Association Rule Mining with R *

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*Chapter 9 - Association Rules, in *R and Data Mining: Examples and Case Studies*.
http://www.rdatamining.com/docs/RDataMining-book.pdf

Outline

Association Rules

- To discover association rules showing itemsets that occur together frequently [Agrawal et al., 1993].
- Widely used to analyze retail basket or transaction data.
- An association rule is of the form $A \Rightarrow B$, where A and B are items or attribute-value pairs.
- The rule means that those database tuples having the items in the left hand of the rule are also likely to having those items in the right hand.
- Examples of association rules:
  - *bread* $\Rightarrow$ *butter*
  - *computer* $\Rightarrow$ *software*
  - *age in [25,35] & income in [80K,120K]* $\Rightarrow$ *buying up-to-date mobile handsets*

Association Rules

Association rules are rules presenting association or correlation between itemsets.

$$
\begin{aligned}
\text{support}(A \Rightarrow B) &= P(A \cup B) \\
\text{confidence}(A \Rightarrow B) &= P(B|A) \\
&= \frac{P(A \cup B)}{P(A)} \\
\text{lift}(A \Rightarrow B) &= \frac{\text{confidence}(A \Rightarrow B)}{P(B)} \\
&= \frac{P(A \cup B)}{P(A)P(B)}
\end{aligned}
$$

where $P(A)$ is the percentage (or probability) of cases containing A.

An Example

- Assume there are 100 students.
- 10 out of them know data mining techniques, 8 know R language and 6 know both of them.
- $knows\ R \Rightarrow knows\ data\ mining$

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- confidence =

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- *knows R* $\Rightarrow$ *knows data mining*
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- confidence = support / P(R) = 0.06/0.08 = 0.75

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An Example

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- 10 out of them know data mining techniques, 8 know R language and 6 know both of them.
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- support = P(R & data mining) = 6/100 = 0.06
- confidence = support / P(R) = 0.06/0.08 = 0.75
- lift = confidence / P(data mining) = 0.75/0.10 = 7.5

Outline

Interestingness Measures

- Which rules or patterns are the most interesting ones? One way is to rank the discovered rules or patterns with interestingness measures.
- The measures of rule interestingness fall into two categories, subjective and objective [Freitas, 1998, Silberschatz and Tuzhilin, 1996].
- Objective measures, such as *lift*, *odds ratio* and *conviction*, are often data-driven and give the interestingness in terms of statistics or information theory.
- Subjective (user-driven) measures, e.g., *unexpectedness* and *actionability*, focus on finding interesting patterns by matching against a given set of user beliefs.

Interestingness Measures - I

#	Measure	Formula
1	ϕ-coefficient	$\frac{P(A,B)-P(A)P(B)}{\sqrt{P(A)P(B)(1-P(A))(1-P(B))}}$
2	Goodman-Kruskal's (λ)	$\frac{\sum_j \max_k P(A_j,B_k)+\sum_k \max_j P(A_j,B_k)-\max_j P(A_j)-max_k P(B_k)}{2-\max_j P(A_j)-\max_k P(B_k)}$
3	Odds ratio (α)	$\frac{P(A,B)P(\overline{A},\overline{B})}{P(A,\overline{B})P(\overline{A},B)}$
4	Yule's Q	$\frac{P(A,B)P(\overline{AB})-P(A,\overline{B})P(\overline{A},B)}{P(A,B)P(\overline{AB})+P(A,\overline{B})P(\overline{A},B)} = \frac{\alpha-1}{\alpha+1}$
5	Yule's Y	$\frac{\sqrt{P(A,B)P(\overline{AB})}-\sqrt{P(A,\overline{B})P(\overline{A},B)}}{\sqrt{P(A,B)P(\overline{AB})}+\sqrt{P(A,\overline{B})P(\overline{A},B)}} = \frac{\sqrt{\alpha}-1}{\sqrt{\alpha}+1}$
6	Kappa (κ)	$\frac{P(A,B)+P(\overline{A},\overline{B})-P(A)P(B)-P(\overline{A})P(\overline{B})}{1-P(A)P(B)-P(\overline{A})P(\overline{B})}$
7	Mutual Information (M)	$\frac{\sum_i \sum_j P(A_i,B_j) \log \frac{P(A_i,B_j)}{P(A_i)P(B_j)}}{\min(-\sum_i P(A_i) \log P(A_i), -\sum_j P(B_j) \log P(B_j))}$
8	J-Measure (J)	$\max\left(P(A,B)\log(\frac{P(B\|A)}{P(B)}) + P(A\overline{B})\log(\frac{P(\overline{B}\|A)}{P(\overline{B})}),\ P(A,B)\log(\frac{P(A\|B)}{P(A)}) + P(\overline{A}B)\log(\frac{P(\overline{A}\|B)}{P(\overline{A})})\right)$
9	Gini index (G)	$\max\left(P(A)[P(B\|A)^2 + P(\overline{B}\|A)^2] + P(\overline{A})[P(B\|\overline{A})^2 + P(\overline{B}\|\overline{A})^2] - P(B)^2 - P(\overline{B})^2,\ P(B)[P(A\|B)^2 + P(\overline{A}\|B)^2] + P(\overline{B})[P(A\|\overline{B})^2 + P(\overline{A}\|\overline{B})^2] - P(A)^2 - P(\overline{A})^2\right)$

