# COMS W4701: Artificial Intelligence

## Lecture 4: Informed Search

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# Today

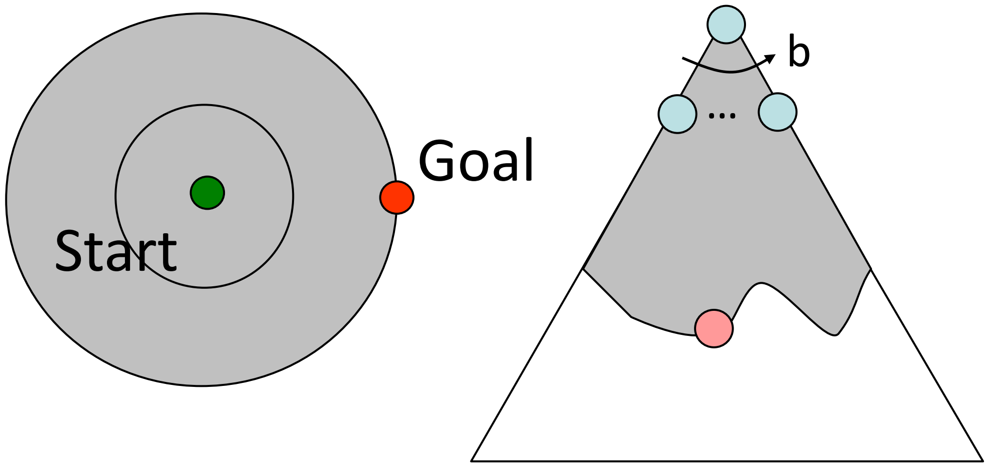
- A* search
- Heuristic function properties
- A* search variations

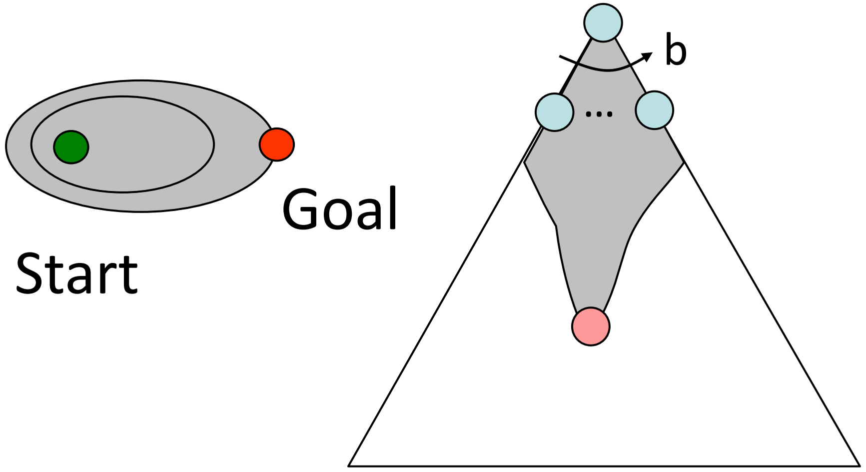
# Informed (Heuristic) Search

- DFS, BFS, UCS were **uninformed** about goal state

- Now suppose we have additional, *domain-specific heuristics* that estimate state cost to goal
- **Heuristic function** $h(n)$: Estimated cost of cheapest path from state at node $n$ to a goal state

