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Graduating Summer 2026

WRITING SAMPLE

Real Business Cycles:
Productivity, Persistence and Plausibility

This paper was originally submitted as an essay for one of my weekly intermediate macroeconomics tutorials. The paper has been typeset using LaTeX and the figures were generated using an RBC simulation which I coded in R using the gEcon package. The model can be accessed via my ***GitHub***.

Question: In a simple Real Business Cycle (RBC) model with stochastic technology, explain how productivity shocks generate business cycles. Why do such models typically assume highly persistent productivity shocks, and how plausible is this assumption and the overall RBC account of business cycles?

RBC models explain cyclical movements in macroeconomic variables as arising from rational, forward-looking households optimally responding to exogenous productivity shocks in a setting of perfectly competitive markets with fully flexible prices and wages. For RBC models to explain the variation observed in business cycle data, productivity shocks must be assumed to be highly persistent. The empirical strategy used to demonstrate the persistence of these shocks, however, is beset with identification problems; furthermore, the simple RBC model generates predictions discordant with well-established labor market data. Taken together, these observations undermine the plausibility of simple variants of RBC models.

In §1, I present and motivate a simple RBC model. In §2, I discuss the role of productivity shocks in the macroeconomic dynamics of the simple model. In §3, I explain why these shocks must be highly persistent for the simple RBC model to account for the variation observed in business cycle data. In §4, I consider a number of empirical challenges facing the simple RBC model.

§1. The Simple RBC Model

Market economies exhibit recurrent fluctuations in output, consumption, employment, interest rates, wages, investment and the capital stock, in addition to numerous regularities among the co-movements of these macroeconomic variables.[1,2] These fluctuations, together with their associated co-movements, are referred to as 'business cycles.' Business cycles can be understood in terms of the following structural equation:

$$Y_t = Y_t^T + u_t. \tag{1}$$

Y_t is the level of output at time t, Y_t^T is trend output at time t and u_t is the residual. Y_t^T is explained by long-run models of economic growth, such as Solow-Swan or endogenous growth models, while

[1] **All variables referred to throughout this essay are in *real* terms.**

[2] Charles I. Plosser, "Understanding Real Business Cycles," *Journal of Economic Perspectives* 3, no. 3 (Summer 1989): 53; Andy W. Mullineux and David G. Dickinson, "Equilibrium Business Cycles: Theory and Evidence," *Journal of Economic Surveys* 6, no. 4 (1992): 321–22.

u_t is the cyclical variation (i.e. 'business cycles') that these long-run models do not provide an account of.[3] Instead, u_t is explained by short-run macroeconomic models such as the Keynesian IS-LM model and (more germane to the purposes of this paper) RBC models.

In contrast to IS-LM models, RBC models provide a *micro-founded* account of short run macroeconomic adjustments. That is, RBC models explain the fluctuations and (co-)movements of macroeconomic variables as arising from the forward-looking, dynamic optimizing behavior of households and firms.[4] One can think of an RBC model as a dynamic Walrasian model of general equilibrium.

In what follows, I present and motivate a simple RBC model. The simple RBC model assumes:[5]

I. Markets are perfectly competitive.
   - $\Pi = \pi_t = 0$.

II. Prices and wages are fully flexible.

III. The economy is modeled by a representative household and firm that operate according to a choice-theoretic, rational expectations framework.

For ease of exposition, I make the additional assumptions IV–X:[6]

IV. The model is in discrete time with an infinite number of periods.

V. During each period, the representative household can work for a wage W_t, consume C_t or invest with a rate of return r_t (note these are *not* mutually exclusive).

VI. The representative household is an expected utility maximizer with time separable utilities.
   - $U = E[\Sigma_{t=0}^{+\infty} \beta^t u_t]$.

VII. The representative household's instantaneous utilities are modeled by CRRA preferences.[7]
   - $u_t = \frac{\left(C_t^{\mu}(1-L_t)^{1-\mu}\right)^{1-\eta}-1}{1-\eta}$.

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3 Mullineux and Dickinson, "Equilibrium Business Cycles," 325.

4 Plosser, "Understanding Real Business Cycles," 53.

5 I follow a similar but not identical RBC model to that of Finn E. Kydland and Edward C. Prescott, "Time to Build and Aggregate Fluctuations," *Econometrica* 50, no. 6 (1982): 1345–70.

6 The general qualitative results of the forthcoming discussion are *not* sensitive to the chosen functional specification. The same results can be arrived at via a number of alternative approaches to setting up the model.

7 μ is a parameter measuring the utility weight given to leisure, while η is a parameter measuring risk aversion.

