Interest Rate Instruments and Related Quantitative Modeling

Agenda

Overview

US interest rate market is one of the largest and most liquid financial markets, serving as the backbone of global fixed income and derivatives trading

- **Money market** (short-term: overnight to 1Y)
  - Federal Reserve/Treasury: Fed Funds (unsecured overnight lending rates), SOFR (secured overnight financing rate); T-bills (1M to 1Y)
  - Banks/Broker-Dealers: Repos (collateralized short-term loans), CDs(certificates of deposits)
  - Corps: CPs
- **Cash instruments** (medium to Long-term: 1Y-30Y)
  - Treasury: T-Notes/Bonds
  - GSE (e.g., FNM): Agency bonds
  - State/local government: muni bonds
  - Corps: corp bonds
- **Derivatives** (market makers: banks/dealers; asset managers/hedge funds, corps)
  - Futures, forwards, interest rate swaps
  - Caps/Floors, Swaptions

- US IR Market: Instruments Across the Maturities

Overnight	**Short-Term (<1Y)**	**Med-Term (1Y-10Y)**	**Long-Term (10Y-30Y)**
Fed Funds	T-Bills	T-Notes	T-Bonds
SOFR	CDs	FRAs	IRSs
Repo	CPs	Futures	

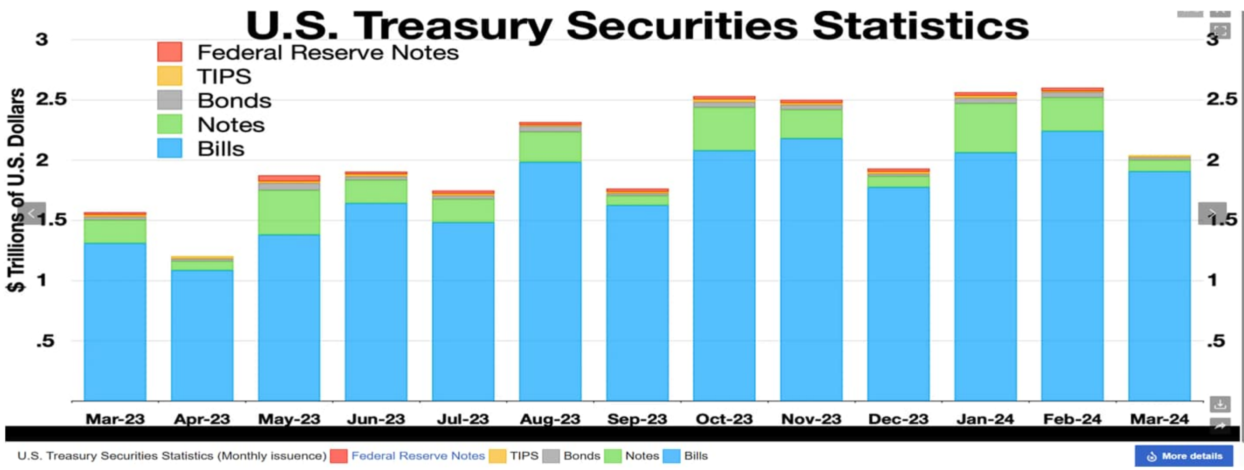
Maturity: Overnight - 30Y

Treasury securities

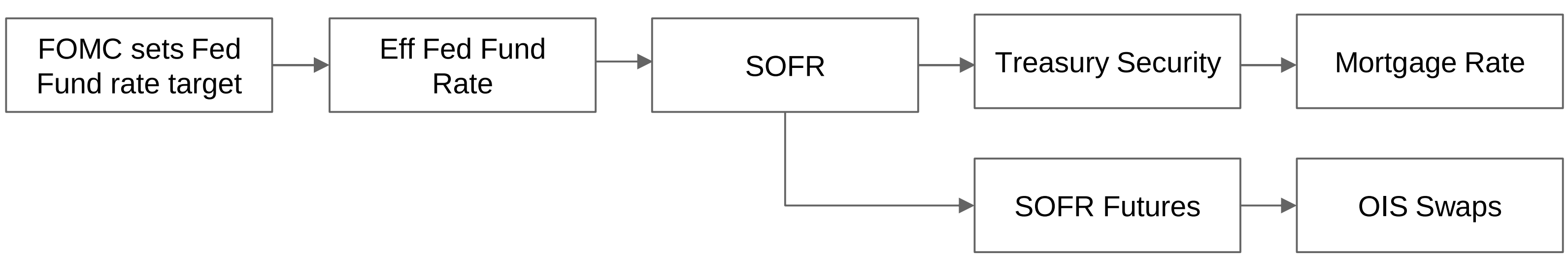
- Treasury securities: influenced by FOMC rate decisions (e.g., inflation expectations), demand for safe assets, etc.

	T-Bills	T-Notes	T-Bonds
Maturity	4wk – 1Y	2Y – 10Y	20Y – 30Y
Coupon	Zero-coupon	Semi-annual coupon	Semi-annual coupon

- Treasury Statistics



- Rate transmission steps



Treasury bill yield calculation*

- Market quotes**

Treasury Yields

NAME	COUPON	PRICE	YIELD
GB3:GOV **3 Month**	0.00	4.07	4.17%
GB6:GOV **6 Month**	0.00	3.89	4.02%

The "price": discount rate

- Price (assume face value = $100)

$$Price = 100 \cdot \left(1 - DiscountRate \cdot \frac{Days\ to\ Maturity}{360}\right)$$

- Yield

$$Yield = \frac{100 - Price}{Price} \cdot \frac{365}{Days\ to\ Maturity}$$

- Note: we can also calculate discount rate using price

$$Discount\ Rate = \frac{100 - Price}{100} \cdot \frac{360}{Days\ to\ Maturity}$$

*: Treasury Calculation Examples 033104 v11.xls

**: United States Rates & Bonds - Bloomberg

Treasury note yield calculation*

- Market quotes

GT2:GOV **2 Year**	3.88	100.31	3.71%
GT5:GOV **5 Year**	3.88	100.46	3.77%

- Price

$$Price = \sum_{i=1}^{n} \frac{Coupon\ Payment}{\left(1 + \frac{Yield}{2}\right)^{i}} + \frac{Face\ Value}{\left(1 + \frac{Yield}{2}\right)^{n}}$$

Note: Coupon Payment is semi-annual coupon determined by coupon rate, e.g., for a 3.88% coupon, $Coupon\ Payment = 3.88\% \cdot \frac{100}{2}$ = 1.94; n is number of semi-annual periods until maturity

Other cash instruments

- Certificated Deposits (CDs)*
  - **Definition**: **Bank** issued time deposit with a **fixed maturity** and **interest rate,** low-risk investments, insured by FDIC up to certain limits
    - Example: $500+, 6mon, nominal annual rate: 5%, APY 5.06%
    - Yield calculation (assume 6mon = 182 days)

$$APY = \left(1 + \frac{0.05 \cdot 182}{365}\right)^{\frac{365}{D}} - 1 \approx 5.06\%$$

  - **Risks**: credit risk, liquidity risk, and interest rate risk

- Repurchase Agreements (Repos)
  - **Definition: short-term collateralized loan**; one party sells securities to another party with an agreement to repurchase them at a later date and a pre-determined price
  - **Purpose:** liquidity management, financing, and monetary policy
  - **Repo type**
    - Overnight, term repo (e.g., 1wk, 1mon), open repo
    - Tri-party repo, reverse repo
  - **Underly collateral**: treasuries, agency MBS, corporate bonds, equities
  - **Risk**: counterparty credit risk (might incl wrong-way risk), settlement risk, liquidity risk

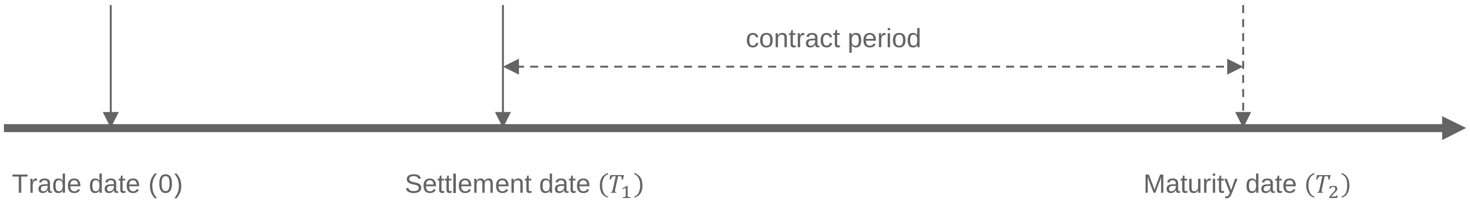
Other cash instruments (cont')

- Municipal bonds (Munis)
  - **Definition:** Debt securities issued by states, cities, and other local entities
    - usually tax-exempt at federal level, sometimes at state levels
  - **Purpose:** Financing infrastructure, transportation, schools, etc.
  - **Type**
    - General Obligations (GO) bonds: back by taxing power of issuing municipality
    - Revenue bonds: back by revenues from a specific project (e.g., toll roads)
  - **Risk**: credit risk, market risk, liquidity risk, interest rate risk

- Corporate bonds
  - **Definition:** Debt securities issued by a corporation
  - **Purpose:** raise capital for various purposes (e.g., M&A)
  - **Type**
    - By seniority: secured, unsecured, subordinated
    - By rating: IG/HY
    - By embedded options: callable, puttable, convertible
  - **Risk**: credit risk, market risk, liquidity risk, interest rate risk

Derivatives – FRA

- **Definition:** an over-the-counter (OTC) derivative where 2 parties agree on an interest rate to be paid/received on a notional for **a specified contract period** at **a prespecified future date**
- **Terms**
  - Notional (N)
  - Contract rate (r_K)
  - Settlement date (T_1)
  - Maturity date (T_2)
  - Refence rate, e.g., SOFR



- **Example:** 1x4 FRA, settlement date is 1mon from now, maturity date is 4 mon from now with 3mon contract period
- **Payoff** (buy floating, pay fix)
  - Reference rate observed at T_1: r_M
  - Cashflow at FRA expiry, T_1

$$Payoff = N \cdot \frac{(r_M - r_K) \cdot (T_2 - T_1)}{1 + r_M \cdot (T_2 - T_1)}$$

Derivatives – FRA (cont')

- **Valuation**

$$PV = E\left[N \cdot \frac{(r_M - r_K)\cdot(T_2 - T_1)}{1 + r_M \cdot (T_2 - T_1)} \cdot P(0, T_1)\right]$$
$$= N \cdot (r(0; T_1, T_2) - r_K) \cdot P(0, T_2)$$

where $r(0; T_1, T_2)$: current forward rate; $P(0, T_2)$: discount factor from T_2.

