

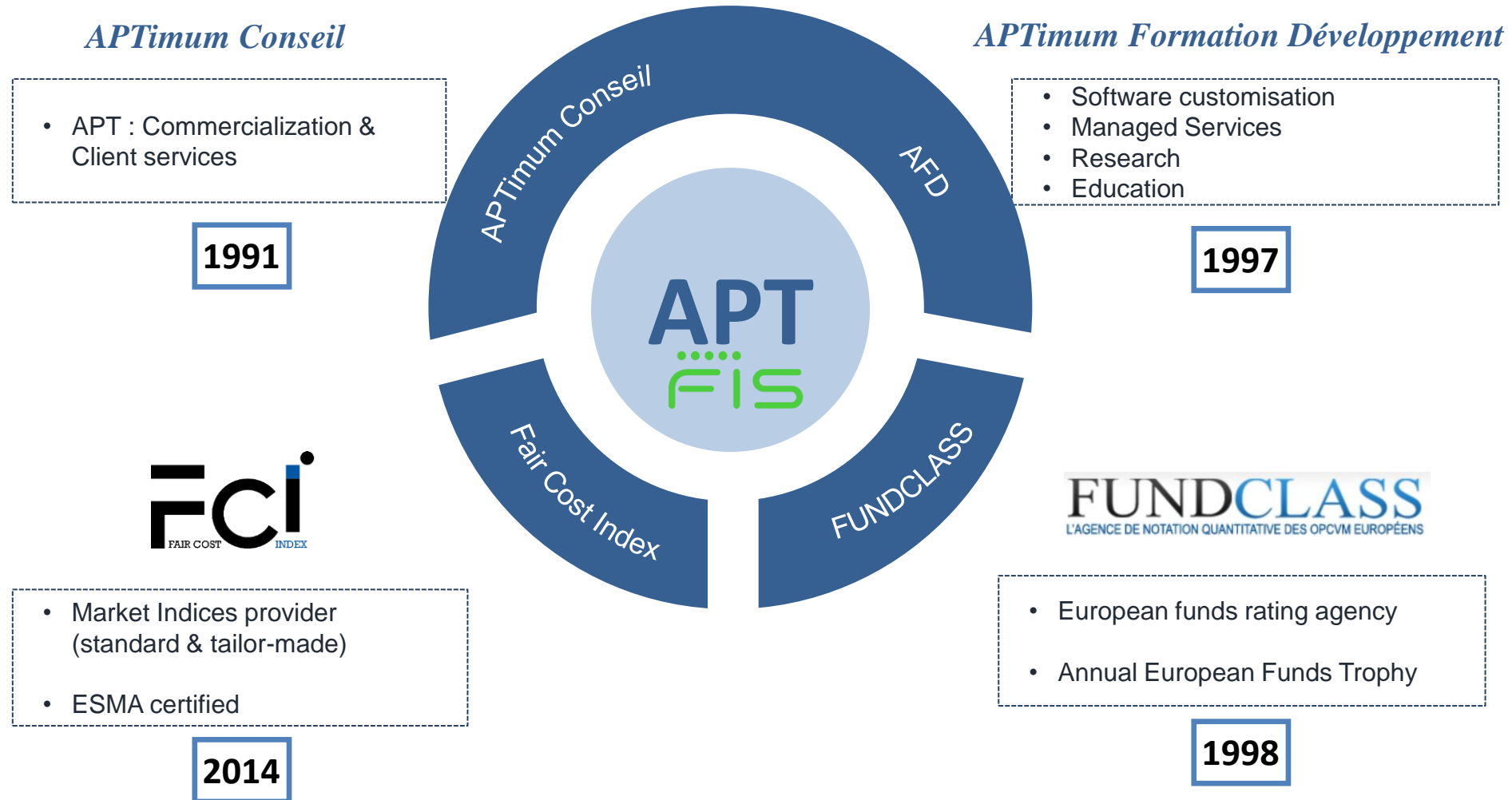
Columbia Seminar I

APT Risk Model : Theory and Practice



Kristen Walters
Karim Jacquelin, CFA, FRM

APTimum



CONTENTS



I. Risk Models

- 1. Variance Covariance Matrix**
- 2. Single Factor Model**
- 3. Prespecified Multi Factor Model**
- 4. APT Statistical Multi Factor Model**

II. The need for Statistical Multi Factor Models

- 1. Risk attribution**
- 2. Risk measurement**

III. Risk Indicators

- 1. Top Level Risk Indicators**
- 2. Position Based Risk Indicators**

IV. Risk Analysis

CONCEPT :: RISK DEFINITION

Market Risk :

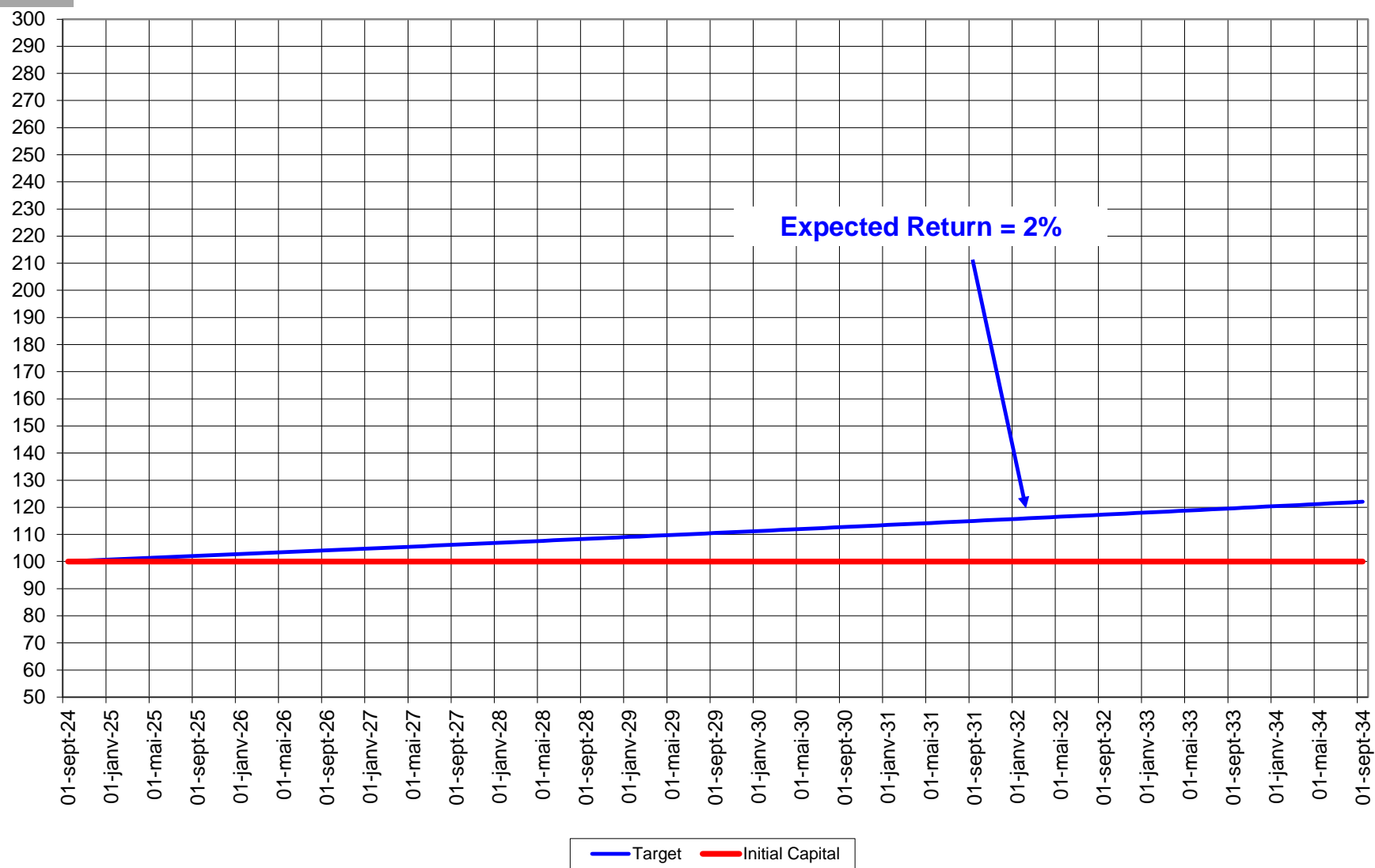
uncertainty in my future wealth value, driven by constantly changing market prices

CONCEPT :: RISK DEFINITION

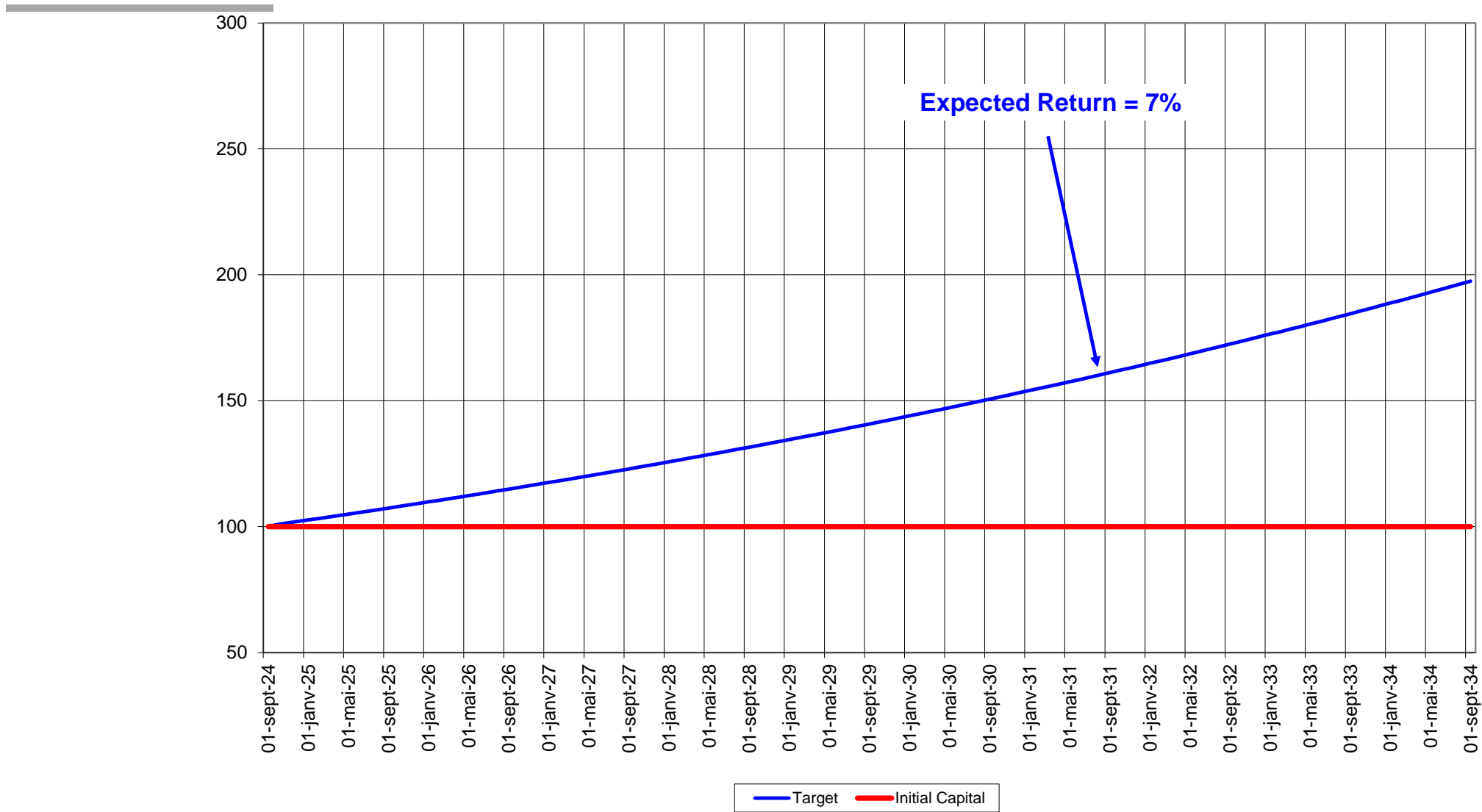
Risk = « *uncertainty in ...* »



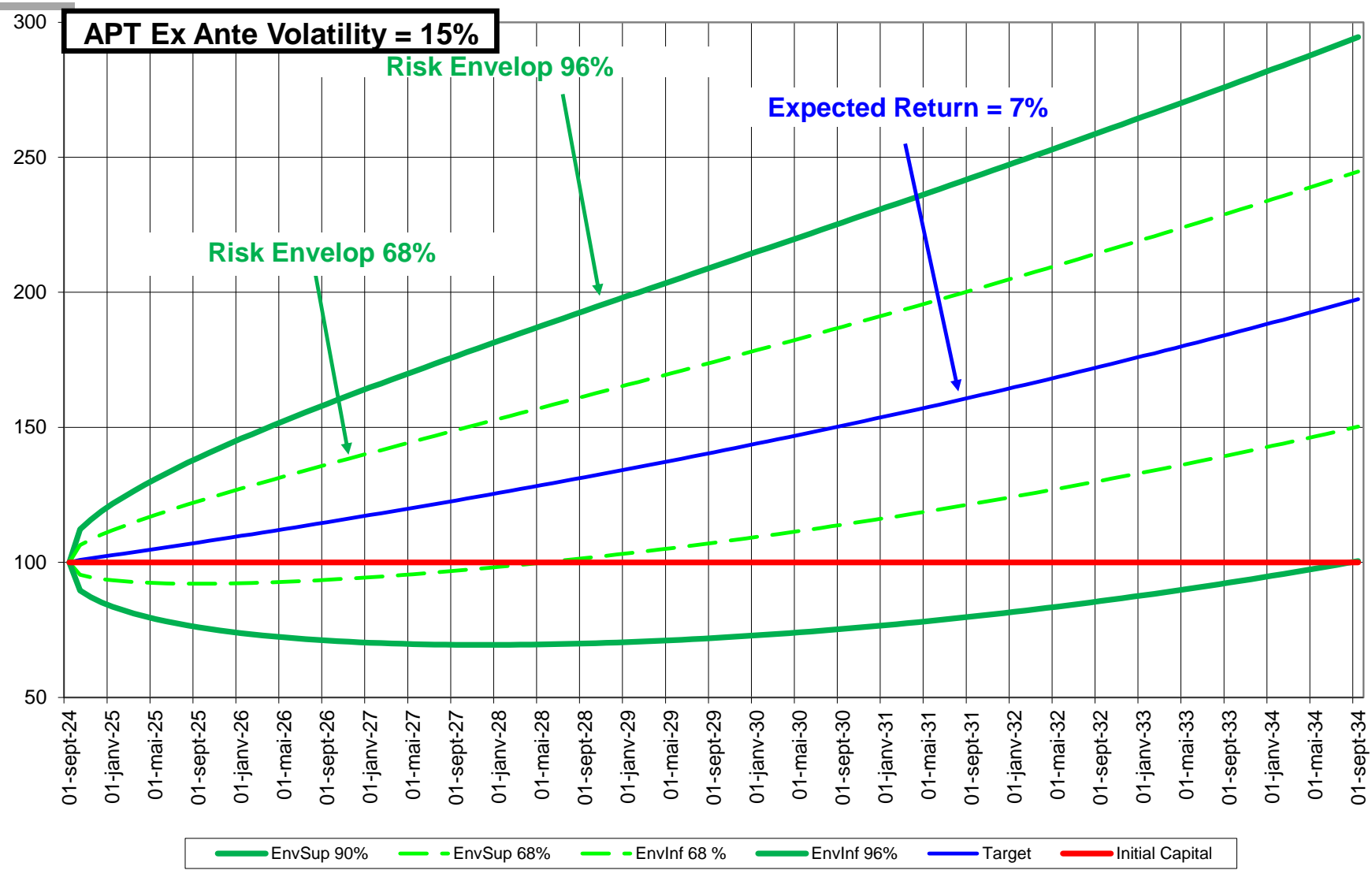
CONCEPT :: RISK ENVELOPS



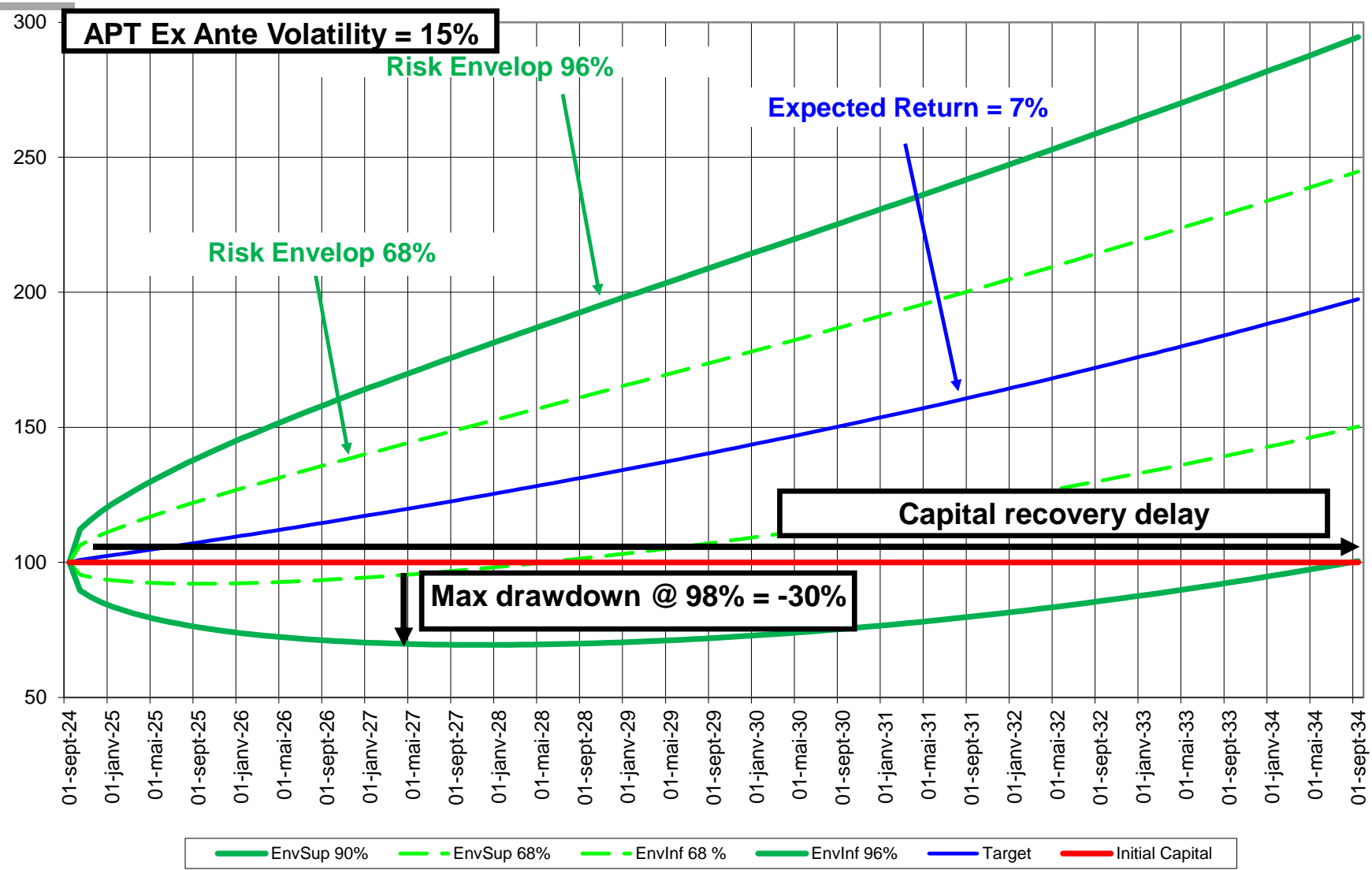
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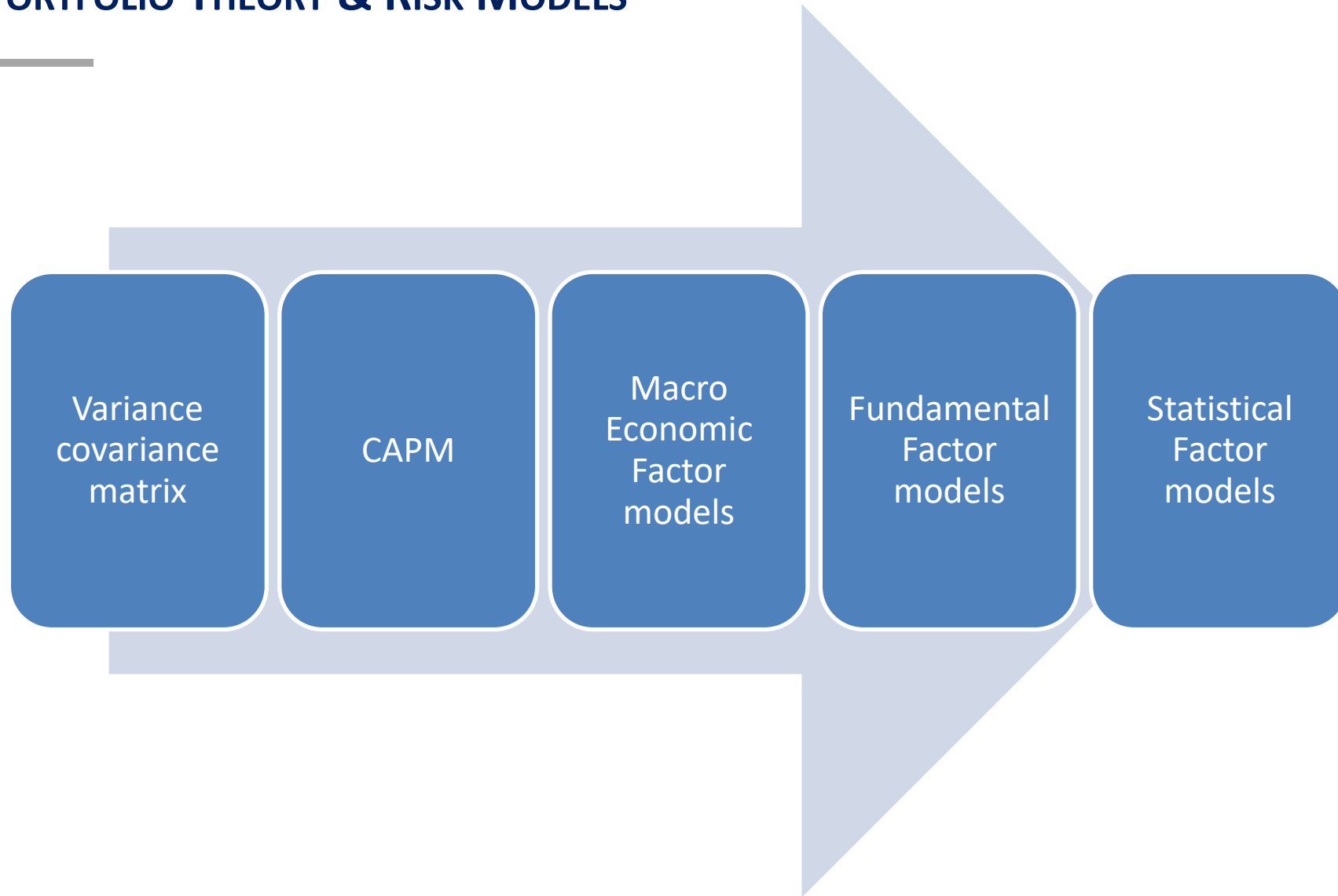
CONCEPT :: RISK ENVELOPS



CONCEPT :: RISK ENVELOPS



MODERN PORTFOLIO THEORY & RISK MODELS



SAMPLE FUNDS

ISIN Code	Fund Name	Date	AUM (Mil)	Fund Style	Benchmark
LU2978126881	Vaughan Nelson U.S. Select Equity	31/12/2024	\$ 137.20	US Equity Blend	S&P 500
LU2423583330	Loomis Sayles US Growth Equity	31/12/2024	\$ 51.30	US Equity Growth	S&P 500 Growth
LU0412075581	Harris Associates U.S. Value Equity	31/12/2024	\$ 1 940.46	US Equity Value	S&P 500 Value
LU1458428320	BNP Paribas Sust US Value Mlt-Fctr Eq	31/12/2024	\$ 1 008.96	US Equity Value	S&P 500 Value

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MODERN PORTFOLIO THEORY (REFRESH)

1934 : Graham and Dodd, *Security Analysis* :

➔ Security selection : « intrinsic value », fundamental analysis

1952 : Markowitz, *Portfolio Selection, Journal of Finance*

1959 : Markowitz, *Portfolio Selection: Efficient Diversification of Investments, Journal of Finance*

➔ Not only focus on expected return but also on risk

➔ Efficient portfolios

➔ Diversification

VARIANCE COVARIANCE MATRIX

Variance Covariance Matrix

$$\Sigma = \begin{pmatrix} \sigma_1^2 & \sigma_{12} & \cdots & \sigma_{1N} \\ \sigma_{21} & \sigma_2^2 & \cdots & \sigma_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{N1} & \sigma_{N2} & \cdots & \sigma_N^2 \end{pmatrix}$$

Notations:

Σ : Variance-covariance matrix (N×N).

w : Column vector of weights (N×1)

σ_P : portfolio volatility.

Mathematical formula :

$$\sigma_P = \sqrt{w^T \Sigma w}$$

Explicit expansion for N assets :

$$\sigma_P = \sqrt{\sum_{i=1}^N w_i^2 \sigma_i^2 + \sum_{\substack{i=1 \\ i \neq j}}^N \sum_{\substack{j=1 \\ i \neq j}}^N w_i w_j \text{cov}(i, j)}$$

VARIANCE COVARIANCE MATRIX

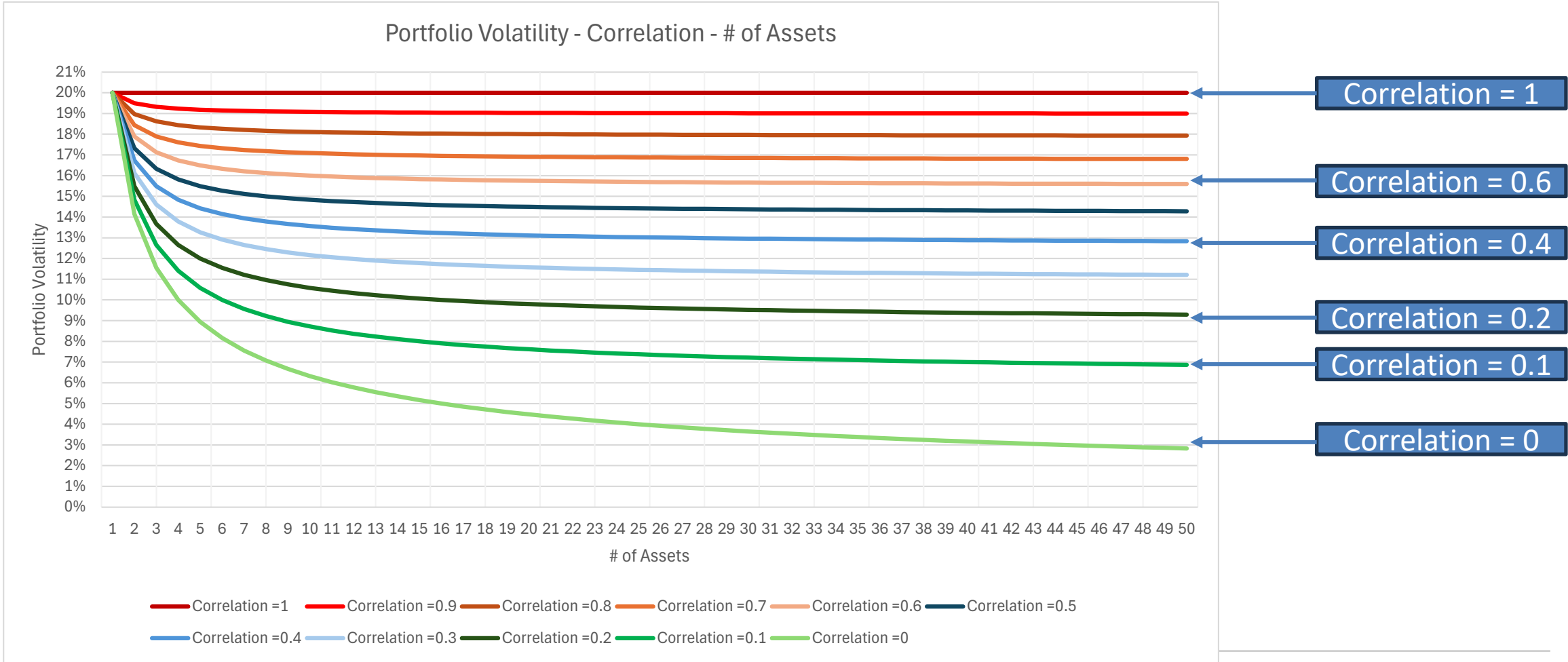
CASE STUDY 1 : Compute volatility for Vaughan Nelson US Equity

VARIANCE COVARIANCE MATRIX

Fund Name	Fund Style	Annualised Vol
Vaughan Nelson U.S. Select Equity	US Equity Blend	22.5%
Loomis Sayles US Growth Equity	US Equity Growth	24.1%
Harris Associates U.S. Value Equity	US Equity Value	18.3%
BNP Paribas Sust US Value Mlt-Fctr Eq	US Equity Blend	13.8%