VIII. The representative firm is modeled by Cobb-Douglas production technology.[8]

- $Y_t = A_t K_t^{\alpha} L_t^{1-\alpha}, \quad 0 < \alpha < 1.$

IX. The representative firm supplies one unique good with the price normalized to 1.

- $p_Y = 1.$

X. The intertemporal budget constraint is given by the Solow capital accumulation equation.[9]

- $K_{t+1} = Y_t - C_t + (1-\delta)K_t, \qquad I_t := Y_t - C_t.$

Under the assumptions I–X, it follows that the optimization problem solved by the representative household in the simple RBC model is given by:[10],[11]

$$\begin{aligned} \text{Max}_{\{C_t, L_t\}} \quad & E_0[\Sigma_{t=0}^{+\infty} \beta^t u_t] \\ \text{s.t.} \quad & K_{t+1} = r_t K_t + W_t L_t + \pi_t - C_t + (1-\delta)K_t, \quad 0 < C_t, \quad 0 \leq L_t \leq 1. \end{aligned} \tag{2}$$

(2) states that the representative household, in seeking to maximize its expected utility, must decide in period t how much to consume and how much to work for each of the periods $t+1, t+2, \ldots, \infty$ (i.e. all future periods), subject to the constraint that capital in the subsequent period is equal to undepreciated current capital, plus current factor income and profits net of consumption.

The intertemporal allocation chosen by the representative household is determined by two key macroeconomic variables: the prevailing interest rate r_t and wage rate W_t.[12] As the representative household is an expected utility maximizer with exponential discounting, it follows that it will form intertemporally consistent choices; that is, the representative household will not revise its intertemporal consumption and labor allocation unless it receives new information.[13]

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8 Note that in each period the representative household has a unitary time endowment and therefore can allocate between 0 and 1 to L_t.

9 Note capital K_t is assumed to be consumable.

10 Robert G. King and Sergio T. Rebelo, "Resuscitating Real Business Cycles," in *Handbook of Macroeconomics*, ed. John B. Taylor and Michael Woodford (Amsterdam: Elsevier, 1999), 946.

11 Note, under perfect competition and Cobb-Douglas production technology, $Y_t = r_t K_t + W_t L_t + \pi_t$.

12 Plosser, "Understanding Real Business Cycles," 55.

13 King and Rebelo, "Resuscitating Real Business Cycles," 997.

Solving (2) yields:[14]

$$\overbrace{MU_{C_t}}^{\text{Marginal Cost of Saving}} = \underbrace{\beta E_t\big[MU_{C_{t+1}}(1-\delta+r_{t+1})\big]}_{\text{Marginal Benefit of Saving}}. \tag{3}$$

$$W_t = \frac{1-\mu}{\mu}\frac{C_t}{1-L_t}. \tag{4}$$

(3) is the 'consumption Euler equation' and (4) is the 'intratemporal labor-leisure optimality condition.' The consumption Euler equation exhibits the tradeoff that the representative household faces between consumption in the present period t and the subsequent period $t+1$ (i.e. the choice of consuming now vs. saving for later). As the representative household's utility function is concave in C_t, it follows that the household will seek to smooth the marginal utility it derives from consumption across each period in order to maximize its lifetime utility U; that is, the household will act so as to ensure the marginal cost of saving equals the marginal benefit of saving *for all* periods.

The reason why the consumption Euler equation (3) must hold is intuitive; if, for example, the left-hand-side of (3) was greater than the right-hand-side, as exhibited in (3′),

$$MU_{C_t} > \beta E_t\big[MU_{C_{t+1}}(1-\delta+r_{t+1})\big], \tag{3′}$$

then it would be the case that the marginal cost of saving (MCS) were higher than the marginal benefit of saving (MBS), and therefore that the household could obtain additional utility by substituting away from saving toward more current consumption. Since the representative household is a utility maximizer with rational expectations, it would avail of the fact that MCS is higher in the present period by substituting future consumption in period $t+1$ for additional consumption in period t; and it would continue to do so until $MCS = MBS$. The result is that the representative household will always select an intertemporal allocation that satisfies the equality in (3); if it did not, it would not be maximizing expected lifetime utility U, and hence be in violation of assumptions III and VI.

The intratemporal labor-leisure optimality condition given in (4) states that the marginal rate of substitution between leisure and consumption (MRS_{1-L_t,C_t}) will always equal the prevailing wage rate W_t.[15] The parameter μ determines how much weight consumption and leisure receive in the

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[14] See Appendix for full derivation.

[15] Note that as L_t corresponds to hours worked, and the representative household's time endowment for each period is equal to 1, it follows that $1-L_t$ represents the leisure taken by the household for any given period.

representative household's utility function and is set within the bounds 0 and 1. The intuition behind why this equation must hold is similar to that of (3): MRS_{1-L_t,C_t} is equal to the marginal utility of leisure divided by the marginal utility of consumption, multiplied by the weighting given by μ. If the wage rate did not equal the right-hand-side of the equation, then it would hold that the weighted marginal utilities did not equal one another; in other words, that the representative household could increase the marginal utility it derives from either consumption or leisure by substituting one for the other. Again, as the representative household is a utility maximizer, the household will *always* be minded to make this substitution, and hence the equality in (4) *always* holds.

