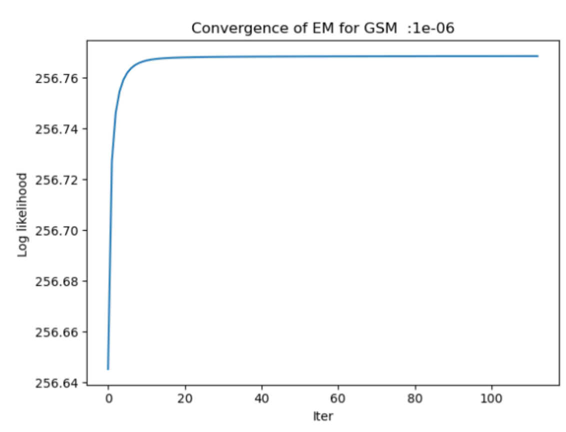
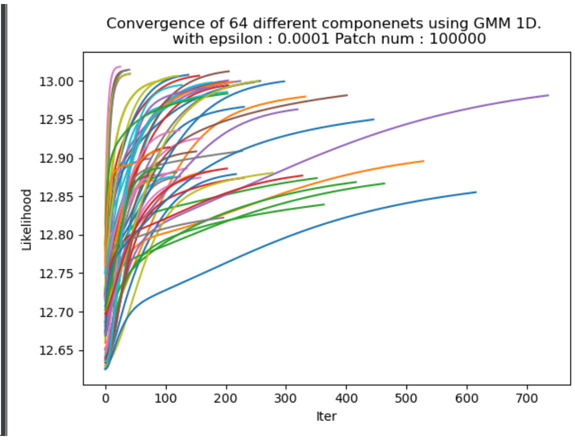




We can clearly see the convergence , the left picture presents the GSM and the right the ICA ,

- A bug\numerical problem i encountered is that sometimes the graph isn't strictly monotonically increasing and he's "dancing" a bit , which contradict the claim that the LL is monotonic increasing , i couldn't find it source.

Evaluating the different models:

Benchmark for comparison : 10 components (k=10) for GMS and k=5 for ICA , 100000 patches , (8,8)patch size , image size =200,200.

Note Different sizes give different denoising quality , for example when i sent a full picture (1900x1400) to the MVN model the result was amazing , and better then anything else at a smaller size level (400,400) , but due to limitations ram wise i couldn't denoise any picutre with the GSM model that's bigger then (400,400) , 1 , my computer \ collab would just crush mid run so i couldn't really compare models at higher resolution.Therefor I make a strong assumption that the quality of the denoising increases linearly with the size of the picture , which could be wrong , maybe an ICA model works better with noise std of 2 at resolution (1200,1800) since he's more localized and the generalization of GMS would actually hurt him when the noise is so severe , or the otherway around and etc.

Run time , Epsilon = 0.001:

The MVN only needs to calculate the covariance matrix and the mean , which is pretty fast resulting in : The MVN is time is pretty short as expected with 0.064 , GMS learning is 1400~ times slower then MVN resulting in 70.94 and the ICA takes the prize with 320.09 seconds , a bit quite longer. i'll explain the disparity using the parameters the model needs to learn in the next paragraph. (epsilion = 0.001)

a GSM model with 10 components needs to learn only $\pi\, r^2$, where $\pi.size =$ 10 and $r^2.size$= 10 on the other hand , an ICA with 10 components needs to learn a different π, r^2 for **every pixel in patch size** , so a if we're working on patches of 64 , we'll get $64 * 20 = 1280$, and if we have 100 components ICA we'll need to learn 7680 parameters compared to 200 parameters for the GMS. So if we assume a patch size of 64 , the amount of parameters with dependence on the competences are :