From [Tan et al., 2002]

Interestingness Measures - II

10	Support (s)	$P(A,B)$
11	Confidence (c)	$\max(P(B\|A), P(A\|B))$
12	Laplace (L)	$\max\left(\frac{NP(A,B)+1}{NP(A)+2}, \frac{NP(A,B)+1}{NP(B)+2}\right)$
13	Conviction (V)	$\max\left(\frac{P(A)P(\overline{B})}{P(A\overline{B})}, \frac{P(B)P(\overline{A})}{P(B\overline{A})}\right)$
14	Interest (I)	$\frac{P(A,B)}{P(A)P(B)}$
15	cosine (IS)	$\frac{P(A,B)}{\sqrt{P(A)P(B)}}$
16	Piatetsky-Shapiro's (PS)	$P(A,B) - P(A)P(B)$
17	Certainty factor (F)	$\max\left(\frac{P(B\|A)-P(B)}{1-P(B)}, \frac{P(A\|B)-P(A)}{1-P(A)}\right)$
18	Added Value (AV)	$\max(P(B\|A) - P(B), P(A\|B) - P(A))$
19	Collective strength (S)	$\frac{P(A,B)+P(\overline{AB})}{P(A)P(B)+P(\overline{A})P(\overline{B})} \times \frac{1-P(A)P(B)-P(\overline{A})P(\overline{B})}{1-P(A,B)-P(\overline{AB})}$
20	Jaccard (ζ)	$\frac{P(A,B)}{P(A)+P(B)-P(A,B)}$
21	Klosgen (K)	$\sqrt{P(A,B)}\max(P(B\|A) - P(B), P(A\|B) - P(A))$

From [Tan et al., 2002]

Outline

Applications

- Market basket analysis
  - Identifying associations between items in shopping baskets, i.e., which items are frequently purchsed together
  - Can be used by retailers to understand customer shopping habits, do selective marketing and plan shelf space
- Churn analysis and selective marketing
  - Discovering demographic characteristics and behaviours of customers who are likely/unlikely to switch to other telcos
  - Identifying customer groups who are likely to purchase a new service or product
- Credit card risk analysis
  - Finding characteristics of customers who are likely to default on credit card or mortgage
  - Can be used by banks to reduce risks when assessing new credit card or mortgage applications

Outline

Association Rule Mining Algorithms in R

- Apriori [Agrawal and Srikant, 1994]
  - a level-wise, breadth-first algorithm which counts transactions to find frequent itemsets and then derive association rules from them
  - `apriori()` in package *arules*
- ECLAT [Zaki et al., 1997]
  - finds frequent itemsets with equivalence classes, depth-first search and set intersection instead of counting
  - `eclat()` in package *arules*