Recall that for the representative household's intertemporal consumption and labor allocation to be revised, the household must receive new information. The information that determines the intertemporal allocation of the representative household comes via two price signals: the interest rate r_t (i.e. the return on capital) and the wage rate W_t (i.e. the return on labor). Following assumptions I and VIII, the equations which describe how these price signals evolve through is given by (5) and (6):[16]

$$r_t = \alpha A_t (\frac{K_t}{L_t})^{\alpha-1} \equiv MPK_t. \tag{5}$$

$$W_t = (1-\alpha) A_t (\frac{K_t}{L_t})^{\alpha} \equiv MPL_t. \tag{6}$$

(5) and (6) convey that r_t and W_t are both positively related to total factor productivity A_t, but that r_t is decreasing in the capital-labor ratio $\frac{K_t}{L_t}$, while W_t is increasing in $\frac{K_t}{L_t}$. For the intertemporal allocation of the representative household to be revised, the values of r_t and W_t must therefore change in a manner that ex ante is not anticipated by the household. From this fact it follows that cyclical fluctuations in the simple RBC model can only occur in the event that unexpected adjustments in r_t and W_t take place.

In the forthcoming section, I elaborate on how RBC models account for these unexpected adjustments in r_t and W_t, and therefore how RBC models explain cyclical macroeconomic fluctuations and business cycles more generally.

[16] See Appendix for full derivation.

§2. The Role of Productivity Shocks in The Simple RBC Model

As it stands, the simple RBC model described by assumptions I–X and equations (2)–(6) will permanently remain at its steady state, as exhibited in figure 1:[17]

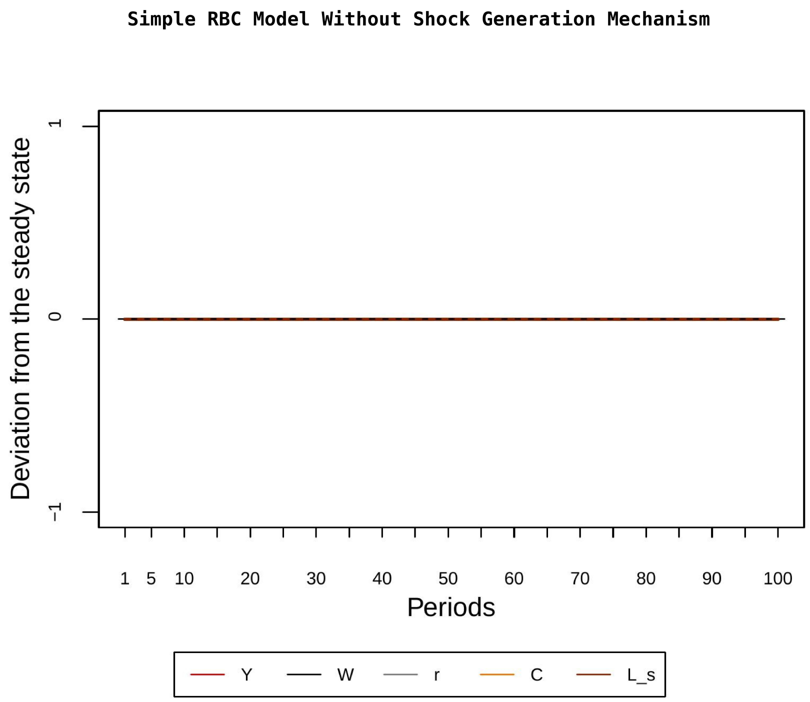

Figure 1.

For the simple RBC model to account for the dynamic macroeconomic fluctuations associated with business cycles, a shock propagation mechanism must be introduced to the model. In the simple RBC model, this mechanism is provided by exogenous stochastic productivity shocks:[18]

$$\ln A_t = \rho \ln A_{t-1} + \epsilon_t, \quad 0 \leq \rho \leq 1. \tag{7}$$

(7) is an $AR(1)$ process that models how productivity evolves through time. In this process, ϵ_t is an exogenous stochastic productivity shock term and ρ is a parameter (the autocorrelation) that measures the persistence of these shocks.

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[17] Figure 1 was generated using a simple RBC model with the structure I-X that I coded in R using the gEcon package. In the simulation, in line with standard quarterly discrete time RBC models, the default parameter values are as follows: $\rho = 0.979$, $\beta = 0.99$, $\mu = 0.3$, $\eta = 1.5$ and $\delta = 0.025$. The model is accessible via my **GitHub**.

For more information regarding the gEcon package, refer to Karol Podemski and Kaja Retkiewicz-Wijtiwiak, *gEcon Users' Guide: General Equilibrium Economic Modelling Language and Solution Framework*, ver. 1.2.3 (Warsaw, 2025); and Karol Podemski, Kaja Retkiewicz-Wijtiwiak, and Grzegorz Klima, *gEcon Reference Manual: General Equilibrium Economic Modelling Language and Solution Framework*, ver. 1.2.3 (Warsaw, 2025).

[18] Mullineux and Dickinson, "Equilibrium Business Cycles," 323–24.

The simple RBC model couples (7) with the assumptions XI and XII:

XI. Productivity shocks cannot be predicted ex ante and have an expected value of zero.

- $\varepsilon_{t\geq 0}$ is an i.i.d. sequence where $E[\varepsilon_t|A_{t-1}] = E[\varepsilon_t] = 0$ and $\mathrm{Var}(\varepsilon_t) = \sigma^2_\varepsilon$.