Note: $P(0, T)$ is supposed to be known at valuation date (0) and $r(0; T_1, T_2)$ can be derived from the relationship below

- Relationship between discount factor, $P(0, T)$, and current forward rate, $r(0; T_1, T_2)$

$$1 + r(0; T_1, T_2) \cdot (T_2 - T_1) = \frac{P(0, T_1)}{P(0, T_2)}$$

- **Risk**
  - DV01: parallel shift on par rates
  - Market risk, counterparty credit risk, liquidity risk

- **P&L Analysis**
  - P&L allocation
  - Unexplained P&L

Derivatives – SOFR futures*

- **Definition:** an exchange-traded derivative whose payoff is linked to the **compounded average SOFR** over **a specified contract period** (1mon and 3mon)
  - SOFR (Secured Overnight Financing Rates): collateralized overnight borrowing rate, published by FRBNY daily
  - Transition from LIBOR to SOFR

- **Quotes**
  - $Future\ Price = 100\ - implied\ forward\ SOFR\ rates$

- **Payoff** – 1mon SOFR future
  - The economic payoff is realized through daily margining, while it can be summarized as a single cashflow at expiration

$$Payoff\ = (FuturePrice_{settle} - FuturePrice_{entry}) \cdot \frac{20.835}{0.005}$$

Note: 20.835 is the value of a 0.005 (1/2bp) tick

$$FuturePrice_{settle} = 100 - arithmetic\ average\ of\ daily\ SOFR\ during\ delivery\ month$$

- **Future rate vs Forward rate**

$$Future\ rate\ = Forward\ rate + convexity\ adjustment$$

*: Understanding SOFR Futures - CME Group

Derivatives – SOFR futures (cont')

SOFR 3-month Future

- Trading Unit: Compounded daily Secured Overnight Financing Rate ("SOFR") interest during contract Reference Quarter, such that each basis point per annum of interest = $25 per contract. Reference Quarter: For a given contract, interval from (and including) 3rd Wed of 3rd month preceding Delivery Month, to (and not including) 3rd Wed of Delivery Month.
- Notional: N.
- Price Basis: Contract-grade IMM Index: 100 minus R. R is the compounded daily SOFR interest during contract Reference Quarter.
- Delivery: Cash settlement, by reference to Final Settlement Price, on first US government securities market business day following Last Day of Trading.
- Delivery month: Nearest 20 March Quarterly months
- Final Settlement Price: Contract-grade IMM Index evaluated on the basis of realized SOFR values during contract Reference Quarter

$$R = \left(\prod_i \left(1 + \frac{d_i}{360} \cdot \frac{r_i}{100}\right) - 1\right) \cdot \frac{360}{D} \cdot 100$$

SOFR 1-month Future

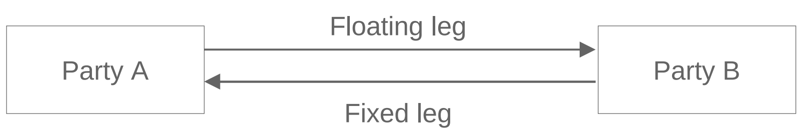
- Trading Unit: Average daily SOFR interest during futures contract delivery month.
- Notional: N.
- Price Basis: Contract-grade IMM Index: 100 minus R. R is the compounded daily SOFR interest during contract Reference Quarter.
- Delivery: Cash settlement, by reference to Final Settlement Price, on first US government securities market business day following Last Day of Trading.
- Delivery month: Nearest 7 calendar months
- Final Settlement Price: Contract-grade IMM Index evaluated on the basis of realized SOFR values during contract Reference month

$$R = \frac{\sum d_i r_i}{\sum d_i}$$

Derivatives – Interest rate swaps (IRS)

- **Definition:** an over-the-counter (OTC) bilateral agreement where two parties **exchange fixed-rate payments for floating-rate payments** on a notional amount
  - Market size: $446trn by notional as of 2024*
  - Maturities: 1y, thru 30y
  - LIBOR → SOFR
  - Besides single currency swap,
    - basis swaps (floating vs floating), cross currency swaps (fixed vs fixed)
    - amortizing/accreting swaps
    - inflation-linked swaps, constant maturity swaps

- **Payoff**
  - Cashflow



  - At each payment date, t_i
    - Floating leg: $Notional \cdot r_i \cdot \Delta t_i$
    - Fixed leg: $Notional \cdot k \cdot \Delta t_i$

*: OTC derivatives statistics - tables | BIS Data Portal

Derivatives – Interest rate swaps (cont')

- **Valuation**
  - PV of fixed leg

$$PV_{fixed} = E\left[Notional \cdot k \cdot \sum_{i=1}^{N} \Delta t_i \cdot P(0, t_i)\right] = Notional \cdot k \cdot \sum_{i=1}^{N} \Delta t_i \cdot P(0, t_i)$$

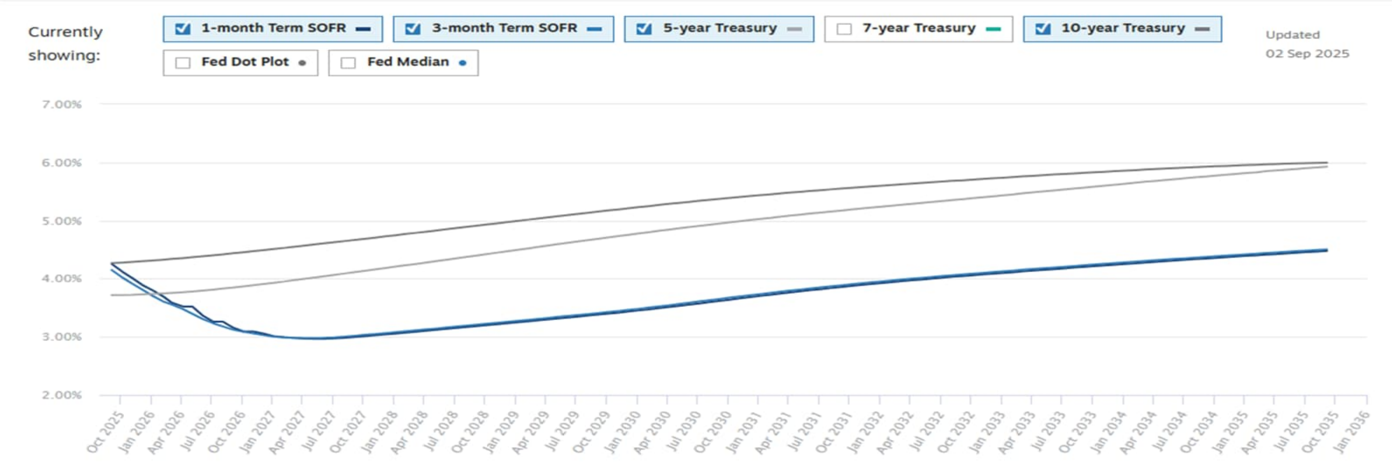
  - PV of floating leg

$$PV_{floating} = E\left[Notional \cdot \sum_{i=1}^{N} r_i \cdot \Delta t_i \cdot P(0, t_i)\right] = Notional \cdot \sum_{i=1}^{N} E[r_i] \cdot \Delta t_i \cdot P(0, t_i)$$

  - PV of an IRS: $PV_{IRS} = PV_{fixed} - PV_{floating}$
  - Single-Curve Framework → Multi-Curve Framework
    - before 2008 financial crisis, LIBOR curve is used to for projection ($E[r_i]$) and discounting ($P(0, t_i)$)
    - post 2008, projection and discounting curves differ. Projection curve depends on reference curve, while discounting curve depends CSA terms
- **Risk**
  - DV01: parallel shift on par rates
  - Market risk, counterparty credit risk, liquidity risk
- **P&L Analysis**

IR curve construction and pricing linear instruments

- **Recap**
  - Multi-curve framework*
    - Forward (projection) curve: project interest rates for cashflow calculation, built using instruments linked to the reference interest rate index (e.g., 3m term SOFR)
    - Discounting curve: calculate present value of future cashflows
  - For each curve
    - Zero rate: $r(t) = r(0; 0, t)$; forward rate over(T_1, T_2): $r(0; T_1, T_2)$
    - Discount factor (or \$1 zero-coupon bond's PV): $P(t) = P(0; t) = \exp(-r(t) \cdot t)$
  - SOFR: collateralized overnight borrowing rate
    - Typically used for fully collateralized derivative discounting
    - Overnight vs Term SOFR vs Treasury rates**



*: Lief A. and Vladmir P., Interest Rate Modeling. Volume 1: Foundations and Vanilla Models

**: Term SOFR and Treasury Forward Curves | Chatham Financial

IR curve construction and pricing linear instruments (cont')

- **Curve construction methodology - SOFR**
  - SOFR OIS is both discounting and reference interest rate index
  - Curve bootstrapping
    - **Principle**: zero rates / discounting curve need to reprice all vanilla SOFR market instruments
    - Interpolation/Smoothing
    - **Market instruments**
      - O/N SOFR
      - SOFR futures
      - SOFR swaps
    - Core formulas

$$P(0;t) = \exp(-r(t) \cdot t)$$

$$r(0;t_1,t_2) = \frac{1}{t_2 - t_1} \cdot \left(\frac{P(0,T_1)}{P(0,T_2)} - 1\right)$$

$$ParRate_N \cdot \sum_{i=1}^{N} \Delta t_i \cdot P(0,t_i) = P(0,0) - P(0,t_N)$$

    - Note: Par swap rate (or par rate) is referred as to the fix rate on a fixed-floating IRS that makes the swap value 0 at inception, $PV_{fixed} = PV_{floating}$

