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Multivariate design of socioeconomic drought and impact of water reservoirs

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ABSTRACT

Socioeconomic drought in association with minimum instream flow against the backdrop of local water supply was investigated. An integrated procedure for design combinations of drought properties, such as duration, severity, and peak, involving the truncation of drought events, the goodness-of-fit of joint and marginal distributions of drought properties, the determination of design combinations of these properties for a given Kendall return period, and the evaluation of uncertainty of the combinations, was developed. In multivariate design of socioeconomic droughts in a case study, univariate, bivariate and trivariate design values of drought properties and their alterations due to the regulation of water reservoirs were computed. Results showed that any two properties of drought exhibited a high positive dependence. For a given bivariate return period, the pairs of cumulative frequencies of drought properties formed a symmetric curve for truncated samples and a symmetric curving-belt for large quantities of simulated samples. For a given trivariate return period, the pairs of cumulative frequencies of duration and peak showed a symmetry, but the pairs of duration and severity or severity and peak were remarkably scattered on the side of severity and comparatively concentrated on the side of duration or peak. For the confidence interval of probability of 0.95, the range of trivariate design values was larger than that of bivariate design. The differences between drought design values of univariate, bivariate, and trivariate designs were small, which resulted from high correlations of drought properties, the use of Kendall frequency, and approximate identical cumulative frequencies in design combinations. The decrease of socioeconomic drought properties under the regulation of water reservoirs was significant, but the drought spell, total volume, and monthly maximum of water deficit for a 5-year return period still accounted for 3.06–3.27 months, 0.426–0.470 billion m^3, and 0.211–0.217 billion m^3, respectively, provided that the local water supply was met.

1. Introduction

The National Drought Mitigation Center of USA considers drought as a creeping insidious natural hazard, which originates from precipitation deficiency over an extended period of time, and generally refers to water shortage, which results in losses to agriculture and livestock, socioeconomic disruption, ecological environmental imbalance, and even endangerment of human life. Impacts of drought result from the interplay between natural events and water demands of regional socio-economy, and can be exacerbated by human activities. Therefore, drought cannot be treated exclusively as a physical phenomenon. Its definitions vary from field to field and its impacts vary from region to region (Dracup et al., 1980). Wilhite and Glantz (1985) divided droughts into four categories: meteorological, agricultural, hydrological, or socioeconomic. The first three types of droughts are viewed as a physical phenomenon and socioeconomic drought is associated with local water supply, which tracks water demand through socioeconomic systems.

Human impacts on the terrestrial hydrosphere are now so widespread that it is difficult to find a watershed that does not reflect the interaction among human activities and natural hydrologic processes (Wada et al., 2013; Vörösmarty et al., 2010; Van Loon and Van Lanen, 2013; Vogel et al., 2015). The newly emerging discipline of socio-hydrology, entailing links and feedbacks between water resources and society, is receiving much attention these days (Wada et al., 2011; Sivapalan et al., 2012; Madani, 2014; Montanari, 2015; Sivapalan, 2015; Loucks, 2015; Troy et al., 2015; Wheater and Gober, 2015). As population, industries and urbanization grow, the water demand

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increases and socioeconomic drought becomes a major concern (Mehran et al., 2015). Socioeconomic drought thereby refers to the condition that water supply cannot satisfy demand, leading to societal and economic disruptions, and environmental impacts (Garen, 1993; Arab et al., 2010; Lapp, 2012; Sivapalan et al., 2012; Hill et al., 2014; Mehran et al., 2015; Wheater and Gober, 2015). Socioeconomic drought, as a seasonal deficit of precipitation combined with the increase of regional water demand due to climate change and regional development, leading to increasing stress on water supply, calls for further studies.

For measuring socioeconomic drought, statistical indices, e.g. reliability, resilience, and vulnerability, first introduced by Hashimoto et al. (1982), have been developed (Moy et al., 1986; Jinno, 1995; Vogel and Bolognese, 1995; Srinivasan et al., 1999; Vogel et al., 1999; Ward et al., 2013; Steinschneider and Brown, 2012). However, these indices differ in their computational methodologies (Jain and Bhunya, 2008; Mehran et al., 2015). Integrating resilience and vulnerability for multi-reservoir systems is not easy (Miller et al., 2010). In addition, these indices cannot exhibit the frequency distribution of any of the important drought properties, e.g. duration, severity of water deficit, etc., which are generally concerned with water resources planning and management.

Water supply usually entails withdrawals from rivers. An appropriate flow in a certain section of river is maintained to preserve its ecological environment on one hand and guarantee the safety of various withdrawals for meeting water demands of socioeconomic systems on the other hand. From another perspective, the expected flow in association with water demand determines the occurrence and frequency of socioeconomic drought. The socioeconomic drought properties, including duration, severity, and peak, can be analyzed on the basis of run method, widely applied in hydrological drought analysis (Zelenhasic and Salvai, 1987; Tallaksen et al., 1997; Tu et al., 2016; Zhang et al., 2013, 2015), by truncating a series of events less than the expected streamflow.

In practice, frequency analysis of socioeconomic drought properties is similar to that of hydrological drought. The difference is that the truncation level of hydrological drought is a flow of a certain percentile less than the average value of streamflow, for example from Q50 to Q95, and that of the socioeconomic drought is closely associated with the requirement of instream flow for satisfying withdrawals of regional socioeconomic systems. Apparently, drought properties of the truncated events are not only individual random processes but also have dependences between them. That is, a sequence of events is viewed as a multivariate random process in association with multivariate dependences. Copula functions, which can model random variables and their dependences and their multivariate joint distributions, have been widely applied in multivariate drought studies (Favre et al., 2004; Salvadori and De Michele, 2004, 2007; Shiau, 2006; Shiau et al., 2007; Dupuis, 2007; Modarres, 2007; Kao and Rao, 2010; Song and Singh, 2010; Mishra and Singh, 2010, 2011; De Michele et al., 2013; Chen et al., 2013, 2015; Tu et al., 2016).

In the analysis of joint return period, the Kendall frequency was applied to improve the calculation of joint probability (Salvadori et al., 2011, 2013; Chen et al., 2013; Jiang et al., 2015). As shown by Volpi and Fiori (2014) for flood events occurring in a reservoir catchment, the Kendall frequency is not necessarily the safety standard for evaluating the joint distribution of multiple variables. Indeed, it is a transfer of probability by defining a multivariate joint frequency as a random variable. The transferred probability exceeding the percentile is larger than the original probability, that is, using the transfer becomes safer for design droughts. From another point of view, design values of multivariate return periods using the Kendall frequency are equally comparable to those of individual univariate return periods. In recent years, multivariate design has received significant attention (Salvadori et al., 2011, 2013; Volpi and Fiori 2012, 2014; Vandenberghe et al., 2012; Serinaldi, 2013; Serinaldi and Kilsby, 2015). As a joint distribution of drought properties is determined by truncated samples, how to determine a combination of drought properties for a given joint return period is perceived as a design realization of drought properties. There is a one-to-one correspondence between return period and design value for an individual design property, but a given joint return period actually leads to many combinations of drought properties and these combinations are generally not equivalent in practice. However, for drought management and mitigation of water deficit, one may be expected to determine just one combination. A most-likely weight function with respect to a product of joint and marginal probability densities was proposed to exclusively determine one design combination of a given joint return period (Salvadori et al., 2011). Then, using the confidence interval (CI) that includes critical events with a significant probability level, the uncertainty of a design combination can be evaluated (Volpi and Fiori, 2012, 2014; Vandenberghe et al., 2012).

In addition, reservoirs play a key role in managing water supply and reducing the impact of socioeconomic droughts (Nalbantis and Koutsoyiannis, 1997; Nalbantis and Tsakiris, 2009; Van Loon and Van Lanen, 2013; Mehran et al., 2015). At the end of the last century, approximately 20% of the total global annual river discharge was controlled by man-made reservoirs, and 70% of global freshwater withdrawals came from these reservoirs (Fekete et al., 1999; Shiklomanov et al., 2000; Vörösmarty and Sahagian, 2000), indicating the importance of reservoirs in providing resilience for human water use globally (Zhang et al., 2014b; Mehran et al., 2015).

The Dongjiang River of South China is the main source for water supply of several booming megalopolises in the Pearl River Delta, e.g. Hongkong, Guangzhou, Shenzhen and Dongguan. Due to its south subtropical humid climate zone, the Dongjiang River basin is rich in water resources, resulting from average annual precipitation of 1770 mm. With the rapid development of local socio-economy, the imbalance of water supply and demand has been increasing in the 21st century. In addition, streamflow processes in the Dongjiang River have been significantly altered by human activities, in particular the regulation of water reservoirs (Tu et al., 2012, 2015; Zhang et al., 2014a).

In the Dongjiang River basin, several studies have paid significant attention to hydrological droughts in association with a certain percentile of streamflow below the mean value (Chen et al., 2013; Zhang et al., 2013; Tu et al., 2016). However socioeconomic droughts, involving water demand of local socioeconomic systems, become more important for local water resources management.

Therefore, in the Dongjiang River basin as a case study, multivariate design of socioeconomic drought in association with a minimum instream flow of water management was investigated in this paper. Drought events were truncated for monthly streamflows by using the run method. The multivariate distribution of drought properties was determined by using the copula function. The joint probability of drought properties exceeding the percentile was transferred by using the Kendall frequency. An integrated procedure of multivariate design of socioeconomic drought involving the evaluation of uncertainty on the basis of the most-likely weight was proposed. Then, univariate, bivariate, and trivariate design values of individual drought properties were compared, and the impacts of water reservoirs on socioeconomic drought were evaluated.