XII. The economy described by the simple RBC model begins in equilibrium.

- $LnA_0 = 0$.

Assumptions XI and XII create a channel through which shocks that perturb the steady state of the model can be propagated. The manner in which households and firms respond to these perturbations is what gives rise to macroeconomic fluctuations in the simple RBC model. When ϵ_t takes on a non-zero value, both r_t and W_t take on new values in subsequent periods (note, as conveyed in (5) and (6), both r_t and W_t are functions of A_t). The representative household responds to these new price signals for r_t and W_t by revising its intertemporal consumption and labor allocation. The macroeconomic adjustments engendered by the representative household's revised intertemporal allocation is what generates business cycles in the simple RBC model. Figure 2 presents simulated time-series data for the simple RBC model in the setting of (7) and XI–XII with random shocks.[19]

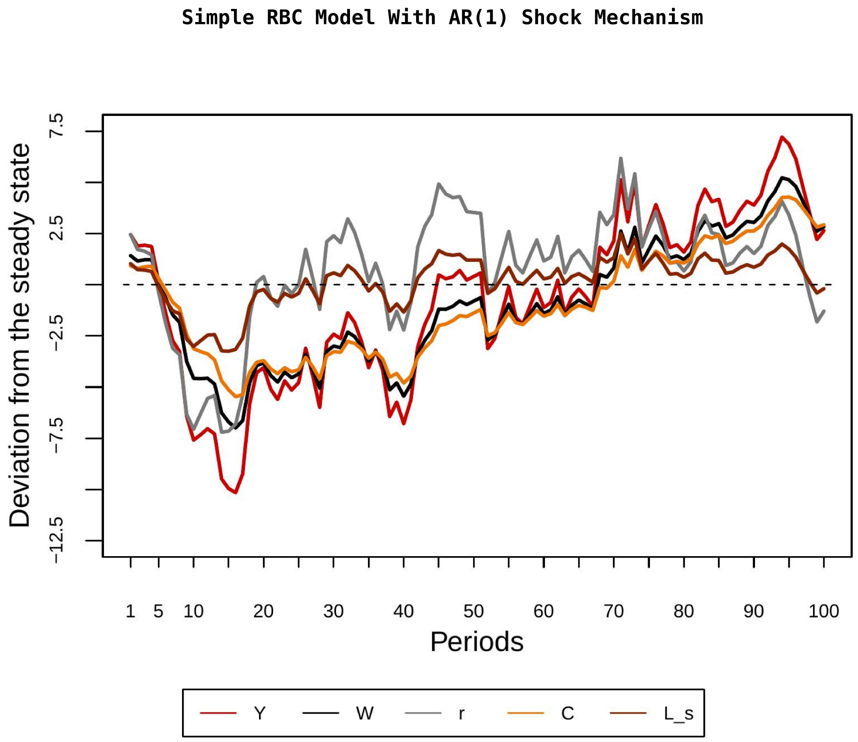

Figure 2.

[19] Figure 2 was generated using a simple RBC model with the structure I-XII that I coded in R using the gEcon package. In the simulation, in line with standard quarterly discrete time RBC models, the default parameter values are as follows: $\rho = 0.979, \beta = 0.99, \mu = 0.3, \eta = 1.5$ and $\delta = 0.025$. The model is accessible via my **GitHub**.

The simple RBC model therefore explains business cycles as phenomena arising from forward looking households optimally responding to unanticipated changes in productivity, thereby inducing dynamic adjustments in macroeconomic variables.[20] Productivity shocks introduce both income and substitution effects to the representative household's optimization problem. The magnitudes and signs of each of these effects, however, depend on whether the shock in question is positive or negative; how persistent the shock is (i.e. the value of ρ); and the household's preferences over consumption and leisure.

The simple RBC model implies two strong results:[21]

i. The economy self-equilibrates in response to exogenous shocks.

ii. The macroeconomic fluctuations associated with business cycles are both Pareto efficient and welfare maximizing.

The first result follows from the assumptions of fully flexible prices and wages, perfect competition and a rational, forward-looking household. That is, as there are *no* nominal rigidities in the model, once a productivity shock occurs, factor prices adjust immediately, and the representative household revises its intertemporal allocation accordingly. This, in turn, ensures that the economy remains in equilibrium, and places it on a trajectory back toward a steady state (in the absence of further shocks). The second result is, similarly, a consequence of the model assuming perfect competition, combined with the absence of externalities or missing markets. Taken together, they imply the First Fundamental Theorem of Welfare economics; namely, that any competitive equilibrium allocation will satisfy Pareto efficiency.[22] As all realized allocations in the setting of the simple RBC model are competitive, it follows that the allocations induced by exogenous shocks, and the macroeconomic adjustments associated with them, are Pareto efficient. The fact that the economy assumes a single representative household further entails that the solution to the maximization problem in (2) will coincide with the allocation chosen by a social planner, and hence all realized allocations in the simple RBC model are welfare maximizing in addition to being Pareto efficient. The combination of ii and iii therefore imply:

iii. Fiscal and monetary interventions cannot yield Pareto or Kaldor-Hicks improvements.[23]

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[20] Plosser, "Understanding Real Business Cycles," 53.