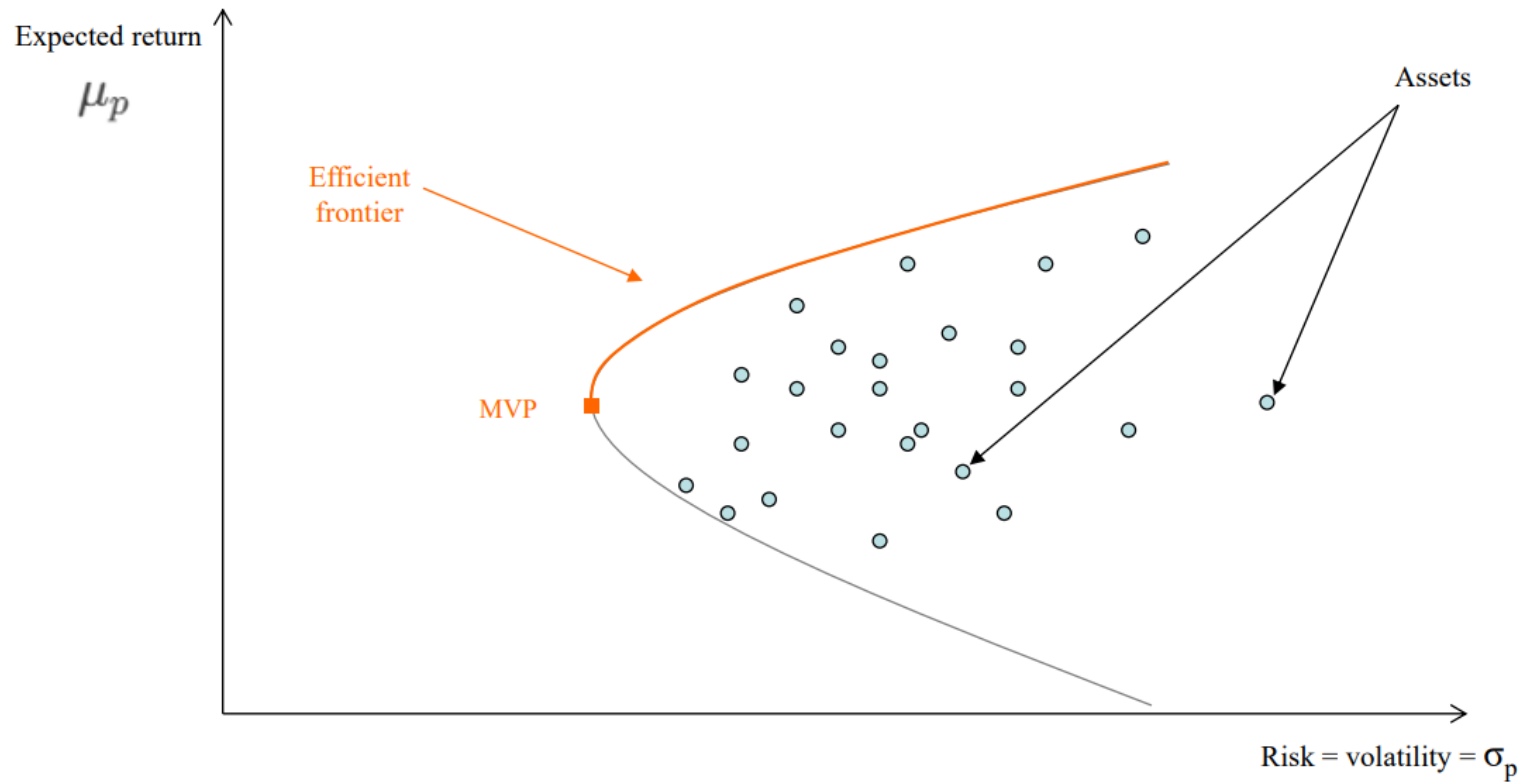
VARIANCE COVARIANCE MATRIX

Diversification



VARIANCE COVARIANCE MATRIX

Efficient frontier : risk-return tradeoff



$$\begin{aligned} &\underset{\mathbf{w}}{\text{minimize}} && \mathbf{w}^T \Sigma \mathbf{w} \\ &\text{subject to} && \mathbf{w}^T \boldsymbol{\mu} = \mu_p \\ & && \sum_{i=1}^N w_i = 1 \\ & && w_i \geq 0 \quad (\text{optional}) \end{aligned}$$

Limitations of the Variance Covariance matrix

- ☐ Dimension
- ☐ Unstability
- ☐ Data mining : no economic interpretation

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SINGLE FACTOR MODEL

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➔ Security selection : « intrinsic value », fundamental analysis

1952 : Markowitz, *Portfolio Selection, Journal of Finance*

1959 : Markowitz, *Portfolio Selection: Efficient Diversification of Investments, Journal of Finance*

➔ Not only focus on return but also risk

➔ Efficient portfolios

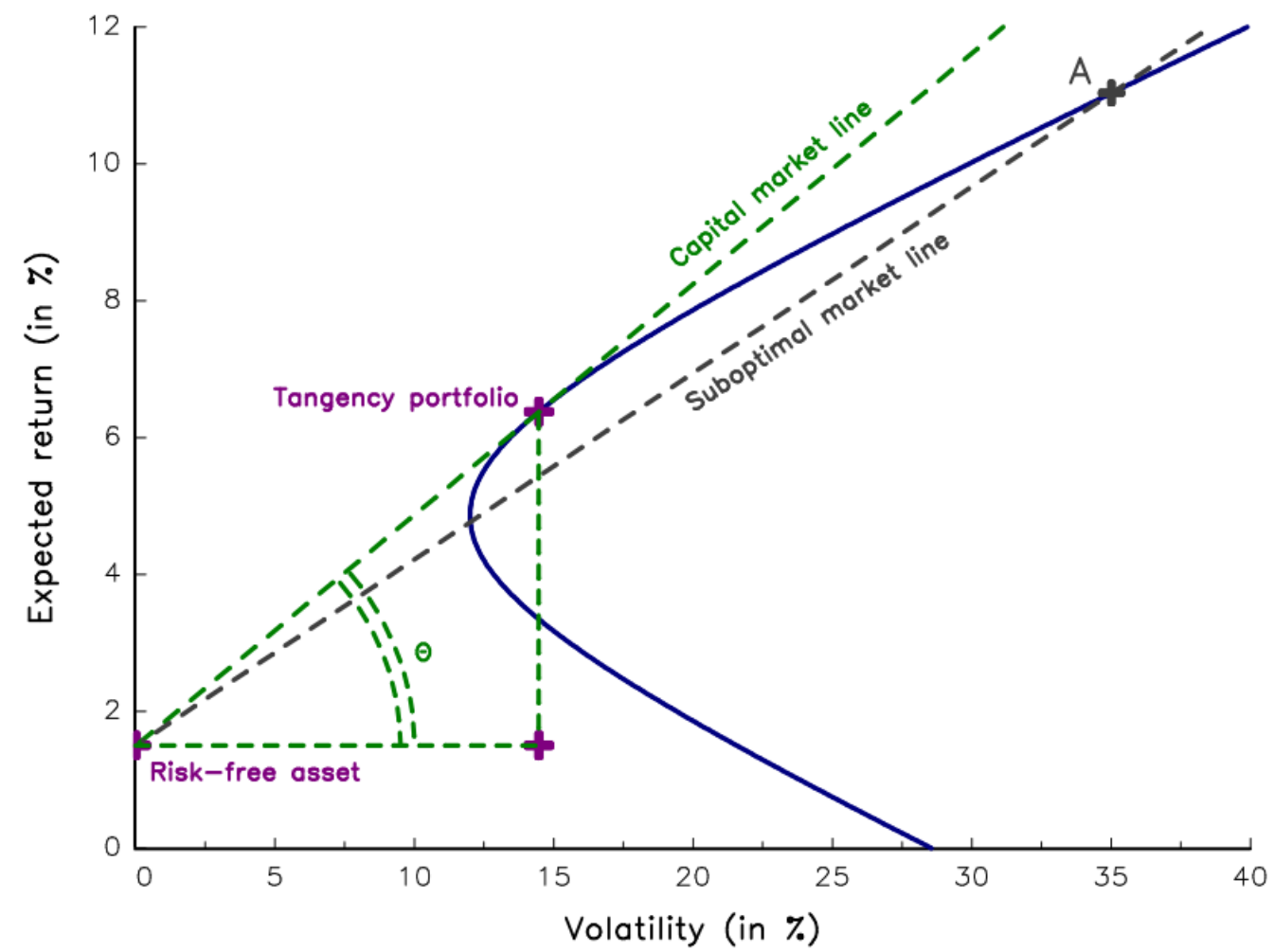
➔ Diversification

1963 : Sharpe, *A simplified Model for Portfolio Analysis*

1964 : Sharpe, *Capital Asset Prices: A theory of Market Equilibrium under Conditions of Risk*

➔ *Capital Asset Pricing Model*

SINGLE FACTOR MODEL



SINGLE FACTOR MODEL

Security Market Line :

$$E(R_i) = R_F + \beta_i (E(R_M) - R_F)$$

$E(R_i)$ = expected return of security i
 R_F = risk-free rate
 β_i = Beta of security i relative to the market

$$\beta_i = \frac{Cov(R_i, R_M)}{Var(R_M)} = \rho_{i, M} \frac{\sigma_i}{\sigma_M}$$

R_M = expected return of the market
 $E(R_M) - R_F$ = Market Risk Premium

SINGLE FACTOR MODEL

$$R_i = R_F + \beta_i (R_M - R_F) + \varepsilon_i$$



Explained by « Market » Factor

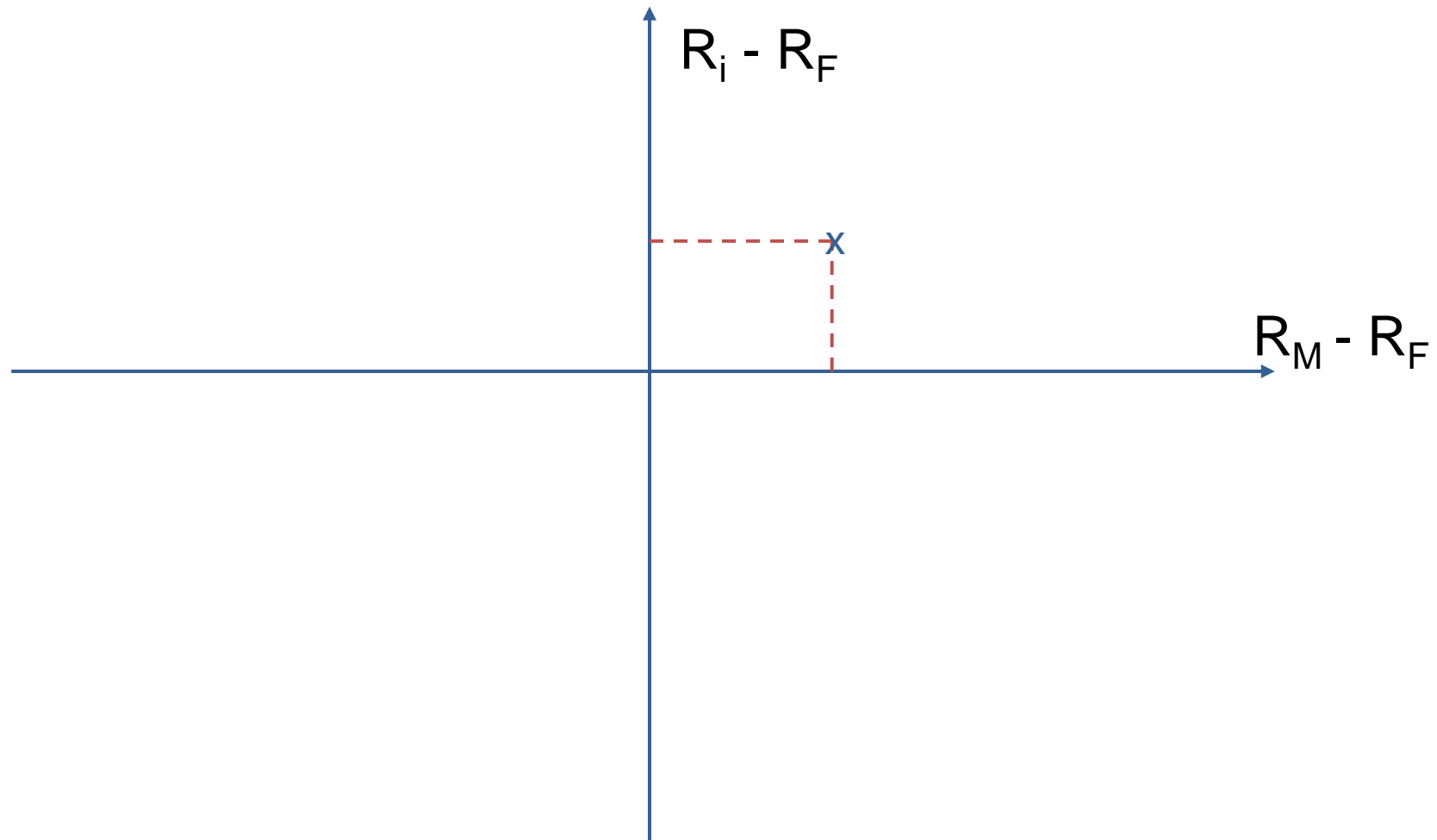
Systematic Return



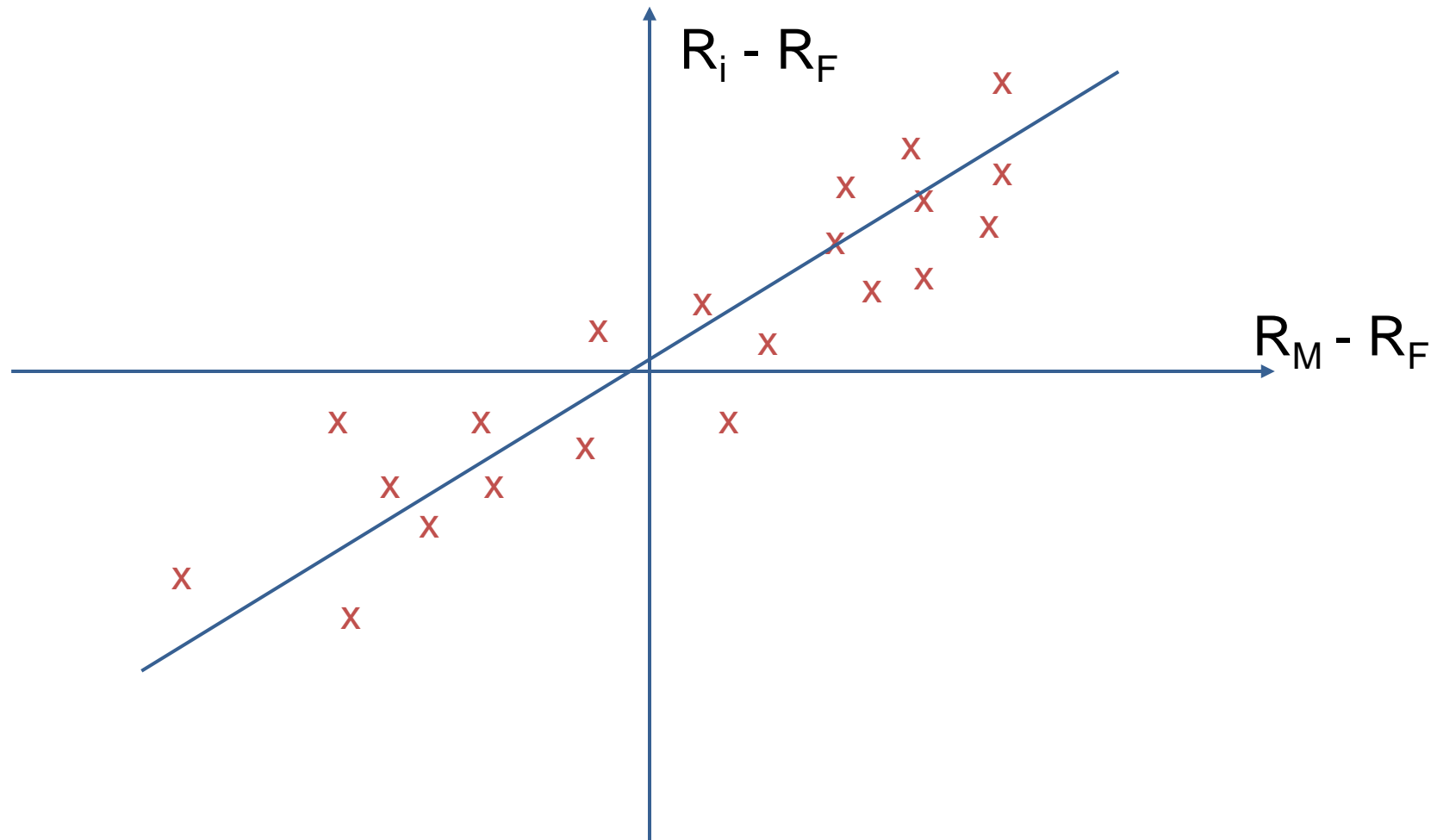
Residual = Unexplained by
« Market » Factor

Specific Return

SINGLE FACTOR MODEL



SINGLE FACTOR MODEL



SINGLE FACTOR MODEL

$$R_i = R_F + \beta_i (R_M - R_F) + \varepsilon_i$$



Explained by « Market » Factor



Residual = Unexplained by
« Market » Factor

Systematic Return

Specific Return

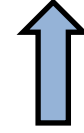
$$\sigma_i^2 = \beta_i^2 \sigma_M^2 + \sigma_{\varepsilon_i}^2$$



Total Risk



Systematic Risk



Specific Risk

SINGLE FACTOR MODEL

$$\sigma_{Total}^2 = \sigma_{Syst}^2 + \sigma_{Spe}^2$$

Model Explanatory Power = R Squared

$$R^2 = \frac{\sigma_{Syst}^2}{\sigma_{Total}^2}$$

SINGLE FACTOR MODEL

CASE STUDY 2 : Apply Single Factor Model for Loomis US Equity Growth

SINGLE FACTOR MODEL

Fund Name	Fund Style	Annualised Vol	Single Factor		
			MARKET	R squared	Residual
Vaughan Nelson U.S. Select Equity	US Equity Blend	22.5%	1.14	90.3%	9.7%
Loomis Sayles US Growth Equity	US Equity Growth	24.1%	1.20	86.5%	13.5%
Harris Associates U.S. Value Equity	US Equity Value	18.3%	0.80	67.7%	32.3%
BNP Paribas Sust US Value Mlt-Fctr Eq	US Equity Value	13.8%	0.65	76.5%	23.5%

SINGLE FACTOR MODEL

Limitations of CAPM

- ❑ Assumptions :
 - Homogeneous expectations
 - Rationale investors with Risk Aversion
 - No taxes or transactions costs
 - unlimited borrowing
- ❑ Non Stationarity
- ❑ Incomplete :
 - positive intercept
 - correlated residual
 - cf empirical tests – market anomalies (Low Beta, size, value..)

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1. Top Level Risk Indicators
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PRESPECIFIED MULTI-FACTOR MODEL

Return Generating Process :

$$R_i = \sum_{j=1}^N \beta_{ij} R_j + \varepsilon_i$$

Explained by Risk Factors Residual = Unexplained by Risk Factors

Systematic Return **Specific Return**

PRESPECIFIED MULTI-FACTOR MODEL

Macro-economic Factor Models :


Factors = observable macro-economic time series

Example :

- Inflation
- economic growth
- interest rates
- exchange rates

PRESPECIFIED MULTI-FACTOR MODEL

Macro-economic Factor Models :

$$R_i = \sum_{j=1}^N \beta_{ij} R_j + \epsilon_i$$


Explained by Macro Economic Factors Residual = Unexplained by Risk Factors

Systematic Risk


Specific Risk

Known parameters

Estimated parameters

PRESPECIFIED MULTI-FACTOR MODEL

Macro-economic Factor Models :

$$R_i = \sum_{j=1}^N \beta_{ij} R_j + \varepsilon_i$$


Explained by Macro Economic Factors Residual = Unexplained by Risk Factors

Systematic Risk

Specific Risk

β_{ij} : estimated by time-series regression

PRESPECIFIED MULTI-FACTOR MODEL

Fundamental Factor Models :

Factors = based on observable company attributes

Example :

Fama – French model:

- Market
- Size (SMB) : “Small Minus Big”
- Value (HML) : “High Minus Low”

PRESPECIFIED MULTI-FACTOR MODEL

CASE STUDY 3 : Apply Fama French Factor Model for Harris Associate US Value

PRESPECIFIED MULTI-FACTOR MODEL

Fund Name	Fund Style	Annualised Vol	Single Factor		
			MARKET	R squared	Residual
Vaughan Nelson U.S. Select Equity	US Equity Blend	22.5%	1.14	90.3%	9.7%
Loomis Sayles US Growth Equity	US Equity Growth	24.1%	1.20	86.5%	13.5%
Harris Associates U.S. Value Equity	US Equity Value	18.3%	0.80	67.7%	32.3%
BNP Paribas Sust US Value Mlt-Fctr Eq	US Equity Value	13.8%	0.65	76.5%	23.5%

Fund Name	Fund Style	Annualised Vol	Multi-Factor prespecified				
			MARKET	HML	SMB	R squared	Residual
Vaughan Nelson U.S. Select Equity	US Equity Blend	22.5%	1.17	-0.27	0.08	91.0%	9.0%
Loomis Sayles US Growth Equity	US Equity Growth	24.1%	1.24	-0.47	0.06	88.0%	12.0%
Harris Associates U.S. Value Equity	US Equity Value	18.3%	0.73	0.62	0.39	88.4%	11.6%
BNP Paribas Sust US Value Mlt-Fctr Eq	US Equity Value	13.8%	0.59	0.55	0.06	87.3%	12.7%

PRESPECIFIED MULTI-FACTOR MODEL

Some securities

							Single Factor			Multi-Factor prespecified				
ISIN	Name	Value score	Size score	\$ Mkt Cap	Capitalization	Vol	MARKET	R squared	Residual	MARKET	HML	SMB	R squared	Residual
US67066G1040	NVIDIA	- 0.33	0.85	3 338 966	Mega Cap	53.8%	2.04	50.2%	49.8%	2.27	-2.38	-0.17	62.4%	37.6%
US0378331005	APPLE	- 0.33	0.89	3 902 904	Mega Cap	26.0%	0.99	51.2%	48.8%	1.01	-0.04	-0.26	53.0%	47.0%
US5949181045	MICROSOFT	- 0.21	0.83	3 420 044	Mega Cap	23.2%	0.89	51.5%	48.5%	0.92	-0.24	-0.42	59.5%	40.5%
US30303M1027	META PLATFORMS	- 0.19	0.51	1 324 895	Mega Cap	47.5%	1.33	27.5%	72.5%	1.39	-0.51	-0.31	29.4%	70.6%
US92826C8394	VISA	- 0.25	0.20	548 393	Mega Cap	22.9%	0.77	39.2%	60.8%	0.70	0.62	0.06	44.0%	56.0%
US0846707026	BERKSHIRE HATHAWAY	0.59	0.23	609 305	Mega Cap	17.7%	0.61	41.5%	58.5%	0.52	0.96	0.03	58.7%	41.3%
US0640581007	BANK OF NEW YORK MELLON	0.68	- 0.61	56 748	Large Cap	24.9%	0.83	39.0%	61.0%	0.75	0.76	0.42	53.5%	46.5%
US7731221062	ROCKET LAB USA	- 0.28	- 1.13	13 292	Large Cap	75.2%	1.92	22.9%	77.1%	2.00	-1.19	1.14	26.3%	73.7%
US46222L1089	IONQ	- 0.26	- 1.24	9 647	Small / Mid Cap	102.7%	2.51	20.9%	79.1%	2.71	-2.41	1.11	23.9%	76.1%
US8404411097	SOUTHSTATE	0.85	- 1.32	7 716	Small / Mid Cap	31.9%	0.77	20.4%	79.6%	0.65	0.84	1.29	62.0%	38.0%
US0028962076	ABERCROMBIE & FITCH CO	- 0.13	- 1.33	7 660	Small / Mid Cap	61.4%	1.16	12.5%	87.5%	1.27	-1.28	0.54	14.8%	85.2%

PRESPECIFIED MULTI-FACTOR MODEL

Fundamental Factor Models :

Example :

Market

+

Country

+

Industry membership

+

Currencies

+

Style : size, book-to-market, momentum...

PRESPECIFIED MULTI-FACTOR MODEL

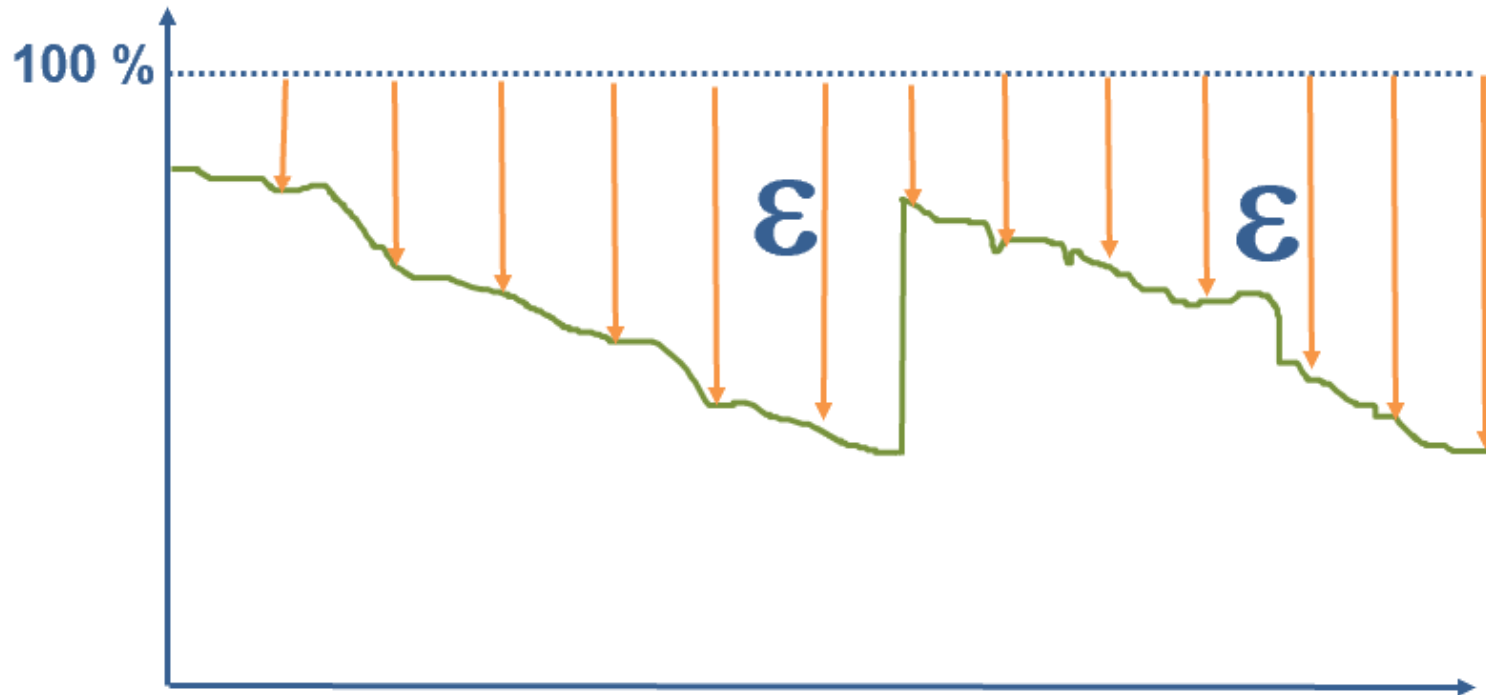
Fundamental Factor Models :

Name	Portfolio Exposure	Contribution to Portfolio Volatility	Marginal Portfolio Volatility
Total		14.881 %	
Systematic		12.840 %	
Beta To Market		9.157 %	
Country		-0.012 %	
Currency Name		3.613 %	
Sector		0.007 %	
Style Score		0.075 %	
Style Score - Dividend Yield (Filtered)	0.433	0.075 %	0.173 %
ALLIANZ	0.020	0.003 %	
BANCO SANTANDER	0.013	0.002 %	
DEUTSCHE TELEKOM	0.046	0.008 %	
ROCHE HOLDING	0.013	0.002 %	
ROYAL BANK CANADA	0.017	0.003 %	
SANOFI	0.019	0.003 %	
TAKEDA PHARMACEUTICAL	0.014	0.002 %	
TELEFONICA	0.020	0.004 %	
TOKYO ELECTRIC POWER	0.031	0.005 %	
TOTAL	0.014	0.002 %	
Filtered Securities		0.039 %	
Style Score - Earnings Variability	0.224	-0.011 %	-0.051 %
Style Score - Growth	-0.035	0.001 %	-0.020 %
Style Score - Leverage	0.154	-0.007 %	-0.048 %
Style Score - Momentum	-0.228	-0.019 %	0.085 %
Style Score - Size	0.307	0.009 %	0.030 %
Style Score - Trading Turnover	-0.295	0.022 %	-0.074 %
Style Score - Value	0.180	0.007 %	0.038 %

PRESPECIFIED MULTI-FACTOR MODEL

Fundamental Factor Models :

Model Explanatory Power = R Squared



Limitations of Prespecified multi factor models

- ❑ Mis-specifications (missing variables, non relevant variables)
- ❑ Collinearity between Factors
- ❑ Decreasing Explanatory Power if new Factors appear

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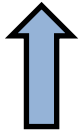
IV. Risk Analysis

RISK MODELS :: STATISTICAL FACTOR MODELS

1976 : Stephen A. Ross, *Return, Risk and Arbitrage*

If markets are efficient $\rightarrow \exists$ Arbitrage factors

$$R_i = \sum_{j=1}^N \beta_{ij} R_j + \epsilon_i$$

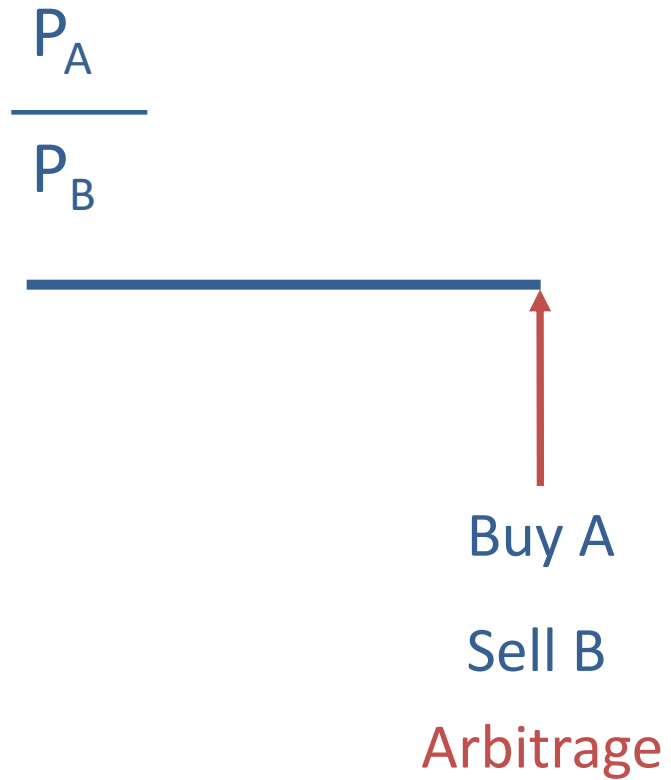
 

Explained by Risk Factors Residual = Unexplained by Risk Factors

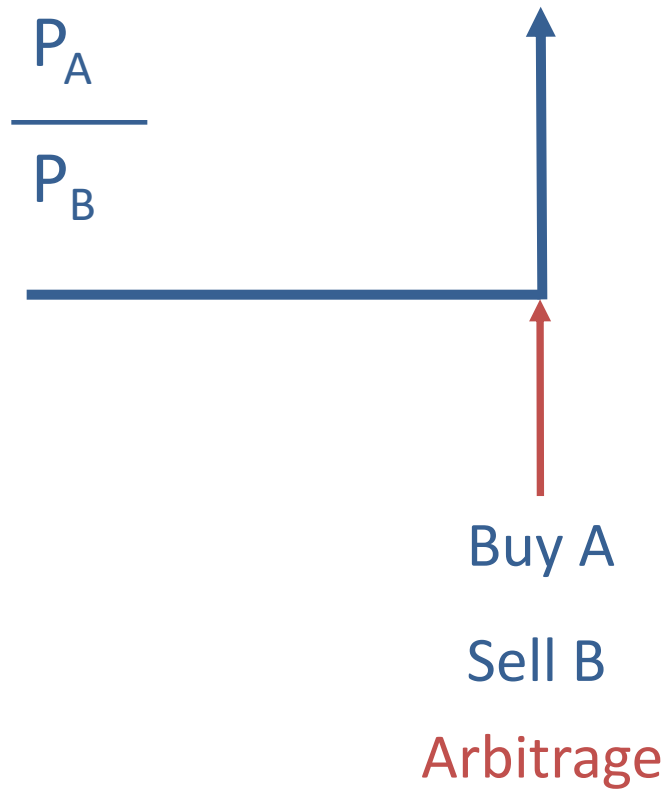
Systematic Return **Specific Return**

Arbitrage?

