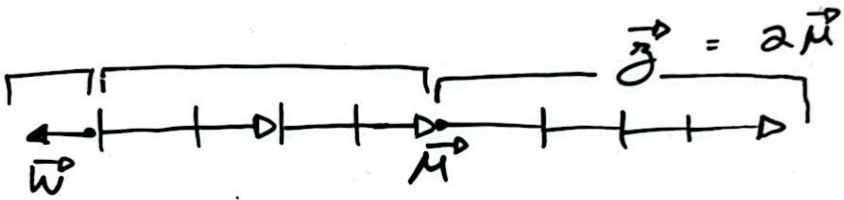
Ângulo entre 2 vetores:

- $\vec{a}$ e $\vec{b}$ → <u>Paralelos</u>
- $\vec{a} \parallel \vec{b}$ → <u>colineares</u>

Vetores paralelos:

$$\vec{v} = \frac{1}{2}\vec{\mu}$$



$w = \frac{1}{4}\vec{\mu}$ $\qquad \|\vec{\mu}\| = 4$

$\vec{\mu}$ com $\vec{z}$ ou $\vec{\mu}$ e $\vec{v}$ são paralelos

Se e só se $\exists k \in \mathbb{R} : \vec{v} = k\vec{\mu}$

$$/\!/ \Leftrightarrow \measuredangle(\vec{\mu} ; \vec{v}) = 0 \vee \measuredangle(\vec{\mu} ; \vec{v}) = 180°$$

(se fizerem 0° graus ou 180° graus, são paralelos)

- São perpendiculares ou ortogonais quando fazem um ângulo de 90° graus.

Ex: 3 ; 6 ; 7.c/d ; 9.

③

$A = (2; 0; 1)$; $B = (3; 0; 1)$; $C = (1, 0, 1)$

$\vec{AB} = B - A$

$\vec{AB} = (3-2) + (0-0) + (1-1)$ ✓

$= (1; 0; 0)$

$\vec{BC} = C - B$

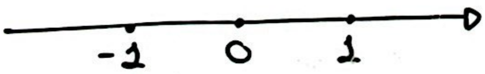
$\vec{BC} = (1-3) + (0-0) + (1-1)$ ✓

$= (-2; 0; 0)$

$\vec{AC} = C - A$

$= (1-2) + (0-0) + (1-1)$ ✓

$= (-1; 0; 0)$



R: são paralelos.

$\vec{AB} \parallel \vec{AC}$

$\vec{AB} = -1 \times \vec{AC}$

$\Leftrightarrow \vec{AB} \parallel \vec{AC}$

5)

$\vec{\mu} = (0; -4; k)$; $\vec{v}\ (0; 3; -1)$

a)

$\|\vec{\mu}\| = 5$

↳ norma de $\vec{\mu} = 5$

$$\|\vec{\mu}\| = \sqrt{\mu_1^2 + \mu_2^2 + \mu_3^2}$$

$(0; -4; k) - (0; 3; -1)$

$\Leftrightarrow \sqrt{0^2 + (-4)^2 + a^2} = 5$

$\Leftrightarrow \sqrt{0 + (+16) + a^2} = 5$

$\Leftrightarrow \left(\sqrt{+16 + a^2}\right)^2 = (5)^2$

$\Leftrightarrow (16 + a^2) = 25$

$\Leftrightarrow 25 = 16 + k^2$

$\Leftrightarrow k^2 = 25 - 16$

$\Leftrightarrow k^2 = 9 \Leftrightarrow k = \sqrt{9}$

$\Leftrightarrow k = 3$

R: $k = 3 \lor k = -3$

b) $\vec{v} \parallel \vec{\mu}$ (v paralelo a μ)

$\Leftrightarrow \vec{v} = \alpha\vec{\mu} \Leftrightarrow (0; 3; -3) = \alpha \cdot (0; -4; K)$

$$\Leftrightarrow \begin{cases} 0 = \alpha \times 0 \\ 3 = \alpha \times (-4) \\ -3 = \alpha \times K \end{cases} \Leftrightarrow \begin{cases} 0 = 0 \\ 3 = -\frac{3}{4} \\ -3 = -\frac{3}{4} \times K \end{cases}$$

$\Leftrightarrow -\frac{3}{4} \times K \Leftrightarrow K = \frac{4}{3}$

7

$\vec{a} = (-4; 0; 3)$

a) $\vec{b} = (2; 0; -2)$ ou $\vec{c} = (8; 0; -6) \parallel \vec{a}$.