[21] Plosser, "Understanding Real Business Cycles," 56; Mullineux and Dickinson, "Equilibrium Business Cycles," 323.

[22] The absence of externalities or missing markets is an artifact of the assumptions that the economy trades one homogeneous good Y, the production of which is characterized by Cobb-Douglas production technology.

[23] Note, in a setting with more than one household, fiscal interventions could theoretically be justified on the grounds of preferences over different allocation distributions. Monetary interventions are, however, inert even in this regard, due to the strong assumption of fully flexible prices and wages.

This latter result has been the focus of much of the criticism of simple variants of RBC models.[24]

§3. Why Productivity Shocks Must be Persistent

For the simple RBC model to accurately fit time-series business cycle data, the model must assume that exogenous productivity shocks are highly persistent (i.e. $\rho \to 1$).[25] To illustrate this point, I briefly review some historical data on business cycle trends, and then consider two cases: when $\rho = 0$ and when $\rho = 0.979$.

Figure 3 presents statistics derived from US quarterly time-series data for the period 1947-1996.[26] Take note of the magnitudes, signs and various co-movements of the macroeconomic variables presented in the figure.

Business cycle statistics for the US Economy (1947-1996)

Macro Variable	s.d.	Relative s.d.	First-order autocorrelation	Contemporaneous correlation with output
Y	1.81	1.00	0.84	1.00
C	1.35	0.74	0.80	0.88
I	5.30	2.93	0.87	0.80
L	1.79	0.99	0.88	0.88
Y/L	1.02	0.56	0.74	0.55
W	0.68	0.38	0.66	0.12
r	0.30	0.16	0.60	−0.35
A	0.98	0.54	0.74	0.78

Figure 3.

In the data, we observe a number of important regularities:

P1. Consumption, investment, employment and productivity are all strongly pro-cyclical.

P2. The interest rate is counter-cyclical.

P3. The wage rate is almost acyclical.

P4. Consumption is considerably less sensitive to changes in output than investment is (note the

24 Plosser, "Understanding Real Business Cycles," 56–57.

25 King and Rebelo, "Resuscitating Real Business Cycles," 963.

26 King and Rebelo, "Resuscitating Real Business Cycles," 938.

relevant metric here is the relative standard deviation). [27]

P5. All macroeconomic variable time-series exhibit substantial persistence.

Although the time-series data is derived from a US context, market economies have, in general, been observed to exhibit similar regularities; a point that proponents of early RBC models (i.e. those akin to the simple model presented here) had argued undermined country-specific explanations of business cycles that invoked fiscal or monetary policy idiosyncrasies.[28]

When $\rho = 0$. If productivity shocks are assumed to have zero persistence, a simulation of the simple RBC model will yield the impulse response functions given in figure 4.[29]

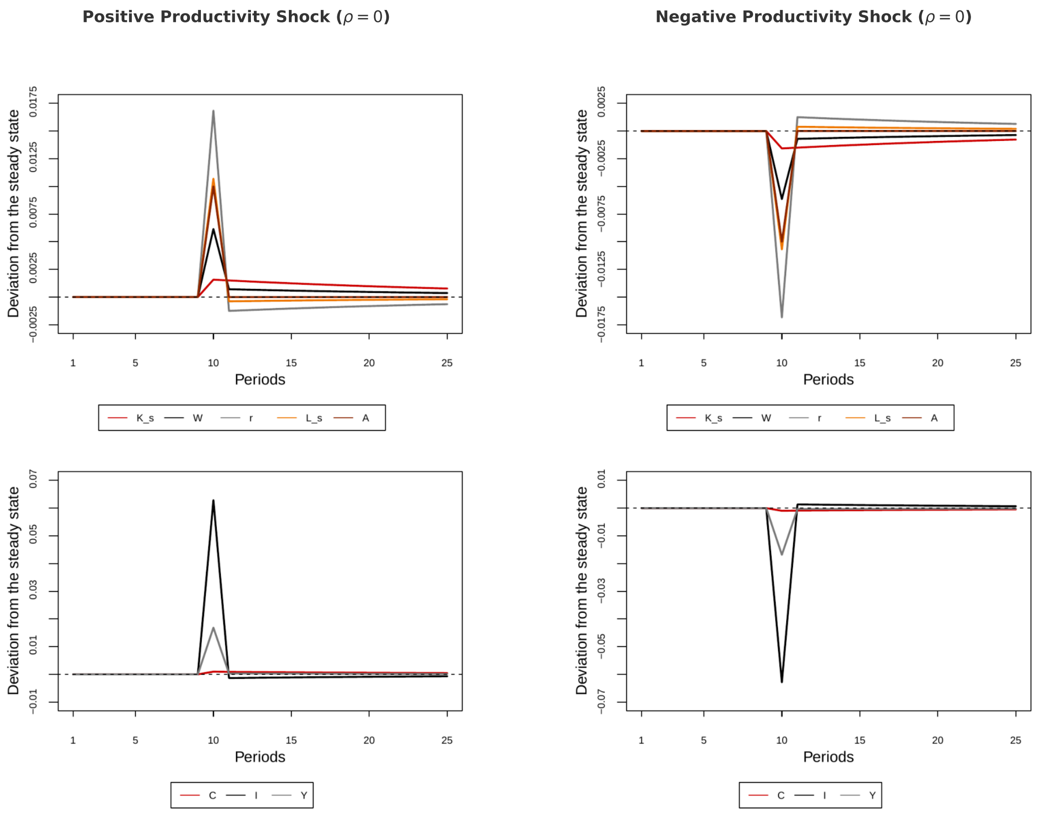

Figure 4.

