University of Salzburg — Lecturer: Roland Kwitt

Machine Learning (911.236)

Exercise sheet **B**

Exercise 1. 2 P.

Let X be a random variable with finite non-zero variance $\mathbb{V}[X] = \mathbb{E}[(X - \mathbb{E}[X])^2] = \sigma^2$. Show that, for $t > 0$, it holds that

$$\mathbb{P}[|X - \mathbb{E}[X]| \geq t] \leq \frac{\mathbb{V}[X]}{t^2} \quad . \tag{1}$$

Exercise 2. 2 P.

Say you have a random variable X that follows an exponential distribution with rate $\lambda > 0$; we write $X \sim \text{Exp}(\lambda)$. Bound $\mathbb{P}[X \geq a]$ by Markov's inequality and compare the bound to the *actual* value of $\mathbb{P}[X \geq a]$. You can find the latter (i.e., $\mathbb{P}[X \geq a]$) by (Riemann) integrating the probability density function (pdf) of the random variable X from a to ∞.

Exercise 3. 3 P.

Let us throw an ideal/fair coin 1,000 times and let the random variable X count how often we observe *heads*. Hence, X follows a *binomial distribution* with $n = 1000$ and success probability of $1/2$; we write $X \sim \text{Binomial}(n, 1/2)$. Bound the probability of observing ≥ 550 times *heads*, using Eq. (1).

Total #points: 7 P.