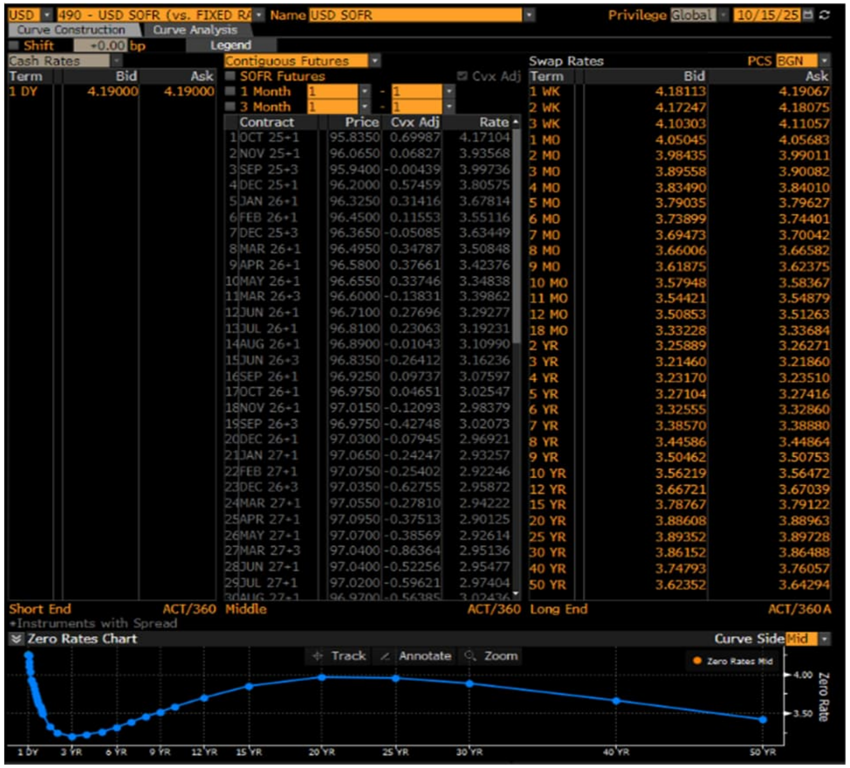
IR curve construction and pricing linear instruments (cont')

- **Pricing linear instruments**
  - Projection curve: Fed-Fund rate, SOFR/Term SOFR, …
  - Discounting curve: SOFR curve
  - Cashflow
    - At each payment date, t_i (assuming $Notional\ =\ \$1$)

$$PV_{fixed,i} = k \cdot \Delta t_i \cdot P(0, t_i)$$

$$PV_{floating,,i} = E[r_i] \cdot \Delta t_i \cdot P(0, t_i) = r(0; t_i, t_{i+1}) \cdot \Delta t_i \cdot P(0, t_i)$$

  - **Example**



Basis swaps, cross currency swaps, swaptions

- **Basis swaps**
  - **Definition:** A swap in which 2 parties agree to **swap floating rates** based on different money market reference rates
  - **Purpose**: mange interest rate **basis** risk, hedge activities in less liquid reference index
  - **Examples**: Fed Fund rate vs SOFR
  - **Payoff**
    - At each payment date, t_i

      $PV_{floating,k,i} = E\left[r_{k,i}\right] \cdot \Delta t_i \cdot P(0, t_i) = r_k(0; t_i, t_{i+1}) \cdot \Delta t_i \cdot P(0, t_i); k = 1{,}2$
    - Note: $r_k(0; t_i, t_{i+1})$ might correspond to a zero curve different from $P(0, t_i)$, which rely on the nature of trade, collateralized vs uncollateralized
  - **Risk**: DV01, basis risk; market, counterparty credit risk, liquidity risk
- **Cross currency swaps**
  - **Definition:** A swap in which 2 parties agree to **swap fixed/floating rates** based on different currencies
  - **Purpose**: mange FX risk, hedge activities in local currency
  - **Examples**: EUR ESTR vs USD SOFR
  - **Payoff**
    - At each payment date, t_i

      $PV_{floating,k,i} = E\left[r_{k,i}\right] \cdot \Delta t_i \cdot P(0, t_i) = r_k(0; t_i, t_{i+1}) \cdot \Delta t_i \cdot P(0, t_i); k = 1{,}2$
  - **Risk**: DV01, FX, basis risk; market, counterparty credit risk, liquidity risk

Basis swaps, cross currency swaps, swaptions (cont')

- **Swaption**
  - **Definition:** A swaption is a derivative that provides **the right**, but not the obligation, to **enter into an interest rate swap** agreement by a specified future date
    - Payer (Receiver) Swaption: buyer has the right to enter into a swap where they **pay** (receive) the **fixed** leg and receive the floating leg
    - 2016, CME first cleared swaption trades*
  - **Swaption style**: European, Bermudan, (and American)
  - **Settlement**: cash/physical settled
  - **Terms (European/cash settlement)**
    - Notional (N)
    - Strike rate (r_K)
    - Expiration (exercise) date (T_1)
    - Underlying swap maturity date (T_2)
    - Other underlying swap terms
    - Example: 1 into 5y payer swaption – right to enter into a 5y payer IRS after 1y from effective date
  - **Payoff (European/cash settlement)**
    - At swaption expiry, T_1

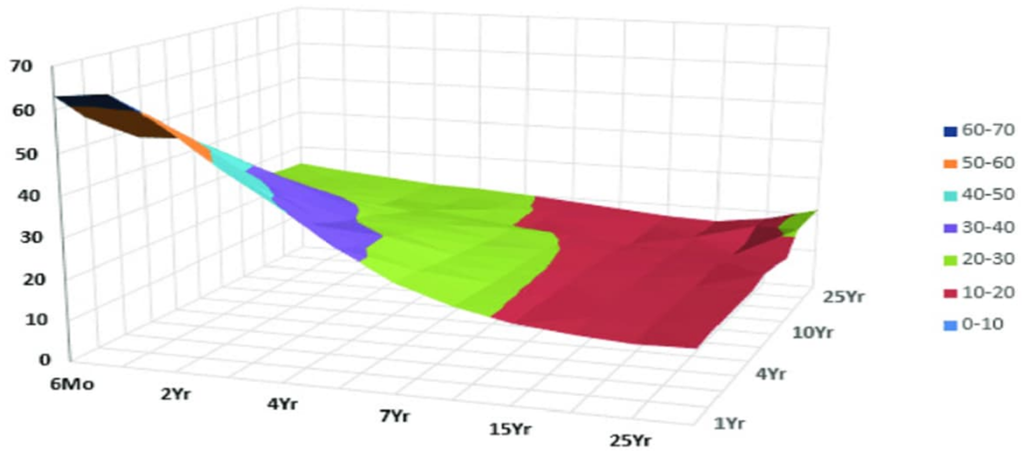
$$Payoff = N \cdot Anuity \cdot \max(0, swap\ rate\ at\ T_1 - strike\ rate)$$

$$Anuity = \sum_{k=i}^{N} \Delta t_i \cdot P(0, t_i)$$

*: CME Group

Basis swaps, cross currency swaps, swaptions (cont')

- **Swaption**
  - **Quotes:** volatility cube (expiry, underlying swap maturity, strike rate)
    - Normal vs lognormal



Swaption expiry: 6m to 25y; Swap maturity: 1y to 25y

  - **Valuation** (assuming log-normal vol, $N(\cdot)$: cumulative normal distribution function)

$$PV = N \cdot Annuity \cdot [r(0; t_1, t_2) \cdot N(d_1) - r_K \cdot N(d_2)]$$

$$d_1 = \frac{\ln(r(0; t_1, t_2)/r_K) - \frac{1}{2} \cdot \sigma^2 \cdot t_1}{\sigma\sqrt{t_1}}, d_2 = d_1 + \sigma\sqrt{t_1}$$

σ: vol corresponds to swaption expiry (t_1), underlying swap maturity (t_2) and strike rate (r_K)

IR volatility construction

- **Recap**
  - Swaption quotes*

Std. Monitor | Export to Excel | Actions | Previous Session | Volatility Monitor

03/05/21 | USD | Mid | SWAPTION | ATM | Quote Normal Vol | Dscnt. OIS | Cntrb. BVOL

Display Annual

Grid | Surface | Chart

	Tenor													
Expiry	1Yr	2Yr	3Yr	4Yr	5Yr	6Yr	7Yr	8Yr	9Yr	10Yr	15Yr	20Yr	25Yr	30Yr
1Mo	19.16	28.12	43.94	59.72	75.00	78.86	82.29	83.91	85.19	86.54	86.78	86.86	86.48	85.37
2Mo	18.58	29.17	45.33	58.76	72.87	76.84	80.60	80.93	83.95	84.51	83.95	83.74	83.23	82.34
3Mo	20.51	28.01	43.35	59.80	72.93	76.03	79.28	80.61	81.54	82.55	81.96	81.60	80.93	79.94
6Mo	22.41	31.84	48.48	58.06	70.72	73.35	75.67	77.87	77.56	78.47	77.84	76.80	76.35	75.71
9Mo	26.12	35.81	48.86	61.13	69.14	71.68	74.26	75.05	75.83	76.63	75.77	75.26	75.18	74.80
1Yr	26.81	39.70	53.28	63.71	68.42	70.65	72.92	73.74	74.54	74.97	73.99	73.25	72.89	72.21
18Mo	38.32	49.44	58.63	66.33	70.42	71.96	73.30	73.67	74.00	74.09	72.91	72.07	71.28	70.47
2Yr	49.74	58.70	64.99	69.42	72.36	72.93	73.43	73.43	73.40	73.25	71.48	70.62	69.57	68.60
3Yr	66.61	70.67	71.64	72.21	72.61	72.69	72.81	72.58	72.32	71.97	70.32	68.99	67.70	66.54
4Yr	71.77	73.69	73.28	72.95	72.34	72.02	71.68	71.23	70.81	70.37	68.70	67.30	65.89	64.91
5Yr	72.99	73.93	73.34	72.73	72.12	71.47	70.78	70.13	69.53	68.98	67.03	65.56	64.28	63.10
6Yr	72.83	73.12	72.14	71.93	70.95	70.20	69.47	68.86	68.28	67.63	65.82	64.39	63.04	61.94
7Yr	72.20	72.23	71.65	70.75	69.70	68.92	68.13	67.56	66.88	66.30	64.57	62.27	60.87	60.64
8Yr	70.13	70.09	69.54	68.61	67.74	67.07	66.39	65.79	65.23	64.74	62.62	61.20	59.78	58.94
9Yr	68.30	67.99	67.40	66.54	65.77	65.21	64.54	64.02	63.57	63.17	60.84	59.40	57.96	57.31
10Yr	66.36	65.79	65.10	64.51	64.00	63.30	62.77	62.38	62.01	61.65	58.73	57.69	56.30	55.67
15Yr	59.83	59.53	59.20	59.22	59.21	58.53	57.88	57.39	56.89	56.41	53.49	52.15	51.20	50.46
20Yr	55.86	55.34	55.34	55.48	54.65	54.14	54.32	53.75	53.19	52.68	49.68	48.62	47.54	46.71
25Yr	53.23	52.59	52.24	51.72	51.65	50.92	51.31	50.60	50.05	49.55	47.37	46.39	45.52	44.57
30Yr	51.40	51.32	50.76	50.44	50.07	49.57	49.06	48.14	47.98	47.77	44.89	44.88	44.10	43.70