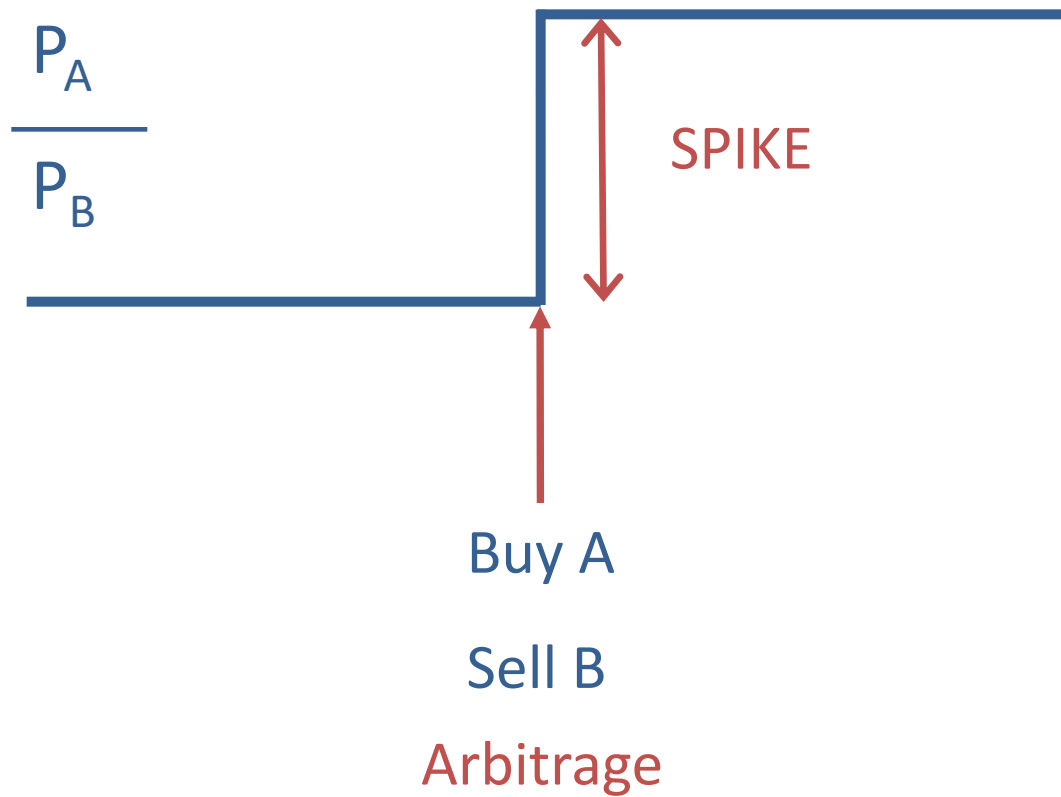
RISK MODELS :: STATISTICAL FACTOR MODELS



RISK MODELS :: STATISTICAL FACTOR MODELS



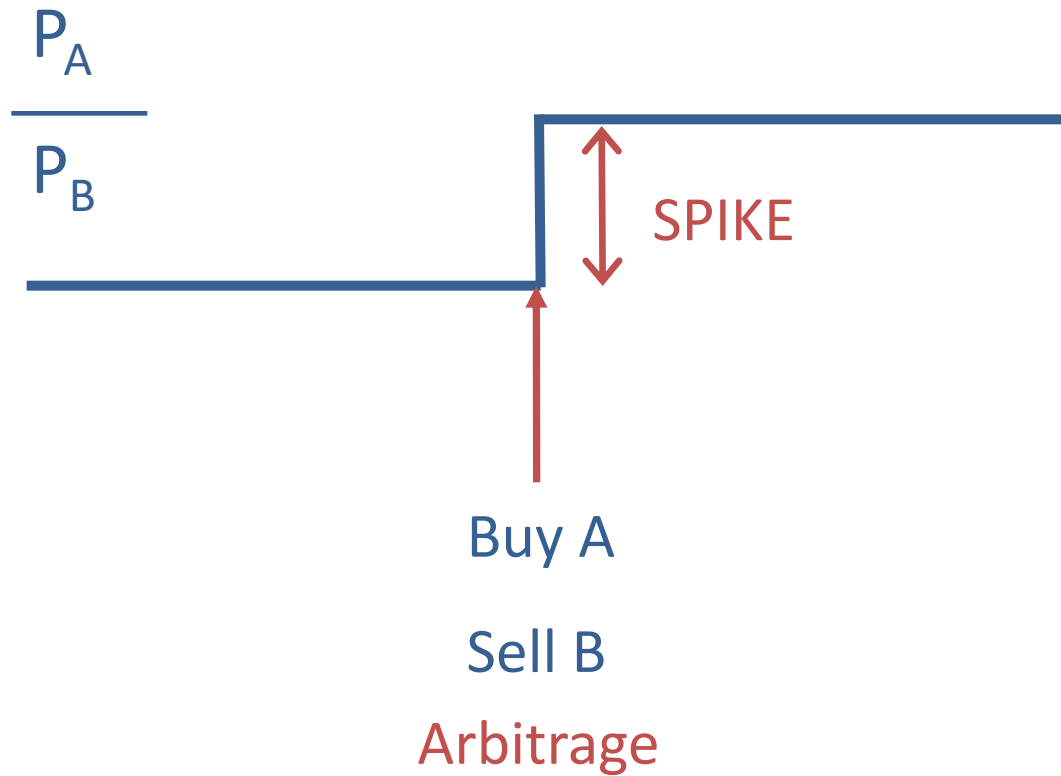
RISK MODELS :: STATISTICAL FACTOR MODELS



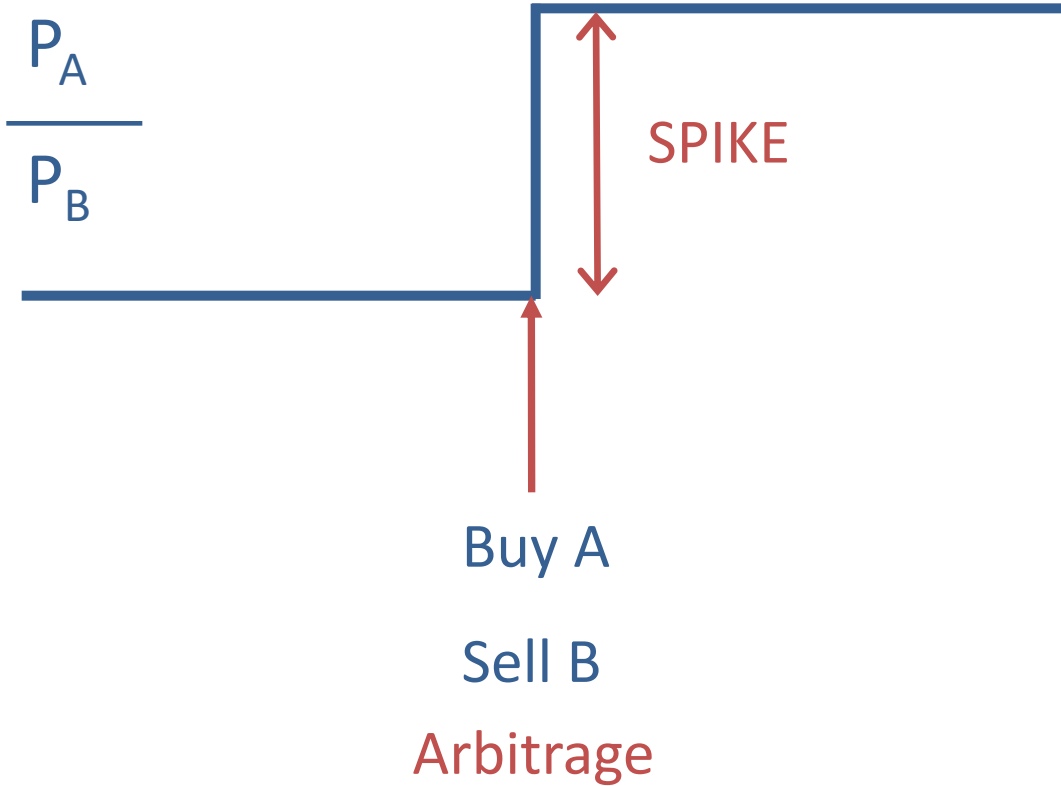
RISK MODELS :: STATISTICAL FACTOR MODELS

Arbitrage → Price Movements → Volatility

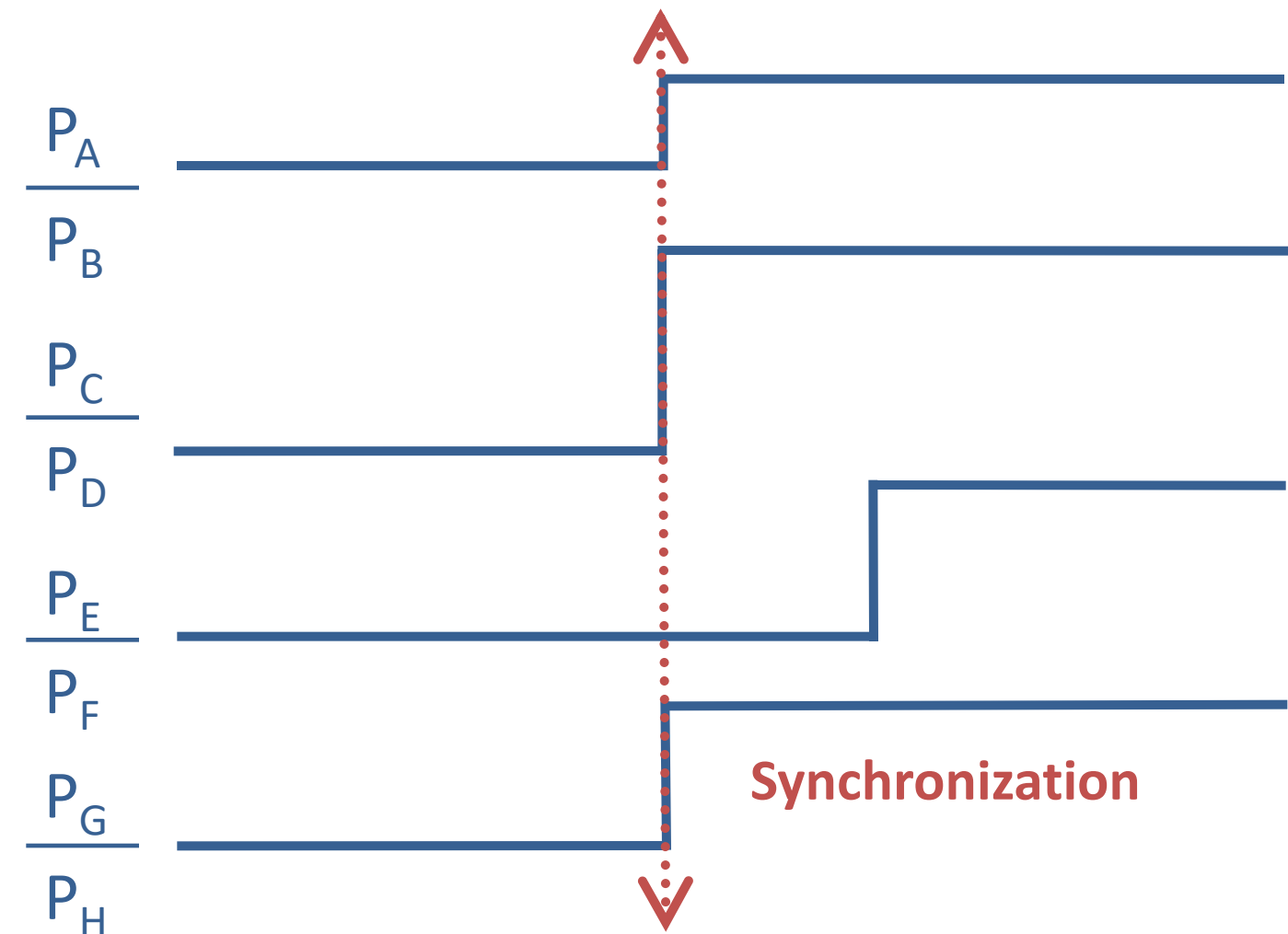
RISK MODELS :: STATISTICAL FACTOR MODELS



RISK MODELS :: STATISTICAL FACTOR MODELS



RISK MODELS :: STATISTICAL FACTOR MODELS



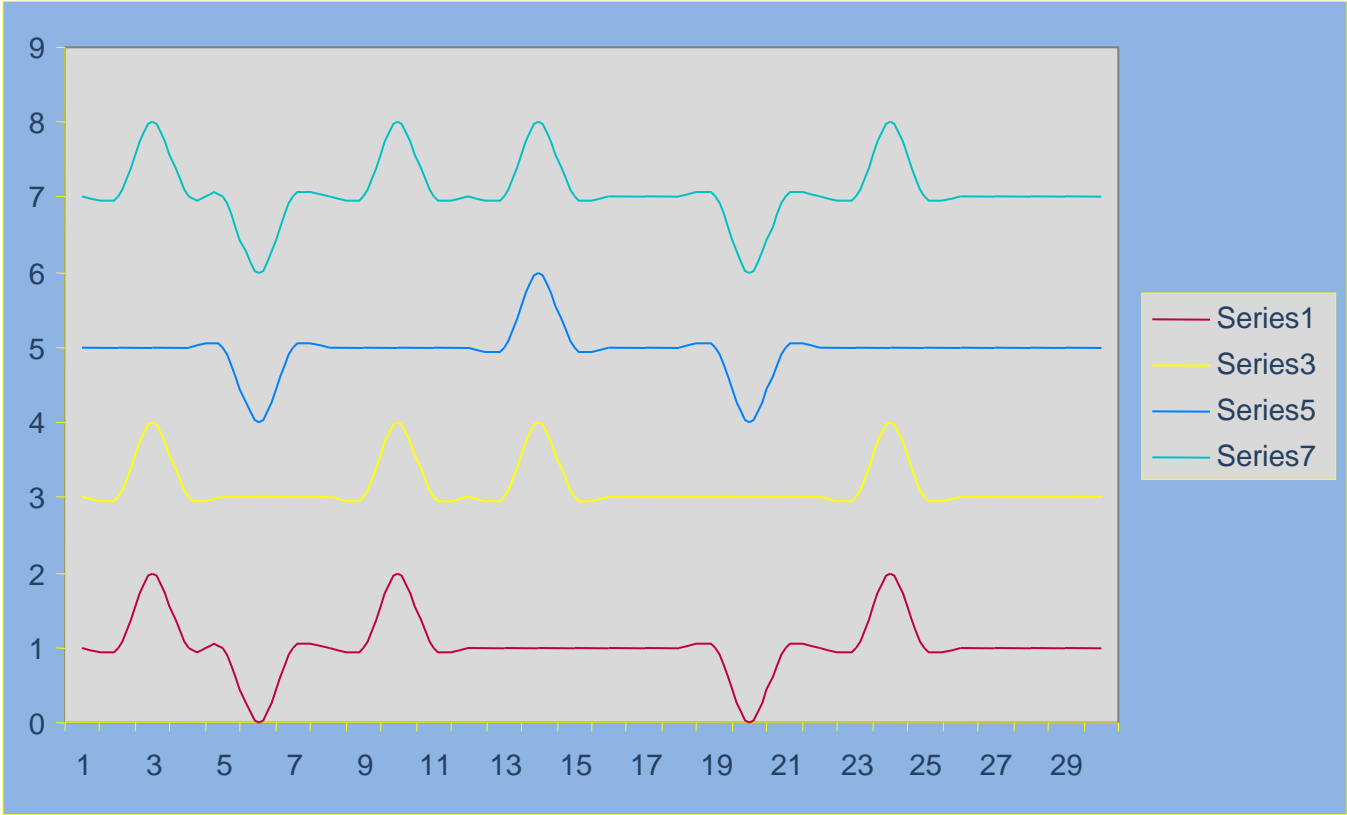
RISK MODELS :: STATISTICAL FACTOR MODELS

Arbitrage ↔ Volatility

APT Factors extraction

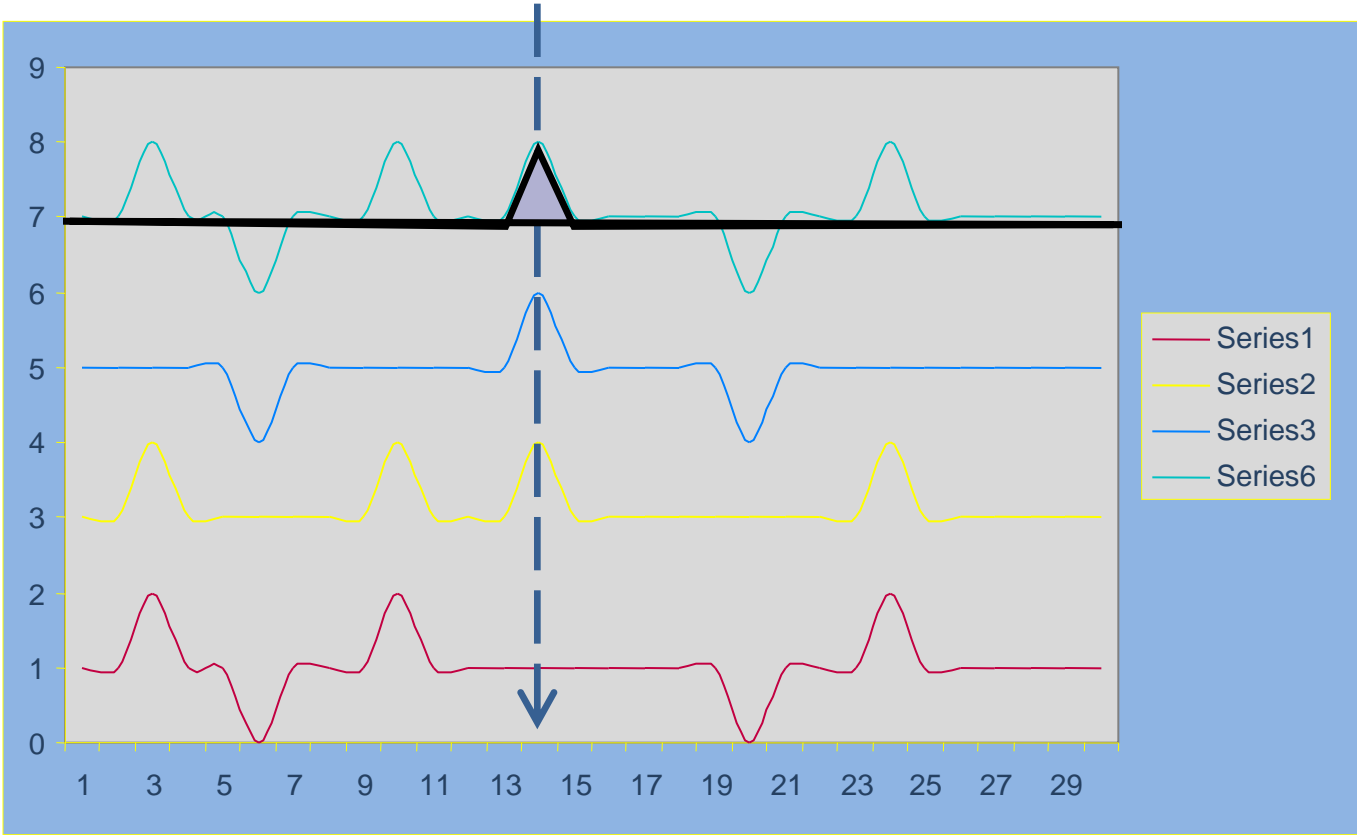
RISK MODELS :: STATISTICAL FACTOR MODELS

n series of relative returns

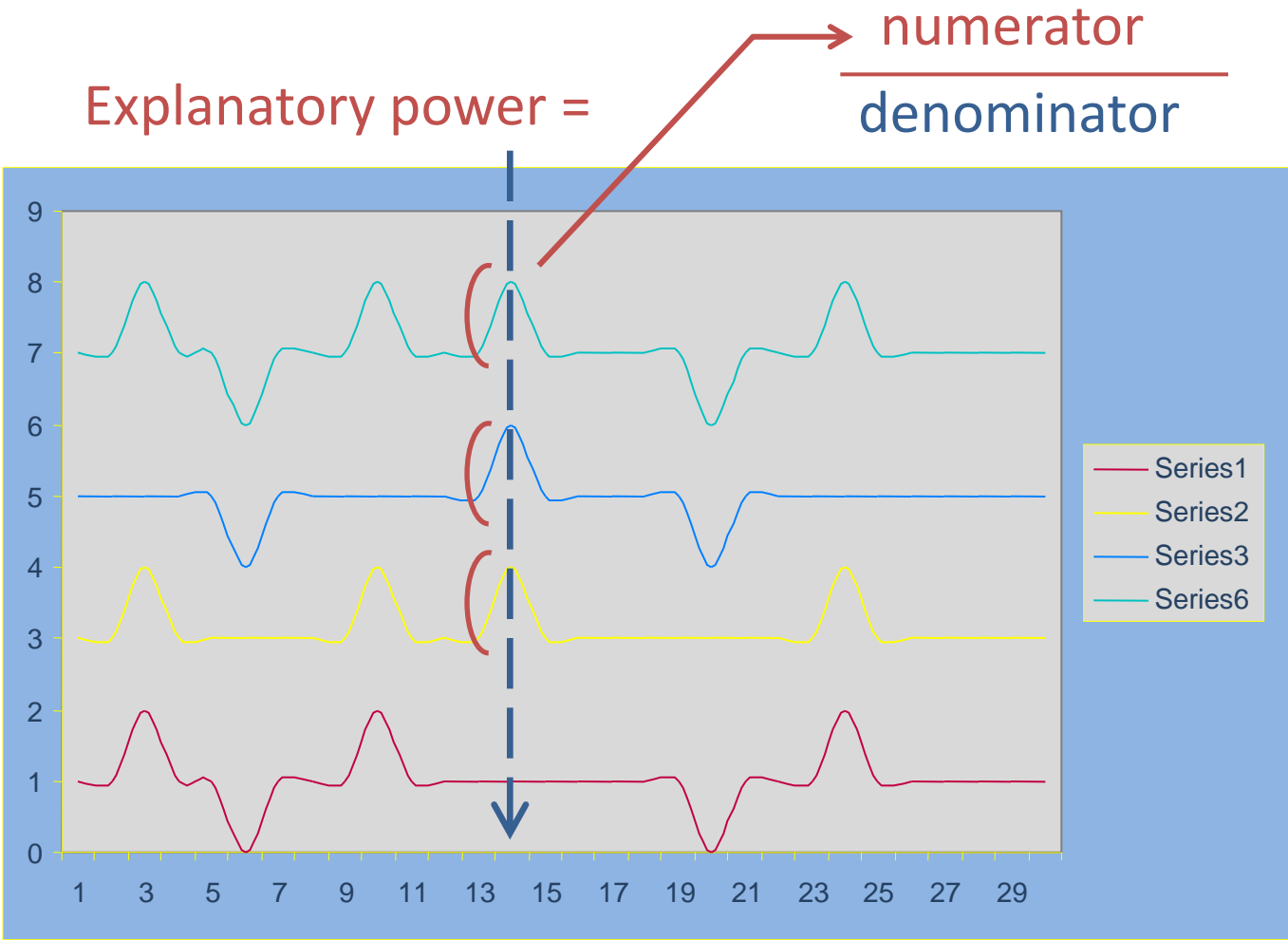


RISK MODELS :: STATISTICAL FACTOR MODELS

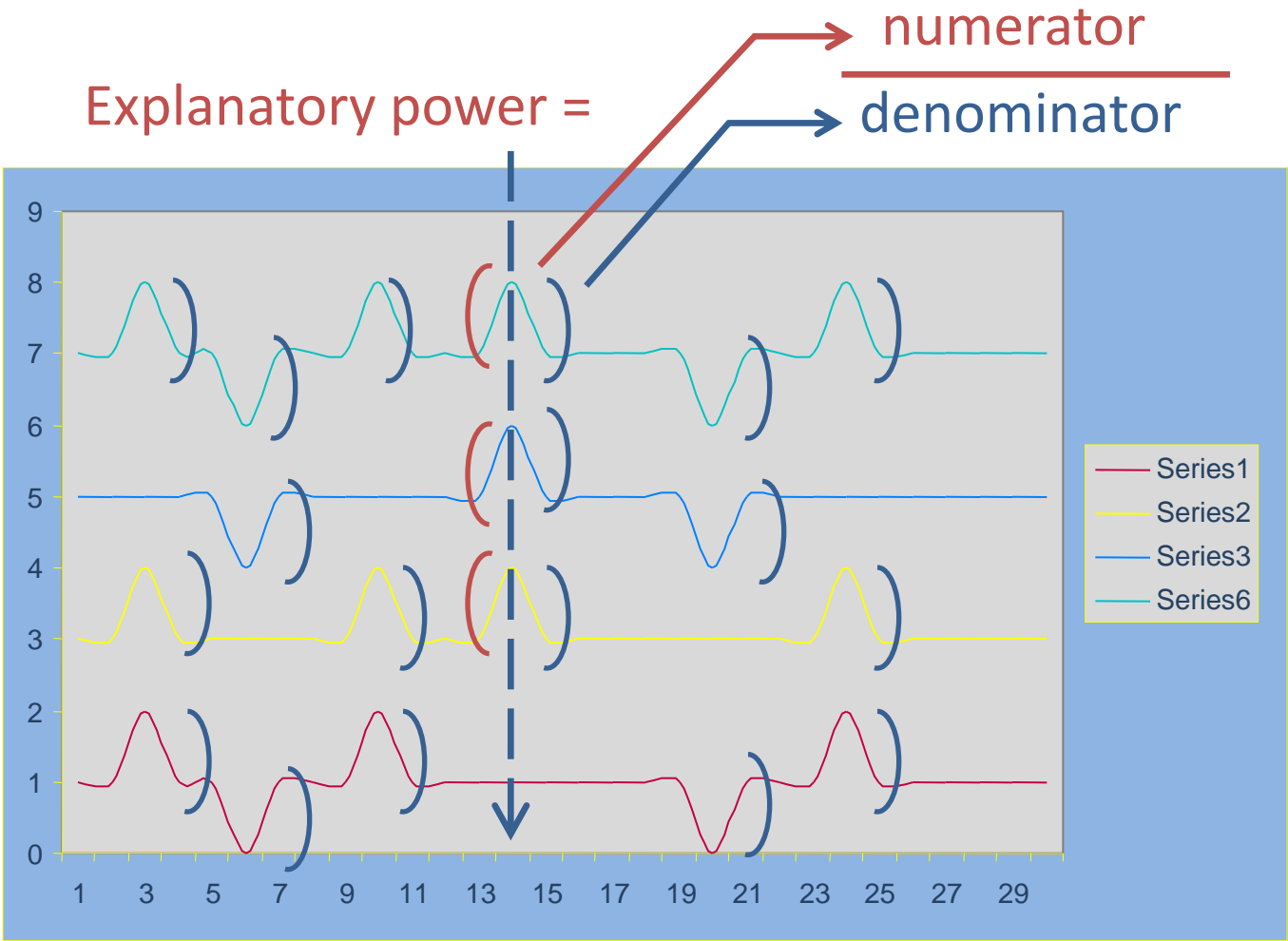
Systematic synchronous price movements



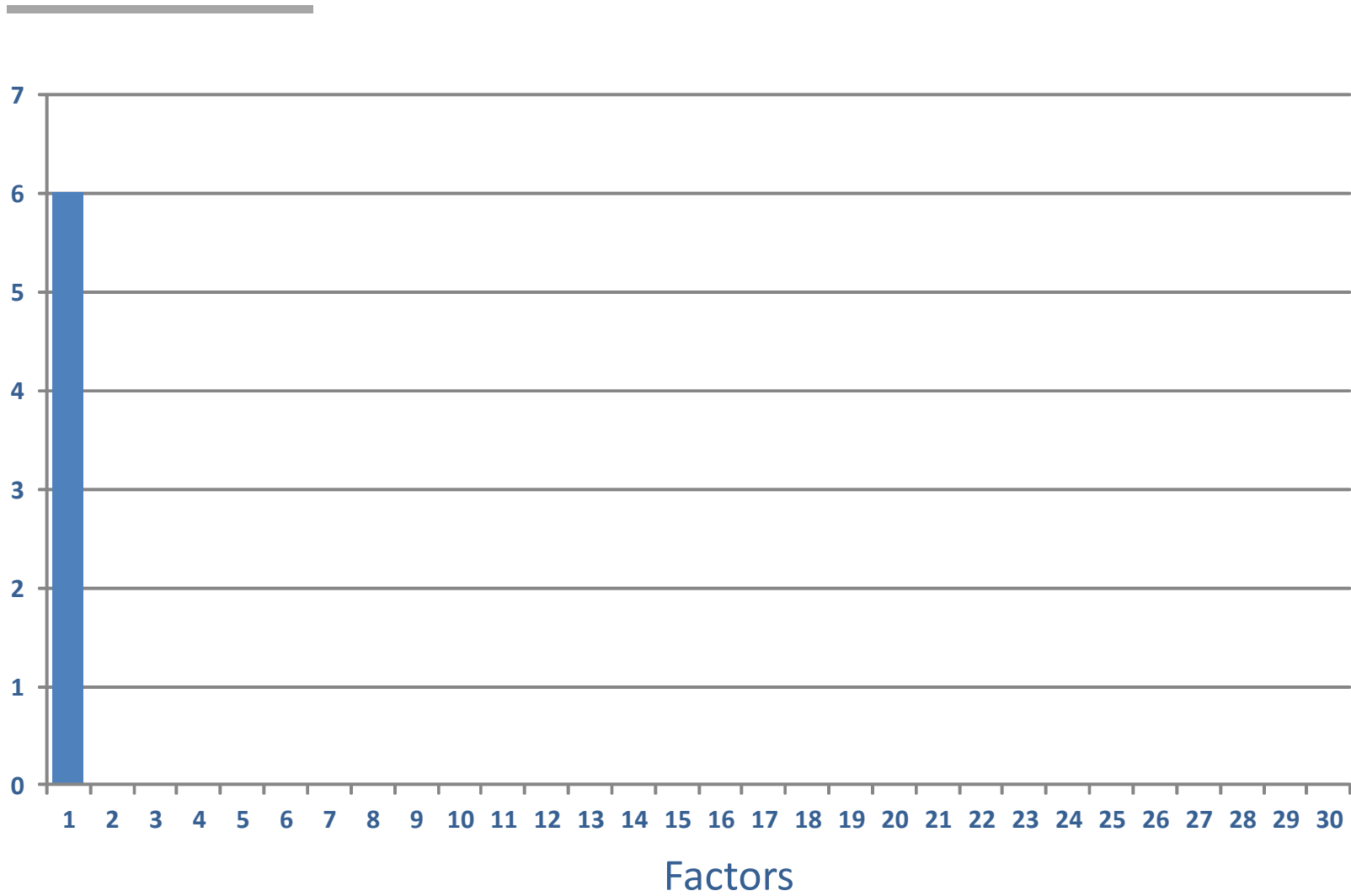
RISK MODELS :: STATISTICAL FACTOR MODELS



