A how to understand Girsanov

1) Closed set - a set is closed under an operation (addition, subtraction, etc.) if performing that operation returns a member of that set. E.g., real numbers are closed under arithmetic operations, but whole numbers are not closed under division.

2) σ-algebra – For a set X, a σ-algebra Σ is a nonempty closed set of subsets of *X* that is closed under union, complement, and intersections, The pair {X, Σ} represents a measurable space.

3) Measurable space - a measurable space is the basic object of measure theory It consists of a set and σ-algebra {X, Σ} defining the sets that will be measured.

4) Measurable function - a measurable function $\mathcal{F}$ is a function defined on {X, Σ}. In our normal discussions X represents the random variable, Σ the outcome space, and $\mathcal{F}$represents the probability distribution

5) Probability Space – a triple of $\{X, \Sigma, \mathcal{F}\}$ representing the outcome space X, the σ-algebra Σ and a probability function (measure), $\mathcal{F}$

6) Radon–Nikodym theorem - is a result in that expresses the relationship between two measures defined on the same measurable space including the probabilities of outcomes on a probability space. Suppose there's a measurable space $\{\mathrm{X}, \Sigma\}$and two measures x and y. If x is absolutely continuous with respect to y then there exists a measurable function $\mathcal{F} : X \to [0, \infty)$, such that for any measurable $A \subset \Sigma$, $y(A) = \int_A \; f dx$

7) Girsanov **-** technically Cameron-Martin-Girsanov works under allows the change in measure from "natural' to "risk neutral." It works easily for financial applications (since most are sub-martingales)

8) Martingales – In brief, a martingale is a fair game. In discrete time a martingale has the properties $E(|X_n|) < \infty$ ∀n, and $E(X_{n+1}|X_1, X_2, X_3 \cdots X_n) = X_n$. Sometimes written as $X_m = E_{\mathcal{F}}(X_s|\Sigma_m) \; for \; s > m$. A sub-martingale has the property that $X_m \geq E_{\mathcal{F}}(X_s|\Sigma_m)$; super-martingales have the property $X_m \leq E_{\mathcal{F}}(X_s|\Sigma_m)$ (weird terminology, right?). A random walk with upward drift would be a sub-martingale. Martingales are "well-behaved"