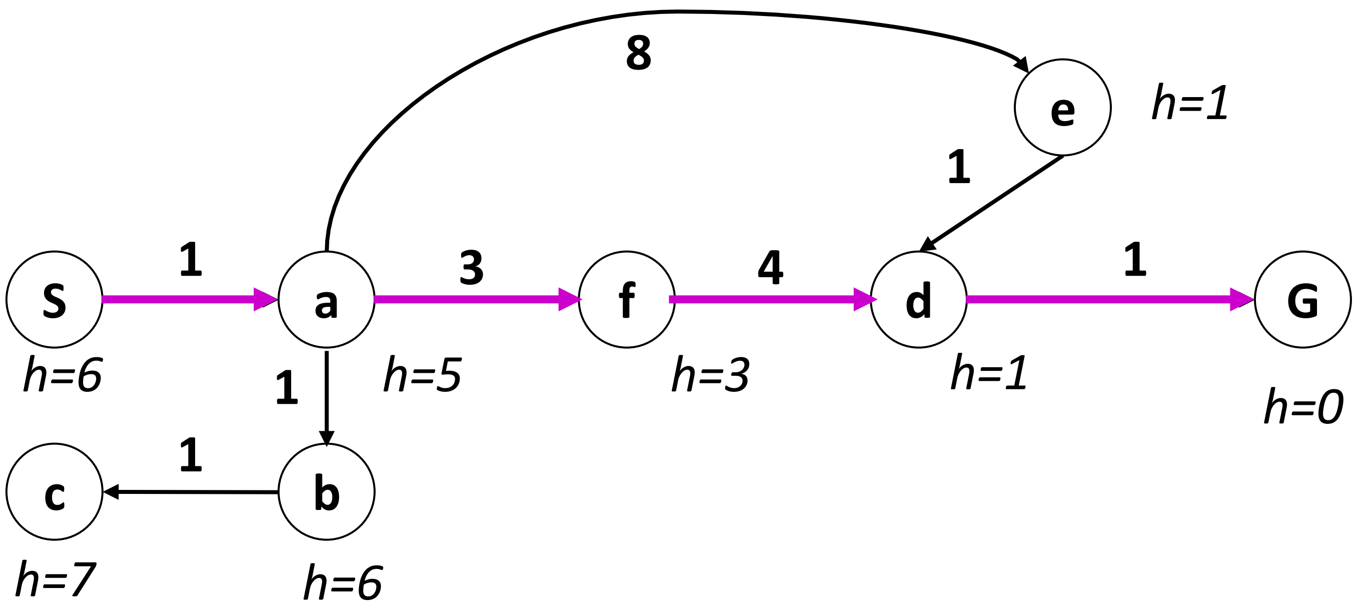
- We can combine heuristics with cumulative path costs
- Can still follow UCS strategy of expanding cheapest paths first, but prune or delay less promising paths



# A* Search

- **A* search**: Evaluate each node based on both cumulative cost and heuristic value
- $f(n) = g(n) + h(n)$: Sum of path cost to $n$ and estimated cost from $n$ to goal
- Implementation: Priority queue using $f(n)$ as before




Frontier (node $n$, $f = g + h$):

1. $[(S, 6)]$
2. $[(a, 6)]$
3. $[(f, 7), (b, 8), (e, 10)]$
4. $[(b, 8), (d, 9), (e, 10)]$
5. $[(d, 9), (c, 10), (e, 10)]$
6. $[(G, 9), (c, 10), (e, 10)]$