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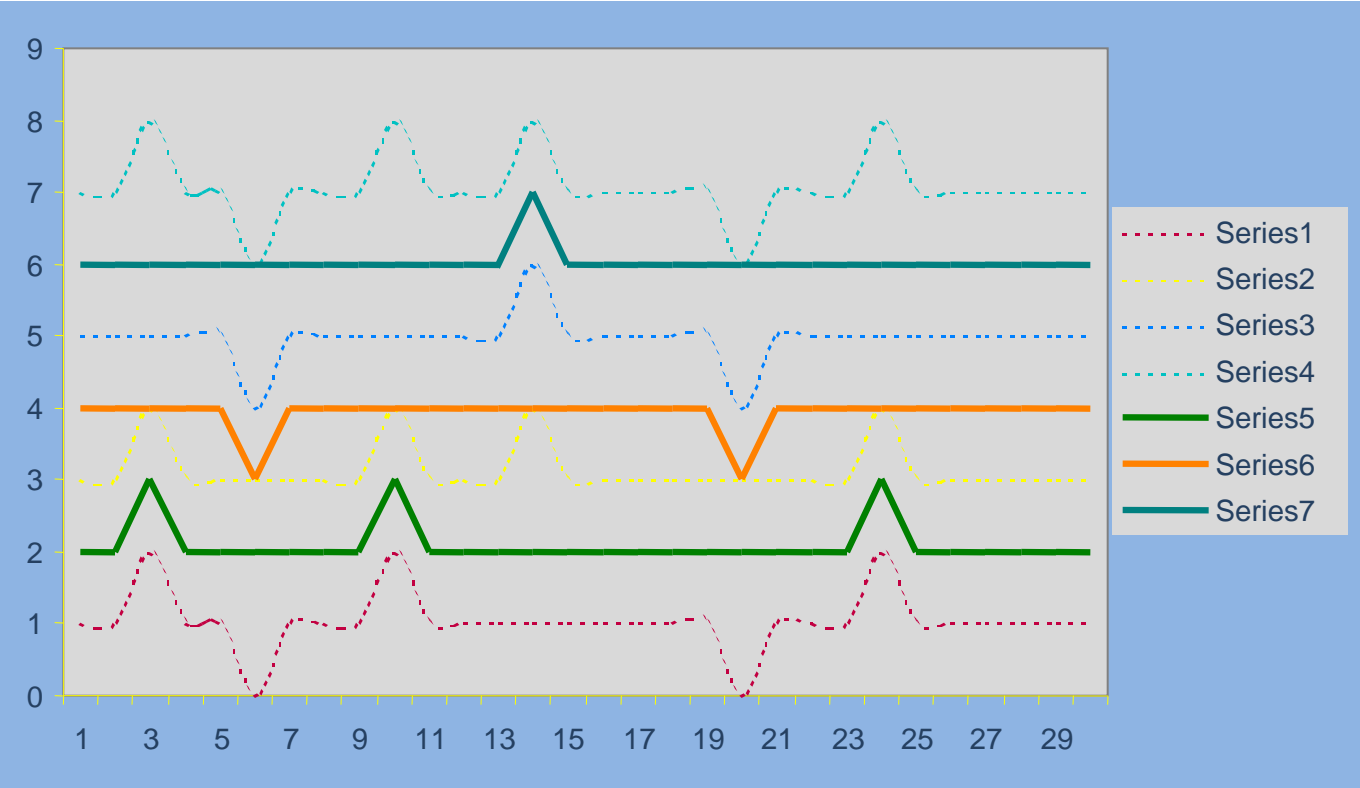


RISK MODELS :: STATISTICAL FACTOR MODELS



RISK MODELS :: STATISTICAL FACTOR MODELS

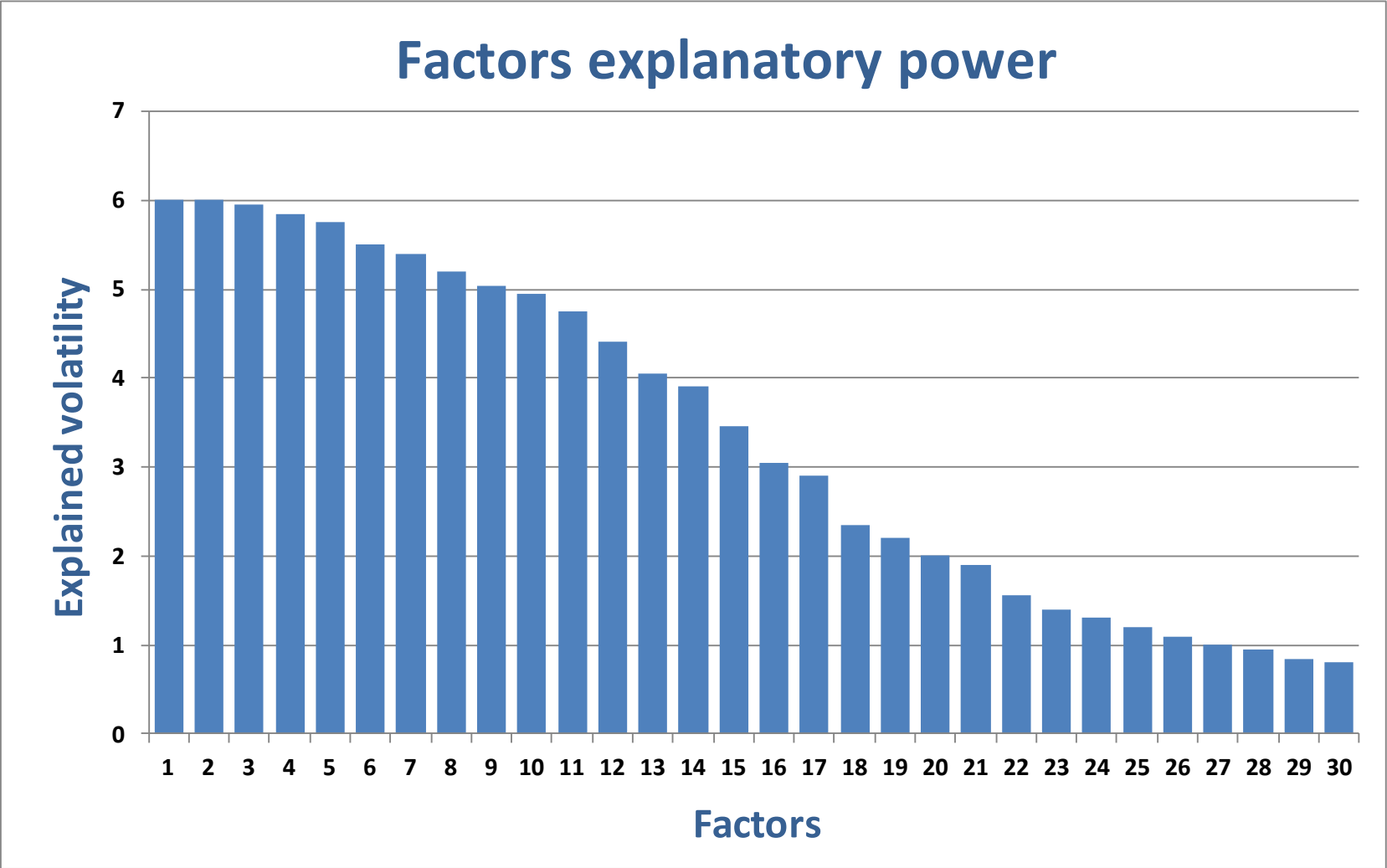
Independent common factors search



RISK MODELS :: STATISTICAL FACTOR MODELS

- ↓
- I. Arbitrage factors are extracted:
Their statistical existence is proven

RISK MODELS :: STATISTICAL FACTOR MODELS



RISK MODELS :: STATISTICAL FACTOR MODELS

- I. Arbitrage factors are extracted:
 - Their statistical existence is proven
- ↓ II. Extracted factors explain the whole systematic volatility

RISK MODELS :: STATISTICAL FACTOR MODELS


I. Arbitrage factors are extracted:

Their statistical existence is proven


II. Extracted factors explain the whole systematic volatility

III. Residuals are consequently statistically independent
from each other

RISK MODELS :: STATISTICAL FACTOR MODELS

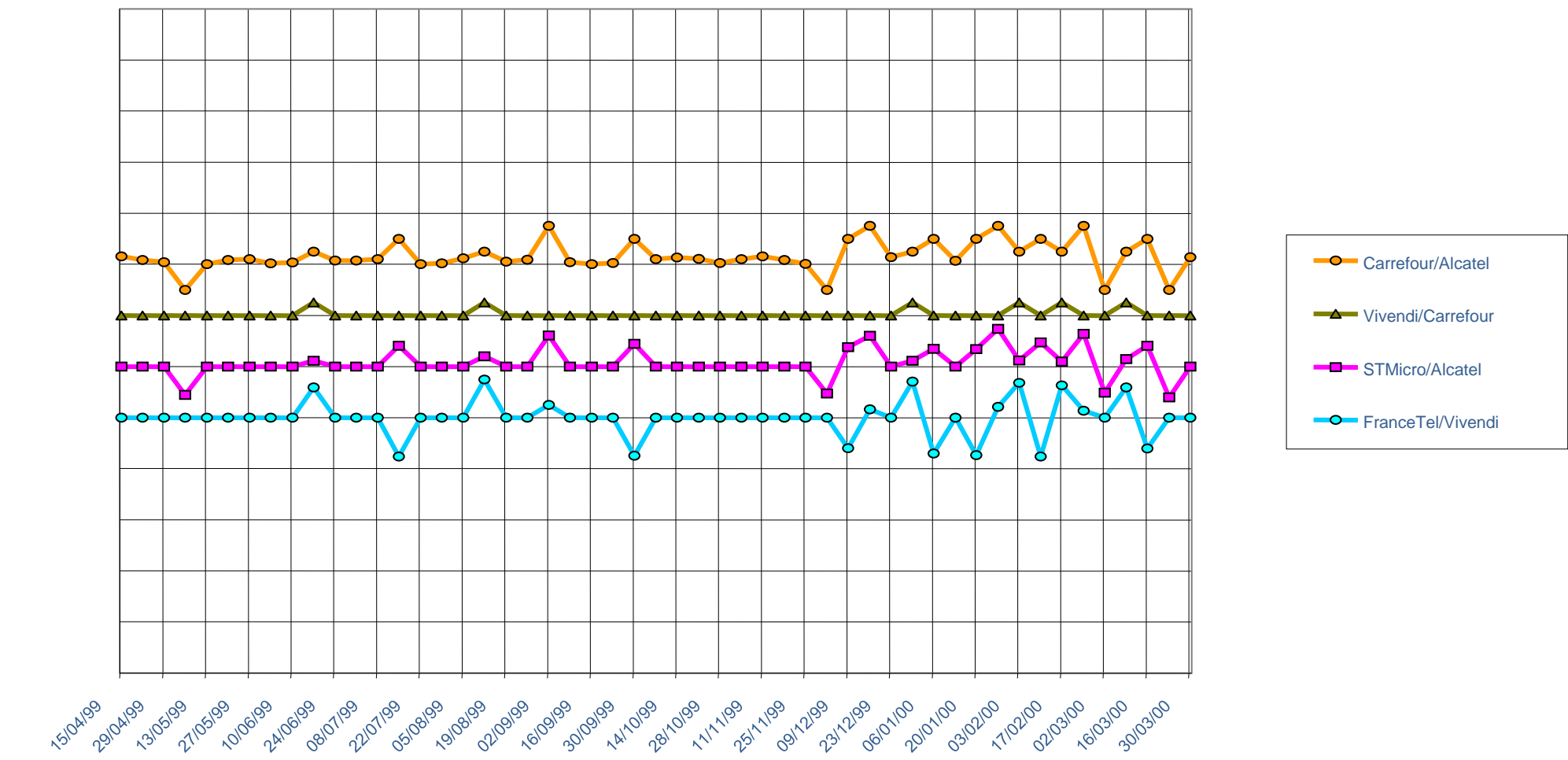
- 
- I. Arbitrage factors are extracted:
Their statistical existence is proven
 - II. Extracted factors explain the whole systematic volatility
 - III. Residuals are consequently statistically independent from each other
 - IV. Factors are by construction statistically independent from each other

RISK MODELS :: STATISTICAL FACTOR MODELS

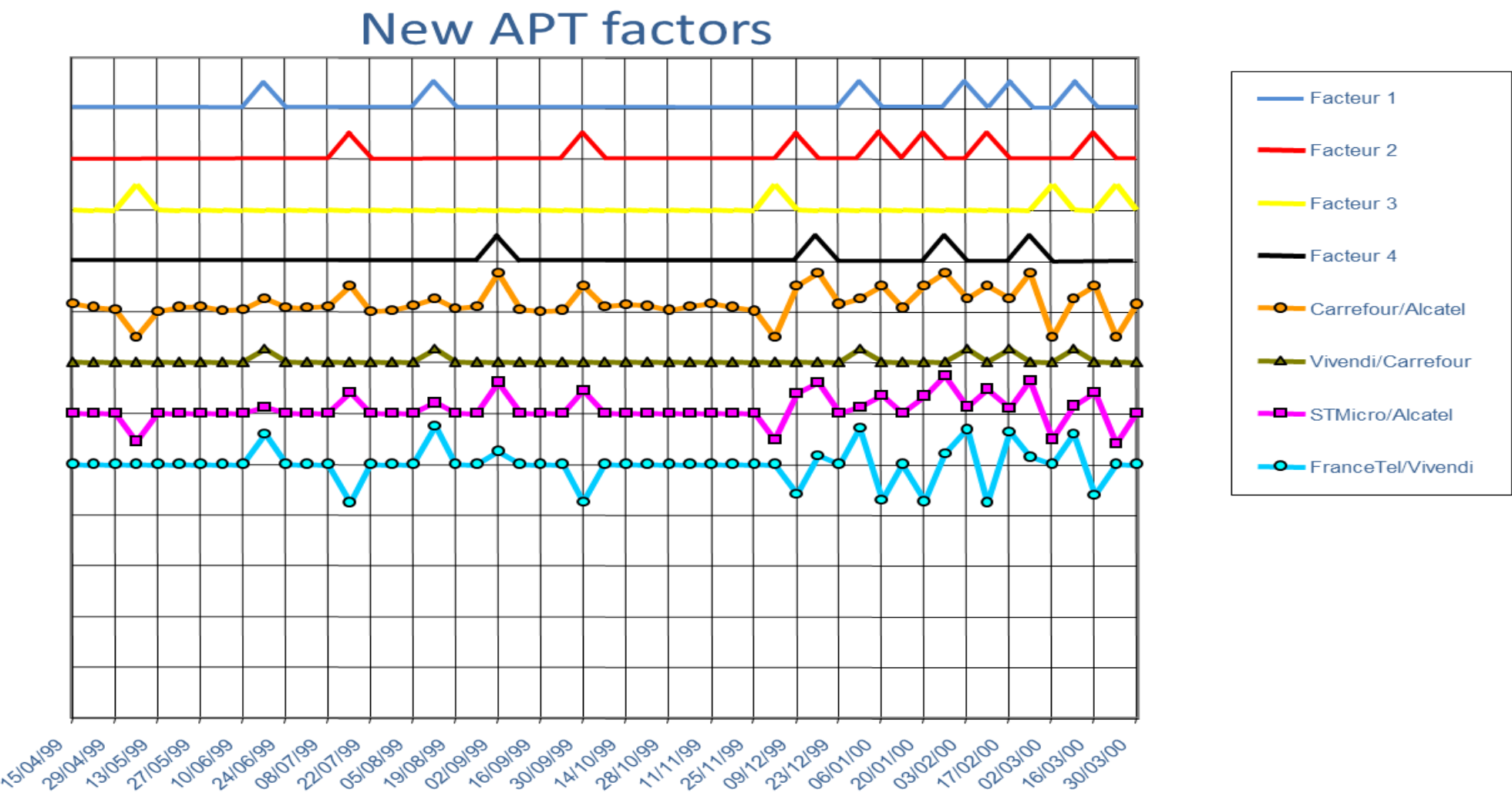
- 
- I. Arbitrage factors are extracted:
Their statistical existence is proven
 - II. Extracted factors explain the whole systematic volatility
 - III. Residuals are consequently statistically independent from each other
 - IV. Factors are by construction statistically independent from each other

RISK MODELS :: STATISTICAL FACTOR MODELS


Relative returns series for Carrefour, Alcatel, Vivendi, etc



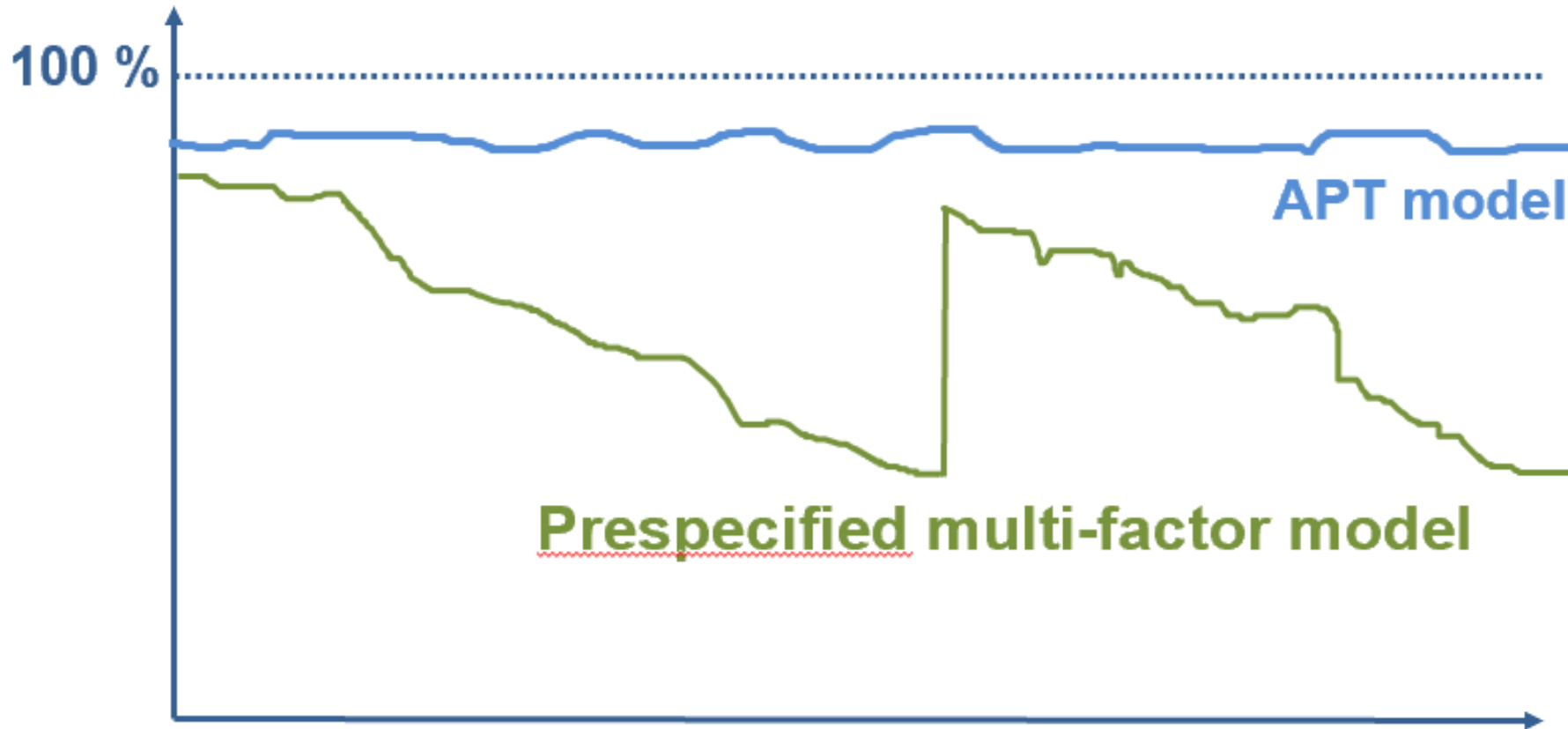
RISK MODELS :: STATISTICAL FACTOR MODELS



RISK MODELS :: STATISTICAL FACTOR MODELS


- 
- I. Arbitrage factors are extracted:
Their statistical existence is proven
 - II. Extracted factors explain the whole systematic volatility
 - III. Residuals are consequently statistically independent from each other
 - IV. Factors are by construction statistically independent from each other
 - V. The model is auto-adaptative
Models are recalculated at each period

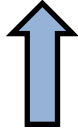
RISK MODELS :: STATISTICAL FACTOR MODELS



RISK MODELS :: STATISTICAL FACTOR MODELS

$$R_i = \sum_{j=1}^N \beta_{ij}^* R_j^* + \varepsilon_i^*$$

 Explained by
Extracted arbitrage Factors
Systematic Return

 Unexplained by
Arbitrage Factors
Specific Return

RISK MODELS :: STATISTICAL FACTOR MODELS

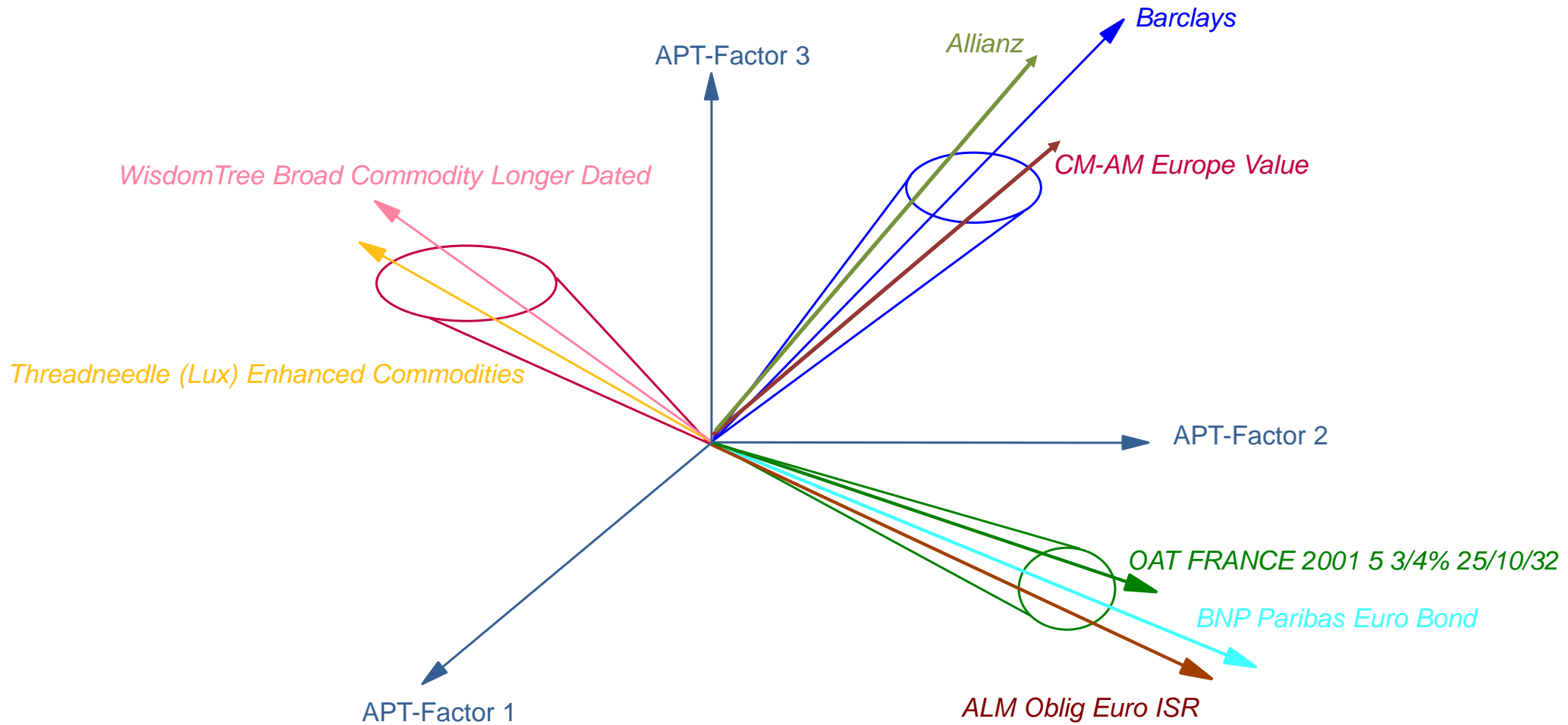
Statistical Factor Models :

	Ticker	Name	Residual	Factor #1	Factor #2	Factor #3	Factor #4	Factor #5	Factor #6	Factor #7	Factor #8	Factor #9
Filter	=MSFT											
1	MSFT	MICROSOFT	0.01980247	-0.00238429	-0.0036002	0.00257395	0.00097124	0.00858416	-0.00327564	-0.00414307	-0.01377416	0.00070867