2. Methodology

2.1. Socioeconomic drought events

A series of drought events using the run method can be truncated with a predefined threshold value of streamflow. The selection of a truncation level depends on the type of drought to be studied (Dracup et al., 1980). For hydrological drought, a certain percentile of streamflow below the mean or median value can be set as the truncation level (Zelenhasic and Salvai, 1987; Clausen and Pearson, 1995; Tallaksen et al., 1997; Tu et al., 2016). For socioeconomic drought, the truncation

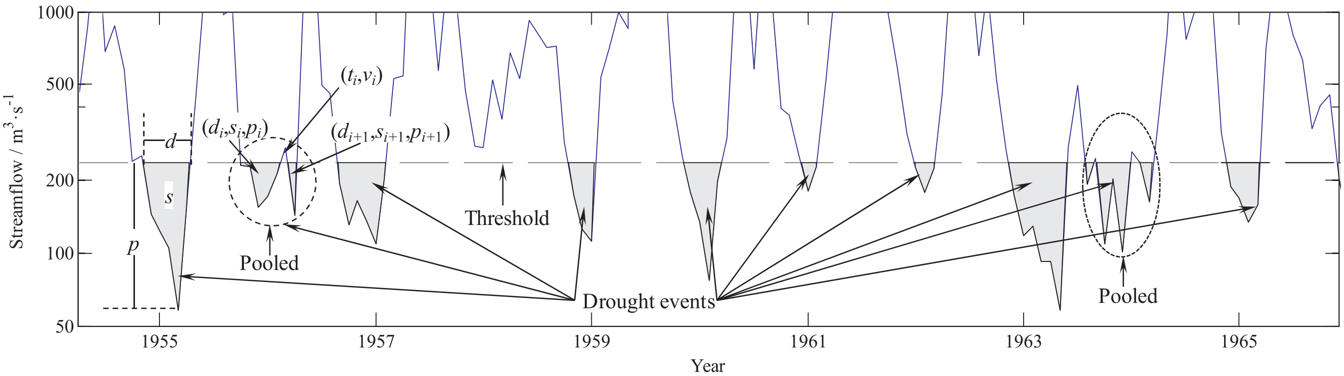

Fig. 1. Truncation and pooling of drought events. Streamflow data are for a certain period from the Boluo gauging station in the Dongjiang River basin, South China.

level was set by minimum instream flow, Q_m, in order to satisfy the regional water supply or considering social-economic development, water environmental protection, etc. (Mathier et al., 1992; Madsen and Rosbjerg, 1995; Tu et al., 2017).

As shown in Fig. 1, a drought event was characterized by duration d, severity s, peak p, which, respectively, refer to drought spell, total volume of water deficit during drought, and maximum deficit for a certain time scale, e.g. monthly. Between two adjacent drought events with longer duration and higher severity, streamflow with shorter duration and smaller magnitude but beyond the threshold occur. The two adjacent drought events were regarded to be subject to a drought, which was split by the shorter streamflow exceeding the threshold. Therefore, in consideration of inter-event time and volume, a criterion for pooling mutually dependent droughts was introduced by Zelenhasic and Salvai (1987). Subsequently, it was revised by Madsen and Rosbjerg (1995), and has been used (Zhang et al., 2013; Tu et al., 2016).

Let (d_i, s_i, p_i) and (d_{i+1}, s_{i+1}, p_{i+1}) be two adjacent droughts with a spell, t_i, and excess volume of spell v_i. If the spell was less than a predefined critical duration t_c, and the ratio ρ_i between the volume and that of the preceding deficit volume s_i was less than a predefined critical value ρ_c, then, (d_i, s_i, p_i) and (d_{i+1}, s_{i+1}, p_{i+1}) were pooled into a new drought calculated as:

$$\begin{cases} d_{\text{pool}} = d_i + d_{i+1} + t_i \\ s_{\text{pool}} = s_i + s_{i+1} + v_i \\ p_{\text{pool}} = \max(p_i, p_{i+1}) \end{cases} \tag{1}$$

where d_{pool}, s_{pool} and p_{pool} refer to the duration, severity, and peak after the two adjacent drought events were pooled, respectively (see a dash cycle in the left part of Fig. 1).

The pooling was ceaselessly repeated for the already pooled droughts by using Eq. (1) if both t_i and ρ_i were less than t_c and ρ_c (see a dash oval in the right part of Fig. 1). Using a monthly time scale, a six-month inter-event time, t_c, can be generally fixed as the critical duration due to half and half of wet and dry seasons within a year in this case study. The critical ratio of excess volume of inter-event time and the preceding deficit volume, ρ_c, can be set as 0.3 (Madsen and Rosbjerg, 1995; Zhang et al., 2013, 2015). In practice, the critical ratio, ρ_c, generally plays a key role in pooling together drought events.

2.2. Marginal and joint distributions of drought properties

Assume that X, which refers to any one of duration d, severity s, peak p of drought, is a continuous random variable. $F(x)$ which refers to its cumulative distribution function can be formulated as:

$$F(x) = P[X \leqslant x] = u \tag{2}$$

where u also refers to a cumulative distribution frequency (CDF) (non-exceedance probability) of duration, severity, or peak.

Several conventional univariate distributions, i.e. two-parameter lognormal distribution (LOGN) and Weibull distribution (WBL), and three-parameter generalized extreme value distribution (GEV), generalized logistic distribution (GLO), and generalized normal distribution (GNO) were fitted to the marginal distributions of drought properties.

Then, using the Sklar theorem (Nelsen, 2006), an m-dimensional joint distribution function of drought properties, $H(x_1,\cdots,x_m)$, can be defined as:

$$H(x_1,\cdots,x_m) = C[F(x_1),\cdots,F(x_m)] = C(u_1,\cdots,u_m) \tag{3}$$

where $C(u_1,\cdots,u_m)$ is the m-dimensional copula function which is the joint distribution function of standard uniform random variables within the range of [0, 1], and $u_i = F(x_i)$, $i = 1, \cdots, m$, refer to individual CDFs of drought properties, and $m = 2$ or $m = 3$ for any two variables or three variables of drought properties, respectively.

Owing to its flexibility, accessibility and simple parameters in association with a correlation coefficient matrix, an m-dimensional *meta*-Gaussian copula was selected for modeling the joint distribution of drought properties. Its theoretical cumulative distribution function, $C(u_1,\cdots,u_m)$, and density function, $c(u_1,\cdots,u_m)$, were deduced as follows (Genest and Favre, 2007):

$$C(u_1,\cdots,u_m) = \int_{-\infty}^{b_1}\cdots\int_{-\infty}^{b_m} g(\omega_1,\cdots,\omega_d)d\omega_1\cdots d\omega_m \tag{4}$$

$$c(u_1,\cdots,u_m) = |\sum|^{-1/2}\exp(-\zeta^T\Sigma^{-1}\zeta/2 + \zeta^T\zeta/2) \tag{5}$$

where

$$g(\omega_1,\cdots,\omega_m) = (2\pi)^{-m/2}|\Sigma|^{-1/2}\exp(-\omega^T\Sigma^{-1}\omega/2) \tag{6}$$

where $b_1 = \Phi^{-1}(u_1)$, $\cdots, b_m = \Phi^{-1}(u_m)$, in which $\Phi^{-1}(\cdot)$ refers to the inverse function of the standard normal distribution; and $\omega = [\omega_1,\cdots,\omega_m]^T$ and $\zeta = [b_1,\cdots,b_m]^T$ are the matrices of variables in the integrand. The correlation coefficient matrix Σ was expressed as:

$$\Sigma = \begin{bmatrix} 1 & \cdots & \rho_{1j} & \cdots & \rho_{1m} \\ \vdots & & \vdots & & \vdots \\ \rho_{i1} & \cdots & \rho_{ij} & \cdots & \rho_{im} \\ \vdots & & \vdots & & \vdots \\ \rho_{m1} & \cdots & \rho_{mj} & \cdots & 1 \end{bmatrix}, \quad \rho_{ij} = \begin{cases} 1, & i = j \\ \rho_{ji}, & i \neq j \end{cases} \tag{7}$$

where $\rho_{ij} \in [-1, 1]$ refers to the correlation coefficient between any two drought properties.

For the goodness-of-fit of marginal and joint distributions of drought properties, the Anderson-Darling's (A-D) test method (Panchenko, 2005; Dobric and Schmid, 2007; Genest and Favre, 2007; Song and Singh, 2010; Ma et al., 2013; Tu et al., 2016) was used.

For the marginal distribution of drought properties, let x be any one of drought properties with a length of n. Then, the A-D statistic A_n^2 was computed as:

$$A_n^2 = -n - \frac{1}{n}\sum_{i=1}^{n}(2i-1)\{\ln F(x_i) + \ln[1-F(x_{n-i+1})]\} \tag{8}$$

where $F(x_i)$ is the CDF derived from the tested distribution.

For the joint distribution of drought properties with an m-dimension, A_n^2, was formulated as:

$$A_n^2 = -n - \frac{1}{n}\sum_{i=1}^{n}(2i-1)\{\ln F(S_i) + \ln[1-F(S_{n-i+1})]\} \tag{9}$$

where S_i, $i = 1, \cdots, n$ refers to the pseudo-observations of drought properties from a χ_2^2 -distribution random variable. For the bivariate or trivariate joint distribution of drought properties, S was, respectively, calculated as:

$$S = [\Phi^{-1}(u_1)]^2 + [\Phi^{-1}(C(u_2|u_1))]^2 \tag{10}$$

or

$$S = [\Phi^{-1}(C(u_1, u_2))]^2 + [\Phi^{-1}(C(u_3|C(u_1, u_2)))]^2 \tag{11}$$

The bootstrap procedure for the goodness-of-fit test statistics of multi-variate joint distribution, A_n^2, was employed (Dobric and Schmid, 2007; Genest et al., 2009; Song and Singh, 2010; Ma et al., 2013).

The critical value at the significance level $\alpha = 0.05$ for the goodness-of-fit test of theoretical individual and joint distributions of drought properties was obtained by the Monte Carlo method with 5000 simulations or more. If a P-value of the test statistic computed from the samples was more than the significance level, the corresponding theoretical distribution was accepted.