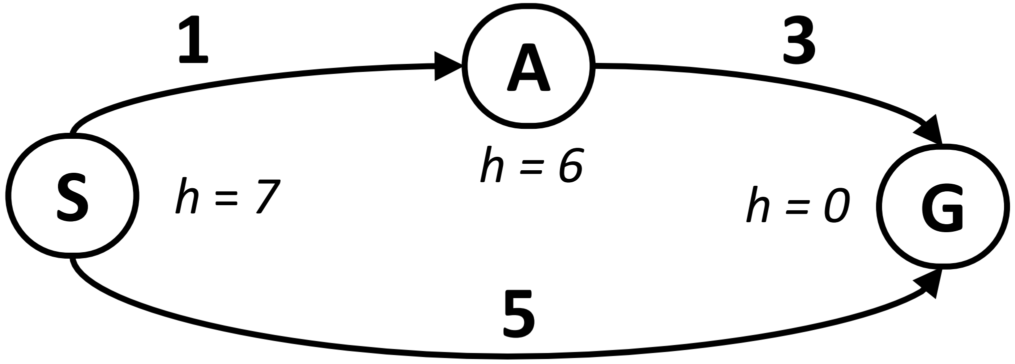
Solution: $S, a, f, d, G$; cost: 9

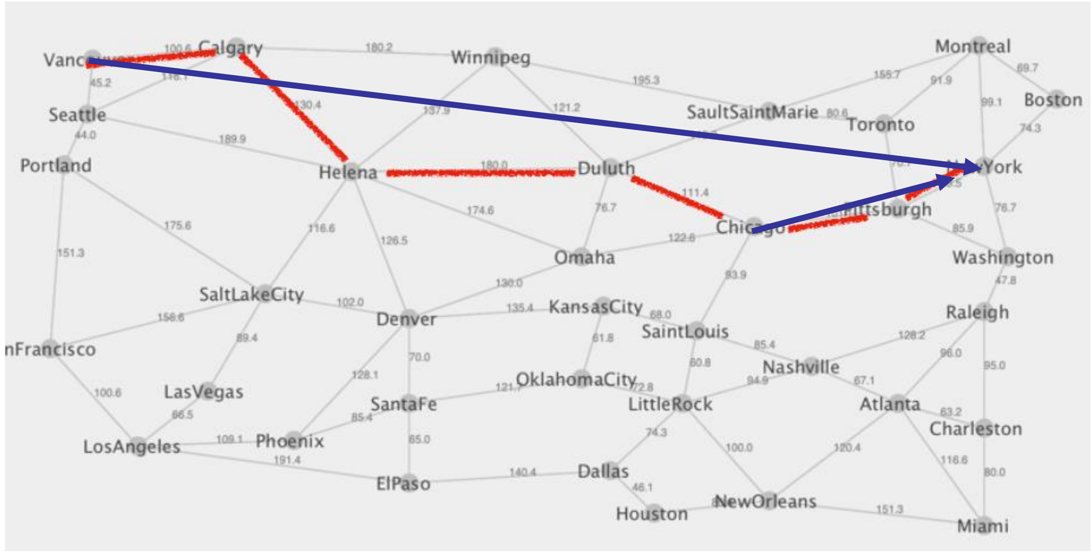
# Admissibility

- In this graph, A* returns suboptimal solution S->G rather than optimal solution S->A->G
- $h(A)$ overestimated the cost from A!

- A heuristic $h$ is **admissible** if $h(n) \leq h^*(n)$ where $h^*(n)$ is true cost from $n$ to goal
- In practice, we usually do not know $h^*(n)$

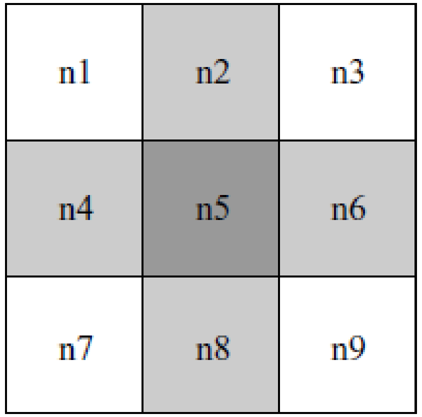
- One strategy to derive admissible heuristics: *relax* problem specifications by removing constraints, making them easier



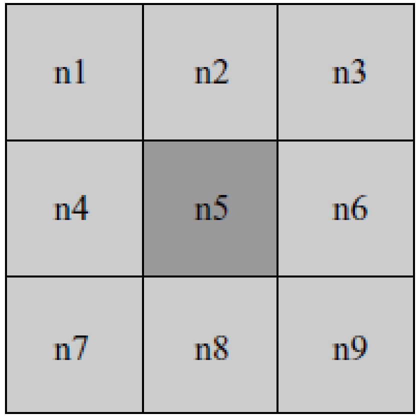



# Example: Grid Distances

- Grid navigation with goal $g = (x_g, y_g)$ and all transitions having cost 1
- **Manhattan distance** ($L^1$ norm): $h_1(x,y) = |x_g - x| + |y_g - y|$
- **Euclidean distance** ($L^2$ norm): $h_2(x,y) = \sqrt{(x_g - x)^2 + (y_g - y)^2}$




- If we have *4-point connectivity* (actions = {up, down, left, right}), both heuristics are admissible ($h_2$ *underestimates* true costs)




- If we have *8-point connectivity* (actions = above + 4 diagonal actions), *neither* is admissible, but $\frac{1}{\sqrt{2}} h_2$ is!