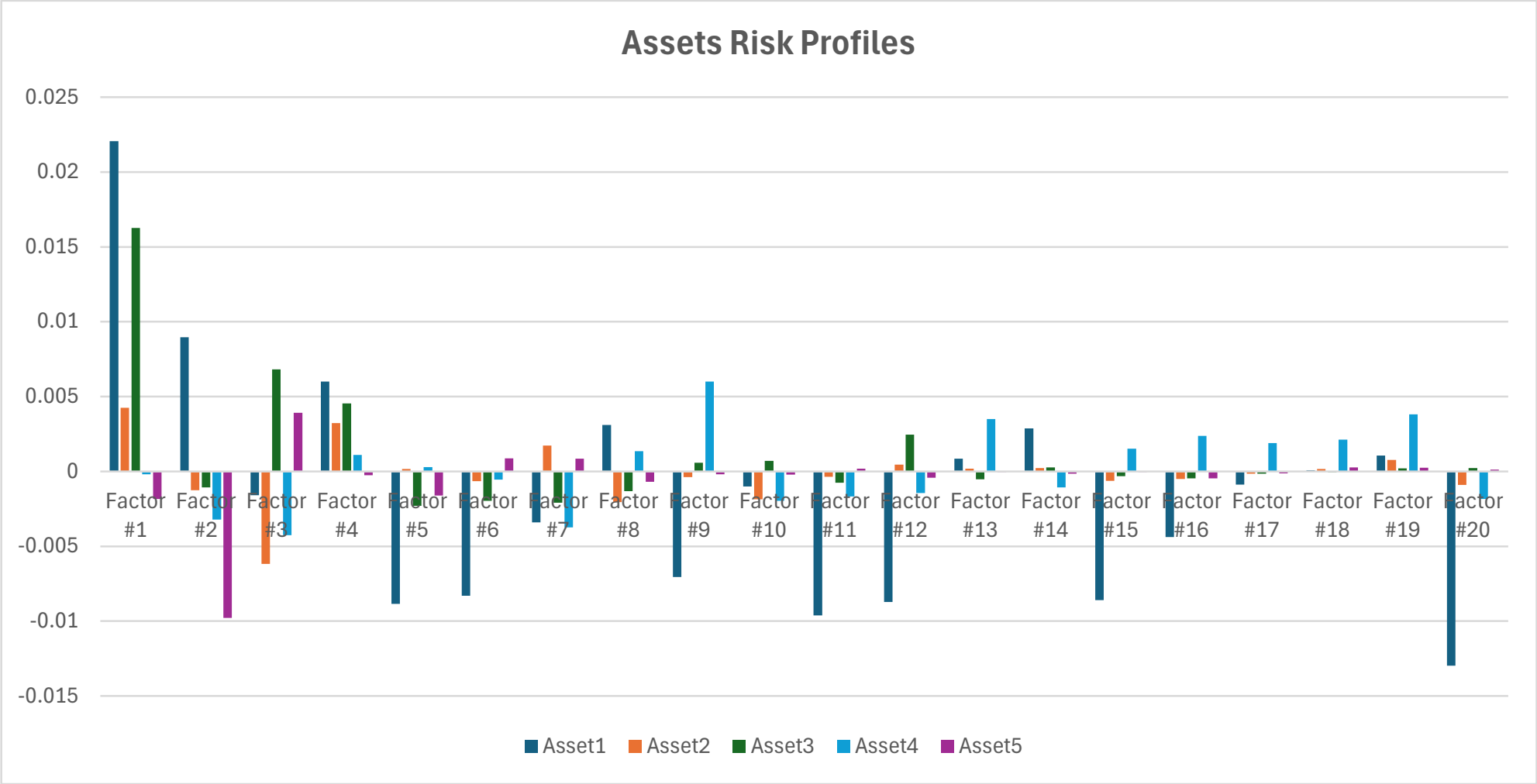
ϵ_i^*

β^*

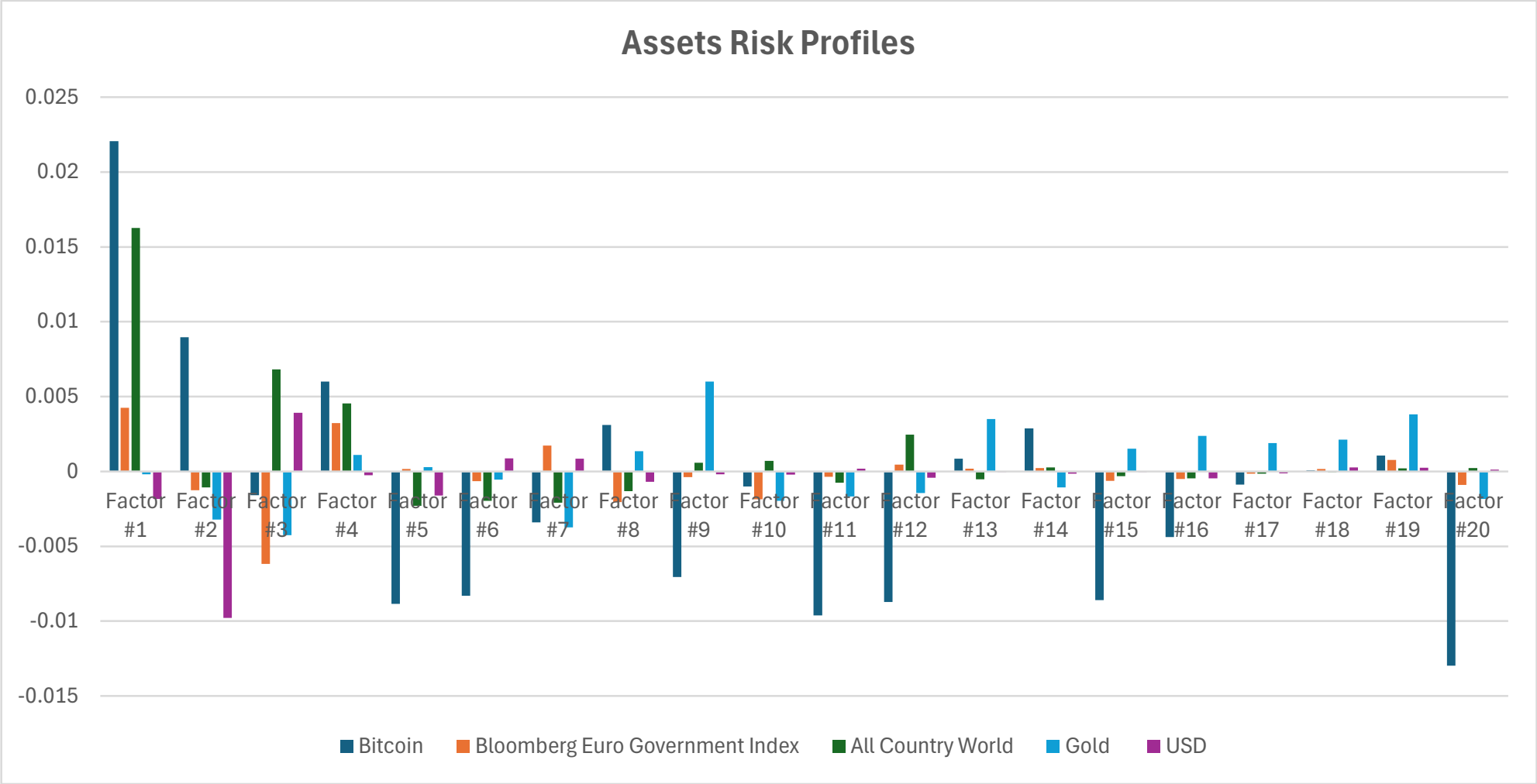
RISK MODELS :: STATISTICAL FACTOR MODELS



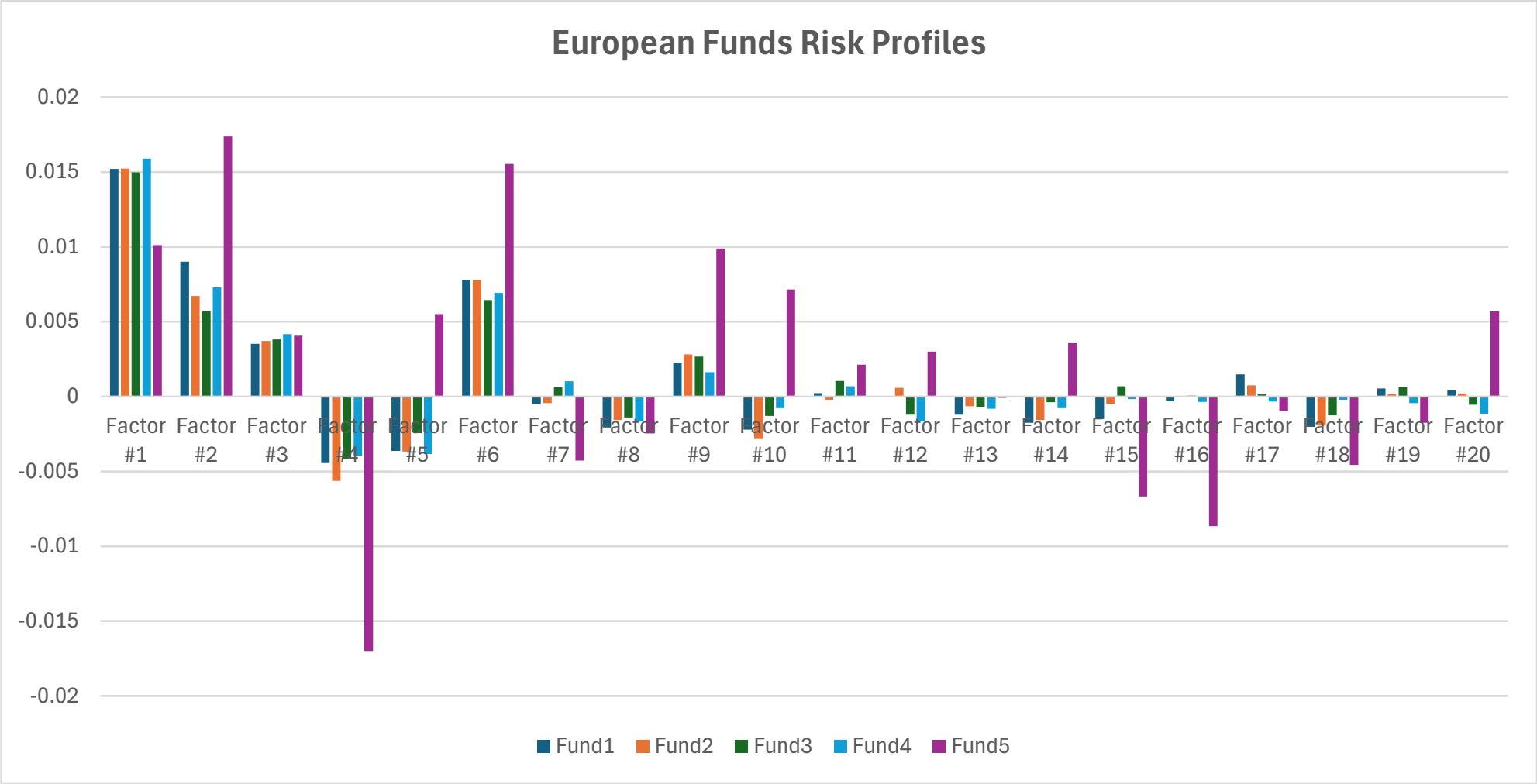
RISK MODELS :: STATISTICAL FACTOR MODELS



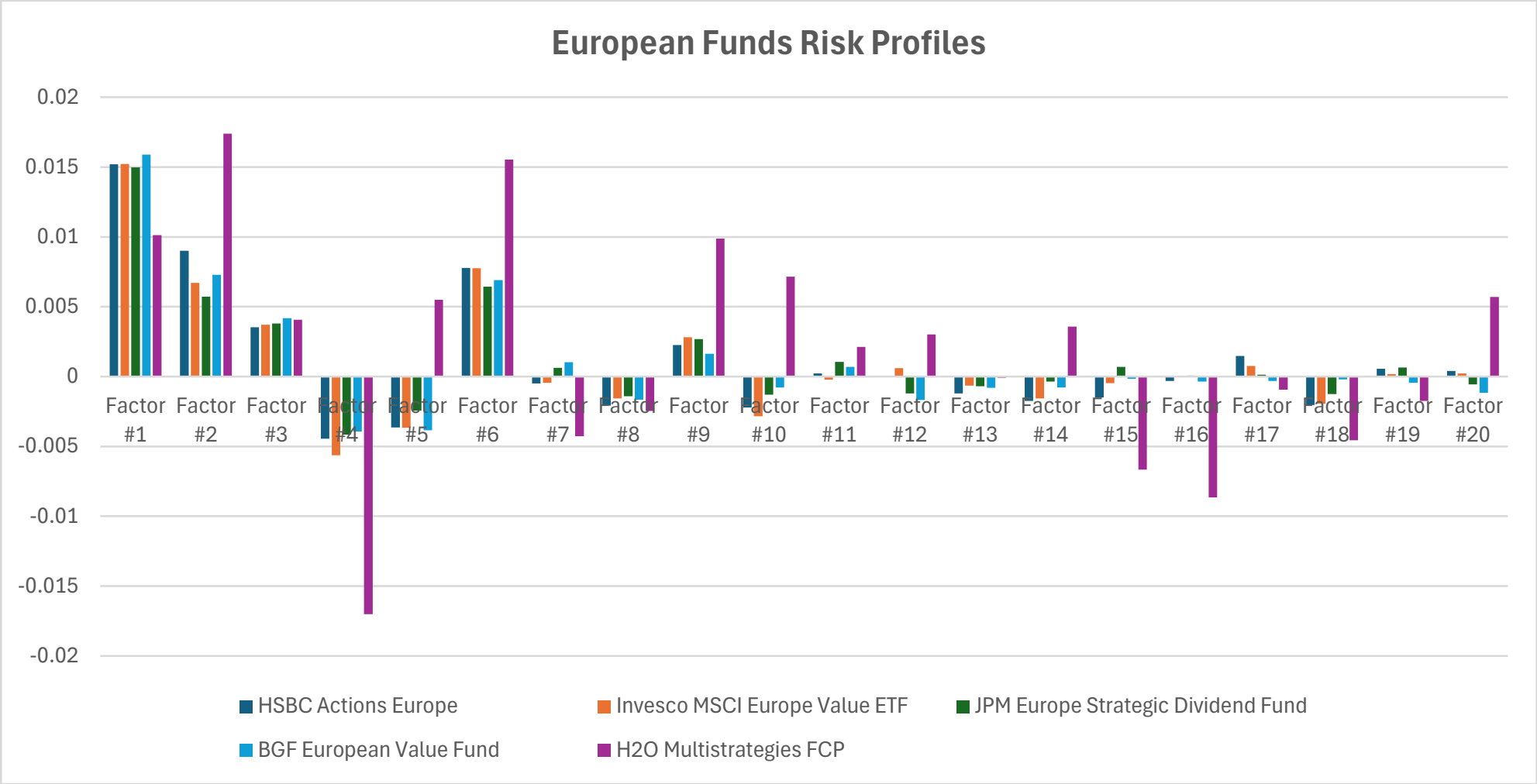
RISK MODELS :: STATISTICAL FACTOR MODELS



RISK MODELS :: STATISTICAL FACTOR MODELS



RISK MODELS :: STATISTICAL FACTOR MODELS



STATISTICAL MULTI-FACTOR MODEL

CASE STUDY 4 : Apply APT Statistical Multi Factor Model for Harris Associate US Value

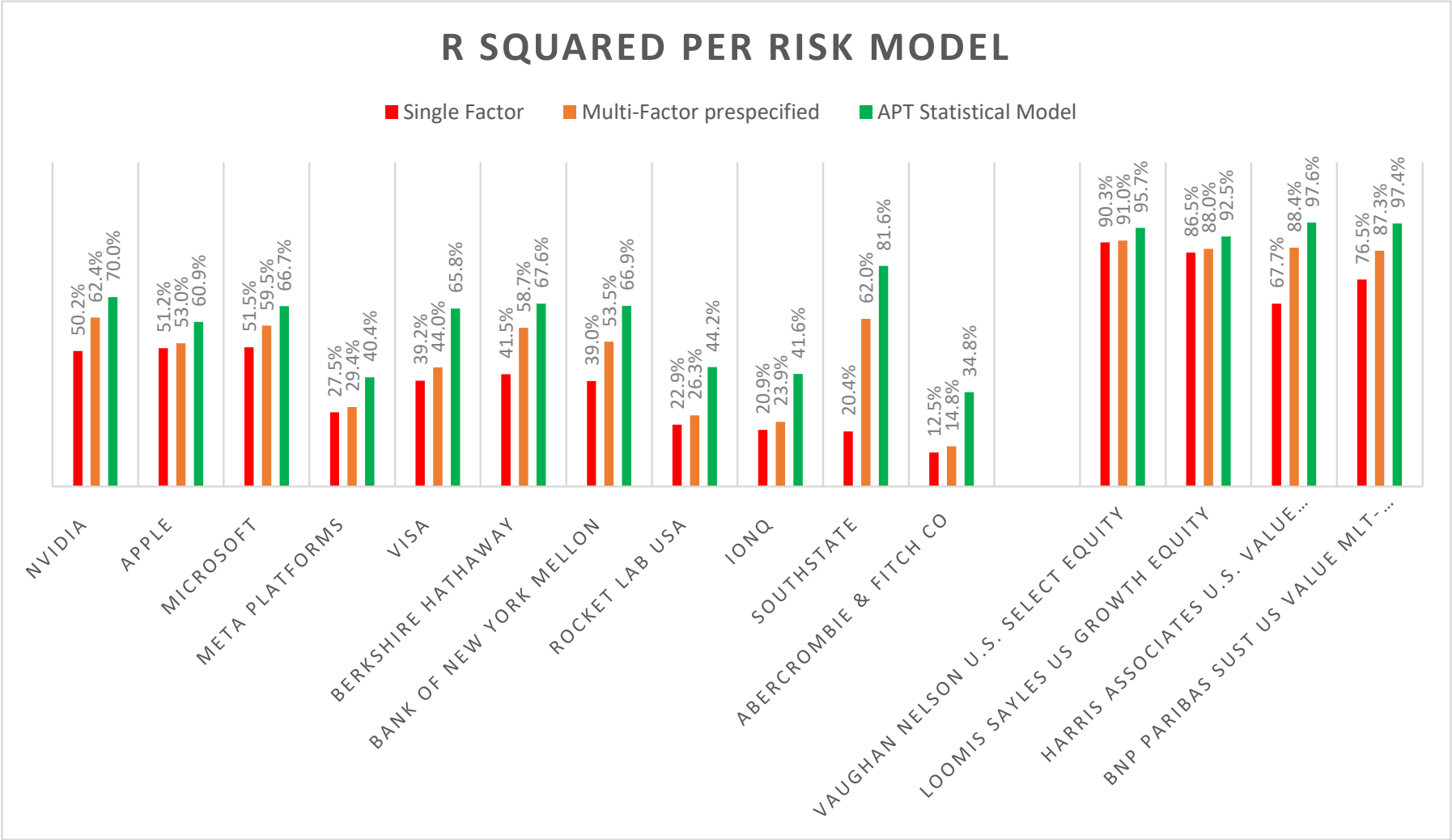
STATISTICAL MULTI-FACTOR MODEL

Fund Name	Fund Style	Annualised Vol	Single Factor		
			MARKET	R squared	Residual
Vaughan Nelson U.S. Select Equity	US Equity Blend	22.5%	1.14	90.3%	9.7%
Loomis Sayles US Growth Equity	US Equity Growth	24.1%	1.20	86.5%	13.5%
Harris Associates U.S. Value Equity	US Equity Value	18.3%	0.80	67.7%	32.3%
BNP Paribas Sust US Value Mlt-Fctr Eq	US Equity Value	13.8%	0.65	76.5%	23.5%

Fund Name	Fund Style	Annualised Vol	Multi-Factor prespecified				
			MARKET	HML	SMB	R squared	Residual
Vaughan Nelson U.S. Select Equity	US Equity Blend	22.5%	1.17	-0.27	0.08	91.0%	9.0%
Loomis Sayles US Growth Equity	US Equity Growth	24.1%	1.24	-0.47	0.06	88.0%	12.0%
Harris Associates U.S. Value Equity	US Equity Value	18.3%	0.73	0.62	0.39	88.4%	11.6%
BNP Paribas Sust US Value Mlt-Fctr Eq	US Equity Value	13.8%	0.59	0.55	0.06	87.3%	12.7%

Fund Name	Fund Style	Annualised Vol	APT Multi-Factor Statistical				
			F1	F2	F3...	R squared	Residual
Vaughan Nelson U.S. Select Equity	US Equity Blend	22.5%				95.7%	4.3%
Loomis Sayles US Growth Equity	US Equity Growth	24.1%				92.5%	7.5%
Harris Associates U.S. Value Equity	US Equity Value	18.3%				97.6%	2.4%
BNP Paribas Sust US Value Mlt-Fctr Eq	US Equity Value	13.8%				97.4%	2.6%

STATISTICAL MULTI-FACTOR MODEL



RISK MODELS :: STATISTICAL FACTOR MODELS

	Theory	Systematic / Specific	Explanatory Power	Stability / Robustness	Auto Adaptative	Multi Asset Class	Intuitive
Variance Covariance Matrix							
Single Factor Model							
Prespecified Multi Factor Model							
Statistical Multi Factor Model							

CONTENTS



I. Risk Models

1. Variance Covariance Matrix
2. Single Factor Model
3. Prespecified Multi Factor Model
4. APT Statistical Multi Factor Model

II. The need for Statistical Multi Factor Models

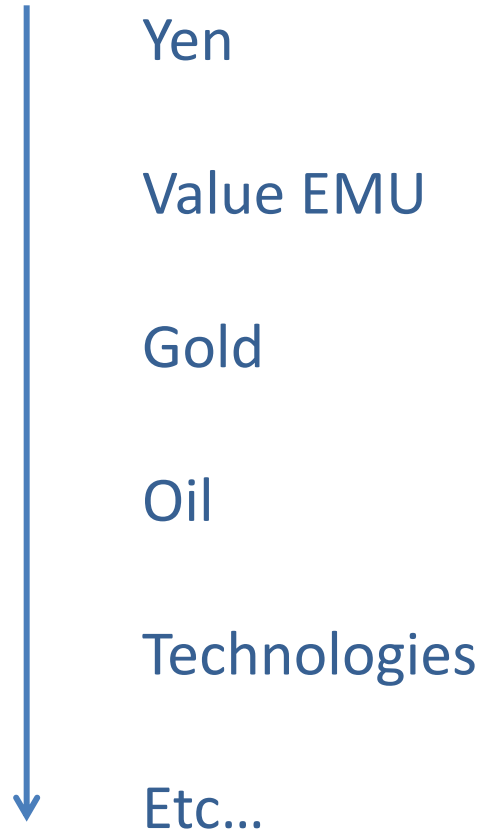
1. Risk attribution
2. Risk measurement

III. Risk Indicators

1. Top Level Risk Indicators
2. Position Based Risk Indicators


IV. Risk Analysis

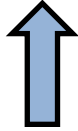
RISK MODELS :: STATISTICAL FACTOR MODELS :: RISK ATTRIBUTION



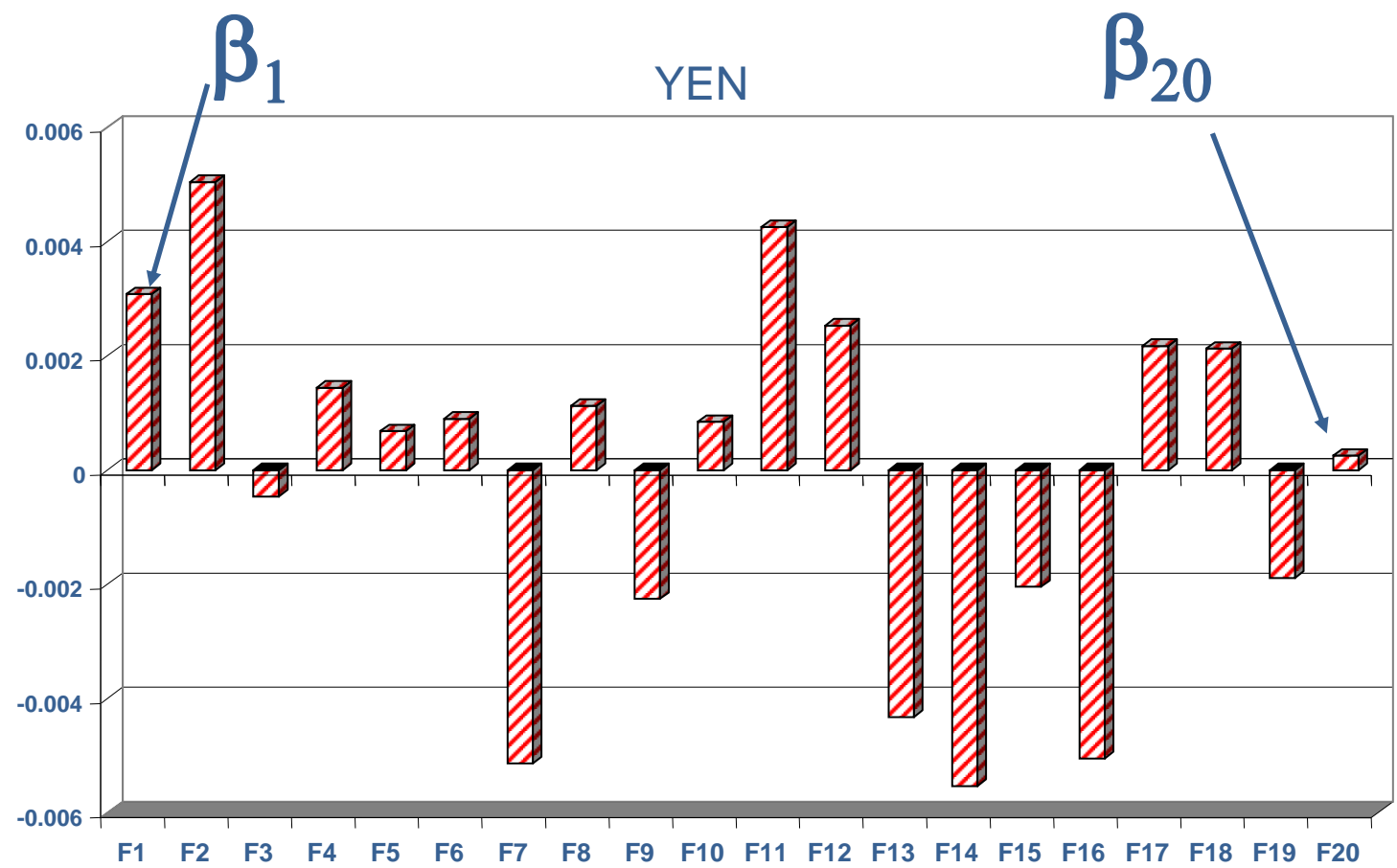
RISK MODELS :: STATISTICAL FACTOR MODELS :: RISK ATTRIBUTION

$$R_i = \sum_{j=1}^N \beta_{ij}^* R_j^* + \varepsilon_i^*$$

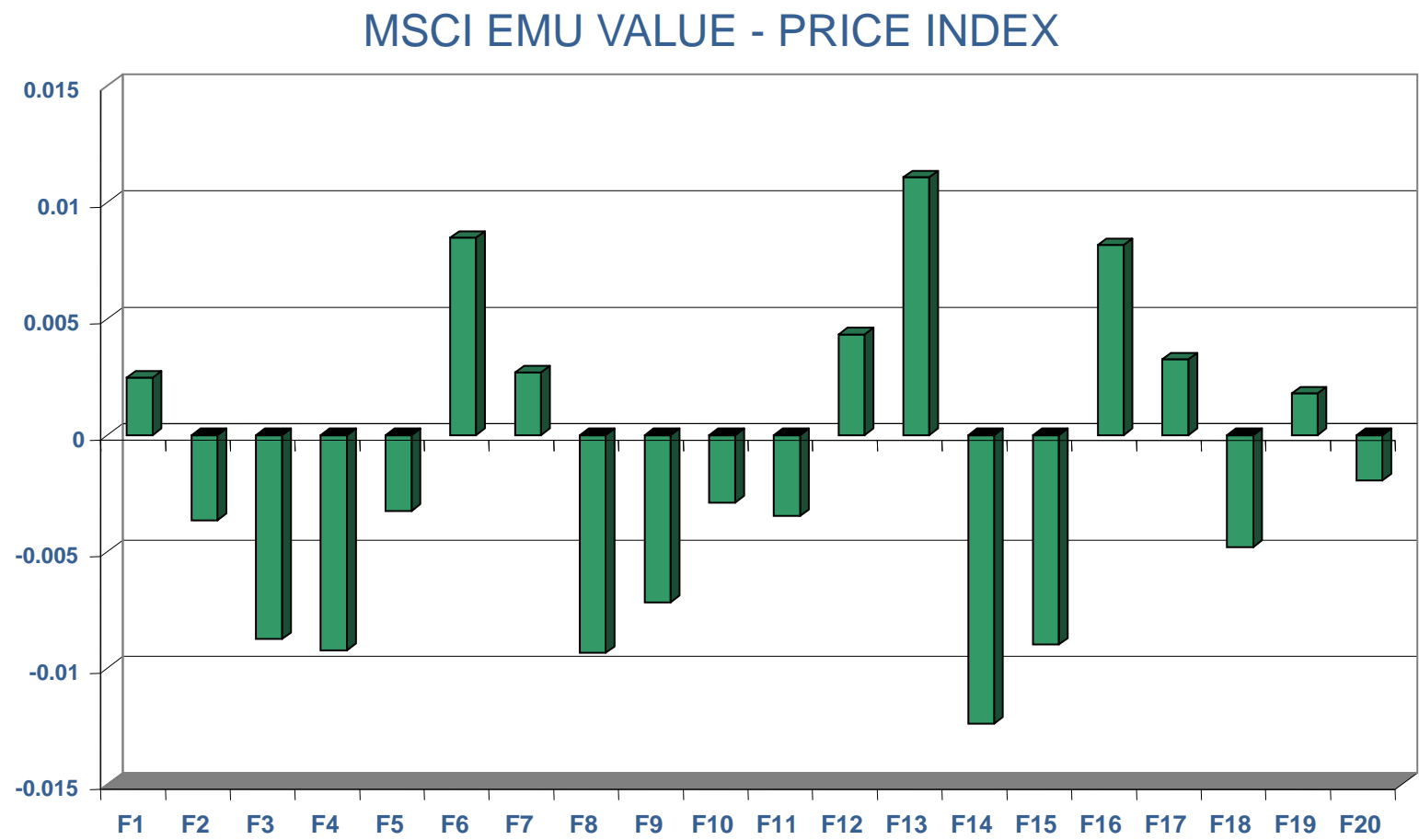
 Explained by
Extracted arbitrage Factors
Systematic Return