Source: Bloomberg Intelligence

  - Volatility (Vol): *(swaption expiry, underlying swap maturity, moneyness) or (swaption tenor, swap tenor, moneyness)*
  - Normal vol vs Log-normal vol
- Purpose of vol construction: pricing, risk, hedging
- History of IR Vol construction
  - Before 2000: swaption vol cube from market observables, largely interpolation for specific moneyness
    - Might violate arbitrage conditions
    - deterministic vol

*: Bond-market selloff suddenly puts focus on rates volatility | Insights | Bloomberg Professional Services

IR volatility construction (cont')

- History of IR Vol construction (cont')
  - Post 2000, **SABR (Stochastic Alpha Beta Rho)** – a stochastic vol model (Hagan et al.*) for swap rate, F_t:

$$dF_t = \sigma_t F_t^{\beta} dW_t^1$$

$$d\sigma_t = \alpha\sigma_t dW_t^2; \ (dW_t^1, dW_t^2) = \rho dt$$

σ_t: volatility; β: elasticity controls "backbone"; ρ: vol-rate correlation; α: vol of vol

- Benefits of SABR model
  - Capture volatility skew and smile
  - Easier to be arbitrage free
  - Closed-from implied volatility approximation
  - Intuitive: σ_t – volatility; ρ – skew; α – smile; β – 1: lognormal, 0: normal

- Typically
  - β is fixed globally
  - (α, ρ) is calibrated per *(swaption tenor, swap tenor)*

- Calibration: for each *(swaption tenor, swap tenor)*, use optimizer to find (α, ρ) that minimize

$$argmin_{(\alpha,\rho,v)} \sum_i \left(\sigma^{model}(K_i) - \sigma^{market}(K_i)\right)^2$$

*: Managing Smile Risk

IR volatility construction (cont')

- No-arbitrage conditions for swaption vol cube
  - Notations
    - Payer swaption (call on swap rate): $Call(T,S,K) = A(T,S) \cdot E[SwapRate(T,S) - K]^+$;
    - Underlying swap: swap inception date (T) with maturity (S), $SwapRate(T,S)$
    - Swap annuity: $A(T,S) = \sum_{i=1} P(0,S_i) \cdot \Delta_i\,; T = S_0 \leq S_1 \leq \cdots \leq S_n = S\ \ and\ \Delta_i = S_i - S_{i-1}$
  - **Monotonicity in strike**: for a payer swaption, swaption price decreases when strike increase

$$Call(T,S,K) \downarrow\ when\ K \uparrow\ \ or\ \partial_K Call(T,S,K) \leq 0$$

  - **Convexity in strike** or **no butterfly arbitrage**. For $K_1 < K_2 < K_3$,

$$Call(T,S,K_2) \leq \frac{1}{2}(Call(T,S,K_1) + Call(T,S,K_3)\ or\ \partial_K^2 Call(T,S,K) \geq 0$$

  - Option value is non-decreasing in expiry, or **no calendar arbitrage**. For $T_2 > T_1$;

$$Call(T_1,S,K)/P(0,T_1) \leq Call(T_2,S,K)/P(0,T_2)$$

Sensitivities and hedging

- **Recap**
  - Linear products: FRAs, Futures, IRS
    - **Sensitivities**: DV01, sensitivities w.r.t. selected tenors (e.g., 2y/5y/10y/30y)
  - Options: Swaptions, Caps/Floors
    - **Sensitivities**: DV01, Gamma, Vega; others: theta, smile dynamics (e.g., SABR parameters)
    - Caps/Floors: a strip of caplets/floorets with strike K, for each caplets/floorets defined on (t_{i-1}, t_i), the payoff at t_i is defined as

$$Caplet: Notional \cdot (t_i - t_{i-1}) \cdot (r(t_i) - K)^+$$

$$Flooret: Notional \cdot (t_i - t_{i-1}) \cdot \left(K - r(t_i)\right)^+$$

    - Note: for SABR model calibration, caps/floors can also be used in calibration
- Exotics interest rate derivatives: cancellable swaps, range accruals, Target Redemption Notes (TARNs), …
- **Hedging**: "*a strategy to **limit investment risks**. Investors hedge an investment by trading in another that is likely to move in the opposite direction*"*
  - Take **an opposite position in a related financial instruments** so that if the original investment losses value, the hedge gains value
  - The reduction in risk provided by hedging also typically results in a reduction in potential profits**.** The goal is **not to make a profit**, but to **stabilize returns**
- **Risks**: market risk (e.g., interest rate risk, FX risk, volatility risk), credit risk, …

*: Hedge: Definition and How It Works in Investing

Sensitivities and hedging (cont')

- Rates hedging
  - IRS hedging: DV01, bucketed DV01 (e.g., 5Y.10Y, 20Y)

$$Hedge\ Ratio: Notional_{Hedge} = \frac{DV01_{Portfolio}}{DV01_{HedgeInstruments}}$$

  - Swaption hedging: Delta, Vega, Gamma, Volga, cross-gamma (e.g., Vanna)
    - Delta-hedging: use swaps/futures to neutralize options/portfolio delta
    - Vega-hedging: use nearest liquid swaptions hedge expiry x maturity vega
  - Portfolio level hedging

$$\Delta Portfolio = DV01 \cdot \Delta r + Vega \cdot \Delta\sigma + \frac{1}{2}\Gamma \cdot \Delta r^2 + \cdots$$

    - Risk: curve – buckets, vega – expiry x maturity grid
    - PnL explains
    - Hedge effectiveness

$$VaR\ reduction = 1 - \frac{VaR\left(\Delta Portfolio_{Hedged}\right)}{VaR\left(\Delta Portfolio_{Uhedged}\right)}$$

$$TrackingError = \sqrt{\frac{1}{T}\sum_{i=1}^{T}\left(\Delta Portfolio_t - \Delta PortfolioHedges_t\right)^2}$$

      - Hedge accounting
        - $DollarOffsetRatio = \frac{\Delta PortfolioHedges}{\Delta Portfolio}$, acceptance criterion: (80%, 125%)

IR exotics and short-rate models

- **Recap**
  - Linear products: FRAs, Futures, IRS
  - Options: Swaptions, Caps/Floors
  - Exotics: cancellable swaps, range accruals*, Targeted Redemption Notes (TARNs)**, …
    - **Range accruals**: coupon depends on how often **a reference interest rate stays within a predefined range** during accrual period
    - **TARNs**: provide investors with **a predefined payoff based on the performance of underlying assets** (e.g., S&P500), **redemption mechanism** that allows for early redemption if certain target levels are met
    - Note: exotic features - path-dependent (e.g., barrier), non-linear coupon basket, hybrid (e.g., IR + Eq)…
- Interest rate simulation models
  - Short rates vs forward rates vs swap rates
  - Purpose
    - Derivative valuation
    - Risk management, e.g., counterparty credit risk
  - History
    - Before 90: short rate model – mean reversion, "positivity"; $r(t; t, t + \Delta t)$
      - Vasicek, Hull-White (HW 1F), Cox-Ingrersoll-Ross (CIR), Black-Karasinski, …
    - Early 90s: instantaneous forward rate model – Heath-Jarrow-Morton (HJM); $r(t; T, T)$
    - Late 90s – now: market models (e.g., LIBOR Market Model or BGM)

*: Range Accrual Options: Types and Calculations

**: Understanding Targeted Accrual Redemption Notes (TARNs) and Their Features

IR exotics and short-rate models (cont')

- Short rate models
  - HW 1F model* (extension of Vasicek model: $dr_t = \kappa \cdot (\theta - r_t)dt + \sigma dW_t$)

$$dr_t = \kappa \cdot (\theta(t) - r_t)dt + \sigma dW_t$$

$$Euler\ Scheme:\ r_{t+1} - r_t = \kappa \cdot \left(\theta(t) - r(t)\right) \cdot \Delta t + \sigma \cdot \epsilon_t$$

    - κ: mean reversion speed
    - $\theta(t)$: mean reversion level, ensure the model fits the observed yield curve
    - σ: volatility

Pros	Cons
Exact match time 0 discount curve, $P(0, t)$	Short rate, can't capture term structure
Analytical tractability, close-form solutions for zero-coupon bond, caps/floors, swaptions	No vol smile/skew, deterministic vol
Intuitive model parameters and efficient calibration	Single factor
Allow negative rates	

  - $r_t \sim \mathrm{N}(\mu(t), v(t))$

$$\mu(t) = r_0 \cdot e^{-\kappa t} + \int_0^t e^{-\kappa(t-s)} \cdot \kappa \cdot \theta(s)ds$$

$$v(t) = \frac{\sigma^2}{2\kappa}(1 - e^{-2\kappa t})$$

*: Pricing Interest-Rate-Derivative Securities

IR exotics and short-rate models (cont')