$$2 * Components = GSM\, learnable\, Parameters$$

$$64 * 2 * Componenets = ICA\, learnable\, Parameters$$

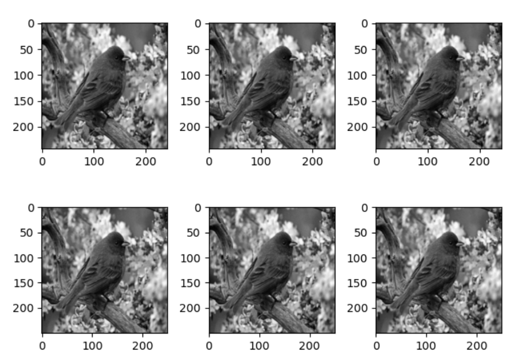
$$MVN\, learnable\, parameters = 0$$

So for our current example , considering the MVN parameter size

$$GSM\, learnable\, paramaters = 20$$

$$ICA\, learnable\, parameters = 1280$$

So more parameters require more time to train which explain the wide difference in run time between the 3 different methods. The following figure is the model reconstruction of the image with



Upper row : Denoised images , MVN , GMS , ICA , left from the right

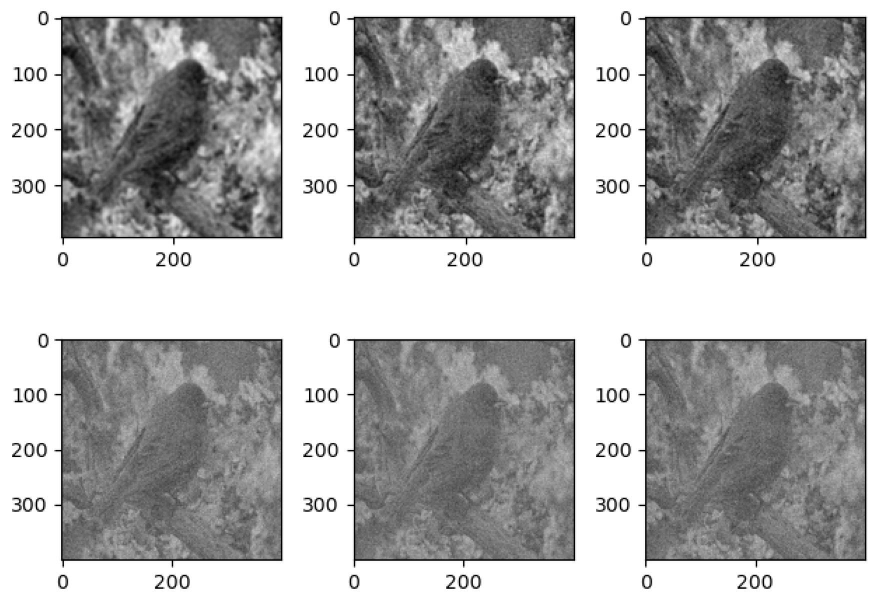
Lower row : Noised images : MVN , GMS , ICA , left from right , Noise level is 0.01

MSE ICA 9.10997512640481e-05

MSE MVN 8.771968120596229e-05

MSE GSM 7.419595622843064e-05

All of the models were able to reconstruct the image on a basic level , which is a good sanity check. The following figure shows the denosiing of the pictures :



Upper row : **Denoised images** , MVN , GMS , ICA , left from the right

Lower row : **Noised images** : MVN , GMS , ICA , left from right , **Noise level is 0.3**

MSE ICA 0.009941623703964163

MSE GSM 0.007241849792936822

MSE MVN 0.005552922661882709

While the MSE from the MVN is the lowest we can see that his denoising quality is the lowest , which make sense since we can see that's he's saving the **original intensity levels** better then the rest , but since the MSE is done in spatial domain it(MSE) isn't linear in the quality of denoising.It only tries to see if the colors match but it doesn't say anything about preserving the high frequencies of the image , therefor as we learned before comparing images in the spatial domain is usually not accurate.A better approach would be to measure the distance of the original picture in Fourier dimension to the denoised picture and check the MSE from the returned image and see if the wavelengths are preserved which would give us (maybe) a more accurate distance measure.