For univariate distributions of drought properties, root-mean-square error (RMSE) and Akaike information criterion (AIC) were used to select the optimal distribution provided that several univariate theoretical distributions were accepted. In addition, empirical and theoretical distributions were compared for assessing the selected marginal and joint distributions of drought properties. The empirical distribution function was defined as (Dobric and Schmid, 2007; Genest et al., 2009; Tu et al., 2016):

$$F(x_1, \cdots, x_m) = \sum_{i=1}^{n} I\{X_1(i) \leqslant x_1, \cdots, X_m(i) \leqslant x_m\}/n \tag{12}$$

where function $I(\cdot)$ is an indicator function, which is equal to one when the enclosed expression is true, but is zero otherwise. In hydrological practice, the empirical probability was usually transformed as (Gringorten, 1963):

$$F_e = \frac{F(x_1, \cdots, x_m) - 0.44}{n + 0.12} \tag{13}$$

where $m = 1$, $m = 2$, or $m = 3$ for any one variable, any two variables or three variables of drought properties, respectively.

2.3. Return periods of drought properties

The selection of probabilities of a random variable below or exceeding the given percentiles depends on different hydrological questions, for example the probabilities exceeding the percentiles are of main concern for drought properties. For the multivariate combination, various types in association with the operators AND, OR, and Conditional can be used (Salvadori and De Michele, 2004). However, in the statistical analysis of drought properties, the individual probabilities of a combination of random variables simultaneously exceeding the percentiles were of main concern. Therefore, for $\bar{u}_i = 1-u_i$, $i = 1, 2, 3$ referring to individual probabilities exceeding the percentiles of three properties of drought, let $\hat{C}(u_1, u_2)$ be the pair of bivariate exceedance probabilities of any two drought properties, defined as:

$$\hat{C}(u_1, u_2) = 1 - \sum_{i=1}^{2} u_i + C(u_1, u_2) \tag{14}$$

Then, let $\hat{C}(u_1, u_2, u_3)$ be the trivariate exceedance probabilities of drought properties, defined as:

$$\hat{C}(u_1, u_2, u_3) = 1 - \sum_{i=1}^{3} u_i + \sum_{i=1}^{2}\sum_{j=i+1}^{3} C(u_i, u_j) - C(u_1, u_2, u_3) \tag{15}$$

Further, a Kendall function, which is a univariate representation of multivariate information (Genest and Rivest, 1993; Barbe et al., 1996; Salvadori et al., 2011, 2013), has been shown to be a good tool for calculating a copula-based joint frequency of multivariate events (Nappo and Spizzichino, 2009; Salvadori et al., 2011), and has been applied in computing the joint return period of hydrological multivariables (Salvadori et al., 2013). The Kendall frequency was estimated as:

$$K_{\hat{C}}(q) = P[\hat{C}(u_1, \cdots, u_m) \leqslant q] = \frac{1}{n}\sum_{i=1}^{n} I(\hat{C}_i \leqslant q) \tag{16}$$

where $q \in (0, 1)$ is a probability level and n refers to the length of observed or simulated samples. The method of solution refers to Algorithms 1 and 2 presented by Salvadori et al. (2011).

Based on the above discussion, T_1 of the univariate return period of drought properties was calculated by individual marginal probabilities exceeding the defined percentile as follows:

$$T_1 = E(L)/\bar{u}_1 \tag{17}$$

where $\bar{u}_1$ refers to the exceedance probability of any one of drought properties.

Then, using the Kendall frequencies, the bivariate and trivariate joint return periods of drought properties, T_2 and T_3, were, respectively, calculated as:

$$T_2 = E(L)/K_{\hat{C}(u_1,u_2)} \tag{18}$$

$$T_3 = E(L)/K_{\hat{C}(u_1,u_2,u_3)} \tag{19}$$

where $K_{\hat{C}(u_1,u_2)}$ and $K_{\hat{C}(u_1,u_2,u_3)}$ refers to the Kendal frequency of joint distribution of any two variables and three variables of drought properties, respectively, and $E(L)$ is the average inter-arrival time of drought events. The inter-arrival time was defined as the time between consecutive arrivals. If N denoted the length of years of sample data and n the lenght of drought events, then $E(L)$ was defined as:

$$E(L) = N/n \tag{20}$$

2.4. Design drought properties

In design practice, using the truncated drought samples, the univariate, bivariate, and trivariate distributions can be determined on the basis of the previously mentioned theoretical distributions. For a given return period, the Monte Carlo method was used to simulate new individual CDFs and combinations of CDFs of drought properties. As is known, a given return period corresponds to a CDF of individual drought properties for the univariate design, but is associated with numerous combinations of CDFs for the bivariate or trivariate design. Therefore, a design realization in association with the most-likely weight function was proposed by Salvadori et al. (2011). The joint design of drought properties was derived as follows:

$$[\breve{u}_1, \cdots, \breve{u}_m] = \arg\max f(x_1, \cdots, x_m) \tag{21}$$

$$f(x_1, \cdots, x_m) = c(u_1, \cdots, u_m)\cdot \prod_{i=1}^{m} f(x_i) \tag{22}$$

where $[\breve{u}_1, \cdots, \breve{u}_m]$ is eventually selected as the design combination of CDFs of drought properties for a given joint return period with $m = 2$ for any two properties of droughts and with $m = 3$ for three properties of droughts; and $f(x_1, \cdots, x_m)$ refers to a product of joint probability density, $c(u_1, \cdots, u_m)$, and their individual marginal probability densities, $f(x_i)$, $i = 1, \cdots, m$.

2.5. Uncertainty of multivariate design of drought properties

In the bivariate design case, a symmetrical curve of the vertex was formed by countless combinations for a given joint return period. These bivariate combinations along the curve may not be equivalent from a practical design view point. Moreover, the distance of each combination along the curve from its vertex can be denoted. The limits of subset extending from two sides of the vertex were measured by evaluating the joint probability density in association with the confidence interval (CI) of a probability, $1-\alpha$ (Volpi and Fiori, 2012). In this case, marginal frequencies of two variables on the vertex of the curve were regarded as identical.

However, in the design realization, the combination with the maximum of products of joint and marginal probability densities should be the core point from the curve for a given return period. The core point may be off the vertex due to different individual marginal distributions for the multivariate design. Therefore, the difference, $S(k)$, $k = 1, \cdots, l$, which for any one combination was far from the core point, was defined as:

$$S(k) = \sqrt{[u_1(k)-\breve{u}_1]^2 + ,\cdots,+[u_m(k)-\breve{u}_m]^2} \tag{23}$$

where $[u_1(k), \cdots, u_m(k)]$, $k = 1, 2, \cdots, l$ and $[\breve{u}_1, \cdots, \breve{u}_m]$ denote those selected combinations and the maximum of the products in the bivariate ($m = 2$) or trivariate ($m = 3$) design, respectively. Then, the uncertain range that included those combinations with the confidence interval of probability $1-\alpha$ were limited based on order statistics of the difference, $S(k)$, $k = 1, \cdots, l$ (Serinaldi, 2013).

2.6. Procedure of multivariate design of drought properties

For calculating design combinations of marginal CDFs of drought properties for a given joint return period, no general analytical formulation can be derived for the Gaussian copula. In addition, the simulated and design combinations for any given Kendall return period were determined by the distribution parameters estimated using the truncated samples. In design practice, the Monte Carlo method was used for repeatedly simulating new combinations of drought properties with the identical length of the truncated samples. The new estimated copula parameters resulted in the changes of simulated and design combinations. That is, the uncertainty of the estimated copula parameter needed to be considered.

Therefore, when the Gaussian copula and the Kendall return period were synthetically applied for the assessment of uncertainty, an integrated procedure for the design combination of drought properties in the face of regional water supply was proposed as follows:

1) Truncating socioeconomic drought events from a streamflow process: Using the run method, drought events including duration, severity and peak, were truncated with a predefined threshold value, which refers to the minimum instream flow for satisfying the withdrawal for local socioeconomic systems. The pooling of drought events was then considered.
2) Fitting joint and marginal distributions of drought properties: For the truncated samples of drought properties, the joint and marginal distributions and their parameters were determined by the goodness-of-fit of the recommended m-dimensional *meta*-Gaussian copula and conventional univariate distributions.
3) Calculating the exceedance CDFs corresponding to a given Kendall return period, T_K: On the one hand, the Kendall CDF, $K_{\hat{C}}$, for the given Kendall return period, can be inversely calculated using Eqs. (18) and (19). On the other hand, using the determined copula parameters fitting the samples, large quantities of combinations of marginal CDFs of drought properties, $[u_1(i), \cdots, u_m(i)]$, $i = 1, \cdots, n_1$, with the length of simulations, n_1, were randomly generated by the Monte Carlo method. Their exceedance joint CDFs, $\hat{C}(i)$, $i = 1, \cdots, n_1$, and Kendall CDFs, $K_{\hat{C}}(i)$, $i = 1, \cdots, n_1$, were subsequently calculated using Eqs. (14)–(16). Among them, the exceedance joint CDF, $\hat{C}(i)$, for which its corresponding Kendall CDF, $K_{\hat{C}}(i)$ was closest to $K_{\hat{C}}$, was selected as $\hat{C}_T$.
4) Acquiring combinations of marginal CDFs for the selected exceedance joint CDF: $[u_1(j), \cdots, u_m(j)]$, $j = 1, \cdots, n_2$, with the length of simulations, n_2, were newly generated by the Monte Carlo method, and $\hat{C}(j)$, $j = 1, \cdots, n_2$, were also calculated. For the selected exceedance joint CDF, $\hat{C}_T$, the allowable relative error, Re, was defined as:

$$\left|\frac{\hat{C}(j)-\hat{C}_T}{\hat{C}_T}\right| \leqslant \mathrm{Re} \tag{24}$$

Then, in the simulated combinations, $[u_1(j), \cdots, u_m(j)]$, $j = 1, \cdots, n_2$, those for which their exceedance CDFs satisfied Equation (24) were selected. That is, the selected combinations, $[u_1(k), \cdots, u_m(k)]$, $k = 1, \cdots, l$ with the length of l, which satisfied $\hat{C}(k) \cong \hat{C}_T$, were found. $[x_1(k), \cdots, x_m(k)]$, $j = 1, \cdots, l$, corresponding to $[u_1(k), \cdots, u_m(k)]$, $k = 1, \cdots, l$, can be inversely calculated.