 Unexplained by
Arbitrage Factors
Specific Return

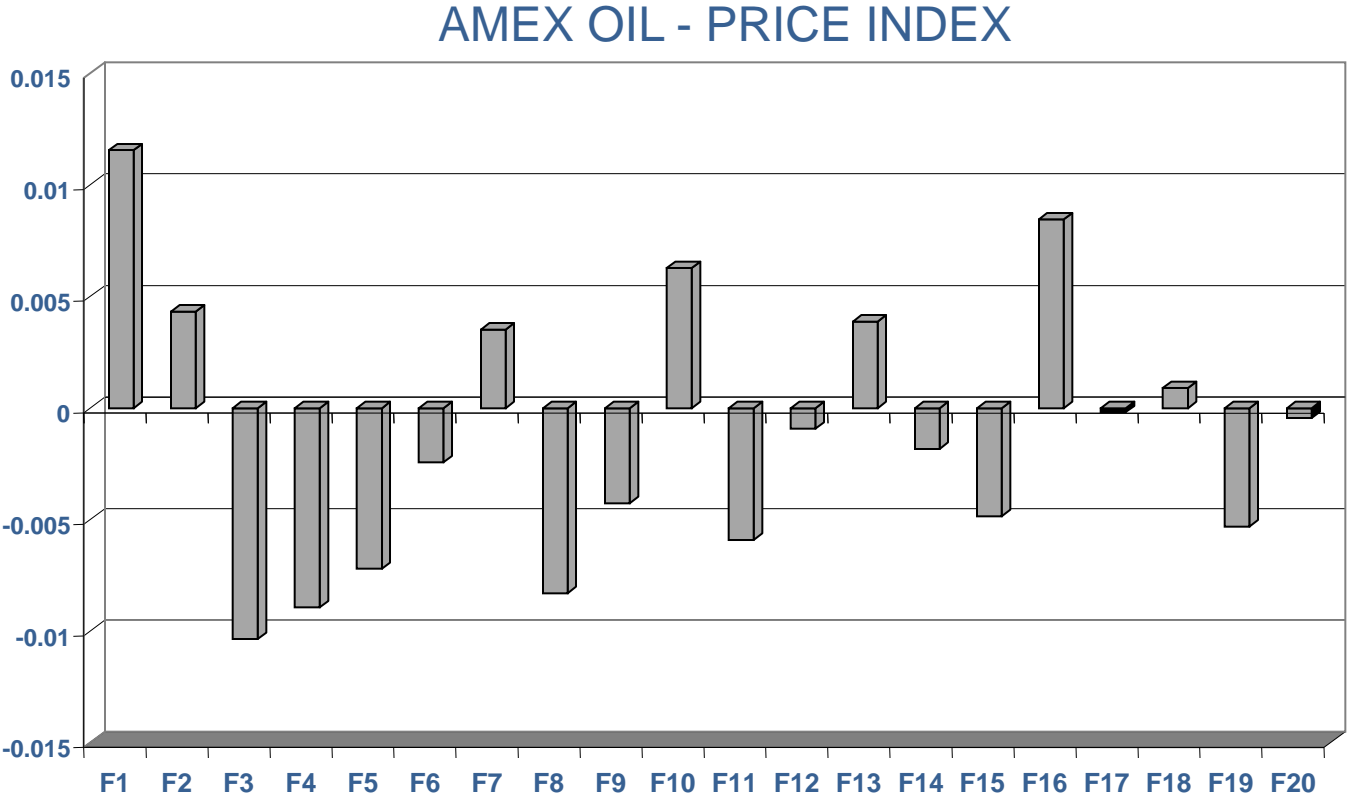
RISK MODELS :: STATISTICAL FACTOR MODELS :: RISK ATTRIBUTION



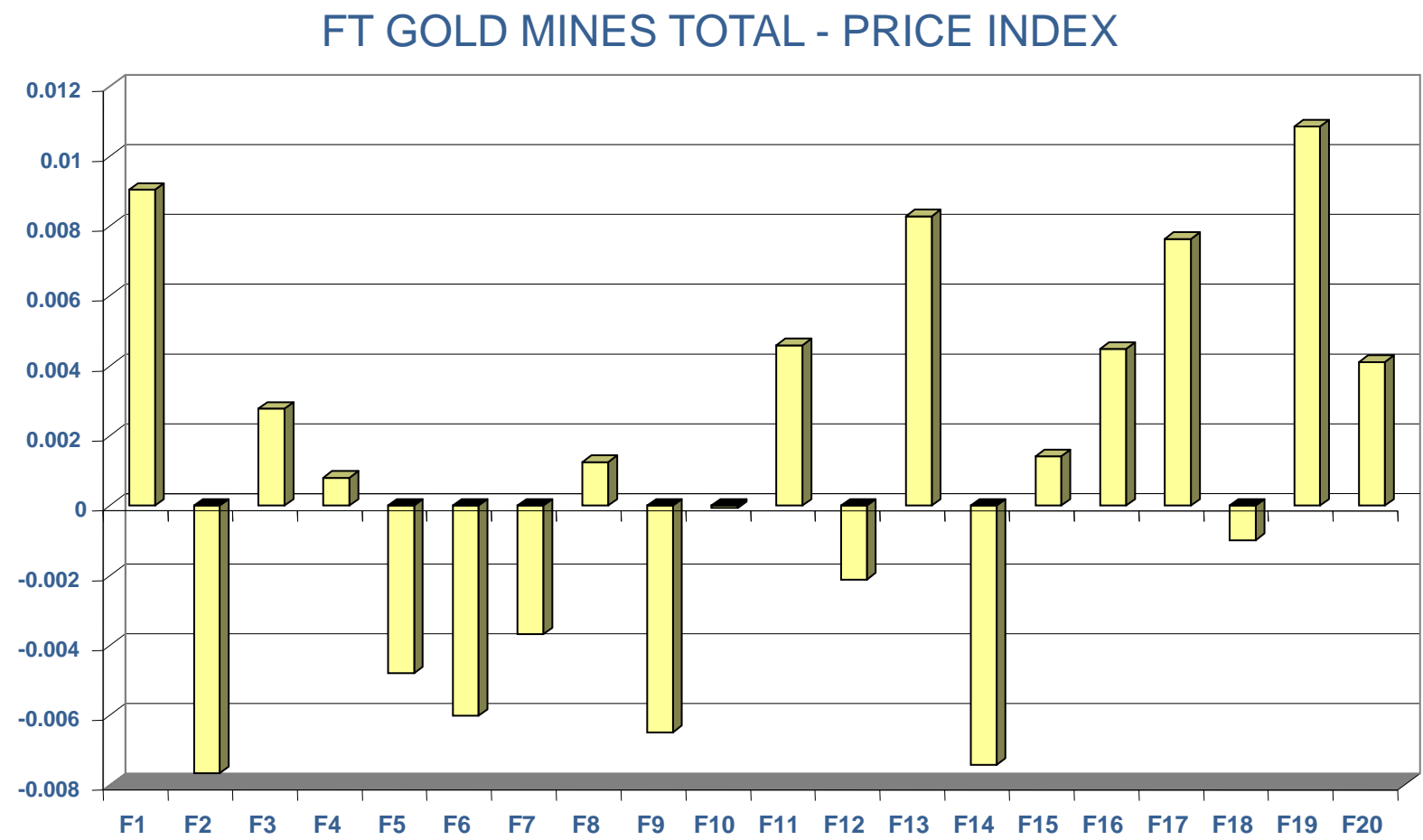
RISK MODELS :: STATISTICAL FACTOR MODELS :: RISK ATTRIBUTION



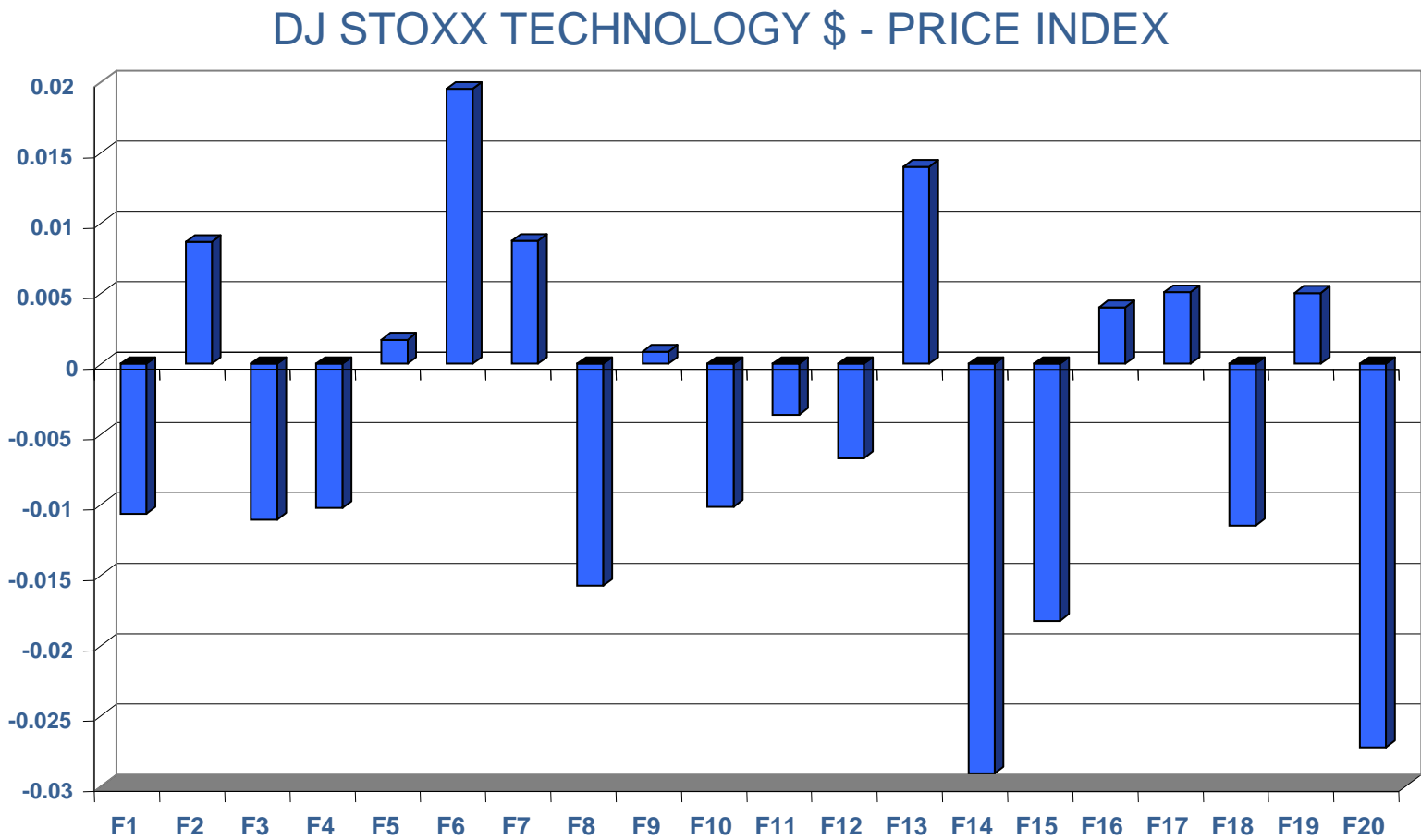
RISK MODELS :: STATISTICAL FACTOR MODELS :: RISK ATTRIBUTION



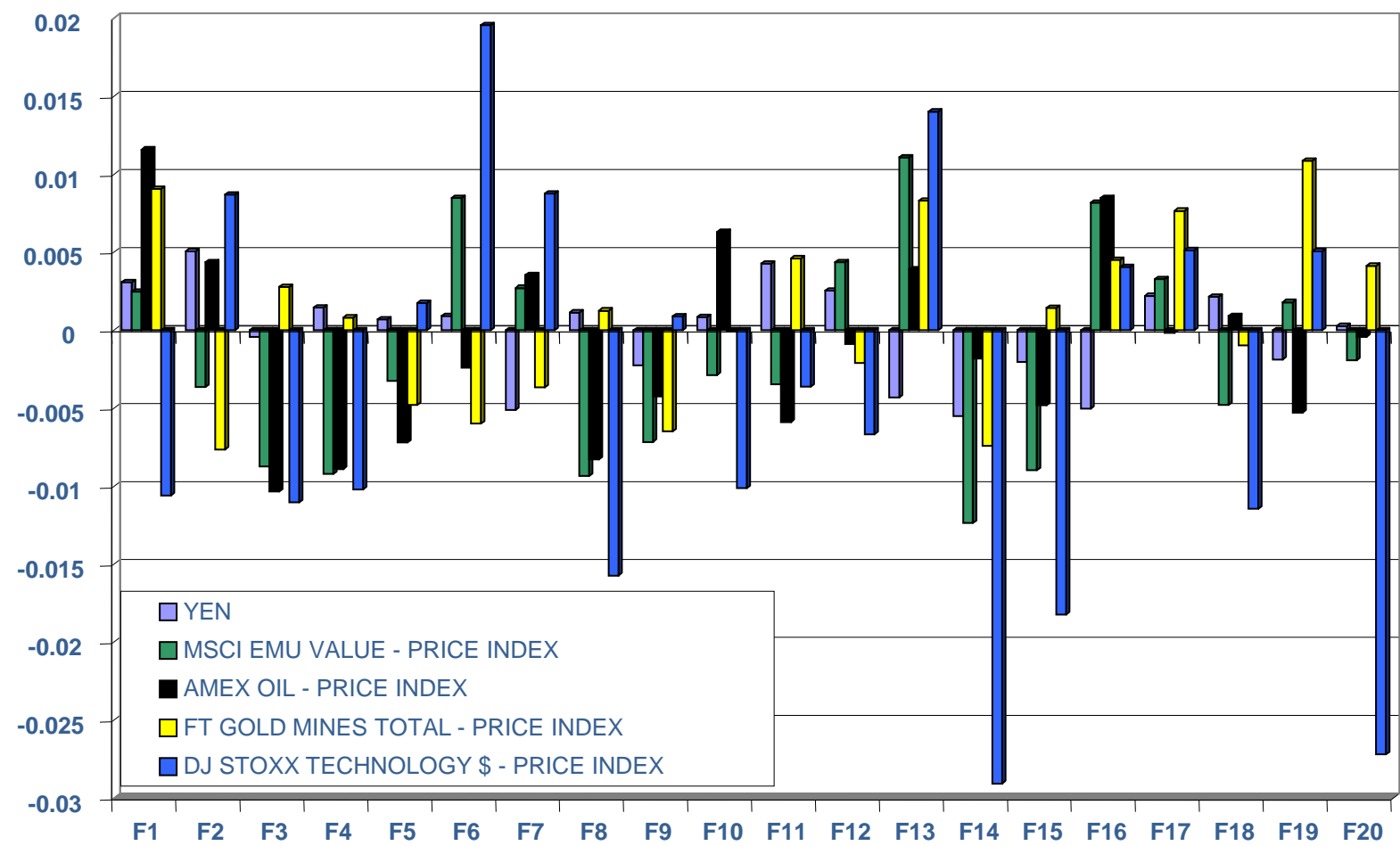
RISK MODELS :: STATISTICAL FACTOR MODELS :: RISK ATTRIBUTION



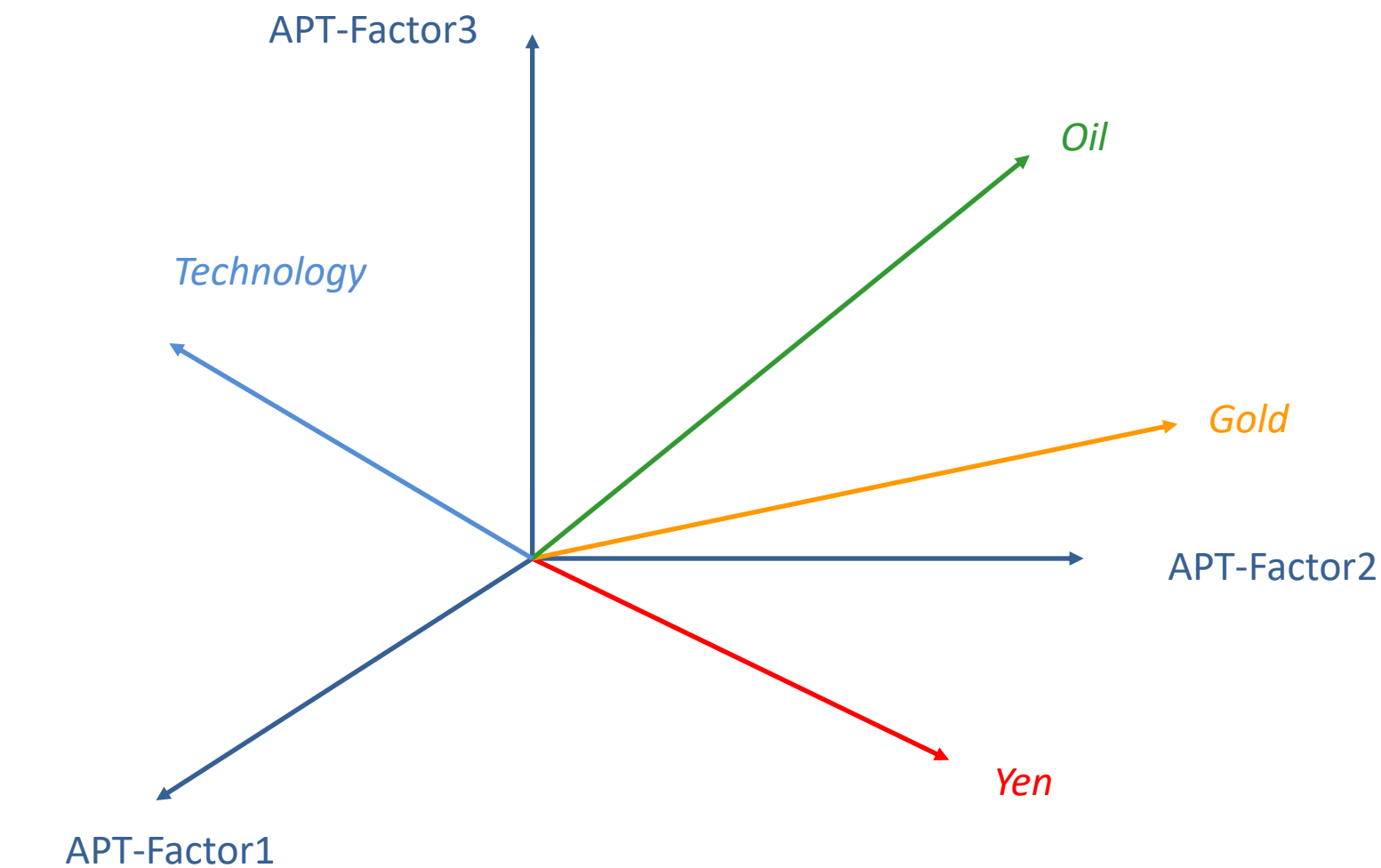
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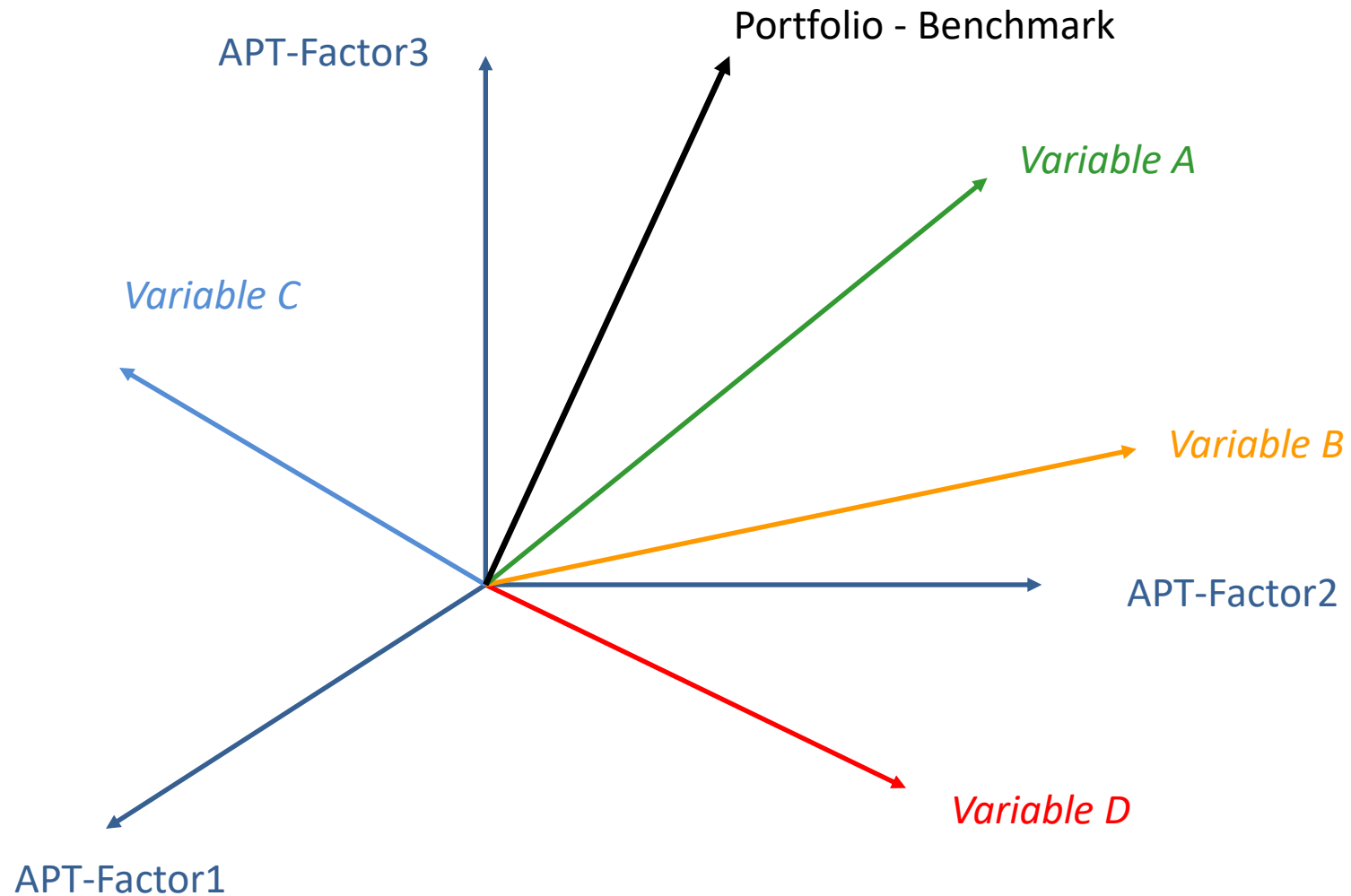
RISK MODELS :: STATISTICAL FACTOR MODELS :: RISK ATTRIBUTION



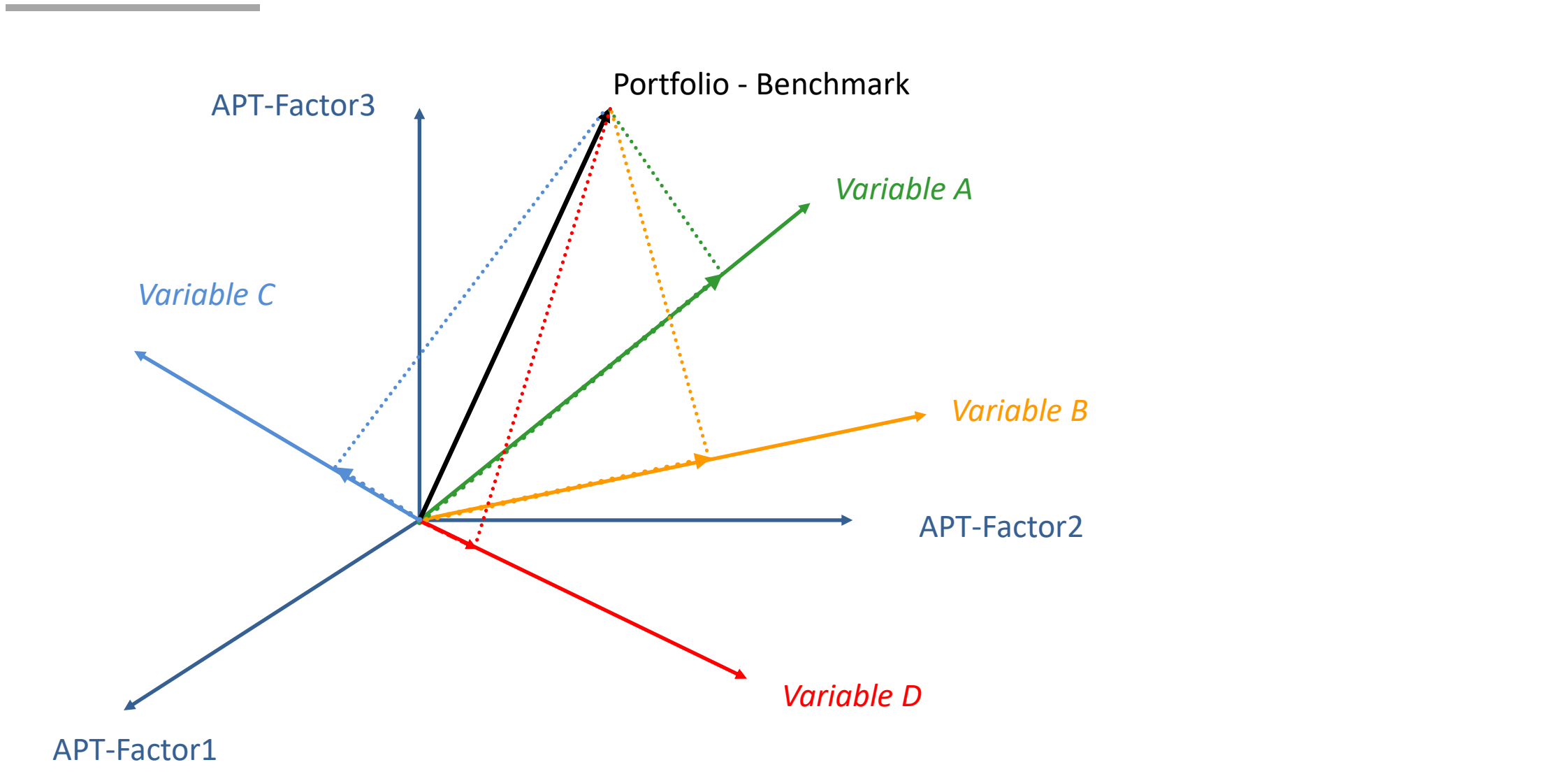
RISK MODELS :: STATISTICAL FACTOR MODELS :: INDEPENDENT ATTRIBUTION



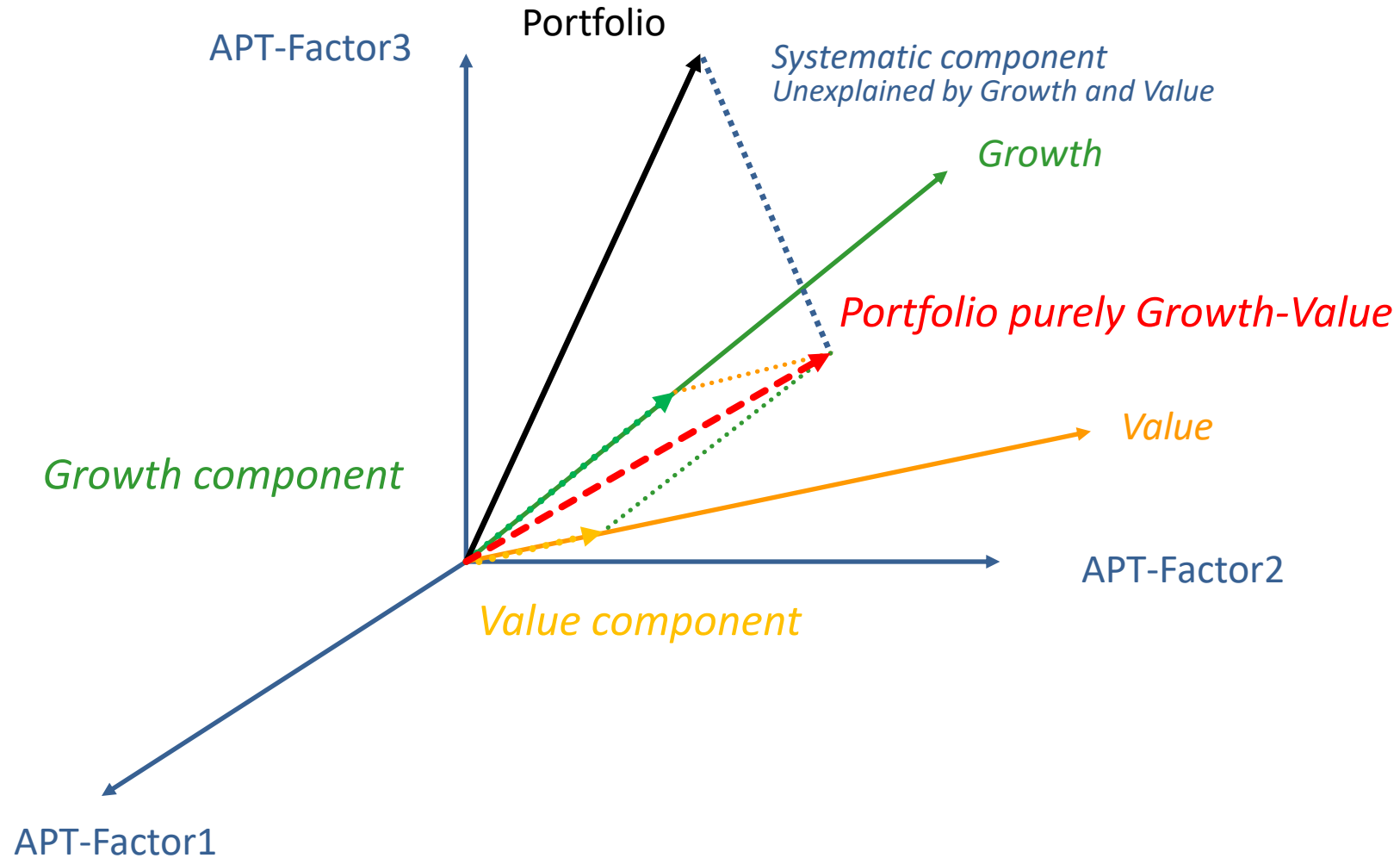
RISK MODELS :: STATISTICAL FACTOR MODELS :: INDEPENDENT ATTRIBUTION



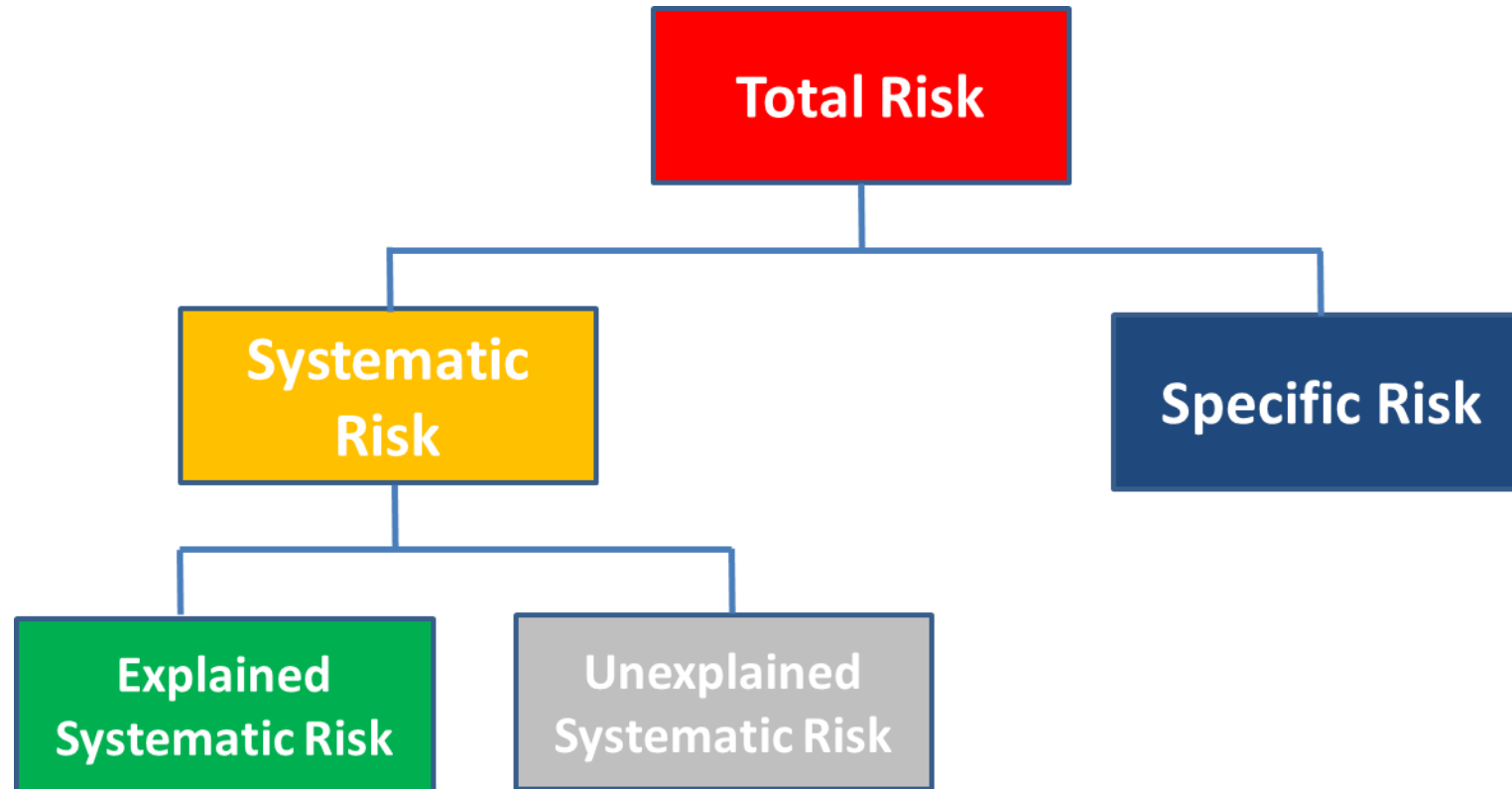
RISK MODELS :: STATISTICAL FACTOR MODELS :: INDEPENDENT ATTRIBUTION



RISK MODELS :: STATISTICAL FACTOR MODELS :: JOINT ATTRIBUTION



RISK MODELS :: STATISTICAL FACTOR MODELS + JOINT ATTRIBUTION



RISK MODELS :: STATISTICAL FACTOR MODELS :: RISK ATTRIBUTION

CASE STUDY 5 : Attribute Risk for Harris Associate US Value

RISK MODELS :: STATISTICAL FACTOR MODELS

CASE STUDY 2 : Apply Single Factor Model for Loomis US Equity Growth

	Theory	Systematic / Specific	Explanatory Power	Stability / Robustness	Auto Adaptative	Multi Asset Class	Intuitive
Variance Covariance Matrix							
Single Factor Model							
Prespecified Multi Factor Model							
Statistical Multi Factor Model							
Statistical Multi Factor Model + Risk Attribution							