- Short rate models
  - Model calibration – risk neutral
    - Inputs:
      - Discount curve, $P(0, T)$; Swaption ATM vols
    - Calibration method
      - Derive κ from historical data or expert judgement
      - Derive $\theta(t)$ from discounting curve

$$\theta(t) = f(t) + \frac{1}{\kappa} \cdot \frac{\partial f(t)}{\partial t} + \frac{\sigma^2}{2\kappa^2}(1 - e^{-2\kappa t})$$

$$f(t) = -\frac{\partial}{\partial t} P(0, t)$$

      - Derive σ based on minimizing the following function

$$\min_{\sigma} \sum_{i}^{j} \left(\sigma_i^{HW} - \sigma_i^{Mkt,ATM}\right)^2$$

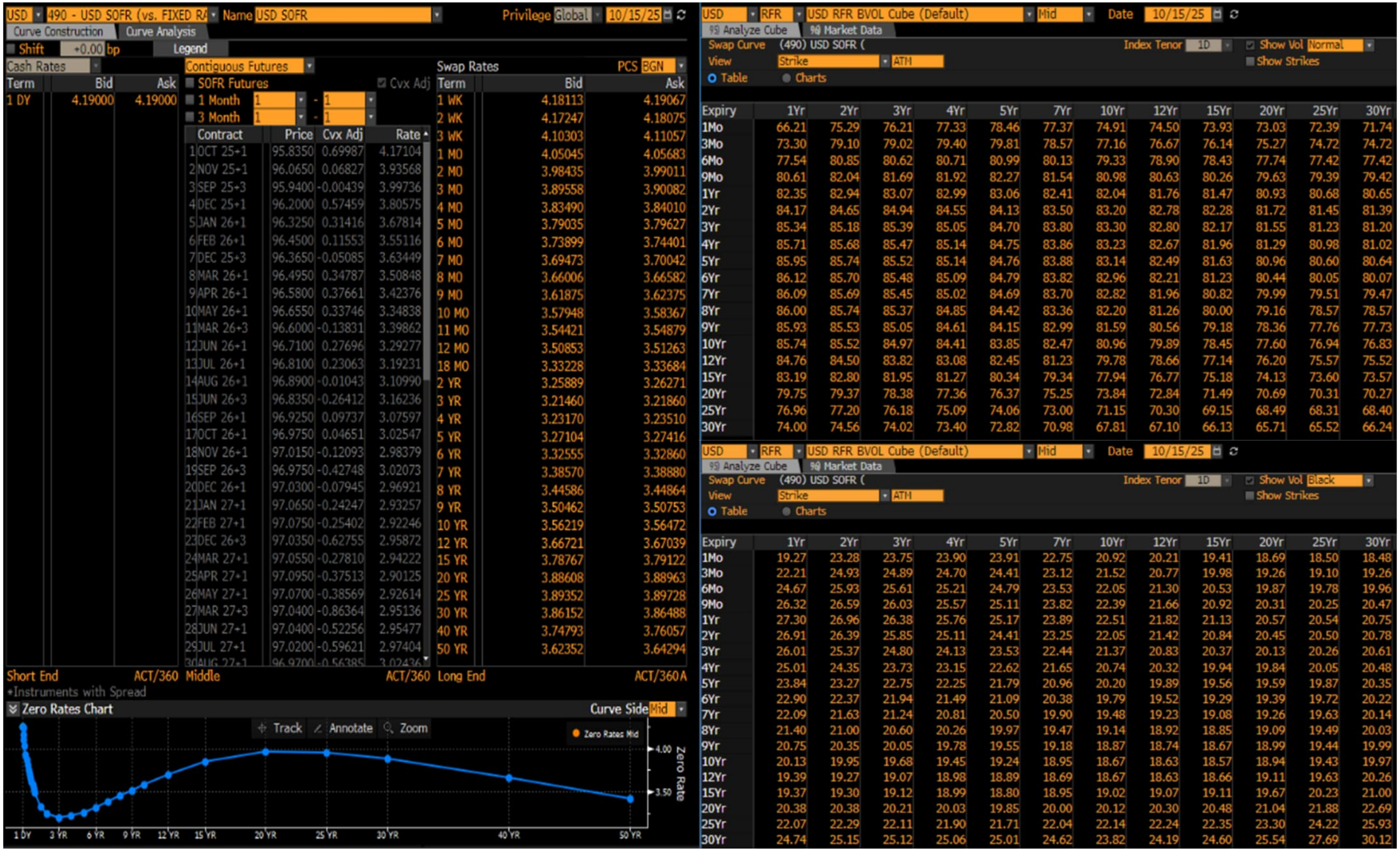
  - Model estimation – real world

$$r_{t+1} - r_t = \kappa \cdot \big(\theta(t) - r(t)\big) \cdot \Delta t + \sigma \cdot \epsilon_t$$

    - Estimation methodology: MLE, Generalized method of moments (GMM)
  - Usages
    - Valuation – risk neutral
    - Risk management – risk neutral / real world

IR exotics and short-rate models (cont’)

- Hull-White model calibration



Term	Bid	Ask
1 DY	4.19000	4.19000

	Contract	Price	Cvx Adj	Rate
1	OCT 25+1	95.8350	0.69987	4.17104
2	NOV 25+1	96.0650	0.06827	3.93568
3	SEP 25+3	95.9400	-0.00439	3.99736
4	DEC 25+1	96.2000	0.57459	3.80575
5	JAN 26+1	96.3250	0.31416	3.67814
6	FEB 26+1	96.4500	0.11553	3.55116
7	DEC 25+3	96.3650	-0.05085	3.63449
8	MAR 26+1	96.4950	0.34787	3.50848
9	APR 26+1	96.5800	0.37661	3.42376
10	MAY 26+1	96.6550	0.33746	3.34838
11	MAR 26+3	96.6000	-0.13831	3.39862
12	JUN 26+1	96.7100	0.27696	3.29277
13	JUL 26+1	96.8100	0.23063	3.19231
14	AUG 26+1	96.8900	-0.01043	3.10990
15	JUN 26+3	96.8350	-0.26412	3.16236
16	SEP 26+1	96.9250	0.09737	3.07597
17	OCT 26+1	96.9750	0.04651	3.02547
18	NOV 26+1	97.0150	-0.12093	2.98379
19	SEP 26+3	96.9750	-0.42748	3.02073
20	DEC 26+1	97.0300	-0.07945	2.96921
21	JAN 27+1	97.0650	-0.24247	2.93257
22	FEB 27+1	97.0750	-0.25402	2.92246
23	DEC 26+3	97.0350	-0.62755	2.95872
24	MAR 27+1	97.0550	-0.27810	2.94222
25	APR 27+1	97.0950	-0.37513	2.90125
26	MAY 27+1	97.0700	-0.38569	2.92614
27	MAR 27+3	97.0400	-0.86364	2.95136
28	JUN 27+1	97.0400	-0.52256	2.95477
29	JUL 27+1	97.0200	-0.59621	2.97404
30	AUG 27+1	96.9700	-0.56385	3.02436

Term	Bid	Ask
1 WK	4.18113	4.19067
2 WK	4.17247	4.18075
3 WK	4.10303	4.11057
1 MO	4.05045	4.05683
2 MO	3.98435	3.99011
3 MO	3.89558	3.90082
4 MO	3.83490	3.84010
5 MO	3.79035	3.79627
6 MO	3.73899	3.74401
7 MO	3.69473	3.70042
8 MO	3.66006	3.66582
9 MO	3.61875	3.62375
10 MO	3.57948	3.58367
11 MO	3.54421	3.54879
12 MO	3.50853	3.51263
18 MO	3.33228	3.33684
2 YR	3.25889	3.26271
3 YR	3.21460	3.21860
4 YR	3.23170	3.23510
5 YR	3.27104	3.27416
6 YR	3.32555	3.32860
7 YR	3.38570	3.38880
8 YR	3.44586	3.44864
9 YR	3.50462	3.50753
10 YR	3.56219	3.56472
12 YR	3.66721	3.67039
15 YR	3.78767	3.79122
20 YR	3.88608	3.88963
25 YR	3.89352	3.89728
30 YR	3.86152	3.86488
40 YR	3.74793	3.76057
50 YR	3.62352	3.64294

Expiry	1Yr	2Yr	3Yr	4Yr	5Yr	7Yr	10Yr	12Yr	15Yr	20Yr	25Yr	30Yr
1Mo	66.21	75.29	76.21	77.33	78.46	77.37	74.91	74.50	73.93	73.03	72.39	71.74
3Mo	73.30	79.10	79.02	79.40	79.81	78.57	77.16	76.67	76.14	75.27	74.72	74.72
6Mo	77.54	80.85	80.62	80.71	80.99	80.13	79.33	78.90	78.43	77.74	77.42	77.42
9Mo	80.61	82.04	81.69	81.92	82.27	81.54	80.98	80.63	80.26	79.63	79.39	79.42
1Yr	82.35	82.94	83.07	82.99	83.06	82.41	82.04	81.76	81.47	80.93	80.68	80.65
2Yr	84.17	84.65	84.94	84.55	84.13	83.50	83.20	82.78	82.28	81.72	81.45	81.39
3Yr	85.34	85.18	85.39	85.05	84.70	83.80	83.30	82.80	82.17	81.55	81.23	81.20
4Yr	85.71	85.68	85.47	85.14	84.75	83.86	83.23	82.67	81.96	81.29	80.98	81.02
5Yr	85.95	85.74	85.52	85.14	84.76	83.88	83.14	82.49	81.63	80.96	80.60	80.64
6Yr	86.12	85.70	85.48	85.09	84.79	83.82	82.96	82.21	81.23	80.44	80.05	80.07
7Yr	86.09	85.69	85.45	85.02	84.69	83.70	82.82	81.96	80.82	79.99	79.51	79.47
8Yr	86.00	85.74	85.37	84.85	84.42	83.36	82.20	81.26	80.00	79.16	78.57	78.57
9Yr	85.93	85.53	85.05	84.61	84.15	82.99	81.59	80.56	79.18	78.36	77.76	77.73
10Yr	85.74	85.52	84.97	84.41	83.85	82.47	80.96	79.89	78.45	77.60	76.94	76.83
12Yr	84.76	84.50	83.82	83.08	82.45	81.23	79.78	78.66	77.14	76.20	75.57	75.52
15Yr	83.19	82.80	81.95	81.27	80.34	79.34	77.94	76.77	75.18	74.13	73.60	73.57
20Yr	79.75	79.37	78.38	77.36	76.37	75.25	73.84	72.84	71.49	70.69	70.31	70.27
25Yr	76.96	77.20	76.18	75.09	74.06	73.00	71.15	70.30	69.15	68.49	68.31	68.40
30Yr	74.00	74.56	74.02	73.40	72.82	70.98	67.81	67.10	66.13	65.71	65.52	66.24