5) Determining the design combination of marginal CDFs of drought properties: From Eqs. (21) and (22), for all selected combinations, the products of probability densities, $f[x_1(k), \cdots, x_m(k)]$, $k = 1, \cdots, l$, on the basis of joint probability densities, $c[u_1(k), \cdots, u_m(k)]$, $k = 1, \cdots, l$ and individual marginal probability densities, $\{f[x_1(k)], \cdots, f[x_m(k)]\}$, $k = 1, 2, \cdots, l$, were calculated, and $[\breve{u}_1, \cdots, \breve{u}_m]$ with the maximum of the product from $[u_1(k), \cdots, u_m(k)]$, $k = 1, 2, \cdots, l$ was found as the design combination for the given Kendall return period of drought properties, T_K.
6) Calculating design values for the determined combination of marginal CDFs of drought properties: Design values of each drought property were inversely calculated, based on their individual marginal distributions.
7) Assessing the uncertainty for design combinations of drought properties: New samples of drought properties with the identical length of truncated drought samples with n_3 times by using the Monte Carlo method were simulated, and their copula parameters were estimated. Then, when the procedure from 3) to 6) was repeatedly executed for each new combination of samples, the design combinations, $[\breve{u}_1(k), \cdots, \breve{u}_m(k)]$, $k = 1, \cdots, n_3$ with the length of n_3, were acquired. From Eq. (24), design combinations that included the confidence interval with probability $1-\alpha$ were limited based on order statistics (Serinaldi, 2013). Among them, $[\breve{\breve{u}}_1, \cdots, \breve{\breve{u}}_m]$, the maximum product of joint probability densities was eventually selected as the design combination from simulated samples.

2.7. Impact of water reservoirs

As water reservoirs of a basin are constructed and operated, observed streamflow substantially differs from its pristine version. A usual method is that streamflow in the period influenced by reservoirs should be entirely returned (Van Loon and Van Lanen, 2013). Let Q_O be the observed streamflow. Then, the naturalized streamflow, Q_N, was defined as:

$$Q_N = Q_O + \Delta Q = Q_O + Q_{in}-Q_{out} \tag{25}$$

where ΔQ is the alteration of streamflow due to the operation of reservoirs, and Q_{in} and Q_{out} refer to the inflow and outflow of reservoirs, respectively.

3. Case study and data

The Dongjiang River basin is located in South China with clear wet

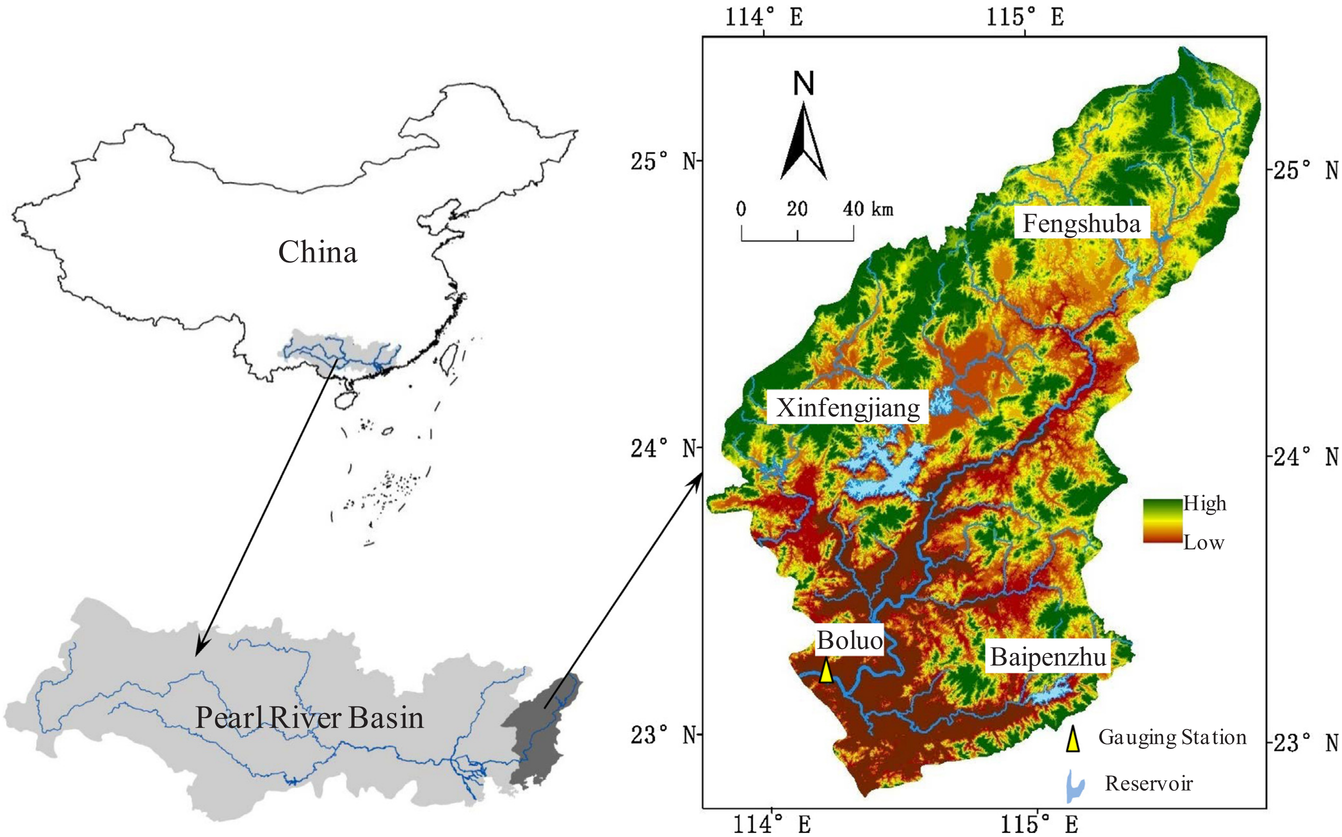

Fig. 2. A gauging station and water reservoirs in the Dongjiang River basin, South China.

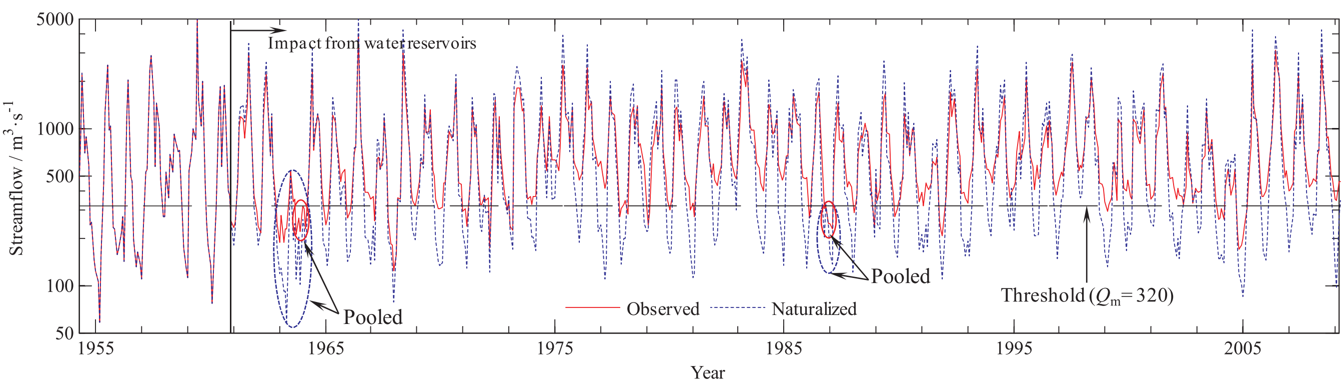

Fig. 3. Streamflow processes and socioeconomic droughts at the Boluo station of Dongjiang River basin.

and dry seasons (shown in Fig. 2). Three major reservoirs, namely, Xinfengjiang, Fengshuba and Baipenzhu, with storages of 6.49, 1.25 and 0.39 billion m^3, have been in operation since 1961, 1974 and 1986, respectively.

Monthly streamflow data from the Boluo station as the control station of the basin with a catchment area of 25 325 km^2, were used. They covered a common period of 56 hydrological years from April 1954 to March 2010. Since the 1960 s, surface water resources in the Dongjiang River basin have been well developed (Tu et al., 2012). Streamflow in the main stream of the basin has been apparently influenced by the regulation of the above reservoirs (Tu et al., 2012, 2015, 2017). On the basis of differences between inflow and outflow of individual reservoirs in terms of monthly time scale, the observed series of streamflow at the Boluo station can be returned to the naturalized series.

Then, with the rapid increase of water withdrawals of local socioeconomic systems, the imbalance between water supply and demand has been growing increasingly in the 21st century. In order to alleviate the imbalance, in particular to secure water supply for the important cities of Pearl River Delta, such as Hong Kong, Guangzhou, and Shenzhen, the minimum instream flow of the Boluo station has been implemented in recent years. According to the *Water Resources Allocation Scheme of the Dongjiang River Basin in Guangdong Province* (2011), the minimum instream flow at the Boluo station, Q_m, was set at 320 $m^3.s^{-1}$, approximately a percentile of 13.1% or 36.2% of the observed or naturalized monthly streamflow, respectively.

