MINE SHAFT REPORT

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ABSTRACT

The purpose of this report is to measure the vertical depth of the deepest mine on Earth by dropping a 1 kilogram mass down the shaft and measuring how long it would take to reach the bottom to approximate the depth of the mine accounting for drag and the speed dependence of drag, the effect of the Earth's rotation on the falling mass as it travels through the tunnel and the adjusted time it will then take to reach the bottom. This report also proposes a trans-planetary shaft from pole to pole of the Earth and takes into account different variations of density profiles for distributions of Earth mass density. Then we consider a shaft on the moon of similar proportions and variations of fall time due to variations in factors and structure of the moon in comparison to the Earth.

I. Introduction

To prove that the deepest tunnel on Earth is 4 kilometers, simulations have been run to determine if this is close to the true value of its depth. The following simulations have taken into account multiple factors that determine and affect the trajectory of a test mass which we have determined to have a weight of 1 kilogram, this test mass will serve as a probe to determine the true depth of the shaft. Factors that can affect this test mass are variable gravity, as the test mass travels down the shaft, gravity is not constant, therefore we must assume that gravity will change as depth changes and will affect the overall velocity of the mass, drag affects the fall time of the mass and determines its terminal velocity of 50 m/s that extends the time needed for the mass to reach the bottom of the shaft, the coriolis effect, which is due to the Earth's rotation, will make the test mass bump into the walls of the shaft as it falls. Simulations have also been performed to determine what would happen to the test mass if the shaft was infinitely deep.

Geology tells us that Earth's mass is not uniformly distributed and thus we must account for what this means for the test mass as it falls through the Earth in the theorized infinitely deep shaft. Taking into account this non-uniform density, we must account for the changes of the forces as the test mass falls through the Earth. A proposed trans-lunar mine shaft has also been simulated in which tests have been performed to see how the same 1 kilogram mass will behave in its fall through the moon.

II. Depth of Mine Shaft

Using plotting technology such as *python's matplotlib library* that allows us to simulate the 1 kilogram mass falling down the shaft with different factors taken into account. We can also use *python's numpy library* and functions from *python's scipy library* to integrate and solve differential equations.

The following differential equation dictates how a projectile moves with constant gravity and drag:

$$\frac{d^2y}{dt^2} = -g + \alpha\left|\frac{dy}{dt}\right|^{\gamma} \qquad (1)$$

Where t is time, y is the height, g is the gravitational acceleration, α is the drag coefficient, and γ is the speed dependence of the drag coefficient. Using this differential equation we can calculate the fall time using *scipy*'s *solve_ivp* function to be able to simulate the test mass falling in different environments and their times to hit the bottom.

	No drag, Constant g	No drag, Linear g	Drag, Linear g
Time (seconds)	28.6	28.6	83.5

The difference between constant and linear gravity is nearly unnoticeable due to the shaft depth only being a fraction of Earth's radius. Applying drag allows for the mass to reach a terminal velocity of 50 m/s, this drastically impacts the time it takes for the mass to reach the bottom.

III. Coriolis Effect

Due to the rotation of the Earth, as the mass falls through the Earth, it experiences a movement as it falls down the shaft, this effect is described by the following equation describes the coriolis force F_c:

$$F_c = -2m(\Omega\, x\, v) \tag{2}$$

where Ω is the Earth's rotation rate which is approximately half a kilometer per second, v is the tangential velocity of the object, and m is the mass of the object. The mass will eventually hit the wall due to this force before reaching the bottom. The mass hits the wall with drag at a depth of 2697.1 m at a time of 29.6 s and with no drag at a depth of 1650.4 m at a time of 21.9 s. Drag makes a significant difference in the time and depth until striking the wall. Due to this effect, we can see an evident issue with continuing with this depth measuring technique as we cannot assume that the mass will not hit the shaft wall, this means that the total travel time will be much longer than originally predicted.

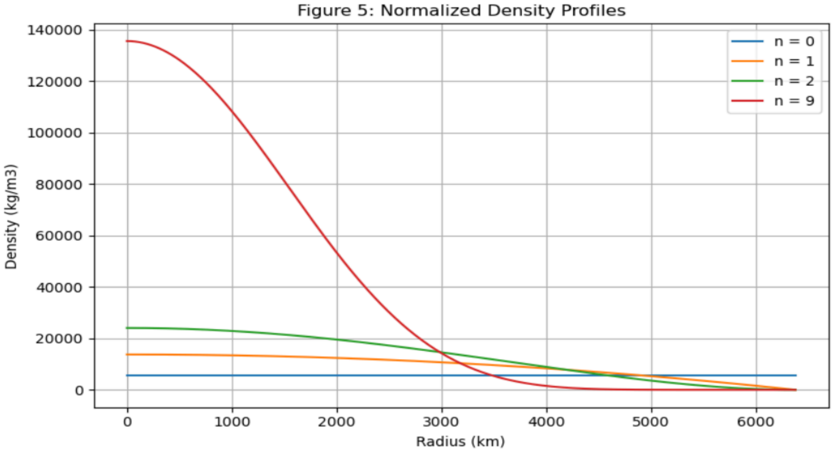
IV. Non-Uniform Density and Trans-planetary Tunneling

We must consider in our calculations that the Earth's density isn't uniformly distributed meaning that forces vary depending on your depth inside Earth. To do this we must consider the following equation describing the change in density ρ as a function of radius r from the center of the Earth:

$$\rho(r) = \rho_n\left(1 - \frac{r^2}{R_\oplus^2}\right)^n \tag{3}$$

Where n is some number, ρ_n is a density constant, and $R_\oplus$ is Earth's radius. This function gives us how the density changes as the radius from the Earth so we therefore calculate the change in density over different n exponents, we will test n = {0, 1, 2, 9} to obtain the following data of density distribution within the Earth:

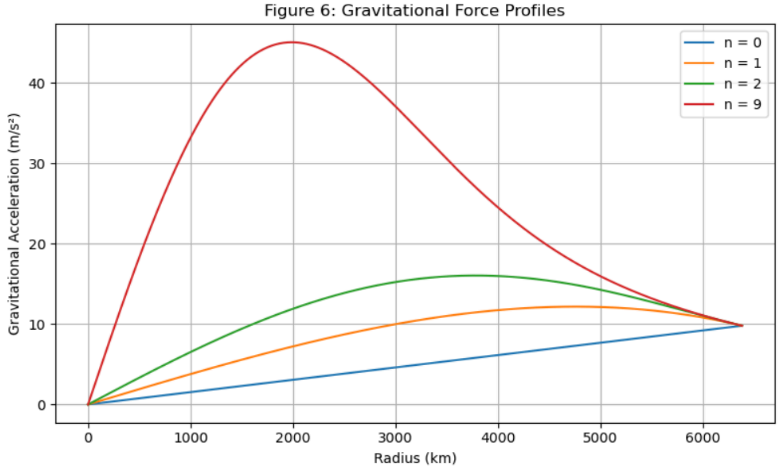
Figure 5: Normalized Density Profiles



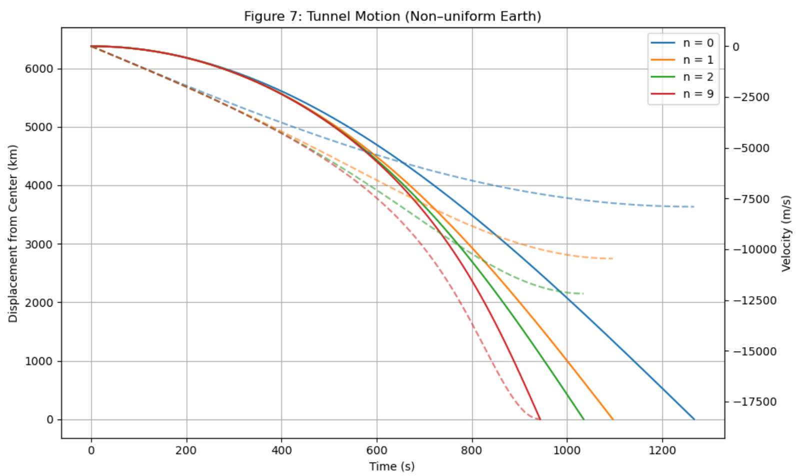
Here we see how the difference in exponents dictate how the Earth mass is distributed, where n = 2 is closer to the actual value of density distribution, this helps visualize how density inside the Earth changes.

Using the data received above we can then calculate the varying forces within the Earth due to the non-uniform density:

Figure 6: Gravitational Force Profiles



Here we see how the gravitational acceleration varies with depth from the center of the Earth, we see that they all converge on 9.81 m/s^2 due to that being the surface gravity we observe. Two extreme cases we observed are when n=0 and 9, at 0 we can assume that the density is uniformly distributed which is not true, and at 9 we assume that density is concentrated near the center affecting the forces drastically. We can then observe with these new forces due to non-uniform density how the 1 kilogram mass will behave in a tunnel that reaches the core of the Earth:



We can see how the times, velocities, and positions vary from each other due to the difference in density profiles. We observe the time and velocity at the center for n = 0 to be 1267.3 s & 7905.3 m/s. n=1 as 1096.5 s & 10457.7 m/s. n=2 as 1035.1 s & 12182.9 (m/s). n=9 as 943.8 s & 18370.7 (m/s) all respectively for time and velocity. Then we consider a trans-planetary and trans-lunar tunnel that reaches pole-to-pole for the Earth and the Moon, we take note of the crossing times (when the mass reaches the other side/pole) as they are vastly different from one another. For the Earth assuming uniform density we take note that the crossing time is 4976.5 s while the Moon has a crossing time of 6500.5 s to reach the other side, this is due to their varying forces, since on the moon there's little to no atmosphere and therefore little to no drag, the moon is also lighter than the Earth and therefore much less gravitational force. We can therefore see that there is a relationship between the crossing times between the orbital period and the crossing time of the mass; The time it would take for an object to fall from one pole of the Earth to another would be equal to the time it would take for an object to orbit around the Earth. This time proportional to $\frac{1}{\sqrt{\rho}}$ meaning that the time it would take to cross is proportional to the density of the body.

V. Conclusion and Future Work

To find the depth of the mine shaft we have to take into account multiple effects that may affect the travel time of the 1 kilogram mass, effects such as the coriolis effect, drag, and non-uniform density all have an effect on how the mass will behave during its trajectory to hit the bottom. During this experiment, we have assumed that the Earth is perfectly spherical and thus we assumed that the density is distributed evenly from the core to the surface, ignoring any imperfections (Mountains, valleys, etc). We also assumed that mining straight through the Earth would be a feasible project, realistically this is not feasible. The assumption is made that the 1 kilogram mass will in fact reach a terminal velocity at 50 m/s which may not be true. Lastly we neglect other forces that may affect the travel time of the mass, such as friction from bumping on the walls of the shaft. To make this experiment more realistic, we must consider that the Earth is evidently not a perfect sphere, and that other physical forces are affecting the mass as it falls down the shaft.