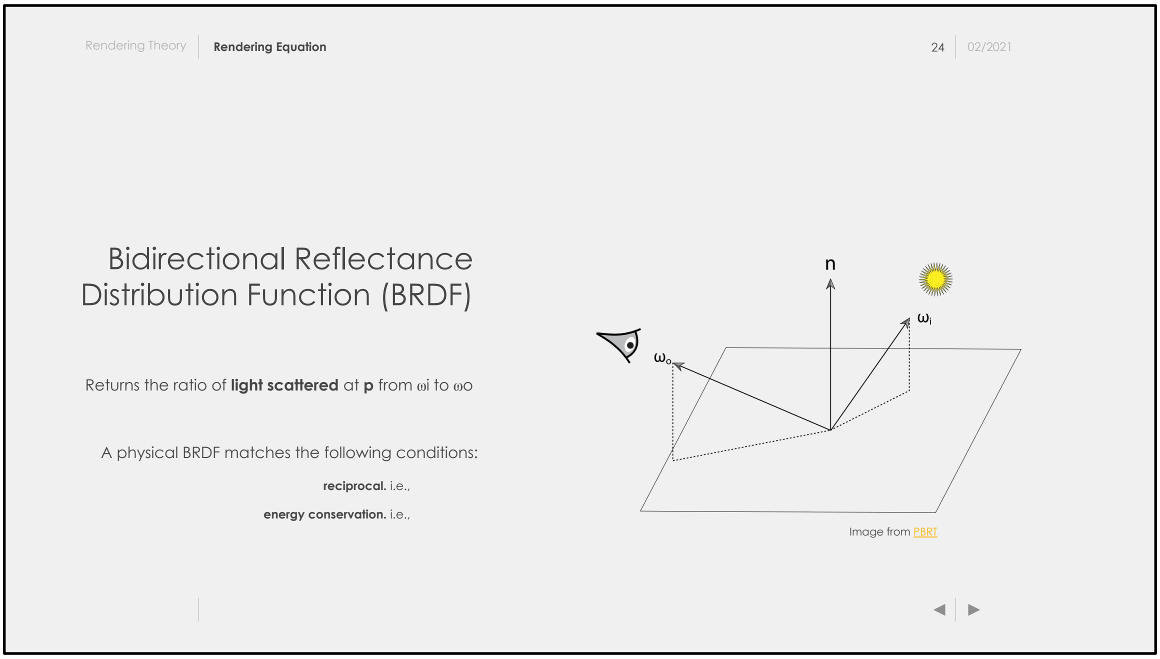
Additional links will be given for readers that want to go further!



The above equation gives the radiance leaving in the direction ωo, due to illumination at point p
from all directions in the oriented in the hemisphere aligned to the surface normal.

As you can see, this equation is recursive and depends on incoming radiance from all over the hemisphere
oriented around the surface normal. This is why rendering photorealistic images is a computational heavy problem:
you need to compute incoming light from many directions **recursively** to determine the final color of a pixel.

We will first break down together this equation, and see how it can be retrieved.
There exist several mathematical tools to describe how light is reflected and transmitted. However,
as mentioned at the beginning of this course, we will only focus on opaque surfaces with interactions
occuring at the object surfaces.