Therefore, in terms of monthly streamflow, both the naturalized series from April of 1954 to March of 2010 and the observed series from April of 1961 to March of 2010 were used in truncating socioeconomic drought events. By comparing to the differences between univariate, bivariate and trivariate design values of individual drought properties on the basis of two series, impacts of water reservoirs on socioeconomic droughts were determined.

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Table 1
Statistical characteristics of drought properties for samples.

Characteristic	Observed			Naturalized			Relative change/%		
	d/month	$s/10^8$ m^3	$p/10^8$ m^3	d/month	$s/10^8$ m^3	$p/10^8$ m^3	d	s	p
Maximum	6.00	17.82	5.06	17.00	57.71	6.80	−64.7	−69.1	−25.5
Mean	2.48	3.75	1.76	4.25	13.00	4.15	−41.6	−71.1	−57.6
Median	2.00	1.96	1.49	4.00	12.57	4.41	−50.0	−84.4	−66.1
Minimum	1.00	0.03	0.03	1.00	0.46	0.46	0.0	−93.3	−93.3

4. Results and discussion

4.1. Drought properties from samples

Observed and naturalized series are illustrated in Fig. 3. The monthly observed and naturalized flows, in terms of the average value with a small difference, accounted for 741 and 758 m^3.s^{-1}, respectively. However, under the regulation of water reservoirs since 1961, low streamflows have significantly increased but drought events have remarkably decreased. Socioeconomic droughts of naturalized streamflows occurred 56 times during the period from 1954 to 2009 with a duration of 1.02 years, and those of observed streamflows decreased to 28 times during the period from 1961 to 2009 with a duration of 0.58 year.

As shown in Table 1, for naturalized flows, the maximum drought lasted 17 months with the severity of 5.77 billion m^3 and the peak of 0.68 billion m^3. The average duration, severity and peak accounted for 4.25 months, 1.3, and 0.415 billion m^3, respectively. The severity of the minimum drought with a duration of a month accounted for 46 million m^3. For observed flows, the duration, severity, and peak of the maximum drought decreased to 6 months, 1.78, and 0.51 billion m^3, respectively. The average duration, severity, and peak accounted for 2.48 months, 0.375 and 0.176 billion m^3, respectively. The severity of the minimum drought with a month accounted for 3 million m^3.

Due to the regulation of water reservoirs, the duration, severity, and peak of the maximum drought decreased by 64.7%, 69.1%, and 25.5%, respectively. The average duration, severity, and peak decreased by 41.6%, 71.1%, and 57.6%, respectively. The severity of the minimum drought decreased by 93.3%. In addition, compared to the average values, the median values of duration and severity for the naturalized series slightly decreased, but the decreases for the observed series were large. It implied that there were more events with shorter duration and lower severity under the regulation of water reservoirs.

4.2. Individual and joint distributions of drought properties

The general statistical boxes of socioeconomic drought properties of observed and naturalized series are shown in Fig. 4. Except that the maxima of duration and severity of the naturalized series were far from the extended boundary and the top three durations of the observed series were out of the extended boundary, most samples of drought properties fell in the range of extended boundaries, which were generally viewed as normal.

The goodness-of-fit test results of univariate distributions are shown in Table 2. For the severity and peak of two series, the P -values of five univariate distributions were greater than the significant level of 0.05 except for the LOGN distribution of the peak of the naturalized series. For the duration, the P -values of LOGN, WBL and GNO distributions for the observed series and only that of the WBL distribution for the naturalized were greater than the significant level. These showed that the five univariate distributions almost fitted the severity and peak of drought, but only the WBL distribution fitted all drought properties for the observed and naturalized series. In addition, comparing the RMSE and AIC values of the accepted distributions, the optimal distribution of duration was WBL and that of the peak was GNO for the two series. The optimal distribution of severity was LOGN and GNO for the observed and naturalized series, respectively. On the whole, among the accepted distributions, the RMSE and AIC values slightly differed and the P -values of WBL for all properties of two series were the greatest. As illustrated in Fig. 5, using the WBL distribution, most points fell near the diagonal of 1:1, which implied small differences between the empirical and theoretical CDFs of individual drought properties. Therefore, for convenience of comparison, it was better to select WBL for modelling all marginal distributions of drought properties.

Then, between observed and naturalized series, the WBL parameters of individual drought properties distinctly differed (see Table 3). As illustrated in Fig. 6, comparing the naturalized series, the design curve of univariate return period of duration, severity, and peak of drought of the observed series remarkably moved down. It denoted that univariate design values of drought properties decreased under the regulation of water reservoirs.

As shown in Table 4, Kendall τ, Spearman ρ, and Pearson γ of any two drought properties were greater than 0.5, implying that the dependences between them were high positive. The differences of dependences between the two series in the combinations of duration and severity and duration and peak were small, but were comparatively large in a combination of severity and peak.

As seen from Fig. 7, most points of the empirical and theoretical CDFs of the bivariate and trivariate distributions of drought properties fell closely to the diagonal of 1:1 except for those for the duration of observed series in the lower CDFs with a large drift, which was attributed to more small droughts with a shorter duration for the observed series.

However, the results of goodness-of-fit of joint distributions for drought properties showed that all P -values were greater than the significant level of 0.05 (see Table 5). It demonstrated that using Gaussian copula to fit the bivariate and trivariate distributions of drought properties was appropriate. The estimated parameters of the Gaussian copula on the samples of two series are shown in Table 5.

4.3. Modelling and design of drought properties based on samples

Based on the selected marginal and joint distributions of the truncated samples of drought properties, the equivalent scatter diagrams of the given bivariate Kendall return periods exceeding the percentiles, e.g. 5 years, 10 years, 20 years, and 50 years, are shown in Fig. 8. It is seen that the scatter points of a certain joint return period of any two variables of drought properties almost formed a symmetric curve of the diagonal of 1:1. Then, the differences of joint return period between observed and naturalized series were distinct. Compared to the naturalized series, the curves of observed series shifted toward southwest. It showed that the joint return period exceeding the percentile under the regulation of water reservoirs significantly increased for the identical bivariate pairs of drought properties. In addition, the design combinations of any two properties of drought were close to the 1:1 diagonal, which implied that the marginal CDFs of any two properties were approximately identical in the bivariate design.

The given trivariate Kendall return periods of the truncated samples of drought properties are shown in Fig. 9 in which subfigures are planar projections of the trivariate combinations. The pairs of duration and

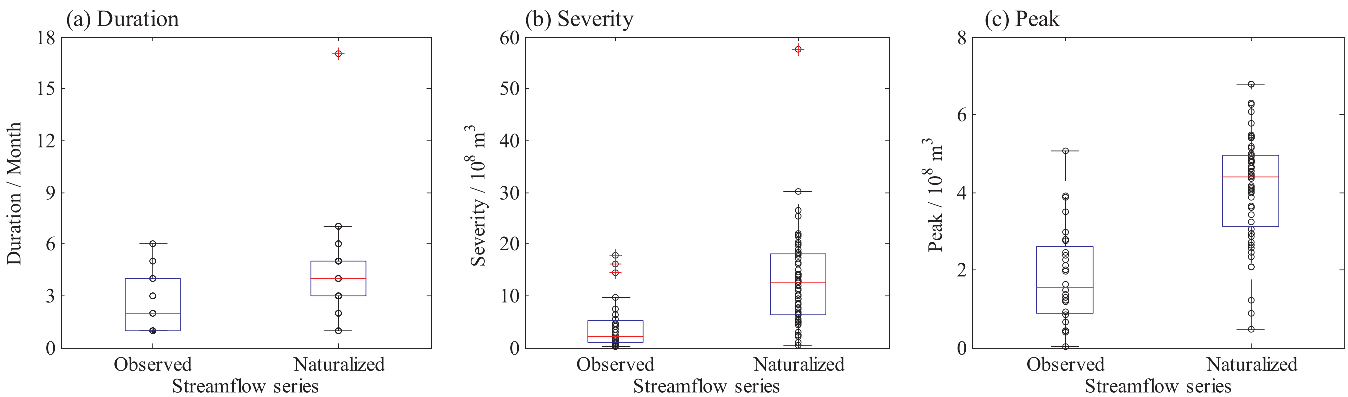

Fig. 4. Box plot of drought properties. The upper and lower boundaries of the box were set to the values of their percentiles as the one and three quarters, q_1 and q_3, respectively. The red solid line refers to the median. The upper and lower boundaries extended along the dash line were further set to the values as $q_1 + 1.5(q_1-q_3)$ and $q_3-1.5(q_1-q_3)$, respectively. Those in the range of extended boundaries were regarded as normal and otherwise abnormal as further marked by the red plus sign. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

peak in the combinations of duration, severity, and peak maintained a good symmetry around the 1:1 diagonal and formed an approximate curve with a tiny drift, which decreased from the center to the two sides. The pairs of duration and severity, and severity and peak in the combinations of duration, severity, and peak maintained an approximate curve better on the side of duration or peak, but were remarkably scattered on the side of severity. The farther the distance to the 1:1 diagonal on the side of severity was, the wider the drift of the pair of CDFs was. The farther the distance to the 1:1 diagonal on the side of duration or peak was, the more concentration on a curve of the pair of CDFs was. These implied that the uncertainty of severity was larger than that of duration and peak for given trivariate joint return period of duration, and severity, and peak. However, the design combinations of any two properties of drought in the trivariate design were also close to the 1:1 diagonal.

4.4. Uncertainty of modeling and design of drought properties

With 5000 simulations of new samples with the same length of truncated samples, their bivariate and trivariate Kendall return periods exceeding the percentiles are illustrated in Figs. 10 and 11, respectively. For a given bivariate return period of any two variables of drought properties (see Fig. 10), the simulated pairs of CDFs and those of the CI of a probability of 0.95, were symmetric about the 1:1 diagonal, but they formed curving belts with a limited width in the direction of northeast–southwest. Then, the width in the CI along the two sides of the 1:1 diagonal differed for different pairs of the two drought properties. The width of duration and peak was larger than that of duration and severity, and that of the latter was larger than that of severity and peak.