The impulse response functions in figure 4 simulate a $\pm 1\%$ productivity shock under the assumption that $\rho = 0$. While the signs and co-movements of the various macroeconomic variables match the

[27] This finding is consistent with the tendency of the representative household, in the setting of the simple RBC model, to smooth marginal utility across each period in order to maximize lifetime utility.

[28] Mullineux and Dickinson, "Equilibrium Business Cycles," 336; King and Rebelo, "Resuscitating Real Business Cycles," 931; Plosser, "Understanding Real Business Cycles," 59.

[29] Figure 4 was generated using a simple RBC model with the structure I-XII that I coded in R using the gEcon package. In the simulation, with each period representing a discrete quarter, the parameter values are as follows: $\rho = 0, \beta = 0.99, \mu = 0.3, \eta = 1.5$ and $\delta = 0.025$. The model is accessible via my **GitHub**.

business cycle statistics given in figure 3, the first-order autocorrelations of productivity, output, investment and labor do not. This is because when $\rho = 0$, there is very little persistence in productivity shocks beyond the period in which they occur, and therefore the substitution effect generated by the shock strongly dominates any income effects.[30] That is, the overall effect of the shock on life-time income for the representative household is negligible: households have little-to-no incentive invest in expanding the capital stock, as the elevated return on capital is entirely transitory, and as the capital accumulation equation is the mechanism within the simple RBC model that propagates shocks through time, the consequence is that the business cycles generated by shocks are abrupt and short lived.[31]

When $\rho = 0.979$. If instead productivity shocks are assumed to be highly persistent, a simulation of the simple RBC model can capture much of the historical variation and co-movements among macroeconomic variables observed in business cycle time-series data.[32]

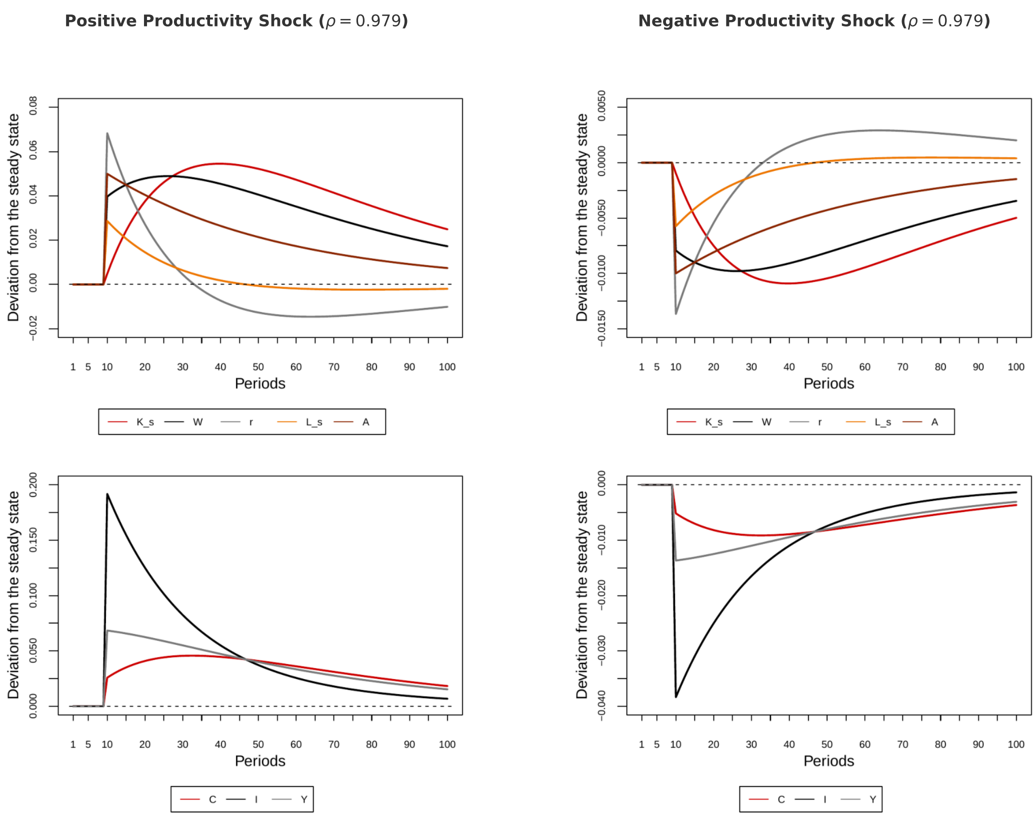

Figure 5.