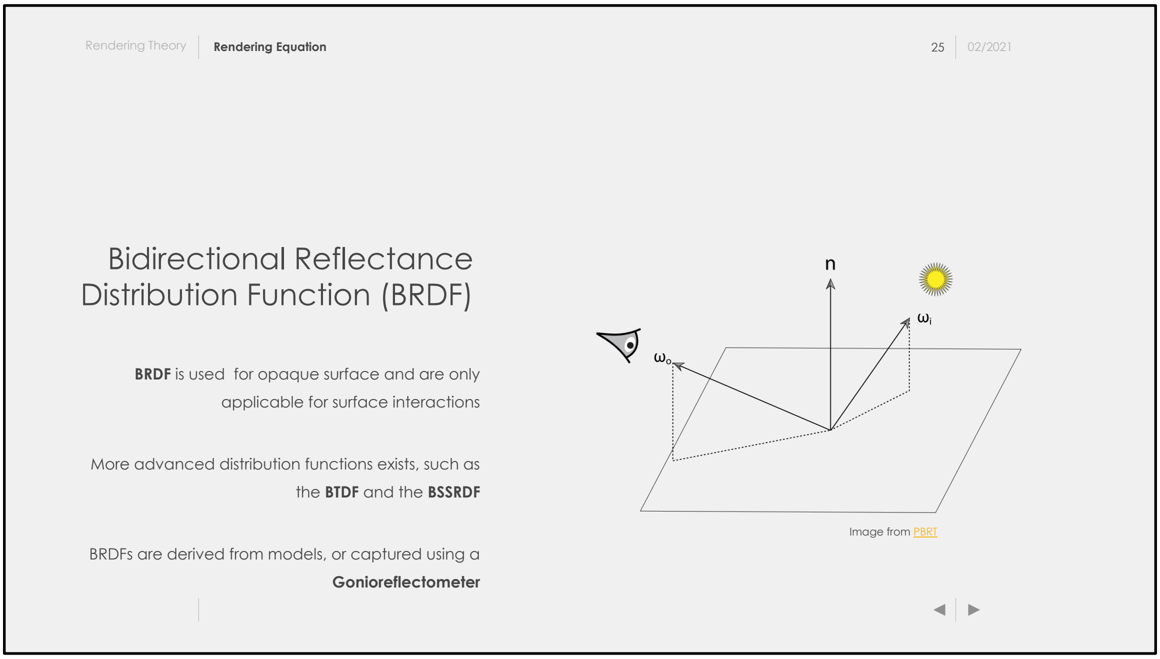


The BRDF is used to weight incoming radiance over the hemisphere. It describes how much light from a direction ωi gets reflected in direction ωo.
Stated differently, the BRDF helps you know how much of the light coming from a particular
direction will contribute to the result direction (ωo).

The function is driven by the material properties and helps define its appearance.

It's really important to remember the two conditions listed above. A BRDF must follow
those rules in order to be physical. The energy conservation rule is especially important
because energy is always transferred between systems.

You may also wish to have a non-physical BRDF to perform some non-realistic rendering, but that's another story.



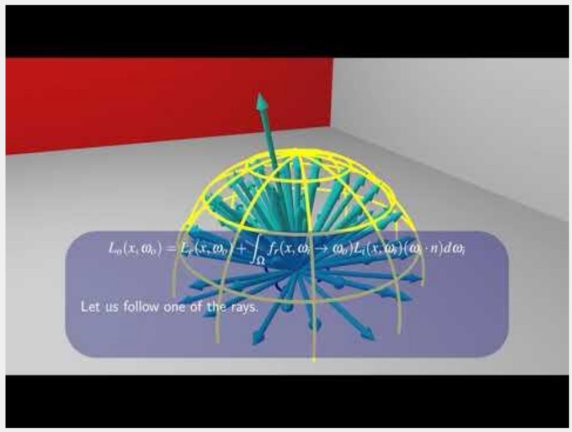
In reality, light penetrates an object and interacts with particles. Lights will scatters inside the material and gets absorbed or eventually leave the material.
"Remaining" wavelengths are what gives objects their colors.

This is why there are other formalism, such as the:

- BTDF: Describes the behavior of transmission
- BSSRDF: Describes the subsurface behavior of light. Light enters at point A but might leave at point B

My goal here is just to present other mathematical tools that can help us describe more complex
behaviors that exist in nature. We mentioned already that we only care about a single point of interaction at the object
surface. So for this entire course, we will **always** focus on **BRDFs** and nothing else.

Rendering Equation Video



Final Notes

Rendering equation uses all quantities we have seen

The rendering equation is what we solve when generating 3D images

A good explanation of how to derive the equation is available in PBRT

References

References

- [Hébert15], M. Hébert, R.D. Hersch, P. Emmel, Fundamentals of Optics and Radiometry for Color Reproduction
- [Kajiya86], J. T. Kajiya, The Rendering Equation
- [Pharr18], M. Pharr, W. Jakob, and G. HUmphreys, Physically Based Rendering: From Theory to Implementation
- [Gui], R. Gui, M. Agopian, Physically Based Rendering in Filament
- [Glassner95], A. S. Glassner, Principles of Digital Image Synthesis

◀ ▶

David Peicho

Thanks

Found an error? Please contact me at david.peicho@gmail.com