Likelihood over test set :

Likelihhod ICA 12.475121749063234

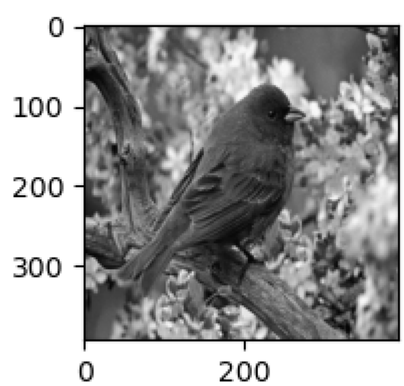
I really dunno how to explain it , maybe the pdf function maxima points are lower because all the of distributions are 1d Gaussian therefor the values are less extreme?

Likelhod GSM 244.3144184889512

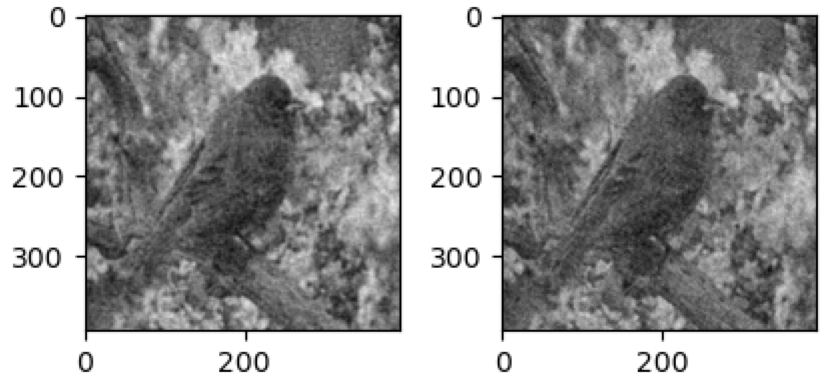
Likelihood mvn 177.6264006379582

Going to the maximal noise level does reveal a few things:

- The MVN is not really compitable with the others , he's only using 1 Gaussian to estimate the image and therefor he's the worse of them.
- GSM and ICA is a bit unclear , but if we look at the beak of the bird we can see it much more sharply , in other words the ICA model preserve the high frequencies of the image at a much better rate then the GSM and therefor he's taking the crown for the current benchmark , but the question if asymptotically can the GMS model perform better then the ICA remains. to comapre , we can see here that the beak(of the bird) is 2 layered with a darker layer on his top side and brighter in his lower , the ICA model managed to capture it while the GSM model couldn't and blurred it all into one big beak with the same color :



Original image



Left : GSM , Right : ICA , We can really see the 2 layered beak in the right picture while it blends in with in the GSM output , which indicate the ability to deal with high frequencies.

Reaching GMS Limit :

In this section I'm checking if the benchmark before was the limit for the GMS model or increasing his training time will result in a better overall image or will it result in an overfit of the parameter? I increased his components number to 20 (40 parameters) , and 30 (60 parameters) ,and made a compression between them :