Expiry	1Yr	2Yr	3Yr	4Yr	5Yr	7Yr	10Yr	12Yr	15Yr	20Yr	25Yr	30Yr
1Mo	19.27	23.28	23.75	23.90	23.91	22.75	20.92	20.21	19.41	18.69	18.50	18.48
3Mo	22.21	24.93	24.89	24.70	24.41	23.12	21.52	20.77	19.98	19.26	19.10	19.26
6Mo	24.67	25.93	25.61	25.21	24.79	23.53	22.05	21.30	20.53	19.87	19.78	19.96
9Mo	26.32	26.59	26.03	25.57	25.11	23.82	22.39	21.66	20.92	20.31	20.25	20.47
1Yr	27.30	26.96	26.38	25.76	25.17	23.89	22.51	21.82	21.13	20.57	20.54	20.75
2Yr	26.91	26.39	25.85	25.11	24.41	23.25	22.05	21.42	20.84	20.45	20.50	20.78
3Yr	26.01	25.37	24.80	24.13	23.53	22.44	21.37	20.83	20.37	20.13	20.26	20.61
4Yr	25.01	24.35	23.73	23.15	22.62	21.65	20.74	20.32	19.94	19.84	20.05	20.48
5Yr	23.84	23.27	22.75	22.25	21.79	20.96	20.20	19.89	19.56	19.59	19.87	20.35
6Yr	22.90	22.37	21.94	21.49	21.09	20.38	19.79	19.52	19.29	19.39	19.72	20.22
7Yr	22.09	21.63	21.24	20.81	20.50	19.90	19.48	19.23	19.08	19.26	19.63	20.14
8Yr	21.40	21.00	20.60	20.26	19.97	19.47	19.14	18.92	18.85	19.09	19.49	20.03
9Yr	20.75	20.35	20.05	19.78	19.55	19.18	18.87	18.74	18.67	18.99	19.44	19.99
10Yr	20.13	19.95	19.68	19.45	19.24	18.95	18.67	18.63	18.57	18.94	19.43	19.97
12Yr	19.39	19.27	19.07	18.98	18.89	18.69	18.67	18.63	18.66	19.11	19.63	20.26
15Yr	19.37	19.30	19.12	18.99	18.80	18.95	19.02	19.07	19.11	19.67	20.23	21.00
20Yr	20.38	20.38	20.21	20.03	19.85	20.00	20.12	20.30	20.48	21.04	21.88	22.69
25Yr	22.07	22.29	22.11	21.90	21.71	22.04	22.14	22.24	22.35	23.30	24.22	25.93
30Yr	24.74	25.15	25.12	25.06	25.01	24.62	23.82	24.19	24.60	25.54	27.69	30.12

PCA, market models and risk management

- **Recap**
  - Yield curve
    - How many "*independent risk factors*" drive a 30+ point yield curve
    - Shape: level, slope, curvature
    - Forward rate: $r(t; T_1, T_2)$
  - Principal Component Analysis (PCA): dimension reduction technique which preserves complex data sets' structure. Assume covariance matrix of "normalized" variables $(x_1, \dots, x_n)$ is S:

$$S = Q \cdot \Lambda \cdot Q^T; \Lambda = diag(\lambda_1, \dots, \lambda_n) \ and \ \lambda_1 \geq \cdots \geq \lambda_n$$

    - Eigenvectors: "shape" of risk factors, and PCs are defined as follows

$$PC = Q^T \cdot (x_1, \dots, x_n)$$

    - Eigenvalues: how much variance each PC explains

$$\frac{\lambda_k}{\sum_i \lambda_i} = fraction\ of\ total\ variance\ explained\ by\ PC\ k$$

    - Typically, 2-4 PCs can hit the target (e.g., 90%)

- Question: how to simulate interest rate yield curve at future time points
  - Option 1: Short rate models (e.g., Hull-White), $r(t; t, t)$
  - Option 2: Instantaneous forward rate model (e.g., HJM), $f(t, T) = \lim_{\Delta \to 0} r(t; T, T + \Delta)$
    - Why not directly simulate $r(t; t_1, t_2)$?
  - Option 3: Swap rate

PCA, market models and risk management (cont')

- HJM model
  - Simulate $f(t, T_i) = \lim_{\Delta \to 0} r(t; T_i, T_i + \Delta) = \lim_{\Delta} -\frac{\log P(t,T_i+\Delta) - \log P(t,T_i)}{\Delta} = -\frac{\partial}{\partial T} \log P(t,T)|_{T=T_i}$
    - $S = Historical\ Correlation(r(t; T_1, T_1 + \Delta), \ldots, r(t; T_n, T_n + \Delta))$
    - How many "*independent risk factors*" drive a 30+ point yield curve – typically 2 – 4 risk factors
  - HJM model spec, $\forall T$

$$df(t,T) = \alpha(t,T)dt + \sigma(t,T)dW_t; \;\; f(0,T) = -\frac{\partial}{\partial T} \log P(0,T)$$

$$\alpha(t,T) = \sigma(t,T) \cdot \int_t^T \sigma(t,s)ds$$

  - In reality

$$d\overrightarrow{f(t,T_i)} = \overrightarrow{\alpha(t,T_i)}dt + \boldsymbol{\sigma}(t,T) \cdot d\overrightarrow{W_t}$$

  Note: Dimension of $\overrightarrow{W_t}$, or more precisely, "# of independent drivers" depends on # of PCs

  - Calibration, rely on swaption vols
    - Note: time 0 yield curve is a model input
- Risk management
  - Besides valuation for exotics and XVAs, HJM model is widely used for counterparty credit risk (CCR) management for IR simulation
  - Valuation – bucket delta/vega
  - CCR – Potential Future Exposure (PFE)

Nelson-Siegel model

- Recap
  - Zero curve, $r(0;0,t)$

  - Parametric form (?)
- Nelson-Siegel*

$$y(t) = \beta_0 + \beta_1 \cdot \frac{1 - e^{-\lambda t}}{\lambda t} + \beta_2 \cdot \left(\frac{1 - e^{-\lambda t}}{\lambda t} - e^{-\lambda t} \right)$$

  - β_0: level, or long-term interest rate asymptotic level that the zero rate curve approaches when maturity increases
  - β_1: slope, controls short-term behavior, usually negative (?)
  - β_2: curvature, controls hump-shaped behavior for medium term maturities
  - λ: decay factor, how quickly the effects of β_1 and β_2 decay with maturity
- Model estimation (using zero rates)

$$\min_{\beta_0, \beta_1, \beta_2, \lambda} \sum_{i=1}^{n} \left(y^{obs}(t_i) - y(t_i; \beta_0, \beta_1, \beta_2, \lambda) \right)^2$$

- Note: (1) can use zero coupon bond price; (2) λ can be fixed or constrained

*: Long-Term Behavior of Yield Curves on JSTOR (1988)

Nelson-Siegel model (cont')

- Nelson-Siegel (NS) model's extensions
  - Nelson-Siegel-Svensson

$$y(t) = \beta_0 + \beta_1 \cdot \frac{1 - e^{-\lambda_1 t}}{\lambda_1 t} + \beta_2 \cdot \left(\frac{1 - e^{-\lambda_1 t}}{\lambda_1 t} - e^{-\lambda_1 t}\right) + \beta_3 \cdot \left(\frac{1 - e^{-\lambda_2 t}}{\lambda_2 t} - e^{-\lambda_2 t}\right)$$

  - Dynamic Nelson-Siegel (Diebold-Li*)

$$y(t, T) = \beta_0(t) + \beta_1(t) \cdot \frac{1 - e^{-\lambda T}}{\lambda T} + \beta_2(t) \cdot \left(\frac{1 - e^{-\lambda T}}{\lambda T} - e^{-\lambda T}\right)$$

- Key usages: dynamic NS – forecast future yield curves for interest rate risk management, CCAR/DFAST (e.g., 9Q projection of interest rate yield curve)
  - CCAR/DFAST usage: $(\beta_0(t), \beta_1(t), \beta_2(t), \lambda) \sim macroeconomic\ variables$

- Pros and Cons of the NS model
  - Pros: parsimonious, only 4 parameters and efficient for estimation; intuitive; good empirical fit
    - Good for risk management (e.g., stress testing)
  - Cons: not arbitrage-free, limited flexibility for complex curves
    - Not ideal for derivative pricing

*: Forecasting the term structure of government bond yields - ScienceDirect (2006)

Hybrid modeling for Equity-Linked Notes

- Equity-Linked Notes (ELNs)*
  - A financial instrument whose pay off depends on an underlying equity index or basket as well as embedded equity option
  - Features
    - Principal protection: none / partial / full
    - Coupon: fixed, step-up / down (e.g., linked with equity index/basket performance)
    - Callability: autocall
    - Basket: e.g., best-of / average
    - Maturity: could be short-term (e.g., 6mon) to longer-term (e.g., 20year)
  - Registered Index Linked Annuities (RILAs) - Insurance
    - Annuities vs Variable Annuities vs RILAs
- Valuation
  - Risk factors
    - Interest rates – projection and discounting; IR vol
    - Equity and equity vol
  - Valuation methodology – Hybrid modeling, e.g.,
    - IR: Hull-White (or Cox-Ingersoll-Ross)

$$dr_t = \kappa(\theta(t) - r_t)dt + \sigma dW_t$$

    - Equity: local vol** (deterministic drift vs stochastic drift)

$$dS_t / S_t = r_t dt + \sigma(S_t, t) dW_t$$

*: Equity-Linked Note (ELN) - Overview, Features, Benefits

**: Dupire - Pricing With A Smile (1994) | PDF | Black–Scholes Model | Option (Finance)

Hybrid modeling for Equity-Linked Notes (cont')