The Titanic Dataset

- The Titanic dataset in the *datasets* package is a 4-dimensional table with summarized information on the fate of passengers on the Titanic according to social class, sex, age and survival.
- To make it suitable for association rule mining, we reconstruct the raw data as `titanic.raw`, where each row represents a person.
- The reconstructed raw data can also be downloaded at `http://www.rdatamining.com/data/titanic.raw.rdata`.

```r
## load dataframe titanic.raw
load("./data/titanic.raw.rdata")
## draw a sample of 5 records
idx <- sample(1:nrow(titanic.raw), 5)
titanic.raw[idx, ]
```

```
##       Class    Sex   Age Survived
## 1585   1st   Male Adult      Yes
## 1021  Crew   Male Adult       No
## 1398   2nd Female Adult       No
## 512    3rd   Male Adult       No
## 1216  Crew   Male Adult       No
```

```r
summary(titanic.raw)
```

```
##   Class         Sex           Age        Survived
##  1st :325   Female: 470   Adult:2092   No :1490
##  2nd :285   Male  :1731   Child: 109   Yes: 711
##  3rd :706
##  Crew:885
```

Function apriori()

Mine frequent itemsets, association rules or association hyperedges using the Apriori algorithm. The Apriori algorithm employs level-wise search for frequent itemsets.

Default settings:

- minimum support: supp=0.1
- minimum confidence: conf=0.8
- maximum length of rules: maxlen=10

```
library(arules)
rules.all <- apriori(titanic.raw)

## Apriori
##
## Parameter specification:
##  confidence minval smax arem  aval originalSupport maxtime
##         0.8    0.1    1 none FALSE            TRUE       5
##  support minlen maxlen target   ext
##      0.1      1     10  rules FALSE
##
## Algorithmic control:
##  filter tree heap memopt load sort verbose
##     0.1 TRUE TRUE  FALSE TRUE    2    TRUE
##
## Absolute minimum support count: 220
##
## set item appearances ...[0 item(s)] done [0.00s].
## set transactions ...[10 item(s), 2201 transaction(s)] don...
## sorting and recoding items ... [9 item(s)] done [0.00s].
## creating transaction tree ... done [0.00s].
## checking subsets of size 1 2 3 4 done [0.00s].
## writing ... [27 rule(s)] done [0.00s].
## creating S4 object  ... done [0.00s].
```

```r
inspect(rules.all)
```

```
##      lhs               rhs             support   confidence...
## [1]  {}             => {Age=Adult}    0.9504771  0.9504771...
## [2]  {Class=2nd}    => {Age=Adult}    0.1185825  0.9157895...
## [3]  {Class=1st}    => {Age=Adult}    0.1449341  0.9815385...
## [4]  {Sex=Female}   => {Age=Adult}    0.1930940  0.9042553...
## [5]  {Class=3rd}    => {Age=Adult}    0.2848705  0.8881020...
## [6]  {Survived=Yes} => {Age=Adult}    0.2971377  0.9198312...
## [7]  {Class=Crew}   => {Sex=Male}     0.3916402  0.9740113...
## [8]  {Class=Crew}   => {Age=Adult}    0.4020900  1.0000000...
## [9]  {Survived=No}  => {Sex=Male}     0.6197183  0.9154362...
## [10] {Survived=No}  => {Age=Adult}    0.6533394  0.9651007...
## [11] {Sex=Male}     => {Age=Adult}    0.7573830  0.9630272...
## [12] {Sex=Female,                                         ...
##       Survived=Yes} => {Age=Adult}    0.1435711  0.9186047...
## [13] {Class=3rd,                                          ...
##       Sex=Male}     => {Survived=No} 0.1917310  0.8274510...
## [14] {Class=3rd,                                          ...
##       Survived=No}  => {Age=Adult}    0.2162653  0.9015152...
## [15] {Class=3rd,                                          ...
##       Sex=Male}     => {Age=Adult}    0.2099046  0.9058824...
## [16] {Sex=Male,                                           ...
##       Survived=Yes} => {Age=Adult}    0.1535666  0.9209809...
```

```r
# rules with rhs containing "Survived" only
rules <- apriori(titanic.raw,
                 control = list(verbose=F),
                 parameter = list(minlen=2, supp=0.005, conf=0.8),
                 appearance = list(rhs=c("Survived=No",
                                         "Survived=Yes"),
                                   default="lhs"))
## keep three decimal places
quality(rules) <- round(quality(rules), digits=3)
## order rules by lift
rules.sorted <- sort(rules, by="lift")
```

```
inspect(rules.sorted)

##      lhs               rhs             support confidence  lift
## [1]  {Class=2nd,
##       Age=Child}   => {Survived=Yes}   0.011      1.000 3.096
## [2]  {Class=2nd,
##       Sex=Female,
##       Age=Child}   => {Survived=Yes}   0.006      1.000 3.096
## [3]  {Class=1st,
##       Sex=Female} => {Survived=Yes}   0.064      0.972 3.010
## [4]  {Class=1st,
##       Sex=Female,
##       Age=Adult}   => {Survived=Yes}   0.064      0.972 3.010
## [5]  {Class=2nd,
##       Sex=Female} => {Survived=Yes}   0.042      0.877 2.716
## [6]  {Class=Crew,
##       Sex=Female} => {Survived=Yes}   0.009      0.870 2.692
## [7]  {Class=Crew,
##       Sex=Female,
##       Age=Adult}   => {Survived=Yes}   0.009      0.870 2.692
## [8]  {Class=2nd,
##       Sex=Female,
##       Age=Adult}   => {Survived=Yes}   0.036      0.860 2.663
## [9]  {Class=2nd,
```