For a given trivariate return period of drought properties (see Fig. 11), the pairs of two drought properties formed a similar shape to that of truncated samples. Namely, the pairs of duration and peak maintained a good symmetry about the 1:1 diagonal and formed a curving belt with a limited width of the direction of northeast–southwest. For duration and severity or severity and peak, the range of the pairs scattered on the side of severity was larger than that on the side of duration or peak.

In addition, the design pairs of CDFs of any two properties of each new simulation for a bivariate or trivariate return period deviated from the 1:1 diagonal. However, the pairs of maximum products of joint probability densities among large simulations were close to the 1:1 diagonal.

4.5. Design values of drought properties

Comparison of design values of drought properties between truncated and simulated samples is shown in Fig. 12. On the whole, for each drought property, the lines of scatter points with the CI of probability of 0.95 for the trivariate design were wider than those for the bivariate design. It implied that the uncertainty of the former was larger than that of the latter. Then, most pairs of design values on the basis of

Table 2
Goodness-of-fit test of marginal distribution of drought properties.

Property	Model	Observed				Naturalized			
		P-value	Accepted	RMSE	AIC	*P*-value	Accepted	RMSE	AIC
Duration	LOGN	0.149	Yes	0.152	−99	0.047	No	0.074	−286
	WBL	0.191	Yes	0.147	−101	0.082	Yes	0.099	−253
	GEV	0.003	No	0.148	−98	0.002	No	0.086	−266
	GLO	0.041	No	0.153	−96	0.013	No	0.085	−268
	GNO	0.054	Yes	0.144	−99	0.007	No	0.087	−265
Severity	LOGN	0.935	Yes	0.031	−188	0.181	Yes	0.066	−299
	WBL	0.950	Yes	0.035	−181	0.826	Yes	0.032	−381
	GEV	0.243	Yes	0.043	−167	0.430	Yes	0.026	−401
	GLO	0.227	Yes	0.047	−162	0.455	Yes	0.027	−395
	GNO	0.584	Yes	0.034	−180	0.452	Yes	0.025	−403
Peak	LOGN	0.381	Yes	0.070	−143	0.028	No	0.094	−259
	WBL	0.986	Yes	0.028	−194	0.599	Yes	0.044	−344
	GEV	0.833	Yes	0.026	−195	0.222	Yes	0.030	−383
	GLO	0.597	Yes	0.033	−182	0.547	Yes	0.027	−396
	GNO	0.859	Yes	0.026	−196	0.471	Yes	0.027	−397

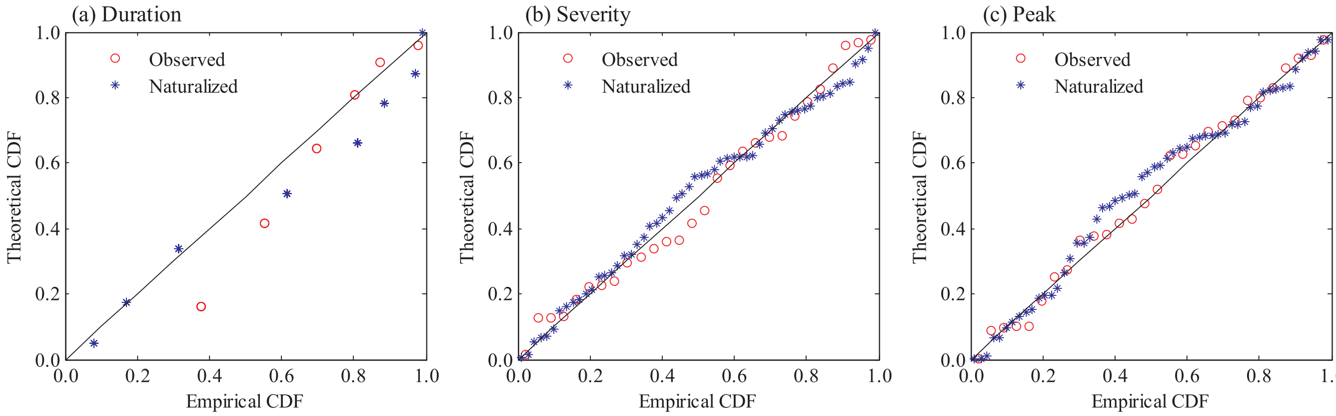

Fig. 5. Q-Q plot of empirical and theoretical CDFs of marginal distributions of drought properties using the WBL.

Table 3
Parameters of the WBL distribution of individual drought properties.

Property	Observed	Naturalized
d	$k = 2.931$ $\lambda = 1.613$	$k = 4.793$ $\lambda = 1.903$
s	$k = 3.943$ $\lambda = 0.889$	$k = 14.387$ $\lambda = 1.491$
p	$k = 2.011$ $\lambda = 1.442$	$k = 4.602$ $\lambda = 3.357$

Table 4
Dependence of drought properties.

Combination	Observed			Naturalized		
	τ	ρ	γ	τ	ρ	γ
$d-s$	0.713	0.850	0.858	0.764	0.896	0.928
$d-p$	0.570	0.711	0.712	0.524	0.672	0.646
$s-p$	0.852	0.957	0.880	0.675	0.840	0.782

truncated and simulated samples were close to the 1:1 diagonal. It demonstrated that the design value of each drought property between truncated and simulated samples slightly differed.

Ultimately, univariate, bivariate and trivariate design values of drought properties for observed and naturalized series are shown in Table 6. For a given univariate, bivariate or trivariate return period, the design value of each drought property of the observed series was conspicuously smaller than that of the naturalized series due to the regulation of water reservoirs.

For example, compared to the naturalized series for the return period of droughts exceeding the percentile of 5 years as a case, the duration, severity, and peak of observed series decreased by 48.8%–51.7% from 6.19 to 6.38 months to 3.06–3.27 months, by 77.3%–79.4% from 1.994 to 2.067 billion m^3 to 0.426–0.470 billion m^3, and by 59.2%–60.5% from 0.532 to 0.542 billion m^3 to 0.211–0.217 billion m^3, respectively.

The impact of water reservoirs on the severity was larger than that on the peak, and the latter was larger than that on the duration. Then, the relative change (absolute value) decreased as the return period of exceeding the percentile increased. It implied that the impact of water reservoirs on the smaller drought was larger than that on the larger droughts.

However, the differences of each drought property among the univariate, bivariate and trivariate designs were slightly small. Compared to the univariate design, the alternations of duration, severity, and peak of multivariate joint design were generally less than 5%. They resulted from a high correlation of drought properties, using the Kendall return period instead of the conventional multivariate joint return period, and the concentration of design combinations of the CDFs of drought properties on the 1:1 diagonal.

5. Conclusions

In this paper, an integrated procedure for design combinations of socioeconomic drought properties against the backdrop of regional water supply, involving the truncation of drought events using the run method, the goodness-of-fit of joint and marginal distributions of the properties using the Gaussian copula and conventional univariate distributions, the determination of design combinations of the properties for a given Kendall return period using the most-likely weight function, and the evaluation of the uncertainty of the combinations using the confidence interval, was developed. Besides, in the Dongjiang River basin, univariate, bivariate and trivariate design values of individual

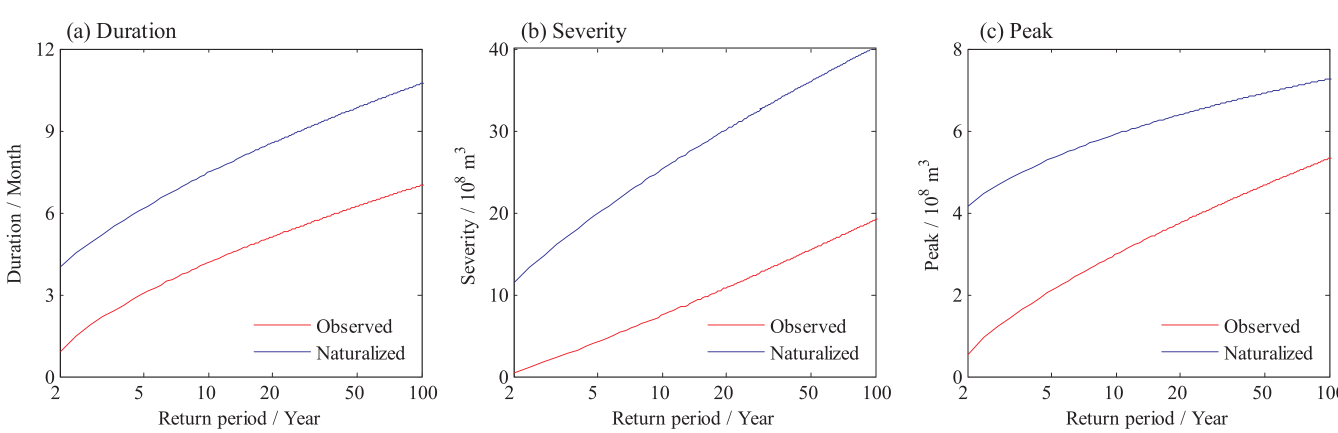

Fig. 6. Design curve of univariate return periods of drought properties.