- Valuation (cont')
  - Valuation methodology – Hybrid modeling, e.g.,
    - Equity: Heston model* (stochastic vol)

$$\frac{dS_t}{S_t} = r_t dt + \sqrt{v_t} dW_t^1$$

$$dv_t = \kappa(\theta - v_t)dt + \xi\sqrt{v_t} dW_t^2$$

  - Calibration
    - Deterministic drift for equity dynamics, calibrated independently
    - Stochastic drift for equity dynamics, calibrate IR first, then equity (incorporate IR vol in equity vol)
  - Numerical implementation: Euler method
- Risk management
  - Key Greeks
    - Equity: Delta, Gamma, Vega, Vanna/Volga
    - Rates: DV01
    - Cross Gamma
    - Correlation
  - Market risk, counterparty credit risk (for hedges), liquidity risk
- Model risk
  - Conceptual soundness – key assumptions
  - Benchmark
  - Ongoing performance monitoring

*: Closed-Form Solution for Options with Stochastic Volatility with Applications to Bond and Currency Options | The Review of Financial Studies | Oxford Academic

Risk management, capital, stress testing and market developments

- Recap
  - Instruments
    - Cash: T-bills/notes/bonds, Munis, Corp bonds, …
    - Derivatives: FRAs, Futures, Swaps; Swaptions, Caps/Floors; exotics
  - Risk management
    - Notional/exposure; DV01 (parallel shift, by tenor – 2y/5y/10y, basis, xccy); 2nd order sensitivities – Gamma/Vega/Vanna/Volga, …
    - P&L attributions
    - Market risk (MR) – VaR
    - Counterparty credit risk (CCR) – PFE
  - Capital: MR capital - VaR/SVaR; CCR – IMM/SA CCR
  - Stress testing: Global Market Shock
- **Fundamental Risk of Trading Book (FRTB) market risk capital***
  - Internal Models Approach (IMA)
    - Expected Shortfall (97.5%)
    - Add-on for non-modellable risk factors (NMRFs)
  - Standardised Approach (SA) – General Interest Rate Risk (GIRR) – "*Parametric VaR*"
    - Delta
    - Vega
    - Curvature
    - Residual Risk Add-on (RRAO): covers risks not captured by delta, vega, or curvature (e.g., exotic underliers, gap risk)

*: Minimum capital requirements for market risk

Risk management, capital, stress testing and market developments (cont')

- SA - Sensitivities-based risk capital charge*: delta and vega
  - The sensitivities-based risk capital charges for delta and vega are calculated for each risk class (e.g., rates, FX, equities) based on granular risk factor sensitivities (delta and vega) and allows for diversification benefits within, but not across, risk classes. The following are calculation steps:

1. Calculation of Weighted Sensitivities within Each Bucket of Each Risk Class *(MAR 21.4 (1-3))*

- Delta and vega sensitivities are sourced for each position and weighted sensitivities are calculated for each bucket with each risk class
- Sample of delta sensitivtiy for the GIRR risk class:

$$s_{k,r_t} = \frac{V_i(r_t + 0.0001, cs_t) - V_i(r_t, cs_t)}{0.0001}$$

Where:

- r_t is the risk-free yield curve at tenor t, cs_t is the credit spread curve at tenor t and v_i is the market value of instrument i as a function of the risk-free interest ate curve and credit spread curve

2. Aggregation per Bucket *(MAR 21.4(4))*

- Aggregation of weighted sensitivities per bucket with prescribed correlations (ρ_{kl})
- Sample of bucket aggregation formula:

$$K_b = \sqrt{\max(0, \sum_k WS_k^2 + \sum_k \sum_{k \neq l} \rho_{kl} WS_k WS_l)}$$

- Delta and vega risk charges are calculated separately, with no diversification benefit recognized between risk factors or buckets

3. Aggregation Across Buckets *(MAR 21.4 (5))*

- Aggregation across buckets with prescribed correlations (γ_{bc})
- Sample of bucket aggregation formula:

$$Delta\ (respectively\ vega) = \sqrt{\sum_b K_b^2 + \sum_b \sum_{c \neq b} \gamma_{bc} S_B S_c}$$

4. Calculation Using Adverse Moves in Correlations *(MAR 21.6)*

- Under the "high correlations" scenario, γ_{bc} and ρ_{kl} are multiplied by 1.25 (subject to a cap at 100%)
- Under the "low correlations" scenario, ρ_{kl} and γ_{bc} are calculated using

$$\rho_{kl}^{low} = \max(2x\ \rho_{kl} - 100\%\ ;75\%\ x\ \rho_{kl}) \text{ and}$$
$$\gamma_{bc}^{low} = \max(2x\ \gamma_{bc} - 100\%\ ;75\%\ x\ \gamma_{bc})$$

5. Total Capital Requirement under SBM *(MAR 21.7)*

- For each of three correlation scenarios, the bank must simply sum up the separately calculated delta, vega and curvature capital requirements for all risk classes to determine the overall capital requirement for that scenario.
- The sensitivities-based method capital requirement is the largest capital requirement from the three scenarios.

*: MAR21 - Standardised approach: sensitivities-based method

Risk management, capital, stress testing and market developments (cont')

▶ SA - Sensitivities-based risk capital charge: curvature*

▶ The curvature risk capital charge is intended to capture the incremental risk not accounted for by the delta risk of price changes in the value of non-linear instruments. It therefore measures the higher order risks, including gamma. The following steps could generally be followed:

1 **Net curvature risk charge $\mathbf{CVR_k}$** (*MAR 21.5(1-2)*): Calculate net curvature charge across instruments for each curvature risk factor *k*. Two stress scenarios involving an **upward shock** and a **downward shock** (size of shock outlined in MAR 21.98-21.99) must be applied to *k*.

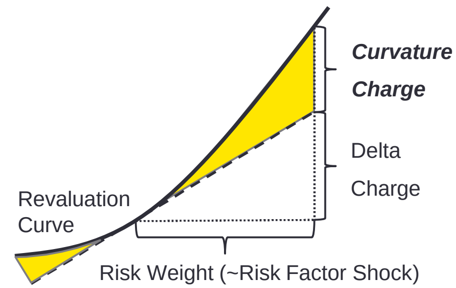
$$CVR_k^+ = -\left(\sum_i V_i\left(x_k^{RW(Curvature)^+}\right) - V_i(x_k) - RW_k^{Curvature} * s_{ik}\right)$$

$$CVR_k^- = -\left(\underbrace{\sum_i V_i\left(x_k^{RW(Curvature)^-}\right) - V_i(x_k)}_{} + \underbrace{RW_k^{Curvature} * s_{ik}}_{}\right)$$

Full valuation-based P&L with shocked delta charge risk weight (up and down) at delta charge risk factor level

–

Sensitivity based P&L with shocked delta charge risk weight (up and down) at delta charge risk factor level



2 **Aggregation within each bucket and across buckets** *(MAR 21.5(3-4))*: Curvature risk exposures must be aggregated within each bucket and across buckets within each risk class, using the corresponding prescribed correlations. The bucket level capital requirement K_b is determined as the greater of the capital requirement under the upward and downward scenario.

Within bucket aggregation:

$$K_b = \max(K_b^+, K_b^-), \text{ where}$$

$$\begin{cases} K_b^+ = \sqrt{\max\left(0, \sum_k \max(CVR_k^+, 0)^2 + \sum_{l \neq k}\sum_k \rho_{kl} CVR_k^+ CVR_l^+ \psi(CVR_k^+, CVR_l^+)\right)} \\ K_b^- = \sqrt{\max\left(0, \sum_k \max(CVR_k^-, 0)^2 + \sum_{l \neq k}\sum_k \rho_{kl} CVR_k^- CVR_l^- \psi(CVR_k^-, CVR_l^-)\right)} \end{cases}$$

Across bucket aggregation:

$$\text{urvature risk} = \sqrt{\max\left(0, \sum_b K_b^2 + \sum_b \sum_{c \neq b} \gamma_{bc} S_b S_c \psi(S_b, S_c)\right)}$$

Where:

- $S_b = \sum_k CVR_k{}^+$ for all risk factors in bucket b, when the upward scenario has been selected for bucket b, $S_c = \sum_k CVR_k{}^-$ otherwise
- $\psi(S_b, S_c)$ takes the value of 0 if S_b and S_c both have negative signs and 1 otherwise

3 **Calculation Using Adverse Moves in Correlations** *(MAR 21.6):* Under the "high correlations" scenario, γ_{bc} and ρ_{kl} are multiplied by 1.25 (subject to a cap at 100%). Under the "low correlations" scenario, ρ_{kl} and γ_{bc} are calculated using:

$$\rho_{kl}^{low} = \max(2 \text{ x } \rho_{kl} - 100\% \,; 75\% \text{ x } \rho_{kl}) \text{ and}$$

$$\gamma_{bc}^{low} = \max(2\text{x } \gamma_{bc} - 100\% \,; 75\% \text{ x } \gamma_{bc})$$

4 **Total Capital Requirement under SBM** *(MAR 21.7):* For each of three correlation scenarios, the bank must simply sum up the separately calculated delta, vega and curvature capital requirements for all risk classes to determine the overall capital requirement for that scenario. The sensitivities-based method capital requirement is the largest capital requirement from the three scenarios.