Outline

Redundant Rules

- There are often too many association rules discovered from a dataset.
- It is necessary to remove redundant rules before a user is able to study the rules and identify interesting ones from them.

Redundant Rules

```
inspect(rules.sorted[1:2])

##     lhs               rhs            support confidence  lift
## [1] {Class=2nd,
##      Age=Child}  => {Survived=Yes}   0.011          1 3.096
## [2] {Class=2nd,
##      Sex=Female,
##      Age=Child}  => {Survived=Yes}   0.006          1 3.096
```

- Rule #2 provides no extra knowledge in addition to rule #1, since rules #1 tells us that all 2nd-class children survived.
- When a rule (such as #2) is a super rule of another rule (#1) and the former has the same or a lower lift, the former rule (#2) is considered to be redundant.
- Other redundant rules in the above result are rules #4, #7 and #8, compared respectively with #3, #6 and #5.

Remove Redundant Rules

```
## find redundant rules
subset.matrix <- is.subset(rules.sorted, rules.sorted)
subset.matrix[lower.tri(subset.matrix, diag = T)] <- NA
redundant <- colSums(subset.matrix, na.rm = T) >= 1
```

```
## which rules are redundant
which(redundant)

##  {Class=2nd,Sex=Female,Age=Child,Survived=Yes}
##                                              2
##  {Class=1st,Sex=Female,Age=Adult,Survived=Yes}
##                                              4
## {Class=Crew,Sex=Female,Age=Adult,Survived=Yes}
##                                              7
##  {Class=2nd,Sex=Female,Age=Adult,Survived=Yes}
##                                              8

## remove redundant rules
rules.pruned <- rules.sorted[!redundant]
```

Remaining Rules

```
inspect(rules.pruned)

##     lhs                rhs             support confidence  lift
## [1] {Class=2nd,
##      Age=Child}   => {Survived=Yes}   0.011      1.000 3.096
## [2] {Class=1st,
##      Sex=Female} => {Survived=Yes}   0.064      0.972 3.010
## [3] {Class=2nd,
##      Sex=Female} => {Survived=Yes}   0.042      0.877 2.716
## [4] {Class=Crew,
##      Sex=Female} => {Survived=Yes}   0.009      0.870 2.692
## [5] {Class=2nd,
##      Sex=Male,
##      Age=Adult}   => {Survived=No}    0.070      0.917 1.354
## [6] {Class=2nd,
##      Sex=Male}    => {Survived=No}    0.070      0.860 1.271
## [7] {Class=3rd,
##      Sex=Male,
##      Age=Adult}   => {Survived=No}    0.176      0.838 1.237
## [8] {Class=3rd,
##      Sex=Male}    => {Survived=No}    0.192      0.827 1.222
```

Outline

```r
inspect(rules.pruned[1])

##     lhs                            rhs            support confi...
## [1] {Class=2nd,Age=Child} => {Survived=Yes} 0.011   1     ...
##     lift
## [1] 3.096
```

- Did children have a higher survival rate than adults?
- Did children of the 2nd class have a higher survival rate than other children?

```r
inspect(rules.pruned[1])
```

```
##     lhs                        rhs            support confi...
## [1] {Class=2nd,Age=Child} => {Survived=Yes} 0.011   1     ...
##     lift
## [1] 3.096
```

- Did children have a higher survival rate than adults?
- Did children of the 2nd class have a higher survival rate than other children?
- The rule states only that all children of class 2 survived, but provides no information at all about the survival rates of other classes.

