Some Basic Gymnastics

- Forwards in Discrete time

Given r_n, the formula for finding the forward rate covering period j->n, where j < n, $f_{j,n}$, can be found as follows

$$(1 + r_n)^n = (1 + r_j)^j (1 + f_{j,n})^{n-j}$$

Or

$$f_{j,n} = \left(\frac{(1 + r_n)^n}{(1 + r_j)^j} \right)^{\frac{1}{n-j}} - 1$$

- *With Compounding m times per period*

$$(1 + r_n)^n = (1 + r_j)^j (1 + f_{j,n})^{n-j}$$

Becomes

$$\left(1 + \frac{r_n}{m}\right)^{nm} = \left(1 + \frac{r_j}{m}\right)^{jm} \left(1 + \frac{f_{j,n}}{m}\right)^{(n-j)m}$$

In the limit as $\lim_{m \to \infty} \left(1 + \frac{r}{m}\right)^m = e^r$

making

$$e^{rn} = e^{rj} e^{f(j,n)(n-j)}$$