CONTENTS



I. Risk Models

1. Variance Covariance Matrix
2. Single Factor Model
3. Prespecified Multi Factor Model
4. APT Statistical Multi Factor Model

II. The need for Statistical Multi Factor Models

1. Risk attribution
2. Risk measurement

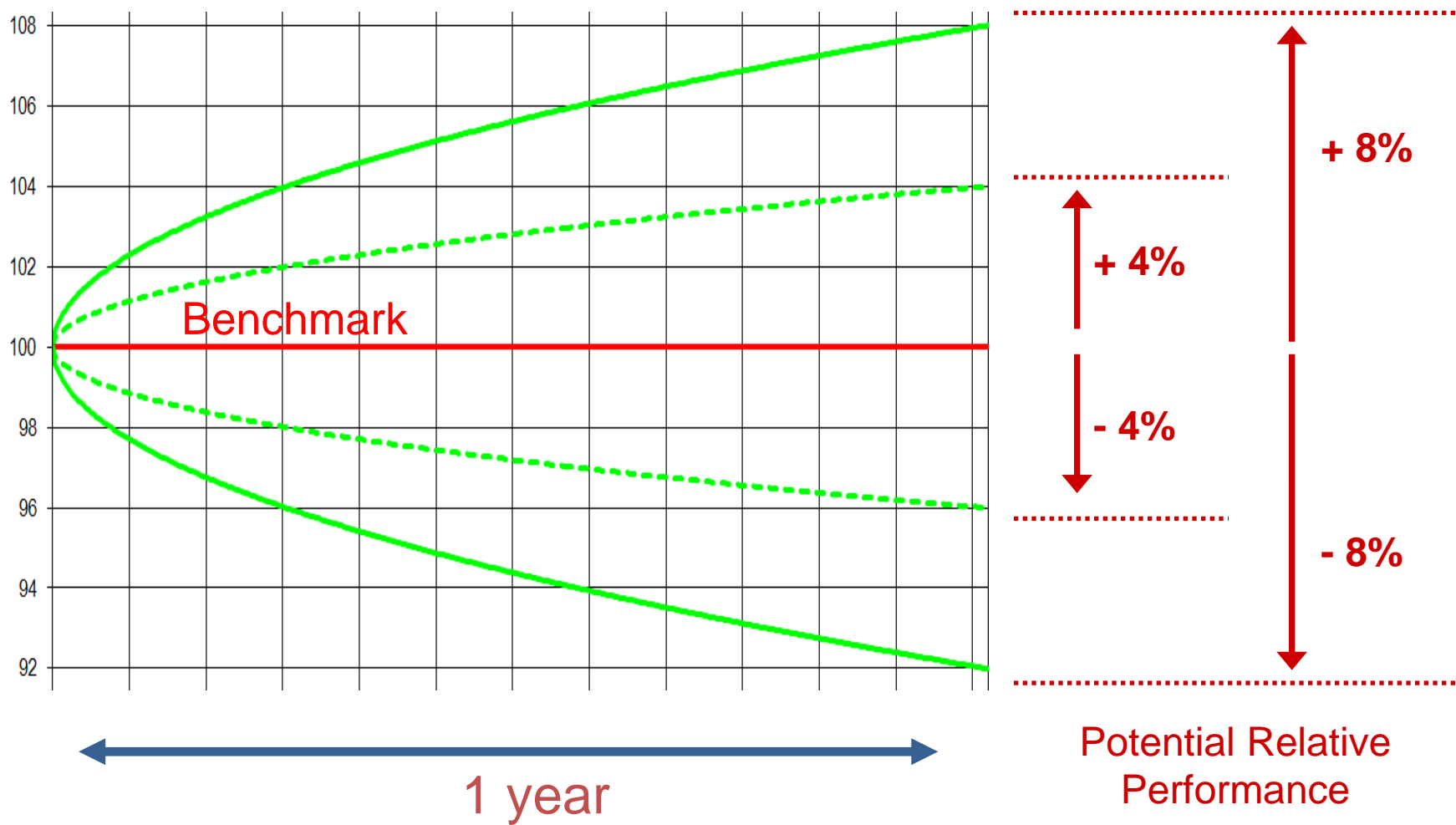
III. Risk Indicators

1. Top Level Risk Indicators
2. Position Based Risk Indicators

IV. Risk Analysis

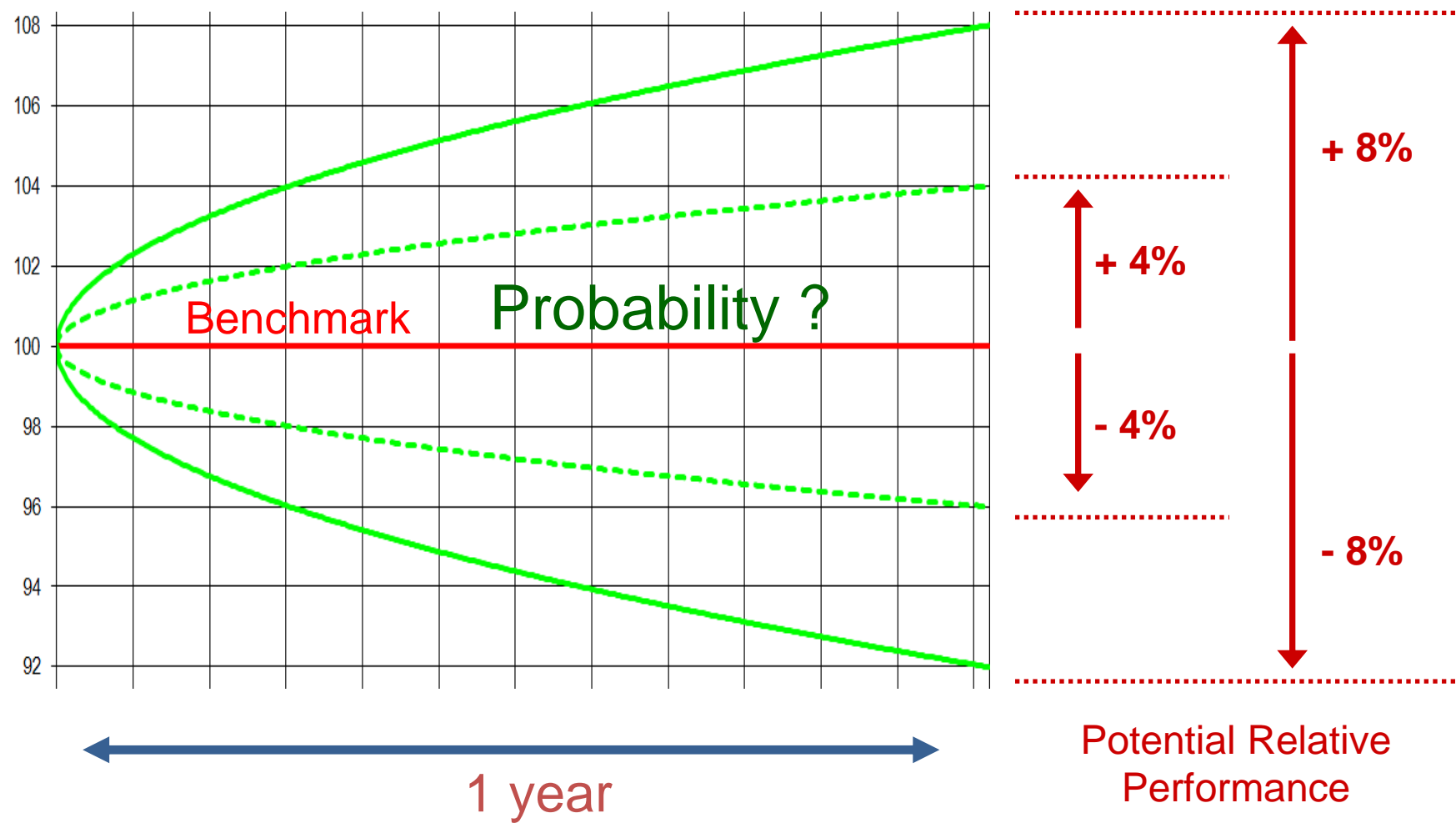
RISK ENVELOPS

Annualized Ex Ante Tracking Error = 4 %



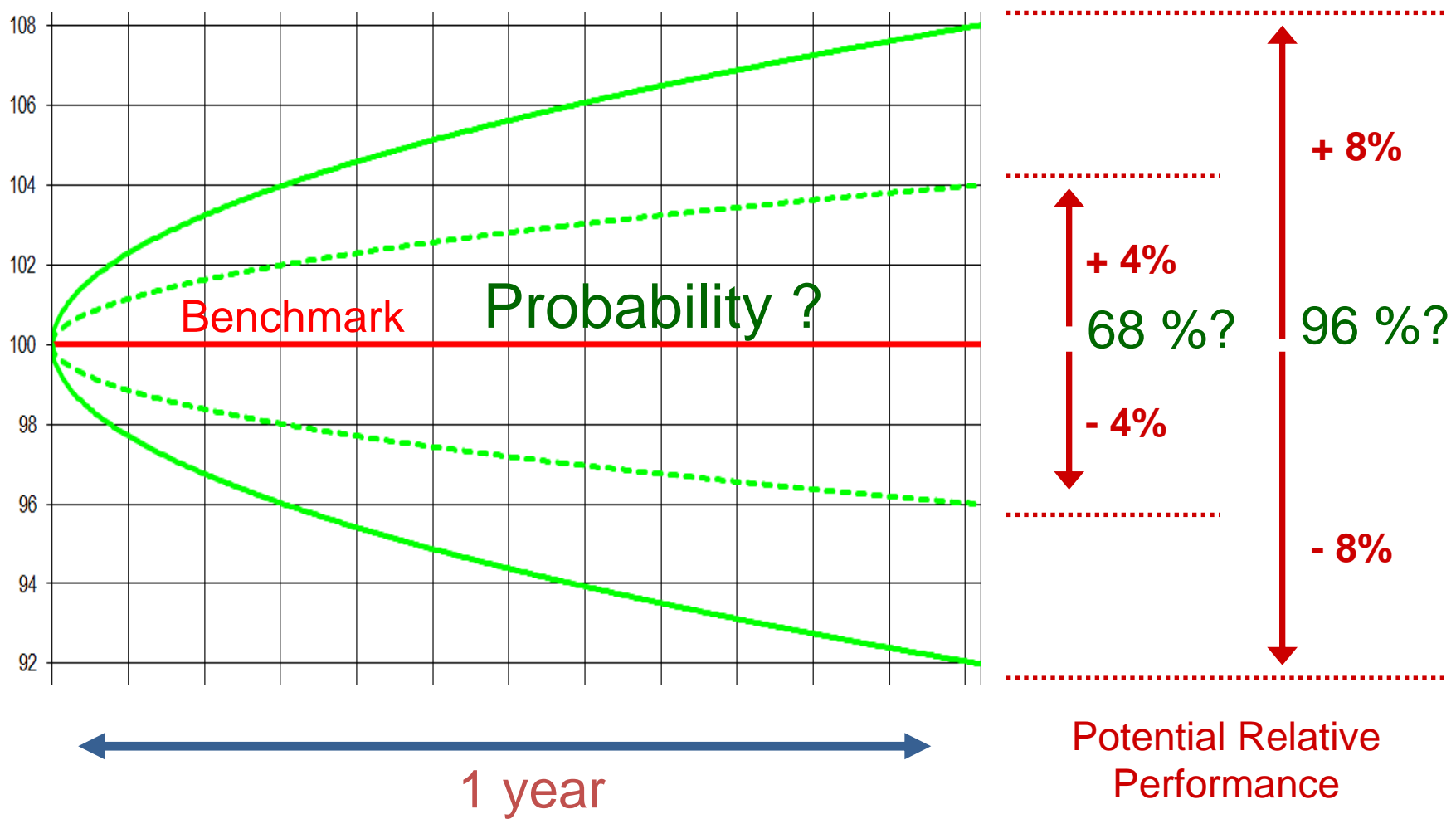
RISK ENVELOPS

Annualized Ex Ante Tracking Error = 4 %



RISK ENVELOPS

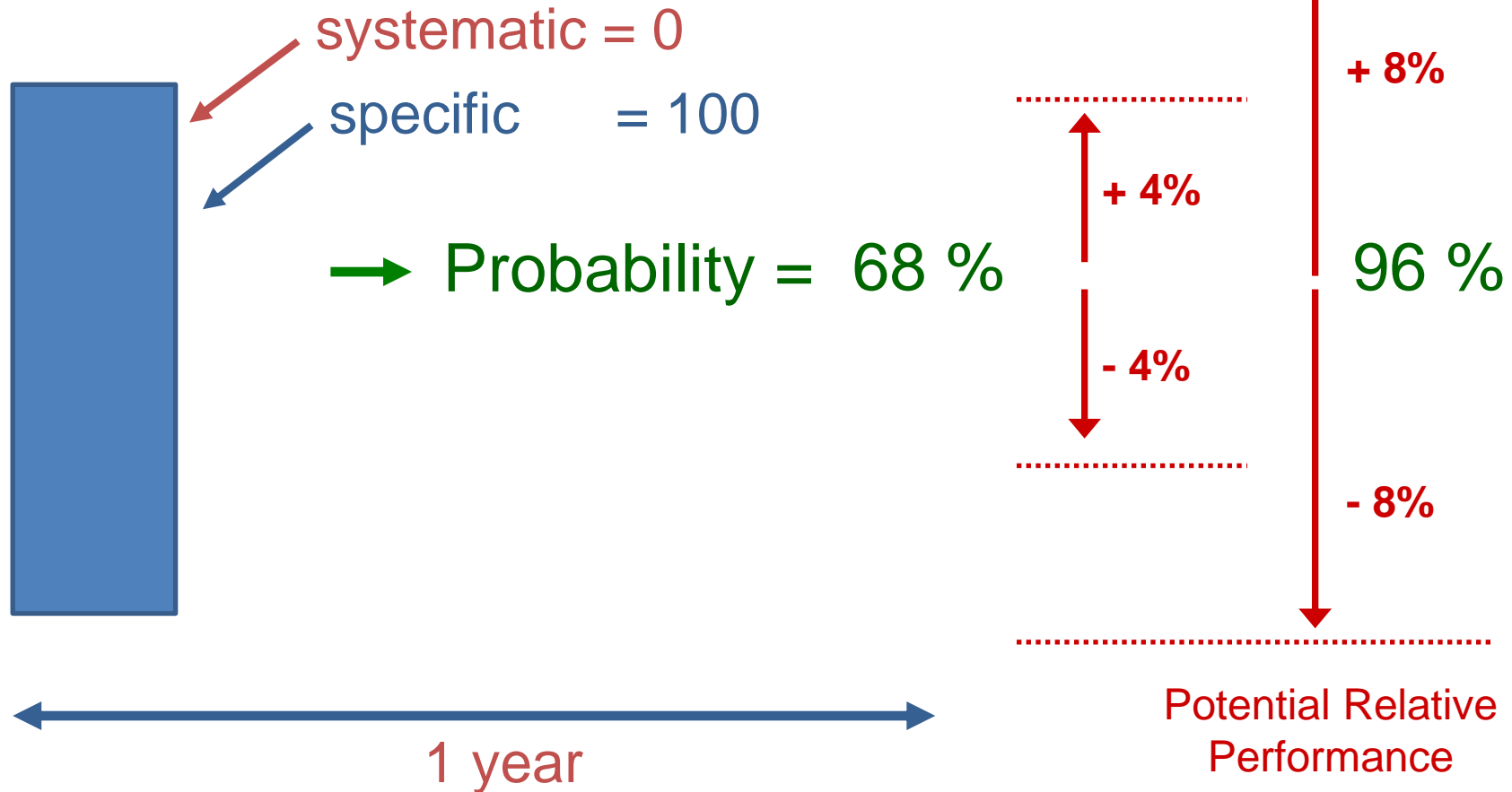
Annualized Ex Ante Tracking Error = 4 %



TRACKING-AT-RISK

Tracking Error = 4%

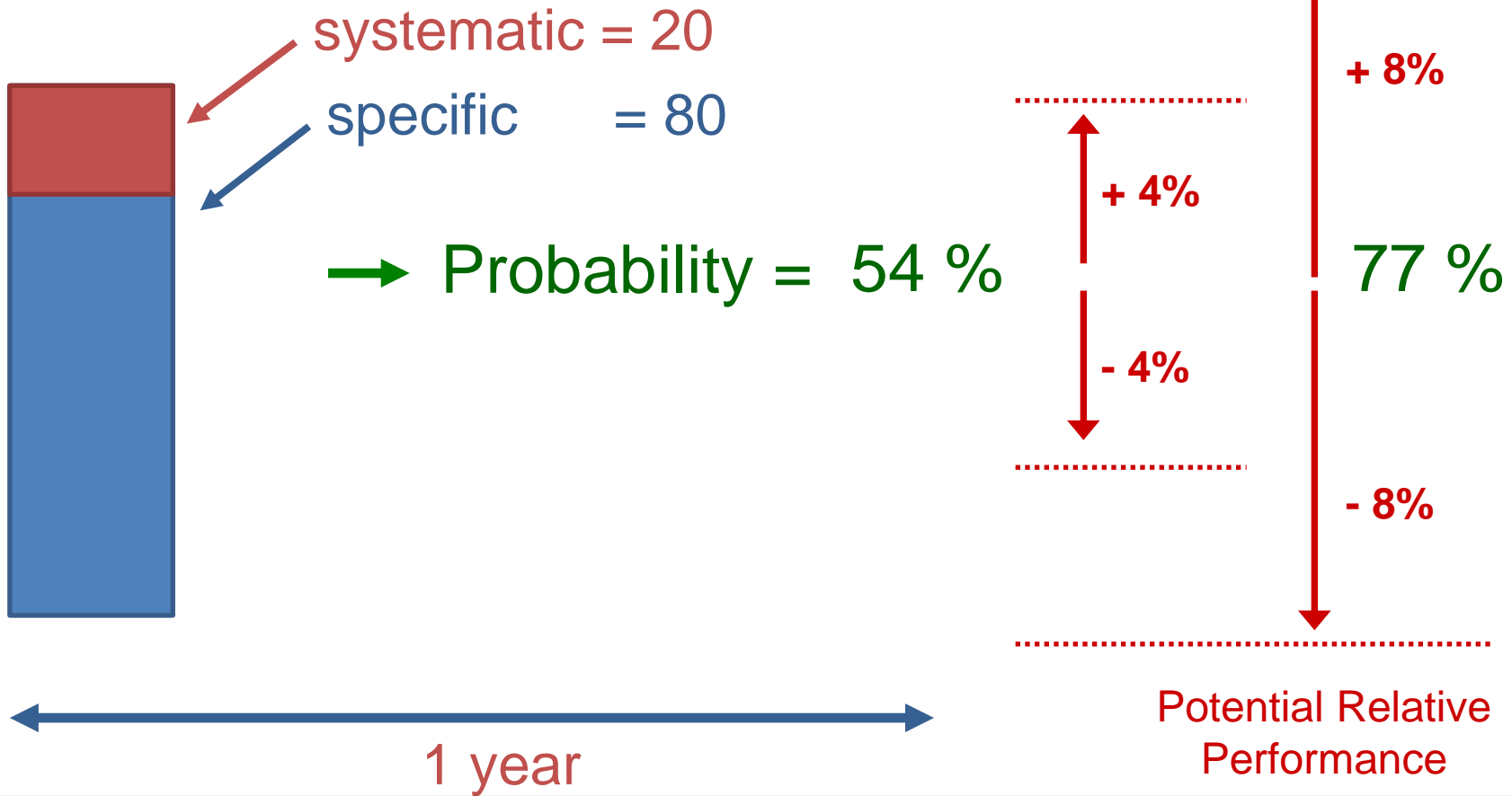
Case 1 :



TRACKING-AT-RISK

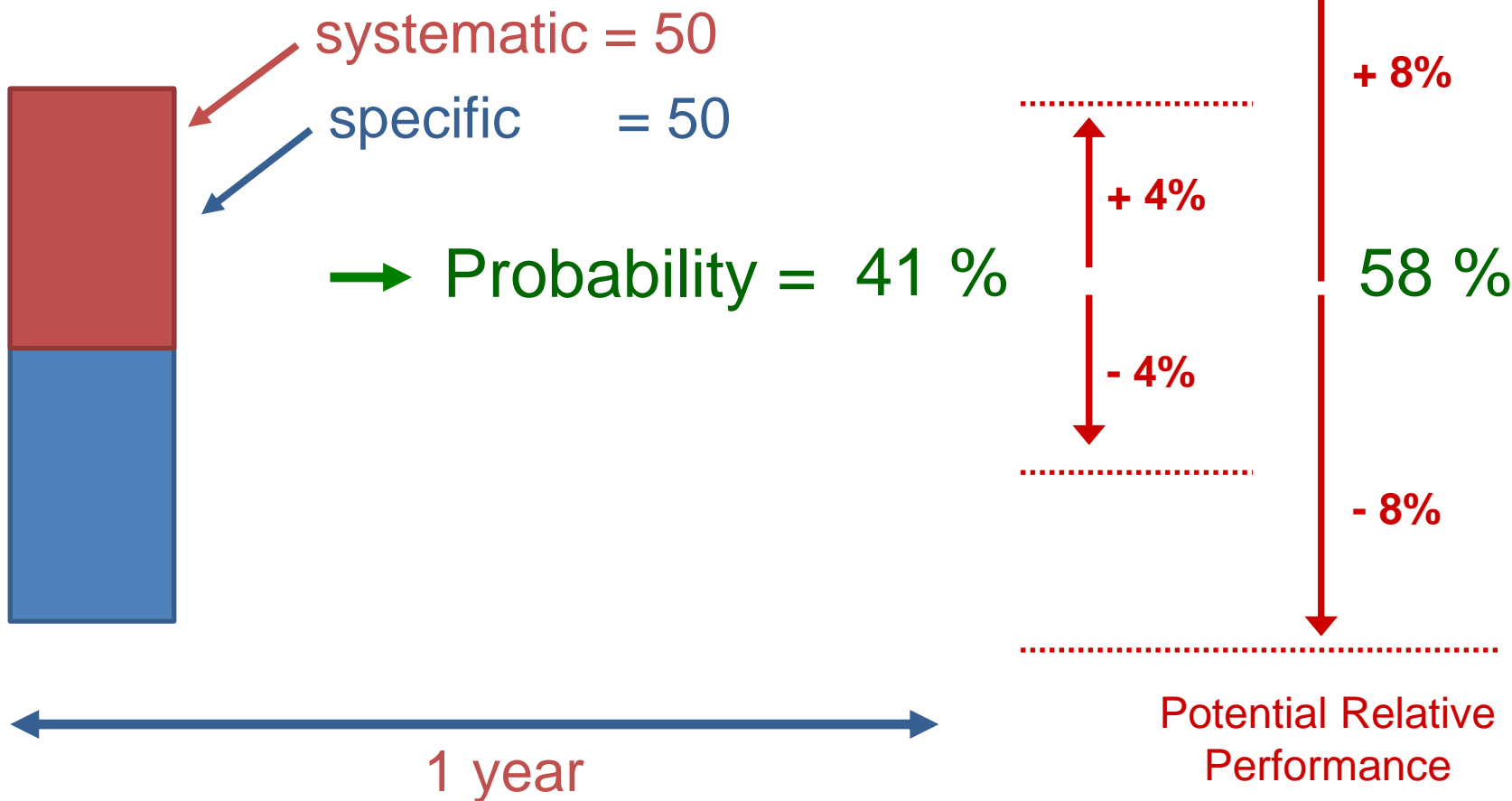
Tracking Error = 4%

Case 2 :



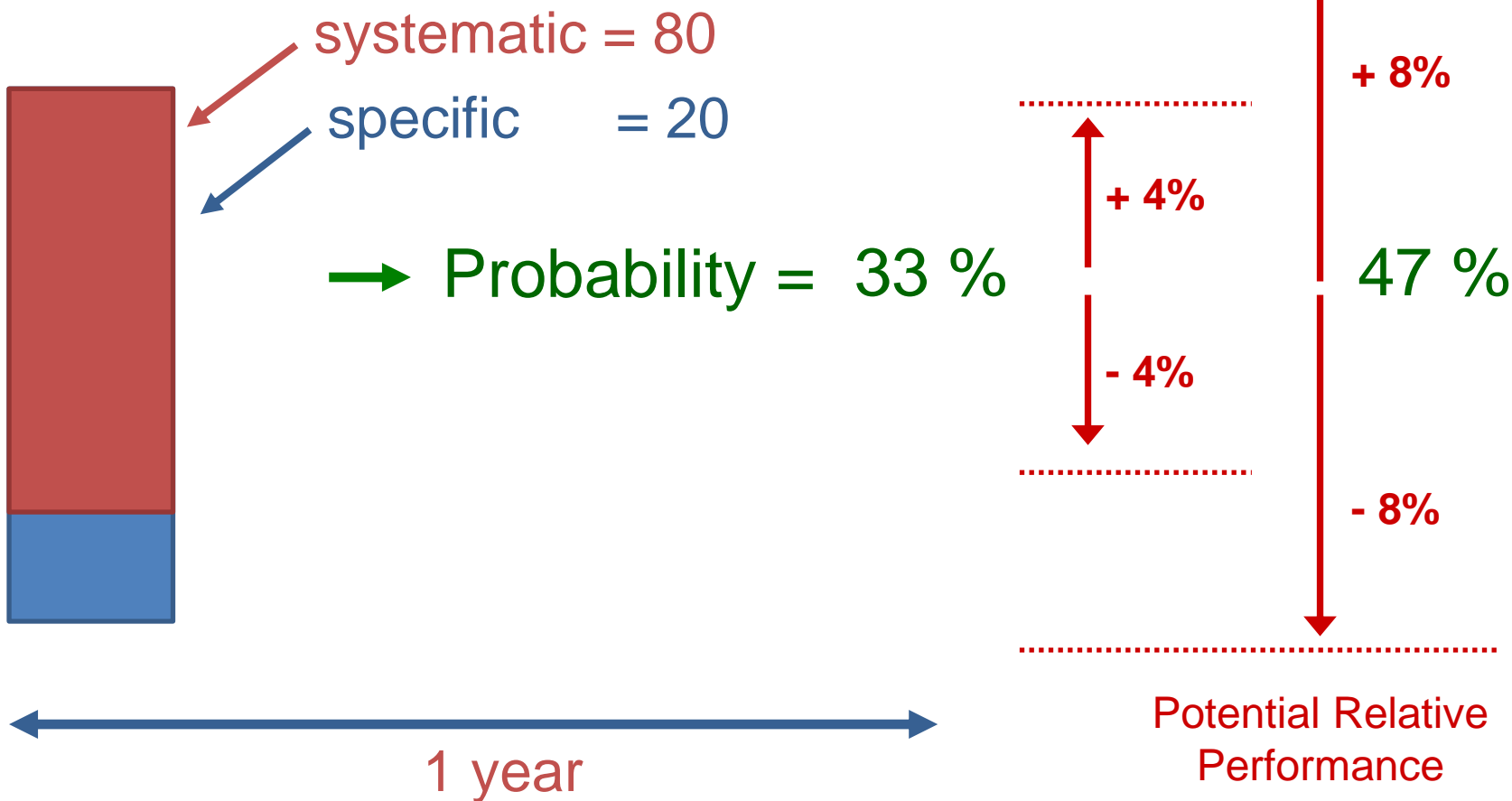
TRACKING-AT-RISK

Tracking Error = 4%
Case 3 :



TRACKING-AT-RISK

Tracking Error = 4%
Case 4 :



