

Please read before using the maple code

David Plenty and Maureen P. Edwards

School of Mathematics and Applied Statistics, University of Wollongong, NSW  
2522, AUSTRALIA

November 13, 2024

## Please read before using the maple code

- We acknowledge that the code was constructed using Maple [2], which is a trademark of Waterloo Maple Incorporated.
- We ask that anyone who uses the code gives appropriate references to Maple according to [1]. We ask that anyone who uses the code appropriately acknowledge David Plenty and Maureen P. Edwards.
- The mathematical formulae for constructing the code in Maple were sourced from Arrigo [3] and Bluman and Anco [4].
- The code has been critically examined for mistakes and thoroughly tested using reaction-diffusion equations.
- The infinitesimals in 1-D are given by  $X$ ,  $T$  and  $U$ . The infinitesimals in 2-D are given by  $X$ ,  $Y$ ,  $T$  and  $U$ . The infinitesimals in 3-D are given by  $X$ ,  $Y$ ,  $Z$ ,  $T$  and  $U$ .
- For each classical code, E is the equation for classical invariance; it is not the system of determining equations! You must obtain the determining equations by plucking out the coefficients of the different derivatives of  $u$  (including the zeroth order derivative  $u$  and 1).
- Each classical and non-classical code solves for the highest derivative, i.e.  $u_{xx}$  for 1-D,  $u_{yy}$  in 2-D and  $u_{zz}$  in 3-D. So, if you wanted a complete classification of an equation, you would need to perform a classical symmetry analysis on the equation, considering the restrictions made when solving for the highest derivative. Care is required because the equation may not be second order if you do this.
- The non-classical code with  $T = 1$  is for the class of evolution equations

$$u_t = F(x, t, u, u_x, u_{xx}). \quad (1)$$

The only usage of the invariant surface condition

$$Xu_x + Tu_t = U \quad (2)$$

is solving for  $u_t$ . If you consider other classes of equations, you may need to incorporate the differential consequences of the invariant surface condition to take care of  $u_{xt}$  and or  $u_{tt}$ . The equation for non-classical invariance is E, but it is not the system of determining equations! You must obtain the determining equations by plucking out the coefficients of the powers of  $u_x$ .

- The non-classical code with  $T = 0$  is for the class of evolution equations (1). The invariant surface condition (2) substitutes expressions for  $u_x$  and  $u_{xx}$ . If you consider other classes of equations, you may need to incorporate the differential consequences of the invariant surface condition to take care of  $u_{xt}$  and or  $u_{tt}$ . The single determining equation is given by E.
- After using the code construct equation E, it is up to the user to ensure that all proceeding mathematical results obtained are well-defined: we are not responsible for any erroneous results that are calculated after the correct equation for E is found.

- Be weary of using Maple's `dsolve` and `pdsolve` commands! They can miss special cases when searching for anti-derivatives. For example, when searching for the anti-derivative of a power law, e.g.  $x^n$  with  $x > 0$  and  $n \in \mathbb{R}$ , it will often miss the case when  $n = -1$  with anti-derivative  $\ln(x) + C$ . It will return the anti-derivative  $\frac{x^{n+1}}{n+1} + C$ , where  $C \in \mathbb{R}$  is the arbitrary constant of integration.

# Bibliography

- [1] Online Help: Author Information. <https://www.maplesoft.com/support/help/Maple/view.aspx?path=author#:~:text=Use%20the%20following%20statement%20to,.%2C%20Waterloo%2C%20Ontario.%22>.
- [2] Maple (2021), Maplesoft, a Division of Waterloo Maple Incorporated; Waterloo, Ontario, 2021.
- [3] D Arrigo. *Symmetry Analysis of Differential Equations*. John Wiley & Sons Inc: Hoboken, New Jersey, USA, 1st edition, 2015. ISBN 978-1118721407.
- [4] G W Bluman and S C Anco. *Symmetry and Integration Methods for Differential Equations*. Springer: New York, New York State, USA, 2nd edition, 2002. ISBN 978-0-387-98654-8.