30 King and Rebelo, "Resuscitating Real Business Cycles," 964.

31 King and Rebelo, "Resuscitating Real Business Cycles," 964.

32 Figure 5 was generated using a simple RBC model with the structure I-XII that I coded in R using the gEcon package. In the simulation, with each period representing a discrete quarter, the parameter values are as follows: $\rho = 0.979$, $\beta = 0.99$, $\mu = 0.3$, $\eta = 1.5$ and $\delta = 0.025$. The model is accessible via my **GitHub**.

Similarly to figure 4, the impulse response functions in figure 5 simulate a $\pm 1\%$ productivity shock, but under the assumption that $\rho = 0.979$. The result is that not only do the simulations correctly predict the signs (except for r_t) and co-movements observed in business cycle data, but also the corresponding first-order auto-correlations. This is because, unlike in the prior case, a high degree of persistence in productivity shocks implies strong income effects, which in turn generate protracted cyclical movements in macroeconomic variables.[33] As the productivity shock is long-lasting, the representative household has an incentive (in the case of a positive shock) to invest in future capital, and thereby substitute current consumption for future consumption.[34] This inclination, recall, is a consequence of the representative household's tendency to smooth marginal utility across each period of the model, so as to maximize lifetime utility.

§4. *Getting Real* about Real Business Cycles

The simple RBC model has been subject to a number of strong criticisms. The assumptions of perfect competition and fully flexible prices and wages have drawn significant scrutiny.[35] Relatedly, critics have questioned whether the simple RBC model can plausibly account for historical episodes of significant downturns in output, such as the Great Depression and Great Financial Crisis.[36] Others have argued that the simple RBC model makes implausible assumptions about the rate at which technological shocks diffuse throughout the economy.[37] Below, I limit my discussion to just two lines of critique that have been directed at the simple RBC model: its discordant predictions regarding labor market data and the identification strategy used to justify the highly persistent shocks assumed by the model.

In effect, the simple RBC model explains unemployment as an artifact of households engaging in intertemporal substitution (e.g. 'wages are higher (lower) today, so on the margin I will work more (less) today and less (more) tomorrow').[38] However, when Millar et al. applied the simple RBC framework to UK labor market data, they found that this intertemporal logic, even when supplemented with various extensions such as indivisible labor supply, cannot account for a number

33 King and Rebelo, "Resuscitating Real Business Cycles," 969–73.

34 King and Rebelo, "Resuscitating Real Business Cycles," 973.

35 Mullineux and Dickinson, "Equilibrium Business Cycles," 338.

36 Lawrence H. Summers, "Some Skeptical Observations on Real Business Cycle Theory," *Federal Reserve Bank of Minneapolis Quarterly Review* 10, no. 4 (Fall 1986): 23.

37 John Muellbauer, "The Assessment: Business Cycles," *Oxford Review of Economic Policy* 13, no. 3 (1997): 9.

38 Stephen Millard, Andrew Scott, and Marianne Sensier, "The Labour Market over the Business Cycle: Can Theory Fit the Facts?" *Oxford Review of Economic Policy* 13, no. 3 (1997): 71.

of salient empirical regularities.[39] First, Millar et al. observed that employment (and unemployment) is more sensitive to *cyclical* changes in output than predicted by the simple RBC model.[40] Second, the observed correlations between wages and employment are much weaker than predicted by the model.[41] Third, persistent unemployment, and the manner in which this persistence evolves through time, cannot be accounted for within the simple RBC model.[42] Furthermore, as observed by Summers, in order for the simple RBC model to match cyclical movements in the labor market generated by relatively small shocks, it must assume a high Frisch intertemporal elasticity of labor supply.[43] However, this generates a dilemma for the model, as assuming a high Frisch elasticity implies that countries experiencing strong growth in trend output over long periods should observe decreases in aggregate working hours.[44] Yet this prediction has not been observed in the high growth episodes of Japan, South Korea, Ireland, or elsewhere.

Another area of difficulty for the simple RBC model is its identification strategy with respect to productivity shocks. Recall that the simple RBC model posits that exogenous stochastic productivity shocks generate business cycles. Fitting the simple RBC model to business cycle data therefore requires the availability of time-series data on productivity shocks. However, productivity shocks are not directly observable in the same way that output, labor supply, consumption and interest rates are. To handle this apparent limitation in the data, proponents of the simple RBC model employed an approach from growth accounting; namely, the Solow Residual measurement technique:[45],[46]

$$\frac{dA_t}{A_t} = \frac{dY_t}{Y_t} - \alpha\frac{dK_t}{K_t} - (1-\alpha)\frac{dL_t}{L_t}. \tag{8}$$

The technique involves taking the log of a Cobb-Douglas production function and expressing it in terms of the log derivative of total factor productivity (TFP). As time-series data is readily available on output, labor supply, the capital stock and the income shares of capital and labor, the left-hand-side of (8) can purportedly be inferred, thereby generating the time-series productivity data necessary to fit

