QRM Addendum 1

Chebychev

Taking what we started with in class, For $Z > 0$

$E(Z) = Pr(Z \geq C)*E[Z|Z \geq C] + Pr(Z < C)*E[Z|Z<C]$, we know that each of the 4 terms on the right are positive, because all Z's are positive, and probabilities are all positive. So if we ignore the $Pr(Z < C)*E[Z|Z<C]$ part, we know we are ignoring something positive, so we have.

$$E(Z) \geq Pr(Z \geq C)*E[Z|Z \geq C];$$

since $E[Z|Z \geq C] > C$, we can write $E(Z) \geq Pr(Z \geq C)*C$

which after rearranging can be written as

$$Pr(Z \geq C) \leq E(Z)/C$$

Now define $(x-\mu)^2 = Z$ which is always positive and let $C = k^2\sigma^2$,

Note: $Pr((x-\mu)^2 > k^2\sigma^2) = Pr(|x- \mu| > k\sigma)$, and $E[(x-\mu)^2] = \sigma^2$

$$Pr(|x- \mu| > k\sigma) \leq 1 / k^2$$

Two-state option pricing

- Let our portfolio be $C_0 + \delta S_0$ at time 0

- This portfolio becomes either $C^u + \delta S^u$ in the 'up' state or $C^d + \delta S^d$ in the 'down' state

- We choose δ^* such that $\delta^* = \frac{C^u - C^d}{S^d - S^u}$ which guarantees that

$C^u + \delta^* S^u = C^d + \delta^* S^d$ and write $C_0 + \delta^* S_0 = PV(C^u + \delta^* S^u)$ (= $PV(C^d + \delta^* S^d)$)

With $S_0 = 100$, $S^u = 120$, $S^d = 95$, $K = 100$, $r = 5\%$, $T = 1$ yr we get

As intermediate steps $C^u = 20$, and $C^d = 0$

Then $C_0 + \frac{20-0}{95-120} 100 = \left(20 + \frac{20-0}{95-120} 120\right) e^{.05*1}$ => $C_0 = \$7.71$