$\vec{AB} = (2; 0; -2) - (-4; 0; 3)$
$= (6; 0; -5)$

$\vec{AC} = (8; 0; -6) - (-4; 0; 3)$
$= (12; 0; -3)$

$\vec{AC} = 2 \times \vec{AB} \Leftrightarrow$ paralelos.

b)

(Direção ; sentido ; comprimento)

$$\hat{a} = \frac{\vec{a}}{\|\vec{a}\|} = \frac{1}{\sqrt{a_1^2 + a_2^2 + a_3^2}}$$

$$= (a_1 ; a_2 ; a_3)$$

$$\hat{a} = \frac{1}{\sqrt{(-4)^2 + 0^2 + 3^2}} = (-4 ; 0 ; 3)$$

$$\Leftrightarrow \frac{1}{\sqrt{16 + 9}} = (-4 ; 0 ; 3)$$

$$\Leftrightarrow \frac{1}{\sqrt{25}} = (-4 ; 0 ; 3)$$

$$\Leftrightarrow \frac{1}{5} = (-4 ; 0 ; 3) \quad \Leftrightarrow \left(-\frac{4}{5} ; 0 ; \frac{3}{5}\right)$$

$$-\hat{a} = \left(\frac{4}{5} ; 0 ; -\frac{3}{5}\right)$$

$4\hat{a}$ e $-4\hat{a}$

$4\hat{a} = 4 \times (-\frac{4}{5} ; 0 ; \frac{3}{5}) = (-\frac{16}{5} ; 0 ; \frac{12}{5})$

$-4\hat{a} = (\frac{16}{5} ; 0 ; -\frac{12}{5})$

9

$\vec{\mu} = (0 ; 4 ; -3) \quad \vec{v} : \vec{v} \parallel \vec{\mu}$ e $\|\vec{v}\| = 2$

$\vec{v} \parallel \vec{\mu} \Leftrightarrow \vec{v} = k\vec{\mu} = k(0 ; 4 ; 3)$

$= (0 ; 4k ; 3k)$

$\|\vec{v}\| = 2 = \sqrt{0^2 + 4k^2 + 3k^2} = (2)$

$\Leftrightarrow \sqrt{16k^2 + 9k^2} = (2)$

$\Leftrightarrow \sqrt{25k^2} = (2)$

$\Leftrightarrow \sqrt{25k^2}^2 = (2)^2$

$\Leftrightarrow 25k^2 = 4$

$\Leftrightarrow k = \pm\sqrt{\frac{4}{25}}$

$\Leftrightarrow k = \pm \frac{2}{5}$

$k = \frac{2}{5} \vee k = -\frac{2}{5}$

$k = \frac{2/5}{\vec{v}} = (0 ; -\frac{8}{5} ; \frac{6}{5})$ ou $\vec{v}$.

$k = (0 ; \frac{8}{5} ; -\frac{6}{5})$

Sejam $\vec{a} = (a_1 ; a_2 ; a_3)$

e $\vec{b} = (b_1 ; b_2 ; b_3)$

e $\theta = \measuredangle(\vec{a} ; \vec{b})$

$$\vec{a} \text{ e } \vec{b} = \begin{cases} \|\vec{a}\| \|\vec{b}\| \text{ com } \measuredangle(\vec{a};\vec{b}) \text{ se } \vec{a}, \vec{b} \neq 0 \\ 0, \text{ caso contrário.} \end{cases}$$

$$= a_1 b_1 + a_2 b_2 + a_3 b_3$$

Exemplo:

$\vec{\mu} = (1; -2; 3)$

$\vec{v} = (7; -3; 5)$

$$\vec{\mu} \cdot \vec{v} = 1 \times 7 + (-2) \times (-3) + 3 \times 5$$
$$= 7 + 6 - 15$$
$$= -2$$

$\vec{a} \cdot \vec{b} = \|\vec{a}$

Ei cos

Ei $\measuredangle$

$\|\vec{\mu}\| = \sqrt{}$

$\|\vec{v}\| = \sqrt{}$

• $\vec{a} \cdot \vec{b}$

$$\vec{a} \cdot \vec{b} = \|\vec{a}\| \cdot \|\vec{b}\| \cos(\measuredangle(\vec{a};\vec{b}))$$

$$\Leftrightarrow \cos(\measuredangle(\vec{a};\vec{b})) = \frac{\vec{a} \cdot \vec{b}}{\|\vec{a}\| \cdot \|\vec{b}\|}$$

$$\Leftrightarrow \measuredangle(\vec{a};\vec{b}) = \arccos\left(\frac{\vec{a} \cdot \vec{b}}{\|\vec{a}\| \cdot \|\vec{b}\|}\right)$$

$$\|\vec{u}\| = \sqrt{1+4+9} = \sqrt{14}$$

$$\|\vec{v}\| = \sqrt{49+9+25} = \sqrt{83}$$

- $\vec{a} \cdot \vec{b} = 0$ se e só se $\vec{a} \perp \vec{b}$