39 Millard et al., "The Labour Market," 88.

40 Millard et al., "The Labour Market," 88.

41 Millard et al., "The Labour Market," 88.

42 Millard et al., "The Labour Market," 88.

43 Summers, "Some Skeptical Observations," 24.

44 Summers, "Some Skeptical Observations," 24.

45 Mullineux and Dickinson, "Equilibrium Business Cycles," 333–34.

46 Note (8) is in continuous, not discrete time. I use continuous time for ease of exposition; a perfectly analogous point can be made in the case of discrete time.

the simple RBC model. This identification strategy, however, is problematic because the structure of (8) imputes to TFP any variation in output that cannot be explained by variation in labor or capital.[47] The identification strategy is therefore very vulnerable to omitted variables bias, in that there may be factors that are correlated to changes in output that are either correlated with TFP (hence engendering an identification problem) or completely unrelated to TFP yet are falsely imputed to it. Moreover, there are a large number of candidates for potential confounders: monetary and fiscal shocks; financial market dynamics; capital under utilization and labor hoarding to name but a few. For these reasons, the Solow Residual is often thought of as 'a measure of ignorance' rather than a reliable measure of productivity.

§5. Conclusion

In the simple RBC model, business cycle fluctuations are driven by exogenous stochastic productivity shocks. The model explains how these shocks propagate throughout the economy by positing households and firms which engage in intertemporal utility-maximizing behavior, hold rational expectations and operate in perfectly competitive markets with fully flexible prices and wages. If the simple RBC model is granted the assumption that these productivity shocks are highly persistent, it can account for a significant amount of the variation in the co-movements, magnitudes, signs and first-order autocorrelation observed in business cycle data. However, the Solow Residual measurement technique used to generate the time series data necessary to calibrate the productivity shocks in the simple model is beset by identification problems. Separately, the simple RBC model generates a number of predictions that are discordant with well-founded observations in the labor market. Therefore, it would appear, contrary to the simple RBC model, that we are not all living in Walras's world after all.

[47] King and Rebelo, "Resuscitating Real Business Cycles," 962.

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Appendix

Deriving the consumption Euler equation (3)

Setting up the Lagrangian $\mathcal{L}$ with multiplier λ_t:

$$\mathcal{L} = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ u(C_t, L_t) + \lambda_t \left[(1 + r_t - \delta)K_t + W_t L_t + \Pi_t - C_t - K_{t+1} \right] \right\}$$

The FOC wrt C_t is:

$$\beta^t \frac{\partial u}{\partial C_t} - \beta^t \lambda_t = 0 \implies \lambda_t = MU_{C_t}$$

The FOC wrt K_{t+1} involves terms from period t and $t+1$:

$$-\beta^t \lambda_t + E_t \left[\beta^{t+1} \lambda_{t+1} (1 + r_{t+1} - \delta) \right] = 0$$

Substituting $\lambda_t = MU_{C_t}$ and $\lambda_{t+1} = MU_{C_{t+1}}$ gives us equation (3):

$$MU_{C_t} = \beta E_t \left[MU_{C_{t+1}} (1 + r_{t+1} - \delta) \right]$$

Deriving the intratemporal labor-leisure optimality condition (4)

The FOC wrt L_t is:

$$\beta^t \frac{\partial u}{\partial L_t} + \beta^t \lambda_t W_t = 0$$

Since utility decreases in labor (increases in leisure), $\frac{\partial u}{\partial L_t} = -MU_{\text{Leisure}}$. Substituting $\lambda_t = MU_{C_t}$:

$$W_t = \frac{MU_{\text{Leisure}}}{MU_{C_t}}$$

Using assumption VII where $u_t = \frac{Z^{1-\eta}}{1-\eta}$ with $Z = C_t^{\mu}(1 - L_t)^{1-\mu}$:

$$MU_{C_t} = Z^{-\eta} \cdot \mu C_t^{\mu-1} (1 - L_t)^{1-\mu}$$
$$MU_{\text{Leisure}} = Z^{-\eta} \cdot (1 - \mu) C_t^{\mu} (1 - L_t)^{-\mu}$$

Expressing it as a fraction gives us the equation (4):

$$W_t = \frac{(1-\mu)C_t^{\mu}(1-L_t)^{-\mu}}{\mu C_t^{\mu-1}(1-L_t)^{1-\mu}} = \frac{1-\mu}{\mu}\frac{C_t}{1-L_t}$$

Deriving the factor price equations (5) and (6)

The firm maximizes profit $\pi_t = Y_t - r_t K_t - W_t L_t$, where $Y_t = A_t K_t^{\alpha} L_t^{1-\alpha}$. The FOCs wrt K_t and L_t are:

$$\frac{\partial \pi_t}{\partial K_t} = \alpha A_t K_t^{\alpha-1} L_t^{1-\alpha} r_t = 0 \implies r_t = \alpha A_t \left(\frac{K_t}{L_t}\right)^{\alpha-1}$$
$$\frac{\partial \pi_t}{\partial L_t} = (1-\alpha) A_t K_t^{\alpha} L_t^{-\alpha} - W_t = 0 \implies W_t = (1-\alpha) A_t \left(\frac{K_t}{L_t}\right)^{\alpha}$$