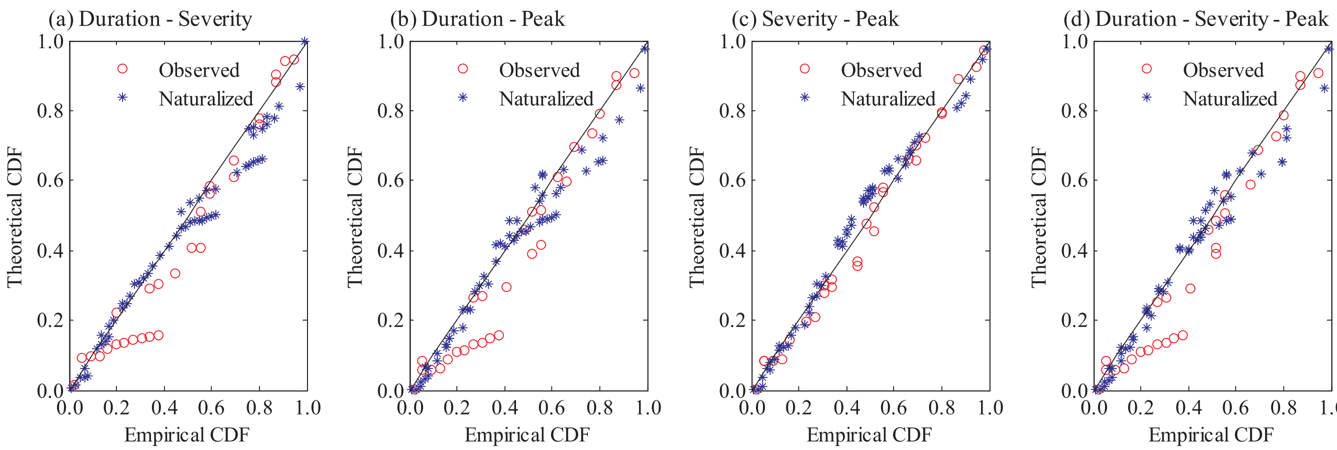

Fig. 7. Q-Q plot of empirical and theoretical CDFs of joint distributions of drought properties.

Table 5
Goodness-of-fit test and parameters of the Gaussian copula for the joint distribution of drought properties.

Type	Combination	Observed		Naturalized	
		P-value	Parameter	*P*-value	Parameter
Bivariable	$d-s$	0.858	0.863	0.203	0.914
	$d-p$	0.791	0.680	0.298	0.707
	$s-p$	0.723	0.940	0.582	0.885
Trivariabe	$d-s-p$	0.902		0.277	

socioeconomic drought properties and their alterations due to the regulation of water reservoirs were investigated. The main conclusions of this study are follows:

(1) For observed and naturalized series, any two drought properties exhibited a high positive dependence. The WBL and Gaussian copula effectively fitted the marginal and joint distributions of drought properties, respectively. The multivariate design pairs of the CDFs of drought properties, including the bivariate and trivariate designs, were close to the 1:1 diagonal. The differences in the design values of duration, severity or peak between univariate, bivariate and trivariate designs were small and their relative changes were less than 5%. They resulted from the high correlation of drought properties, the use of the Kendall return period, and the approximately identical CDFs in design combinations.
(2) For a given bivariate return period of drought properties, the pairs of CDFs of any two drought properties formed a symmetric curve for truncated samples and a symmetric curving belt for large quantities of simulated samples. For a given trivariate return period of drought properties, the pairs of CDFs of duration and peak showed a good symmetry, but those of duration and severity or severity and peak were remarkably scattered on the side of severity and comparatively concentrated on the side of duration or peak. It implied that the uncertainty of severity was larger than that of duration and peak in the trivariate combination.
(3) Design values of drought properties for each combination of new simulated samples partly changed. In terms of the range of CI of probability of 0.95, the change of design value of the trivariate design was larger than that of the bivariate design. It implied that the uncertainty of the former was larger than that of the latter. However, among large quantities of simulated samples, the design values of each drought property of the selected one with the maximum product of joint probability densities were close to those of truncated samples.
(4) In the Dongjiang River basin, South China, the impact of water reservoirs on the socioeconomic drought was very significant. For example, for the return period of droughts exceeding the percentile of 5 years, the duration, severity, and peak decreased by 48.8%–51.7%, 77.3%–79.4%, and 59.2%–60.5%, respectively. But, the drought spell, and total volume and monthly maximum of water deficit for a given return period of 5 years still accounted for 3.06–3.27 months, 0.426–0.470 billion m^3 and 0.211–0.217 billion m^3, respectively, provided that the local water supply was met.

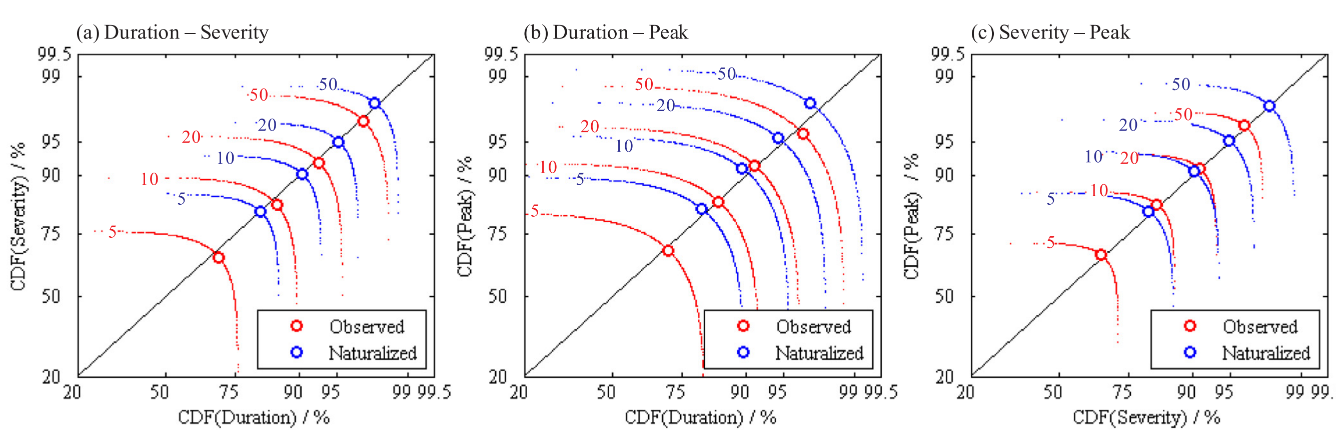

Fig. 8. Bivariate exceedance joint return period of drought properties based on truncated samples. The red/green points refer to all combinations for given joint return periods. The red/green hollow points are the design combinations using the most-likely weight function. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

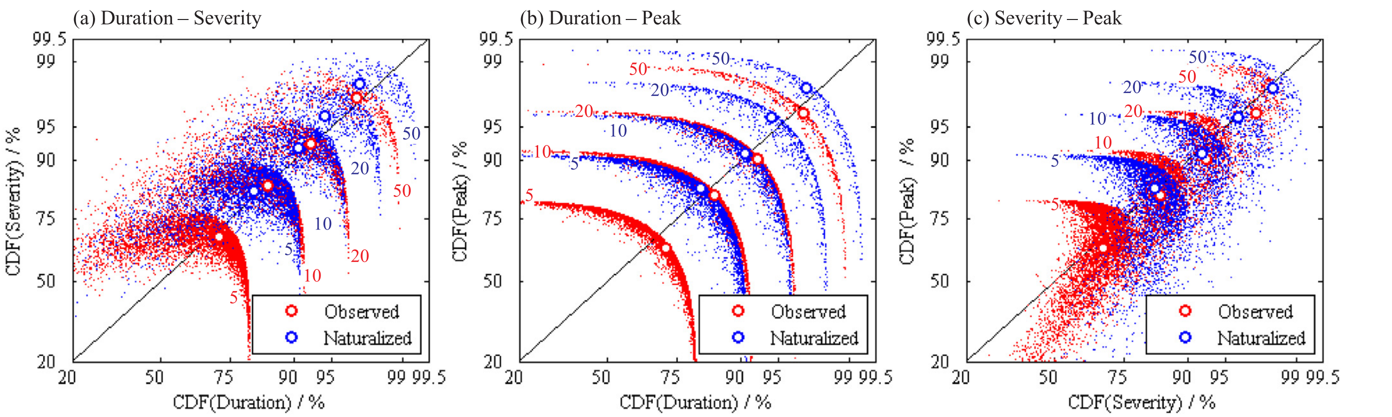

Fig. 9. Trivariate exceedance joint return period of drought properties based on truncated samples in which subfigures are planar projections of trivariate combinations. The red/green points refer to all combinations for given joint return periods. The red/green hollow points are the design combinations using the most-likely weight function. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

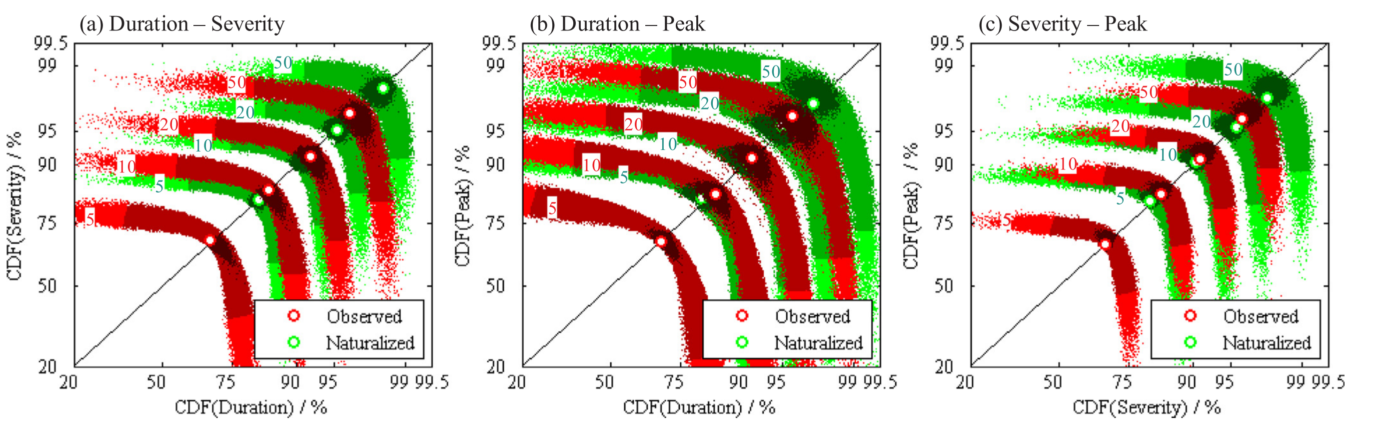

Fig. 10. Uncertainty of bivariate exceedance joint return period of drought properties based on simulated samples. The red/green points from light to dark refer to all simulations, those of the CI of probability of 0.95, and those of the most-likely weight, respectively. The red/green hollow points are the design combinations using the most-likely weight function. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