Epsilion 0.00001

10 Comp learning time GSM learning time : 102.05344152450562

20 Comp learning time GSM learning time : 347.3299403190613

30 Comp learning time GSM learning time : 223.7286777496338

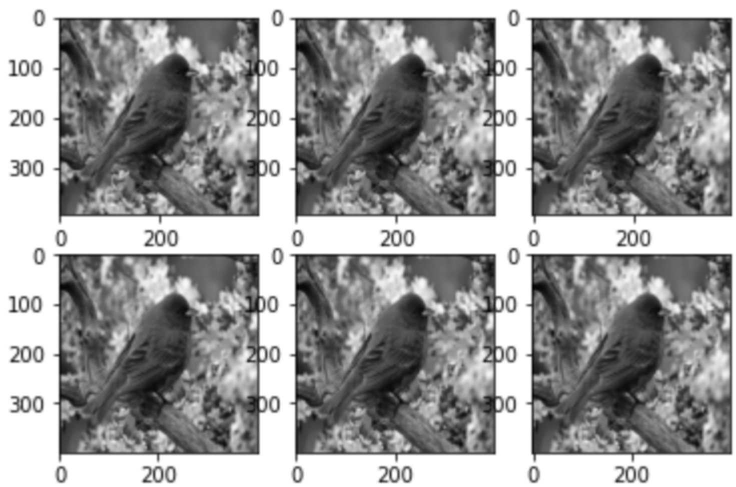
Likelihood 161.10433522848817

Likelihood 20 197.78416280600464

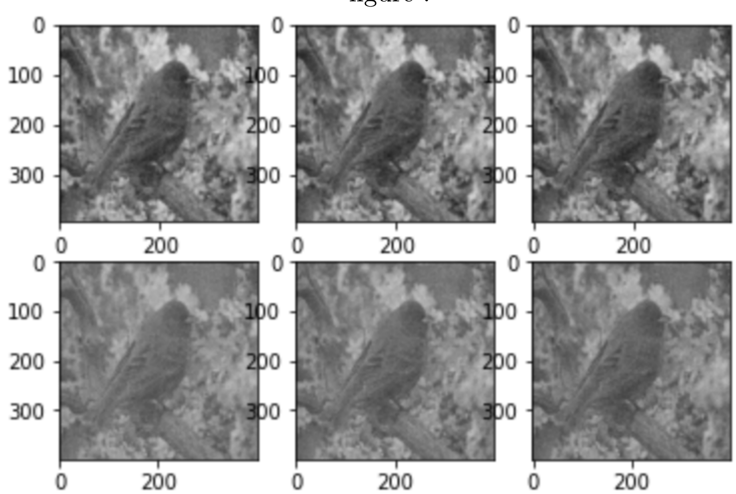
Likelihood 30 200.96604336563422

Conclusion while the likelihood is worse at 20 , he was able to learn the most out of it , but he perhaps also got stuck in a local minima. Since the results aren't so different (below) i'll continiou the final test with the 20 componenets for the GMS. Looking at the reconstruction of the image , with 0.01 we get

:



Upper row is the denoised version : 10 , 20 ,30 , left from right
Lower row is the noised version : 10 ,20 ,30 components , left from
And if we move on to see the results with 0.2 noise with get the following figure :



Upper row is the denoised version : 10 , 20 ,30 , left from right
Lower row is the noised version : 10 ,20 ,30 components , left from right

MSE 10 0.2 noise 0.007510941811677585

MSE 20 0.2 noise 0.007480369207290683

MSE 30 0.2 noise 0.007606872973945596

MSE 10 0.01 noise : 7.738788924137774e-05

MSE 20 0.01 noise 7.917810914180991e-05

MSE 30 0.01 noise 7.853160394503919e-05

We can see that the difference is not major on any account between the MSE of all of them on any level of noise , the lowest being at 20 components for 0.2 noise level , maybe indicating that the best amount of parameters for this task lays there. The denoised image also doesn't show major difference in quality or the type of denoising (blurring denoising or sharp and with some noise) , therefor the difference isn't major when the number of components the GSM have is bigger then 10 and he'll probably behave the same asymptotically for any components number , maybe resulting in over fitting for a bigger number of parameters.Therefor the results from before for the chosen benchmark are an accurate description of the model potential(When the max image size is (400,400)).

Conclusion :

Frequencies restoration The beak demonstrate that the ICA preserves high frequencies the best at the current benchmark , the extensive training of the GSM with more resources explain that he's not getting much better then he already was in the frequencies department and the only was ICA can go is up , which make sense based on the amount of parameters ICA have compared to the GSM. Also the MVN isn't really competitive and therefor the ICA wins here.

Noise removed Both GMS and ICA image still remains with a bit of noise at the same level , therefor they both lose the same here.

Original colors Both GMS and ICA return an image with intensity levels that are not as accurate as the original ones , therefor they both score bad at this department

Therefor his(mine) hard fought battle resulted in winning over the GMS by preserving the high frequencies of the image (the edges) in a higher quality and he's taking the trophy home with pride.