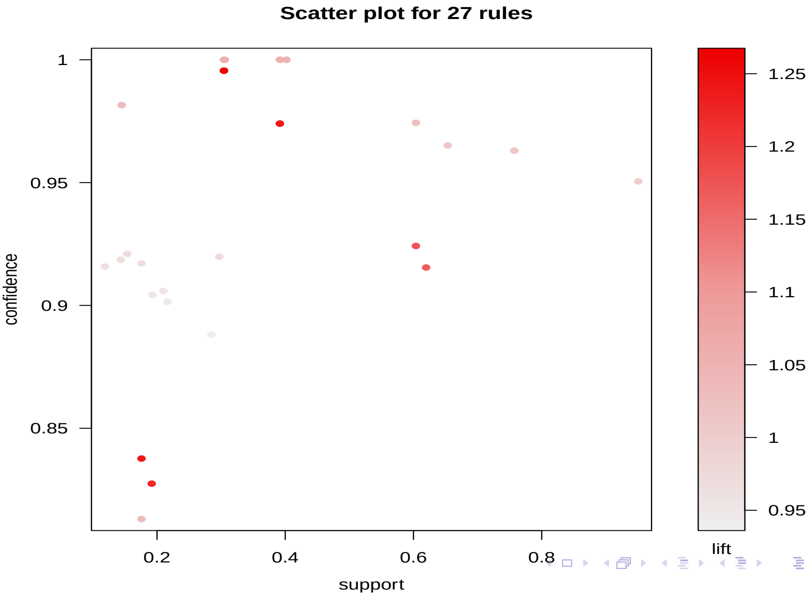
Rules about Children

```
rules <- apriori(titanic.raw, control = list(verbose=F),
     parameter = list(minlen=3, supp=0.002, conf=0.2),
     appearance = list(default="none", rhs=c("Survived=Yes"),
                       lhs=c("Class=1st", "Class=2nd", "Class=3rd",
                             "Age=Child", "Age=Adult")))
rules.sorted.2 <- sort(rules, by="confidence")
inspect(rules.sorted.2)
```

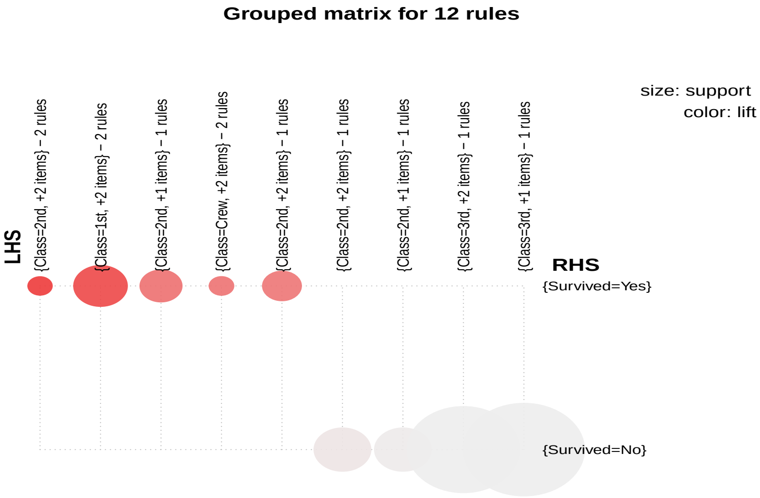
```
##     lhs                          rhs            support
## [1] {Class=2nd,Age=Child} => {Survived=Yes} 0.010904134
## [2] {Class=1st,Age=Child} => {Survived=Yes} 0.002726034
## [3] {Class=1st,Age=Adult} => {Survived=Yes} 0.089504771
## [4] {Class=2nd,Age=Adult} => {Survived=Yes} 0.042707860
## [5] {Class=3rd,Age=Child} => {Survived=Yes} 0.012267151
## [6] {Class=3rd,Age=Adult} => {Survived=Yes} 0.068605179
##     confidence lift
## [1] 1.0000000  3.0956399
## [2] 1.0000000  3.0956399
## [3] 0.6175549  1.9117275
## [4] 0.3601533  1.1149048
## [5] 0.3417722  1.0580035
## [6] 0.2408293  0.7455209
```

