Modelos Estáticos-Anisótropos.

Las ecuaciones de campo

Ecuaciones de Einstein

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R.$$

Elemento de línea

$$ds^2 = e^{2\nu} dt^2 - e^{2\lambda} dt^2 - r^2 dr^2 - r^2 sen^2\theta \, d\phi^2$$

El tensor de Energía-Impulso

$$T_{\mu}^{\nu} = \begin{pmatrix} \rho & 0 & 0 & 0 \\ 0 & -P_r & 0 & 0 \\ 0 & 0 & -P_t & 0 \\ 0 & 0 & 0 & -P_t \end{pmatrix}$$

Las ecuaciones de campo se escriben como:

$$8\pi\rho = \frac{e^{-2\lambda}(2r\lambda' - 1)}{r^2} + \frac{1}{r^2}$$

$$8\pi P_r = \frac{e^{-2\lambda}(2r\nu' - 1)}{r^2} - \frac{1}{r^2}$$

$$8\pi P_t = e^{-2\lambda}\left(\nu'' + \nu'^2 + \left(\lambda' - \frac{1}{r}\right)\nu' - \frac{\lambda'}{r}\right)$$

La ecuación de T.O.V. $\quad T^{\mu}_{1;\mu} = 0$

$$P' + \frac{\left(\frac{m'}{r^2} + P\right)(m + r^3 P)}{r(r - 2m)} - \frac{2}{r}(P_t - P) = 0$$

$m' = 4\pi r^2 \rho$

Propuestas de anisotropía

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1) $$P_t - P = \frac{m' m}{2 r^2 (r-2m)}$$

$$P' + \frac{r^2}{(r-2m)} P^2 + \frac{m' r + m}{r(r-2m)} P = 0$$

In[•]:=
```
ec1 = D[P[r], r] + r^2 / (r - 2 * m[r]) * P[r]^2 +
      (r * D[m[r], r] + m[r]) / (r * (r - 2 * m[r])) * P[r]
```
deriva
deriva

In[•]:=
```
DSolve[ec1 == 0, P[r], r] // FullSimplify
```
resolvedor diferencial   simplifica completamente

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2) $$P_t = \frac{m' m}{2 r^2 (r-2m)} \quad y \quad P_r = \frac{P}{r^2}$$

$$P' + \frac{1}{(r-2m)} P^2 + \frac{m' r + m}{(r-2m)} P = 0$$

In[•]:=
```
ec2 = D[P[r], r] + 1 / (r - 2 * m[r]) * P[r]^2 + (r * D[m[r], r] + m[r]) / (r - 2 * m[r]) * P[r]
```
deriva   deriva

Out[•]= $$\frac{P[r]^2}{r - 2\,m[r]} + \frac{P[r]\,(m[r] + r\,m'[r])}{r - 2\,m[r]} + P'[r]$$

In[•]:=
```
DSolve[ec2 == 0, P[r], r] // FullSimplify
```
resolvedor diferencial   simplifica completamente

Out[•]= $$\left\{\left\{P[r] \to \frac{e^{\int_1^r -\frac{m[K[1]] + K[1]\, m'[K[1]]}{K[1] - 2\, m[K[1]]} dK[1]}}{\mathbb{C}_1 - \int_1^r -\frac{e^{\int_1^{K[2]} -\frac{m[K[1]] + K[1]\, m'[K[1]]}{K[1] - 2\, m[K[1]]} dK[1]}}{K[2] - 2\, m[K[2]]} dK[2]}\right\}\right\}$$

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3) $$P_t = \alpha(r)\, P \quad y \quad P_r = \frac{P}{r^2}$$

$$P' + \frac{1}{(r-2m)} P^2 + \left(\frac{m' r + m}{r(r-2m)} - \frac{2\,\alpha(r)}{r}\right) P + \frac{m' m}{r(r-2m)} = 0$$

Haciendo el siguiente cambio

P=$\tilde{P} - \frac{m}{r}$ y $\alpha(r) = \frac{m' r - m}{2m}$

Obtenemos que:

$$\tilde{P}' + \frac{1}{(r-2\,m)}\,\tilde{P}^2 + \left(\frac{m'\,r+m}{r(r-2\,m)} - \frac{2\,\alpha(r)}{r}\right)\tilde{P} = 0$$

In[•]:=
```
ec3 = D[P[r], r] + 1/ (r - 2 * m[r]) * P[r]^2 +
      deriva
  (r * D[m[r], r] + m[r]) / (r^2 - 2 * r * m[r] - 2 * α (r) / r) * P[r]
       deriva
```

Out[•]=

$$\frac{P[r]^2}{r-2\,m[r]} + \frac{P[r]\,(m[r]+r\,m'[r])}{r^2-2\,\alpha-2\,r\,m[r]} + P'[r]$$

In[•]:=
```
DSolve[ec3 == 0, P[r], r] // FullSimplify
resolvedor diferencial        simplifica completamente
```

Out[•]=

$$\left\{\left\{P[r] \to \frac{e^{\int_1^r \frac{m[K[1]]+K[1]\,m'[K[1]]}{2\,\alpha-K[1]^2+2\,K[1]\times m[K[1]]}\,dK[1]}}{C_1 - \int_1^r -\frac{e^{\int_1^{K[2]} \frac{m[K[1]]+K[1]\,m'[K[1]]}{2\,\alpha-K[1]^2+2\,K[1]\times m[K[1]]}\,dK[1]}}{K[2]-2\,m[K[2]]}\,dK[2]}\right\}\right\}$$

Propuesta con ecuación de estado Politropica

$$P_r = k\,\rho^{\gamma}$$

4) $$P_t = \frac{r^3}{2\,(r-2\,m)}\,P^2 + \left(\frac{m'\,r+m}{r(r-2\,m)} + \frac{\gamma\,r\,m''}{2\,m'} + 1 - \gamma\right)P + \frac{m'\,m}{2\,r^2(r-2\,m)}$$

Lo mismo que en los casos anteriores dada una masa m, que cumpla con todas las condiciones habidas y por haber, tenemos cerrado el sistema.