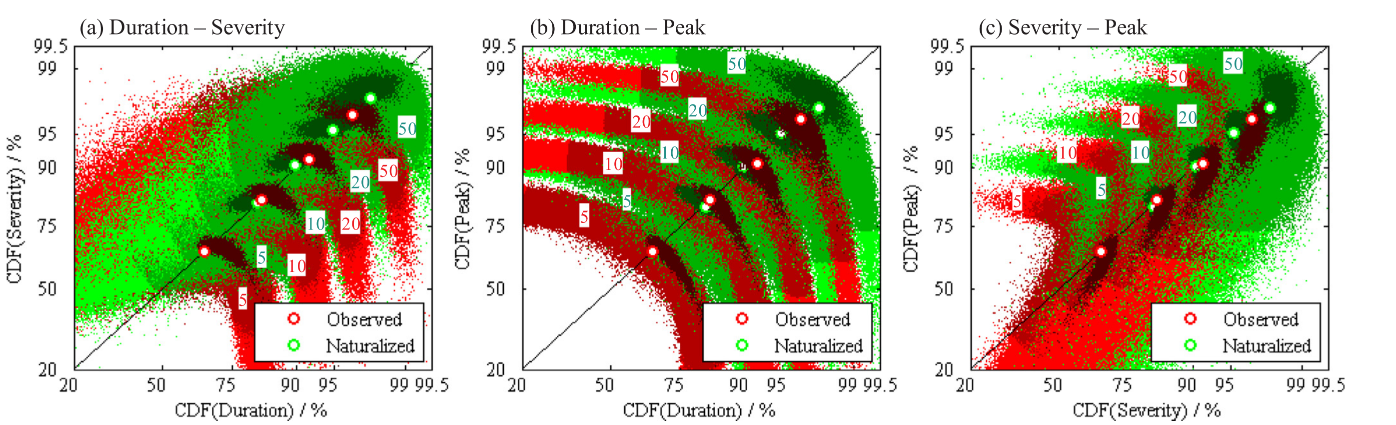

Fig. 11. Uncertainty of trivariate exceedance joint return period of drought properties based on the simulated samples. These subfigures are planar projections of trivariate combination. The red/green points from light to dark refer to all simulations, those of the CI of probability of 0.95, and those of the most-likely weight function, respectively. The red/green hollow points are the selected design combinations using the most-likely weight function. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

All in all, in the multivariate design realization using the Gaussian copula and the Kendall return period, no general analytical formulation for calculating the combination of marginal CDFs of drought properties for a given joint return period could be derived. Therefore, in this paper, the procedure for the most-likely weight function proposed by Salvadori et al. (2011) was further improved and applied to the multivariate design of socioeconomic drought properties, by which the uncertainty of multivariate design was evaluated. In addition, the socioeconomic drought in the case study was still serious even under the regulation of water reservoirs. The regulation of reservoirs in the basin must be further improved and even the inter-basin water transfer can be considered to ensure the safety of increasing local water supply.

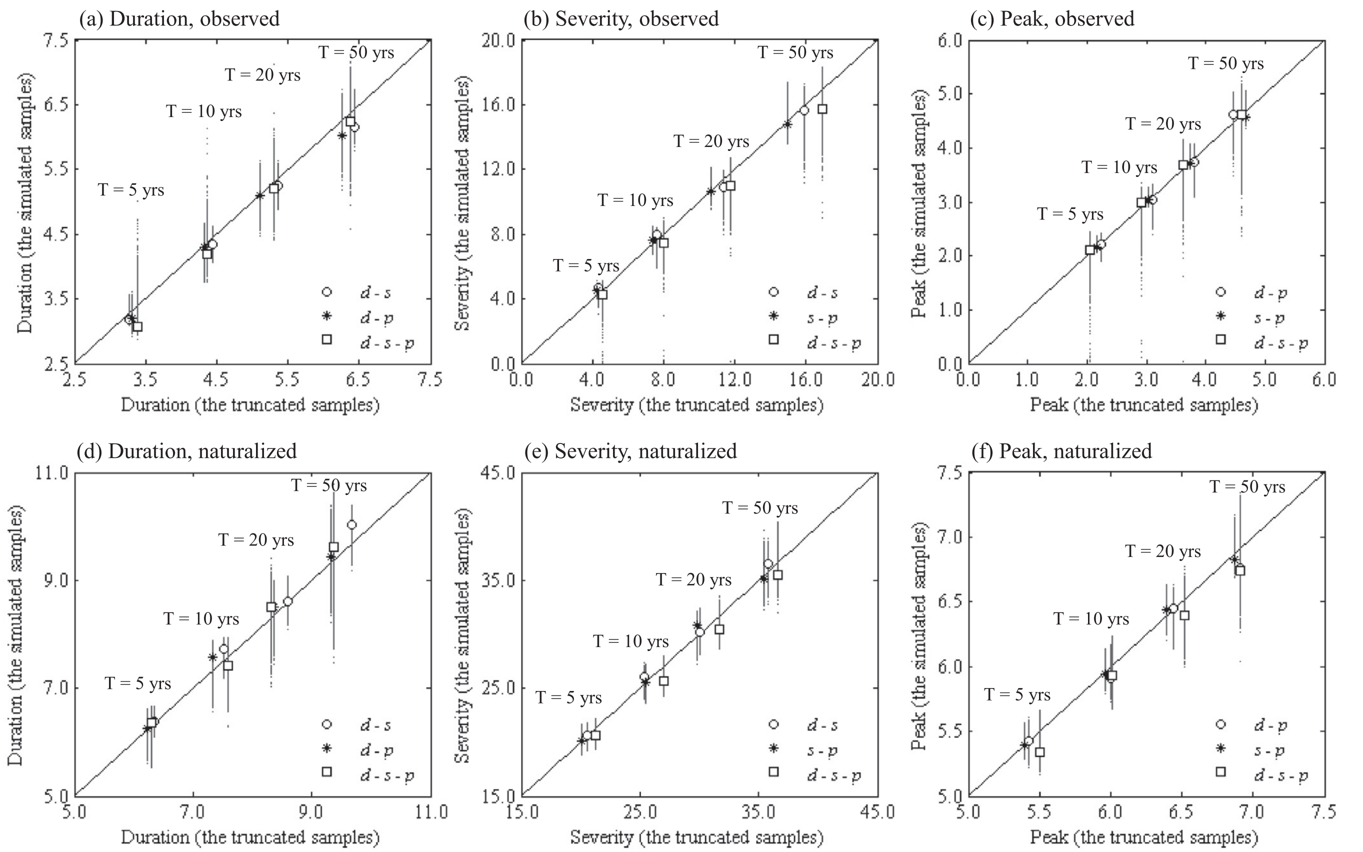

Fig. 12. Comparison of design values of drought properties between truncated and simulated samples. The lines consist of scatter points with the CI of probability of 0.95. The points marked by the circle, star and square are the selected design combinations using the most-likely weight function.

Table 6
Individual and joint design values of drought properties.

Type	Combination	Property	Observed				Naturalized				Relative change/%			
			5 yrs	10 yrs	20 yrs	50 yrs	5 yrs	10 yrs	20 yrs	50 yrs	5 yrs	10 yrs	20 yrs	50 yrs
Univariable		d/month	3.06	4.17	5.12	6.23	6.19	7.46	8.56	9.84	−50.6	−44.1	−40.2	−36.7
		$s/10^8m^3$	4.26	7.46	10.84	15.48	19.94	25.30	30.15	36.03	−78.7	−70.5	−64.1	−57.0
		$p/10^8m^3$	2.11	2.98	3.75	4.67	5.32	5.91	6.39	6.92	−60.4	−49.6	−41.3	−32.4
Bivariable	$d-s$	d/month	3.17	4.34	5.25	6.16	6.38	7.72	8.62	10.03	−50.3	−43.8	−39.1	−38.6
		$s/10^8m^3$	4.70	7.95	10.87	15.69	20.66	26.07	30.14	36.48	−77.3	−69.5	−63.9	−57.0
	$d-p$	d/month	3.20	4.31	5.10	6.02	6.26	7.57	8.49	9.44	−48.8	−43.1	−39.9	−36.3
		$p/10^8m^3$	2.21	3.04	3.73	4.62	5.42	5.91	6.45	6.76	−59.2	−48.6	−42.1	−31.6
	$s-p$	$s/10^8m^3$	4.49	7.66	10.69	14.74	20.14	25.56	30.83	35.14	−77.7	−70.0	−65.3	−58.0
		$p/10^8m^3$	2.17	3.03	3.70	4.57	5.40	5.94	6.44	6.83	−59.7	−49.0	−42.5	−33.1
Trivariable	$d-s-p$	d/month	3.07	4.19	5.20	6.23	6.36	7.41	8.50	9.61	−51.7	−43.5	−38.8	−35.1
		$s/10^8m^3$	4.26	7.45	10.96	15.72	20.67	25.63	30.49	35.47	−79.4	−70.9	−64.0	−55.7
		$p/10^8m^3$	2.11	2.98	3.68	4.62	5.34	5.93	6.39	6.74	−60.5	−49.7	−42.5	−31.5

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