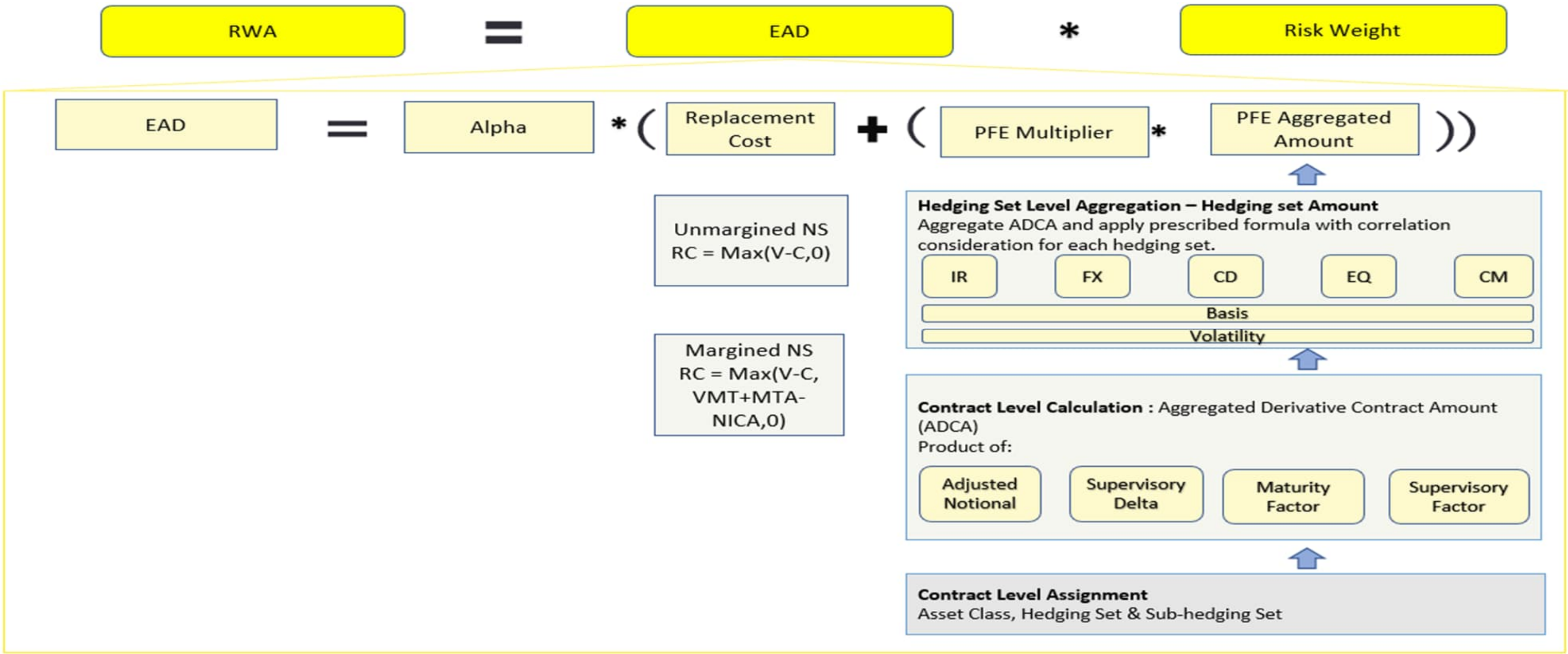
Risk management, capital, stress testing and market developments (cont')

- **Standardized Approach for Counterparty Credit Risk (SA-CCR)***
  - On Mar. 31, 2014, the Basel Committee finalized a standardized method for calculating CCR exposures associated with Over the Counter (OTC) derivatives, Exchange-traded derivatives, Long settlement transactions
  - Exposure at Default (EAD) for a netting set

$$EAD = regulatory\ multiplier \cdot (Replacement\ Cost + PFE)$$

$Replacement\ Cost$: Current exposure;

PFE: Potential Future Exposure (risk sensitivities-based add-on)



*: CRE52 - Standardised approach to counterparty credit risk

Risk management, capital, stress testing and market developments (cont')

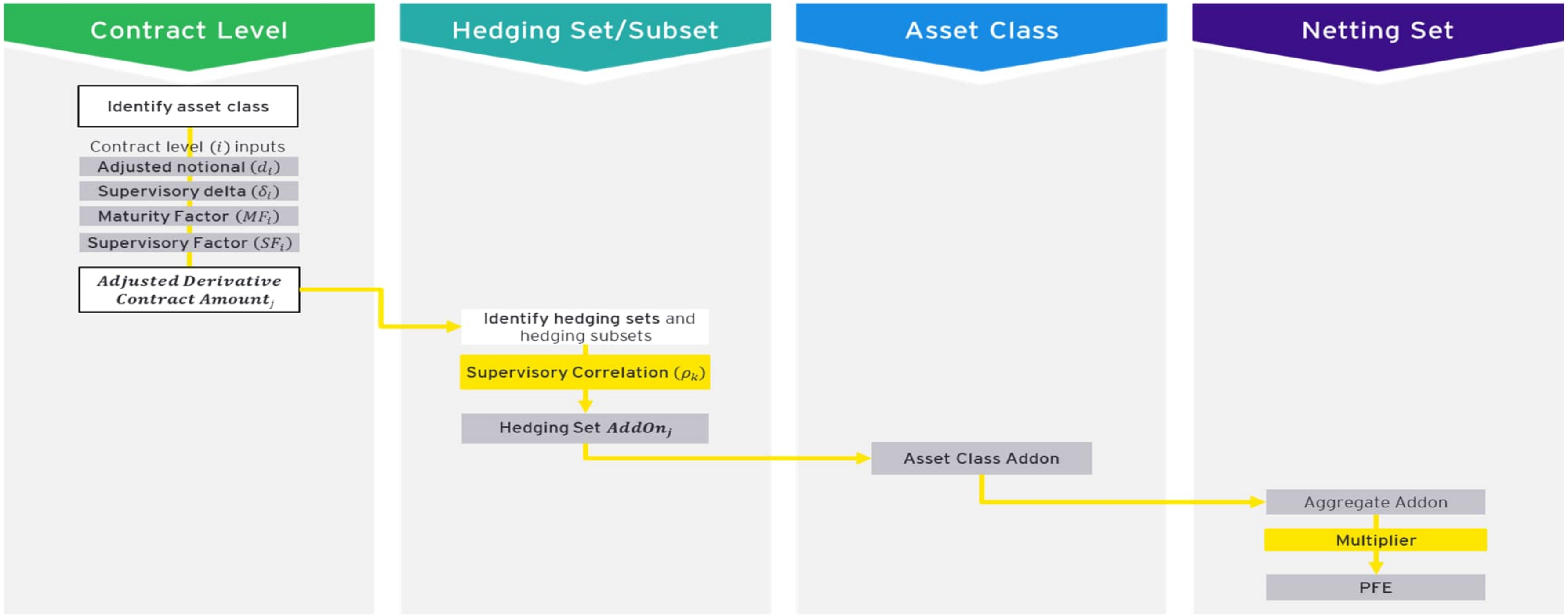
SA-CCR - PFE

$$PFE = AddOn^{aggregate} * multiplier$$

$$AddOn^{Aggregate} = \sum_{AssetClass} AddOn_{AssetClass}$$

$$multiplier = \min\left\{1; Floor + (1 - Floor) * \exp\left(\frac{V - C}{2 * (1 - Floor * AddOn^{aggregate})}\right)\right\}$$

- The PFE calculation consists of:
  - A **multiplier**, scaling down the aggregate add-on to recognize the presence of **excess collateral** or **negative mark-to-market** value of the transactions. The multiplier is floored at 5%.



Risk management, capital, stress testing and market developments (cont')

- **Comprehensive Capital Analysis and Review (CCAR) – Global Market Shock***
  - On Oct. 24, 2025, the FRB published stress testing methodology for comments
  - Key proposed changes
    - introduces ~2,300 risk factor shocks, reduced from ~20,000 risk factors in the 2025 Proposal
    - reduced the liquidity horizon by asset class from 3-12 months to 1 or 3 months

Primary Risk Factors

- Primary risk factors characterize the scenario narratives at a very high level.

Financial Market	Primary Risk Factor
Equity	S&P 500 index return
Credit	Moody's Baa-Aaa credit spread
Interest Rate	U.S. 10-year minus three-month Treasury term spread U.S. 10-year Treasury bond yield
FX	U.S. dollar-to-Euro exchange rate
Commodity	Gold Global Price Index of Energy Global Price Index of Metal

- The Board uses statistical analysis of historical data to specify primary risk factor shocks: 1) percentiles of historical data (table below) and 2) historical simulation method which considers firm's past exposure and tail losses.

Scenario Severity	Percentile
Mild	15^{th}~85^{th}
Moderate	5~15^{th}/85~95^{th}
Large	1~5^{th}/95~99^{th}
Severe	min~1^{th}/99^{th}~max
Unprecedented	Lower than min/higher than max

Secondary Risk Factors

- Around 100 secondary risk factors are selected which can both broadly characterize scenario narratives and are statistically related to the primary risk factors. The set of secondary risk factors is flexible and can be expanded to accommodate various market scenarios, as appropriate.
- In most cases, secondary risk factor shocks are obtained from models with primary risk factor shocks as inputs (see next two pages for model details).

Secondary Risk Factor Asset Class	Related Primary Risk Factor
Public Equity	S&P 500 index return
Agencies/Sovereign credit/Corporate credit/Municipal credit/Securitized products	Moody's Baa-Aaa credit spread
Energy	Global Price Index of Energy
Metals	Global Price Index of Metal
FX	U.S. dollar-to-Euro exchange rate
10-year government bond	U.S. 10-year Treasury bond yield
3-month government bond	U.S. 10-year-minus-3-month Treasury term Spread
10-year swap	10-year government bond yield[1]
3-month swap	10-year-minus-3-month government bond term spread[2]

*: Supervisory Stress Test Documentation Global Market Shock Component - October 2025

Risk management, capital, stress testing and market developments (cont')

- **The U.S. Treasury and Repo Central Clearing rule***
  - In December 2023, the SEC adopted rules requiring the majority of US Treasury market transactions to be cleared through an SEC-approved Covered Clearing Agency (CCA)
    - An average of over $700 billion and $4.5 trillion in cash and financing transactions, respectively, involving Treasury securities**
  - Scope: eligible repo and cash trades in the US Treasury market
  - Clearing agency: Fixed Income Clearing Corporation (FICC) is the sole clearing agency
  - CCAs would be required to collect margin separately for house & customer transactions
  - Timeline
    - 12/31/2026: Compliance by the direct participants of CCAs to clear eligible Cash transactions
    - 6/30/2027: Compliance by the direct participants of CCAs to clear eligible Repo transactions
  - Implications from quantitative risk management perspective
    - Margin calculation
    - Collateral management/optimization
    - Liquidity risk

*: SEC.gov | SEC Adopts Rules to Improve Risk Management in Clearance and Settlement and Facilitate Additional Central Clearing for the U.S. Treasury Market

**: SEC Ruling on Central Clearing | BNY