Outline

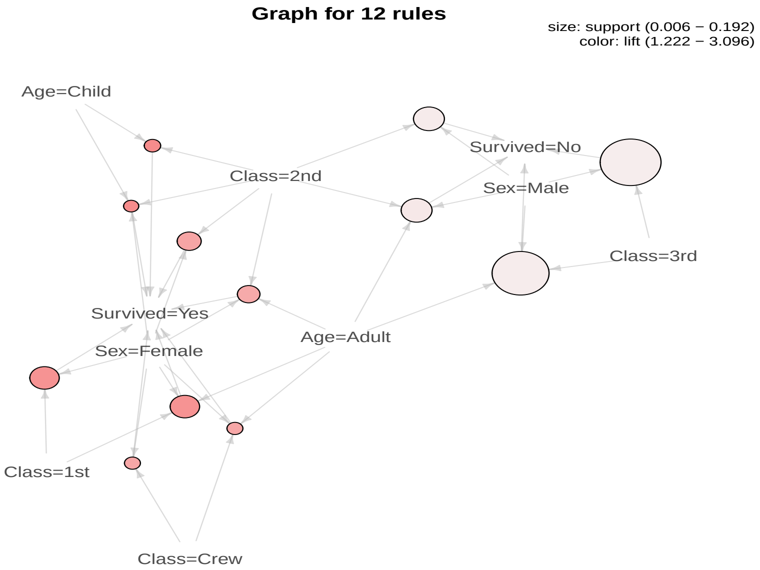
```r
library(arulesViz)
plot(rules.all)
```



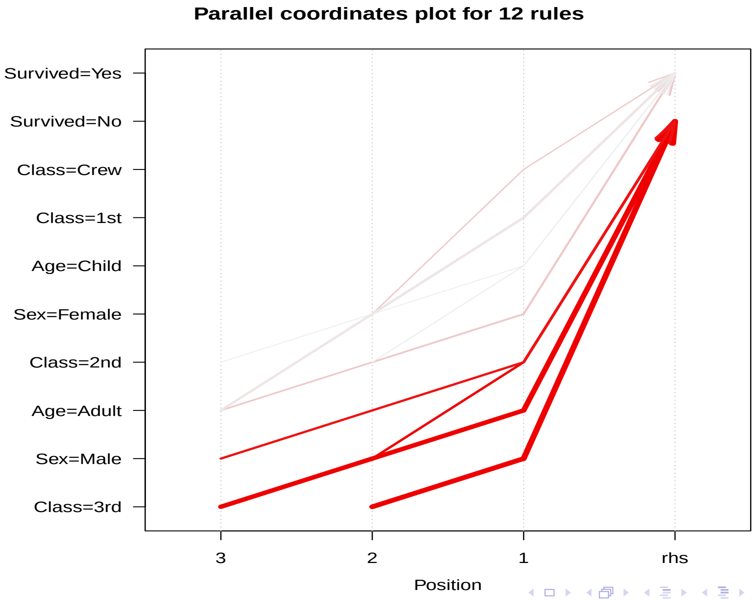
```r
plot(rules.sorted, method = "grouped")
```



```
plot(rules.sorted, method = "graph", control = list(type = "items"))
```



```r
plot(rules.sorted, method = "paracoord", control = list(reorder = T))
```



Outline

Further Readings

- Association Rule Learning

  https://en.wikipedia.org/wiki/Association_rule_learning
- Data Mining Algorithms In R: Apriori

  https://en.wikibooks.org/wiki/Data_Mining_Algorithms_In_R/Frequent_Pattern_Mining/The_Apriori_Algorithm
- Data Mining Algorithms In R: ECLAT

  https://en.wikibooks.org/wiki/Data_Mining_Algorithms_In_R/Frequent_Pattern_Mining/The_Eclat_Algorithm
- Data Mining Algorithms In R: FP-Growth

  https://en.wikibooks.org/wiki/Data_Mining_Algorithms_In_R/Frequent_Pattern_Mining/The_FP-Growth_Algorithm
- FP-Growth Implementation by Christian Borgelt

  http://www.borgelt.net/fpgrowth.html
- Frequent Itemset Mining Implementations Repository

  http://fimi.ua.ac.be/data/

Further Readings

- More than 20 interestingness measures, such as chi-square, conviction, gini and leverage