# Heuristic Domination

- In a 4-point connected grid, $L^1$ norm always $\geq L^2$ norm between two cells
- $h_1$ **dominates** $h_2$ if $h_1(n) \geq h_2(n)$ for all $n$

- A* using $h_1$ will be more efficient and never expand more nodes than $h_2$
- $h_1$ reflects true costs more accurately

- Suppose we have collection of admissible heuristics $h_1, h_2, \ldots, h_m$
- The composite heuristic $h(n) = \max\{h_1(n), \ldots, h_m(n)\}$ is admissible and dominates all other heuristics!

# Completeness and Optimality of A*

- Let $g^*(n)$ be cheapest path cost from start to node $n$
- If optimal goal has cost $C^*$, all nodes along path satisfy $g^*(n) + h^*(n) = C^*$

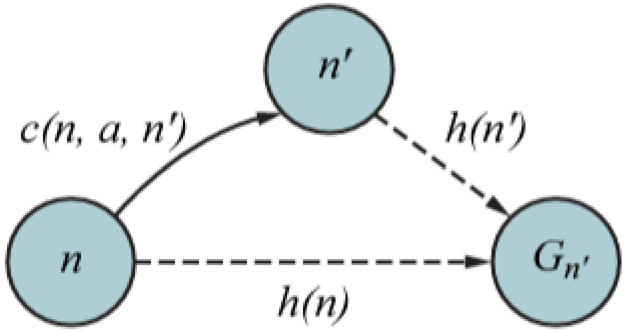
- Now suppose that heuristic function is admissible: $h(n) \leq h^*(n)\ \forall n$
- Then all nodes along optimal path satisfy $f(n) = g^*(n) + h(n) \leq C^*$
- All optimal solution nodes are expanded before any suboptimal goal with cost $C > C^*$

- A* is **complete**: If it exists, a solution will eventually be found and returned
- A* is **optimal**: Optimal solution will be returned before others with $C > C^*$
- A* *improves* upon UCS by skipping “useless” nodes that have $g^*(n) + h(n) > C^*$

# Consistency

- A stronger heuristic property is **consistency** (triangle inequality)
- $h(n) - h(n') \leq c(n, a, n')$ for all $n'$
- Parent heuristic − child heuristic ≤ true cost

- All consistent heuristics are admissible (but not vice versa)
- *Most* admissible heuristics are also consistent in practice




- Consistency ensures that the first expansion of a node is along cheapest path
- Heuristic consistency ensures that A* is **optimally efficient**—it expands the fewest nodes compared to any other optimal algorithm with the same heuristic

# Satisficing Solutions

- Like BFS or UCS, A* may suffer computationally intractable memory requirements
- Idea: Trade off admissibility for more accurate heuristics to reduce computation
- Return **satisficing solutions**—suboptimal, but "good enough"

- **Weighted A* search**: $f(n) = g(n) + \alpha h(n)$
- $\alpha > 1$ focuses the contour of reached states closer to the goal
- Generalizes A* ($\alpha = 1$), UCS ($\alpha = 0$), and greedy best-first ($\alpha \to \infty$)

- Fewer states expanded than A*, but may miss the optimal solution
- Suboptimality: If optimal solution has cost $C^*$, weighted A* solution may cost up to $\alpha C^*$

# Memory-Bounded Search

- We can also consider A* variants that are more memory-efficient

- **Beam search**: Fixed frontier size, only keep $k$ best nodes at any iteration
- Or set threshold for discarding frontier nodes relative to current lowest $f$-value

- **Iterative-deepening A*** (IDA*): Depth-first iterative deepening search, only considering nodes with $f$-value not exceeding current cutoff value
- In each iteration, increment cutoff by *smallest* $f$-value of the skipped nodes

- IDA* yields linear spatial complexity of DFS; each iteration can progress steadily down the tree if $f$-values tend to increase consistently along paths

# Summary

- Domain-specific heuristics can guide search toward goal
- A* search combines true costs and heuristics to evaluate frontier nodes

- Admissible heuristics do not overestimate true costs -> A* is optimal

- Consistent heuristics satisfy triangle inequality -> A* is optimally efficient

- Many other variations of A* to deal with suboptimality, memory limits