  Tan, P.-N., Kumar, V., and Srivastava, J. (2002). Selecting the right interestingness measure for association patterns. In Proc. of KDD '02, pages 32-41, New York, NY, USA. ACM Press.
- More reviews on interestingness measures: [Silberschatz and Tuzhilin, 1996], [Tan et al., 2002], [Omiecinski, 2003] and [Wu et al., 2007]
- Post mining of association rules, such as selecting interesting association rules, visualization of association rules and using association rules for classification [Zhao et al., 2009]

  Yanchang Zhao, et al. (Eds.). "Post-Mining of Association Rules: Techniques for Effective Knowledge Extraction", ISBN 978-1-60566-404-0, May 2009. Information Science Reference.
- Package *arulesSequences*: mining sequential patterns

  `http://cran.r-project.org/web/packages/arulesSequences/`

Online Resources

- Chapter 9 - Association Rules, in book titled
  R and Data Mining: Examples and Case Studies [Zhao, 2012]
  `http://www.rdatamining.com/docs/RDataMining-book.pdf`
- RDataMining Reference Card
  `http://www.rdatamining.com/docs/RDataMining-reference-card.pdf`
- Free online courses and documents
  `http://www.rdatamining.com/resources/`
- RDataMining Group on LinkedIn (22,000+ members)
  `http://group.rdatamining.com`
- Twitter (2,800+ followers)
  @RDataMining

References I

Agrawal, R., Imielinski, T., and Swami, A. (1993).
Mining association rules between sets of items in large databases.
In *Proc. of the ACM SIGMOD International Conference on Management of Data*, pages 207–216, Washington D.C. USA.

Agrawal, R. and Srikant, R. (1994).
Fast algorithms for mining association rules in large databases.
In *Proc. of the 20th International Conference on Very Large Data Bases*, pages 487–499, Santiago, Chile.

Freitas, A. A. (1998).
On objective measures of rule surprisingness.
In *PKDD '98: Proceedings of the Second European Symposium on Principles of Data Mining and Knowledge Discovery*, pages 1–9, London, UK. Springer-Verlag.

Omiecinski, E. R. (2003).
Alternative interest measures for mining associations in databases.
IEEE Transactions on Knowledge and Data Engineering, 15(1):57–69.

Silberschatz, A. and Tuzhilin, A. (1996).
What makes patterns interesting in knowledge discovery systems.
IEEE Transactions on Knowledge and Data Engineering, 8(6):970–974.

Tan, P.-N., Kumar, V., and Srivastava, J. (2002).
Selecting the right interestingness measure for association patterns.
In *KDD '02: Proceedings of the 8th ACM SIGKDD International Conference on Knowledge Discovery and Data Mining*, pages 32–41, New York, NY, USA. ACM Press.

References II

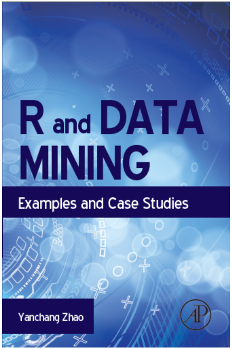
Wu, T., Chen, Y., and Han, J. (2007).
Association mining in large databases: A re-examination of its measures.
In *PKDD'07: Proc. of the 11th European Conference on Principles and Practice of Knowledge Discovery in Databases, Warsaw, Poland, September 17-21, 2007*, pages 621–628.

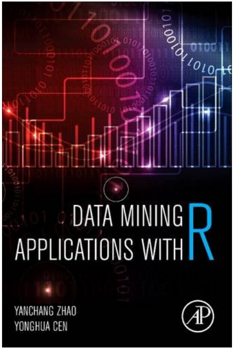
Zaki, M. J., Parthasarathy, S., Ogihara, M., and Li, W. (1997).
New algorithms for fast discovery of association rules.
Technical Report 651, Computer Science Department, University of Rochester, Rochester, NY 14627.

Zhao, Y. (2012).
R and Data Mining: Examples and Case Studies.
Academic Press, Elsevier.

Zhao, Y., Zhang, C., and Cao, L., editors (2009).
Post-Mining of Association Rules: Techniques for Effective Knowledge Extraction, ISBN 978-1-60566-404-0.
Information Science Reference, Hershey, PA.

The End





Thanks!

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