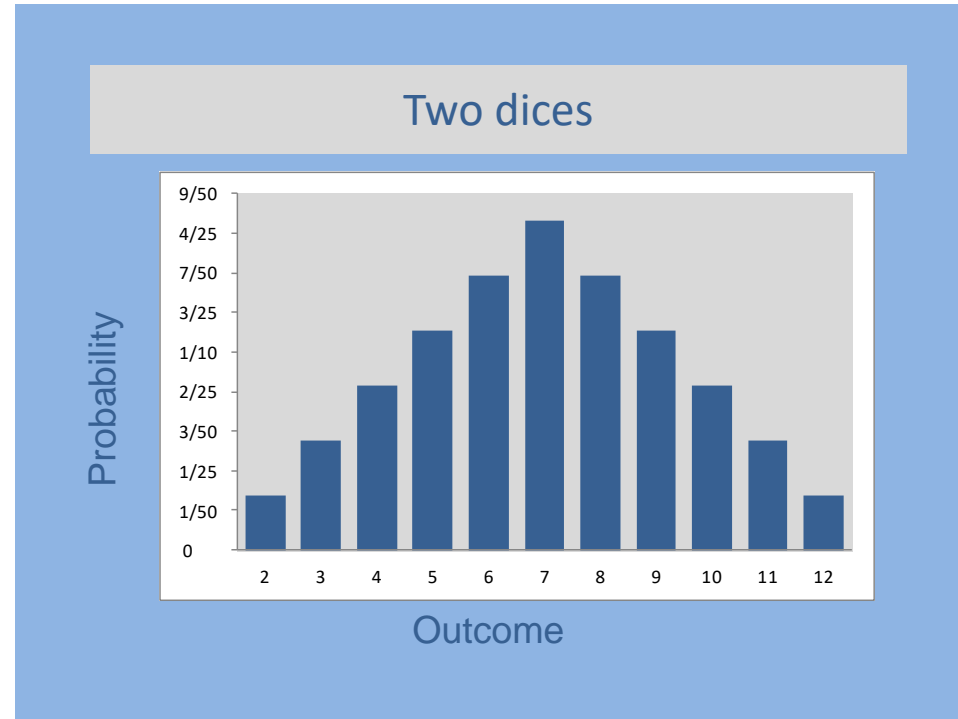
Specific Risk



Systematic Risk

TRACKING-AT-RISK

Specific Risk

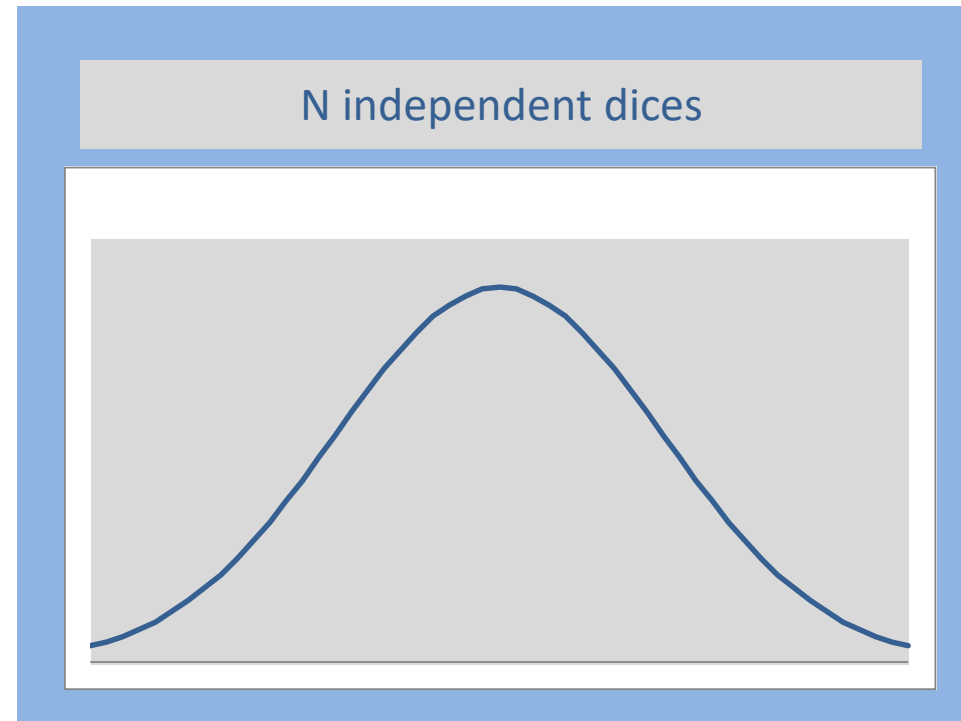


Systematic Risk

TRACKING-AT-RISK

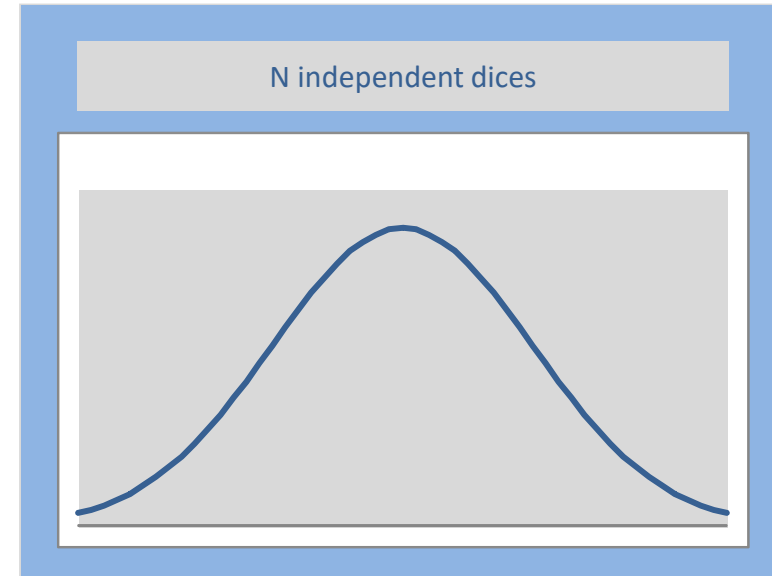
Specific Risk

Systematic Risk

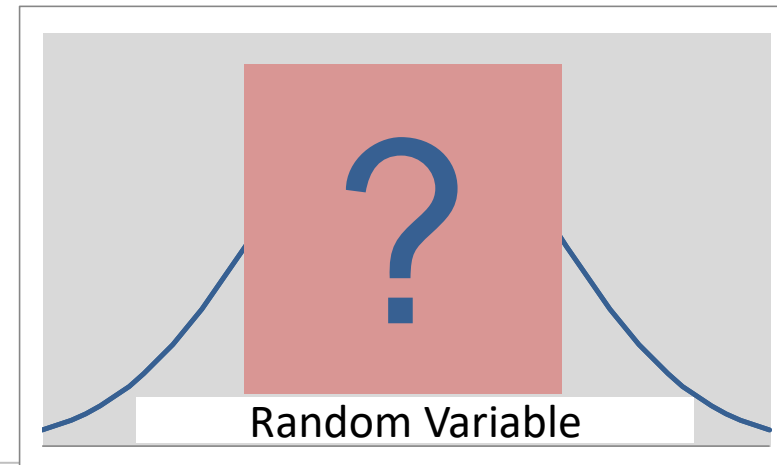


TRACKING-AT-RISK

Specific Risk

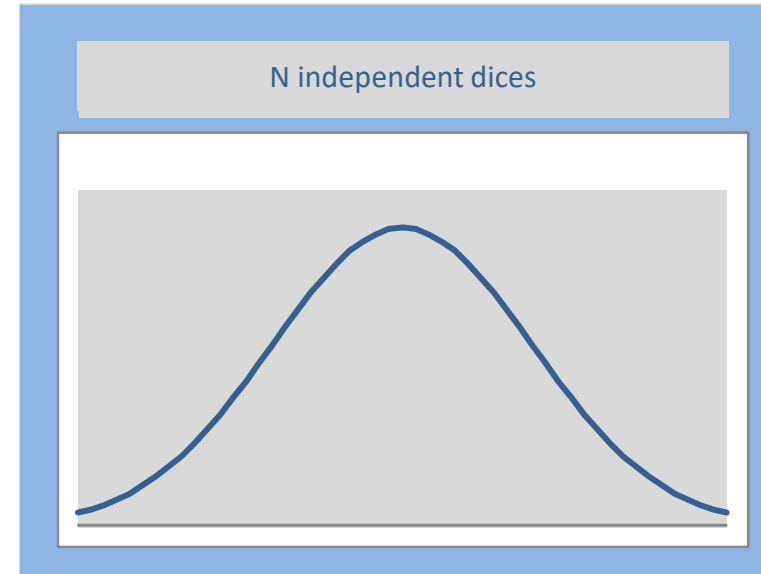


Systematic Risk

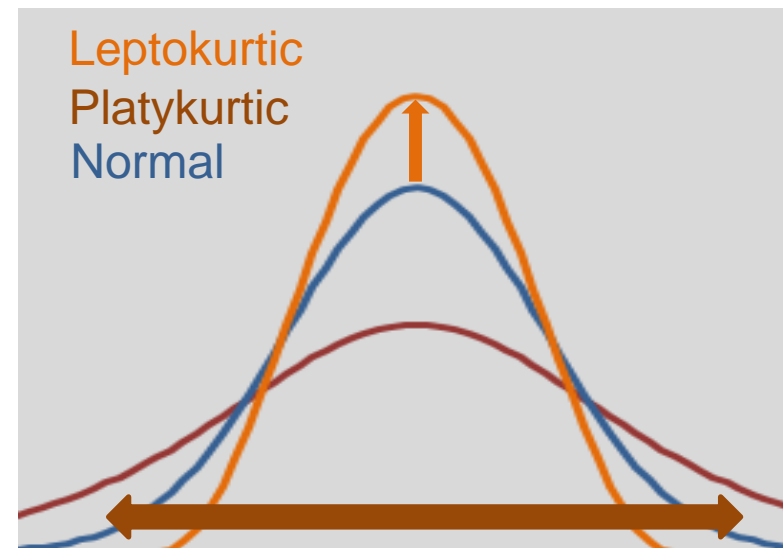


TRACKING-AT-RISK

Specific Risk

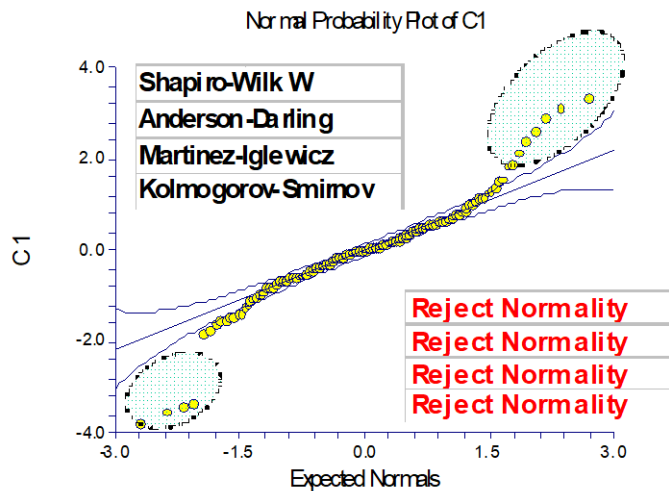


Systematic Risk

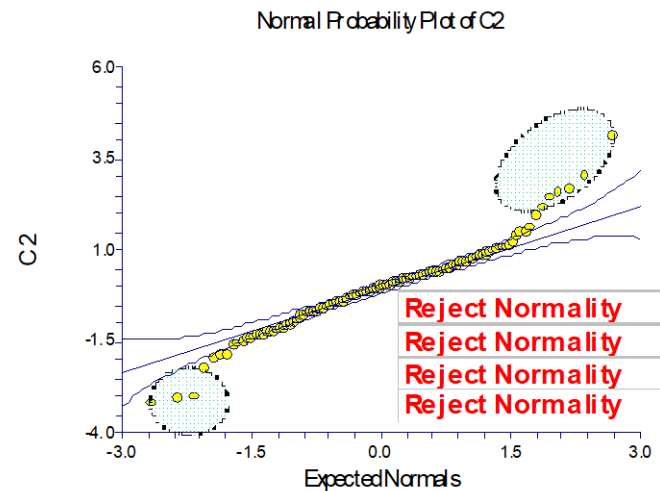


TRACKING-AT-RISK

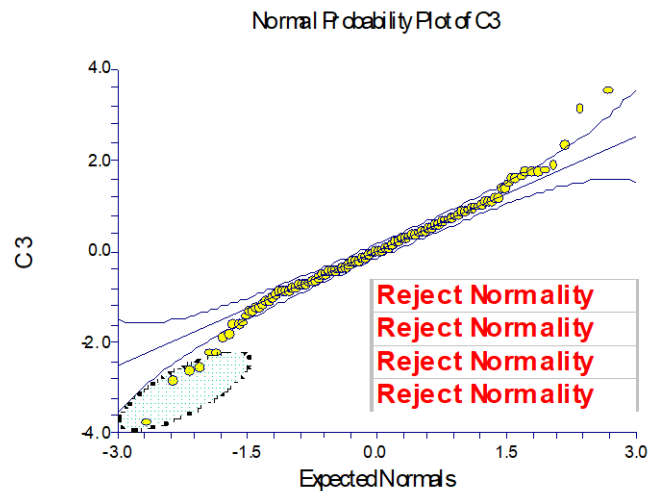
FMP1



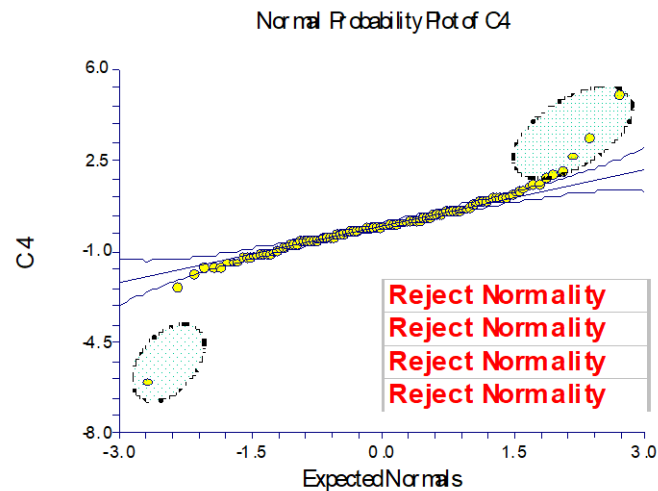
FMP4



FMP2



FMP3



Tracking at Risk

=

Prediction Interval

- probability
- horizon

Tracking at Risk

=

Prediction Interval

- probability : 96%
- horizon : 1 year

(example)

TRACKING-AT-RISK

Tracking Error = 4%

Tracking at Risk (1 year, 96 %) = ?

TRACKING-AT-RISK

Tracking Error = 4%

Tracking at Risk (1 year, 96 %) =

Specific/Systematic Ratio =

<u>100/0</u>→	<u>8.0%</u>
<u>80/20</u>→	<u>10.0%</u>
<u>50/50</u>→	<u>13.0%</u>
<u>20/80</u>→	<u>16.7%</u>

?



TRACKING-AT-RISK

95% Confidence Interval Estimates for Various Breakdowns of Shared vs Specific Risk

		% Tracking Error +>>											
		1%	1.5%	2%	3%	3%	3.5%	4%	4.5%	5%	5.5%	6%	6.5%
Tracking Error Specific	100%	2.0%	2.9%	3.9%	4.9%	5.9%	6.9%	7.8%	8.8%	9.8%	10.8%	11.8%	12.7%
	95%	2.1%	3.1%	4.2%	5.2%	6.3%	7.3%	8.4%	9.4%	10.5%	11.5%	12.5%	13.6%
	90%	2.2%	3.3%	4.4%	5.6%	6.7%	7.8%	8.9%	10.0%	11.1%	12.2%	13.3%	14.4%
	85%	2.4%	3.5%	4.7%	5.9%	7.1%	8.2%	9.4%	10.6%	11.8%	12.9%	14.1%	15.3%
	80%	2.5%	3.7%	5.0%	6.2%	7.5%	8.7%	9.9%	11.2%	12.4%	13.7%	14.9%	16.1%
	75%	2.6%	3.9%	5.2%	6.5%	7.8%	9.2%	10.5%	11.8%	13.1%	14.4%	15.7%	17.0%
	70%	2.7%	4.1%	5.5%	6.9%	8.2%	9.6%	11.0%	12.4%	13.7%	15.1%	16.5%	17.9%
	65%	2.9%	4.3%	5.8%	7.2%	8.6%	10.1%	11.5%	13.0%	14.4%	15.8%	17.3%	18.7%
	60%	3.0%	4.5%	6.0%	7.5%	9.0%	10.5%	12.0%	13.5%	15.0%	16.5%	18.1%	19.6%
	55%	3.1%	4.7%	6.3%	7.9%	9.4%	11.0%	12.6%	14.1%	15.7%	17.3%	18.8%	20.4%
	50%	3.3%	4.9%	6.5%	8.2%	9.8%	11.4%	13.1%	14.7%	16.4%	18.0%	19.6%	21.3%
	45%	3.4%	5.1%	6.8%	8.5%	10.2%	11.9%	13.6%	15.3%	17.0%	18.7%	20.4%	22.1%
	40%	3.5%	5.3%	7.1%	8.8%	10.6%	12.4%	14.1%	15.9%	17.7%	19.4%	21.2%	23.0%
	35%	3.7%	5.5%	7.3%	9.2%	11.0%	12.8%	14.7%	16.5%	18.3%	20.2%	22.0%	23.8%
	30%	3.8%	5.7%	7.6%	9.5%	11.4%	13.3%	15.2%	17.1%	19.0%	20.9%	22.8%	24.7%
	25%	3.9%	5.9%	7.9%	9.8%	11.8%	13.7%	15.7%	17.7%	19.6%	21.6%	23.6%	25.5%
	20%	4.1%	6.1%	8.1%	10.1%	12.2%	14.2%	16.2%	18.3%	20.3%	22.3%	24.3%	26.4%
	15%	4.2%	6.3%	8.4%	10.5%	12.6%	14.7%	16.8%	18.9%	20.9%	23.0%	25.1%	27.2%
	10%	4.3%	6.5%	8.6%	10.8%	13.0%	15.1%	17.3%	19.4%	21.6%	23.8%	25.9%	28.1%
	5%	4.5%	6.7%	8.9%	11.1%	13.4%	15.6%	17.8%	20.0%	22.3%	24.5%	26.7%	28.9%
	0%	4.6%	6.9%	9.2%	11.5%	13.7%	16.0%	18.3%	20.6%	22.9%	25.2%	27.5%	29.8%

Chebyshev's theorem

Chebyshev's theorem gives an upper bound to the probability that a certain random variable X falls far away from its mean *without any assumption on the shape of the probability distribution*. The only requirement is that the variance $\sigma^2 = Var(X)$ of the distribution is finite. In that case Chebyshev's inequality holds [6]:

$$\Pr(|X - \mu| \geq t\sigma) \leq \frac{1}{t^2} \quad (\text{B.4})$$

where $\mu = E(X)$. The result is valid as long as $t > 0$ but it provides non-trivial information only for $t > 1$. Eq. (B.4) states that for any random variable, the probability to fall more than t standard deviation far away from the mean cannot be bigger than $1/t^2$. Chebyshev's inequality (B.4) can be re-shuffled as

$$t \leq \frac{1}{\sqrt{1 - \Pr(|X - \mu| \leq t\sigma)}} \quad (\text{B.5})$$

This form of the inequality helps us answering the question “given a confidence level α , how many standard deviations do we need to go away from the mean to be sure that the variable is within that range with at least probability α ?”. The answer is the t_{\max} for which (B.5) becomes an equality.

This one-tailed variant of eq. (B.4) is known as Cantelli's inequality,

$$\Pr(X - \mu \geq t\sigma) \leq \frac{1}{1 + t^2} \quad (\text{B.6})$$

where $t > 0$ is assumed.

Source: APT Analytics Guide

TRACKING-AT-RISK

Low Volatility Regime



Tracking Error

TE

High Volatility Regime



Tracking at Risk

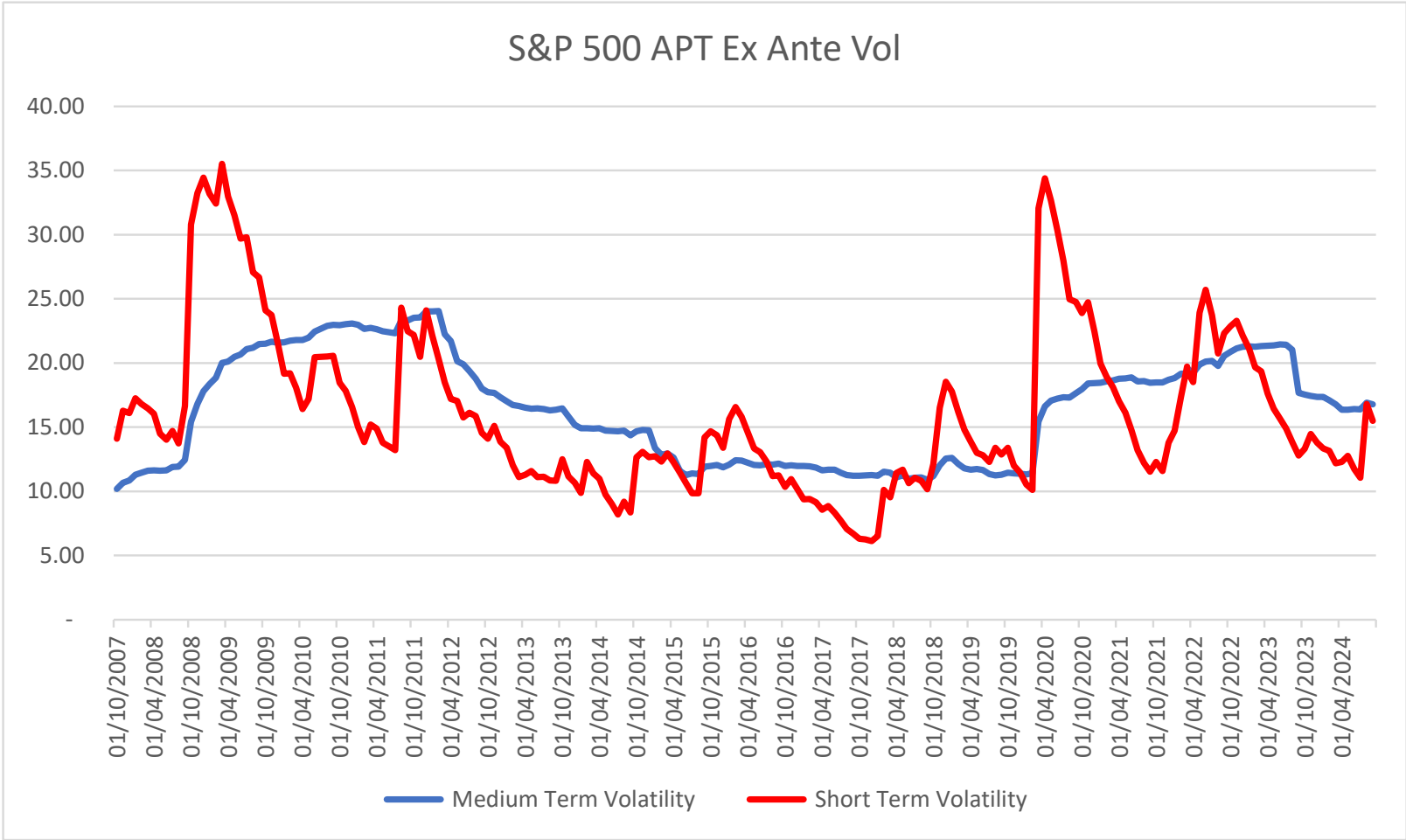
TaR

TRACKING-AT-RISK

Date	SPX Variation	Daily vol	# SD	Gaussian Prob	Chebychev Prob
03/04/2025	-4.84%	1.2%	4.1	0.002%	5.9%
04/04/2025	-5.97%	1.2%	5.1	0.000%	3.9%
08/04/2025	-1.57%	1.2%	1.3	9.129%	56.3%
09/04/2025	9.52%	1.2%	8.1	0.000%	1.5%

ACTIVE RISK MEASUREMENT IN DIFFERENT VOLATILITY REGIMES

DIFFERENT MARKET REGIMES



ACTIVE RISK MEASUREMENT IN DIFFERENT VOLATILITY REGIMES

CORONAVIRUS CRASH 2020 :: ILLUSTRATION WITH 2 PORTFOLIOS

Period & Benchmark:

Period : 2019-12-31 to 2020-12-31

Portfolio : European Equities

Reference Benchmark: FCI Europe 300

ACTIVE RISK MEASUREMENT IN DIFFERENT VOLATILITY REGIMES

ILLUSTRATION WITH 2 PORTFOLIOS



ACTIVE RISK MEASUREMENT IN DIFFERENT VOLATILITY REGIMES

CORONAVIRUS CRASH 2020 :: ILLUSTRATION WITH 2 PORTFOLIOS

Period & Benchmark:

Period : 2019-12-31 to 2020-12-31

Portfolio : European Equities

Reference Benchmark: FCI Europe 300

1 – Portfolio 1:

- *Stock-Picking Approach*
- *APT Ex-Ante TE = 4%*
- *High Active **Specific** Risk*

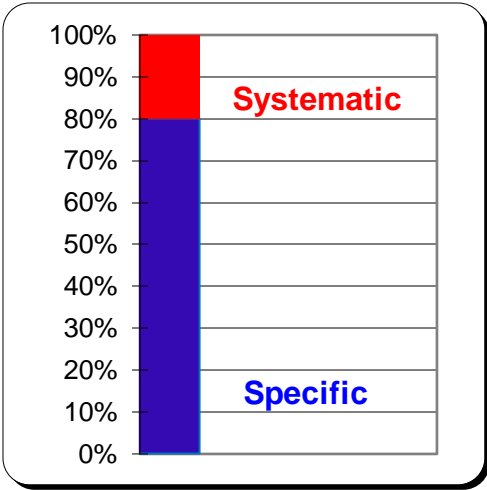
ACTIVE RISK MEASUREMENT IN DIFFERENT VOLATILITY REGIMES

PORTFOLIO 1: RELATIVE RISK AS OF 2019-12-31

Tracking error :	4.00
-Systematic	1.80
-Specific	3.56
Specif/Syst ratio :	80/20

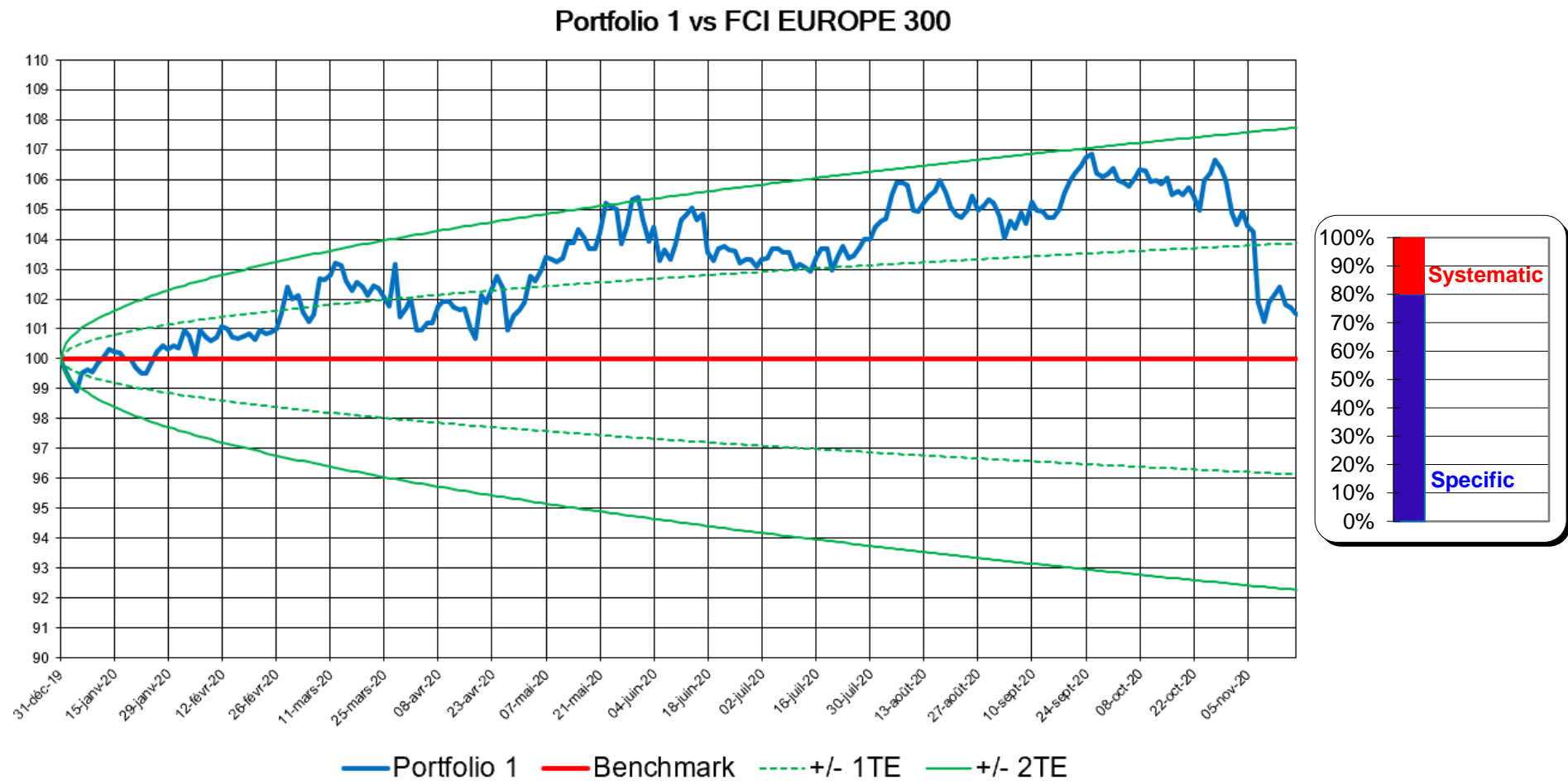
Tracking at Risk (68%)	4.80
------------------------	------

Systematic Beta	1.01
Systematic correlation	99.54%



ACTIVE RISK MEASUREMENT IN DIFFERENT VOLATILITY REGIMES

PORTFOLIO 1



ACTIVE RISK MEASUREMENT IN DIFFERENT VOLATILITY REGIMES

CORONAVIRUS CRASH 2020 :: ILLUSTRATION WITH 2 PORTFOLIOS

Period & Benchmark:

Period : 2019-12-31 to 2020-12-31

Portfolio : European Equities

Reference Benchmark: FCI Europe 300

1 – Portfolio 1:

- *Stock-Picking Approach*
- *APT Ex-Ante TE = 4%*
- *High Active **Specific** Risk*

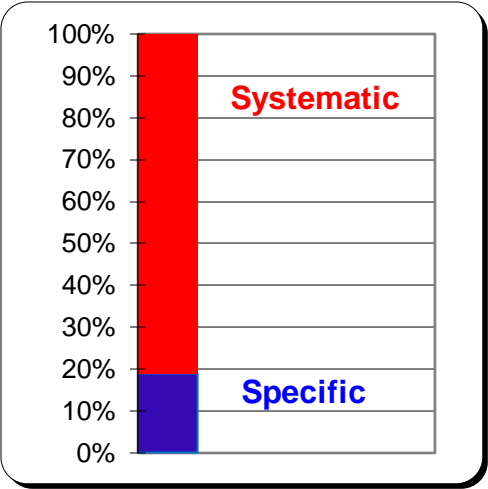
2 – Portfolio 2:

- *Top-Down Approach*
- *APT Ex-Ante TE = 4%*
- *High Active **Systematic** Risk*

ACTIVE RISK MEASUREMENT IN DIFFERENT VOLATILITY REGIMES

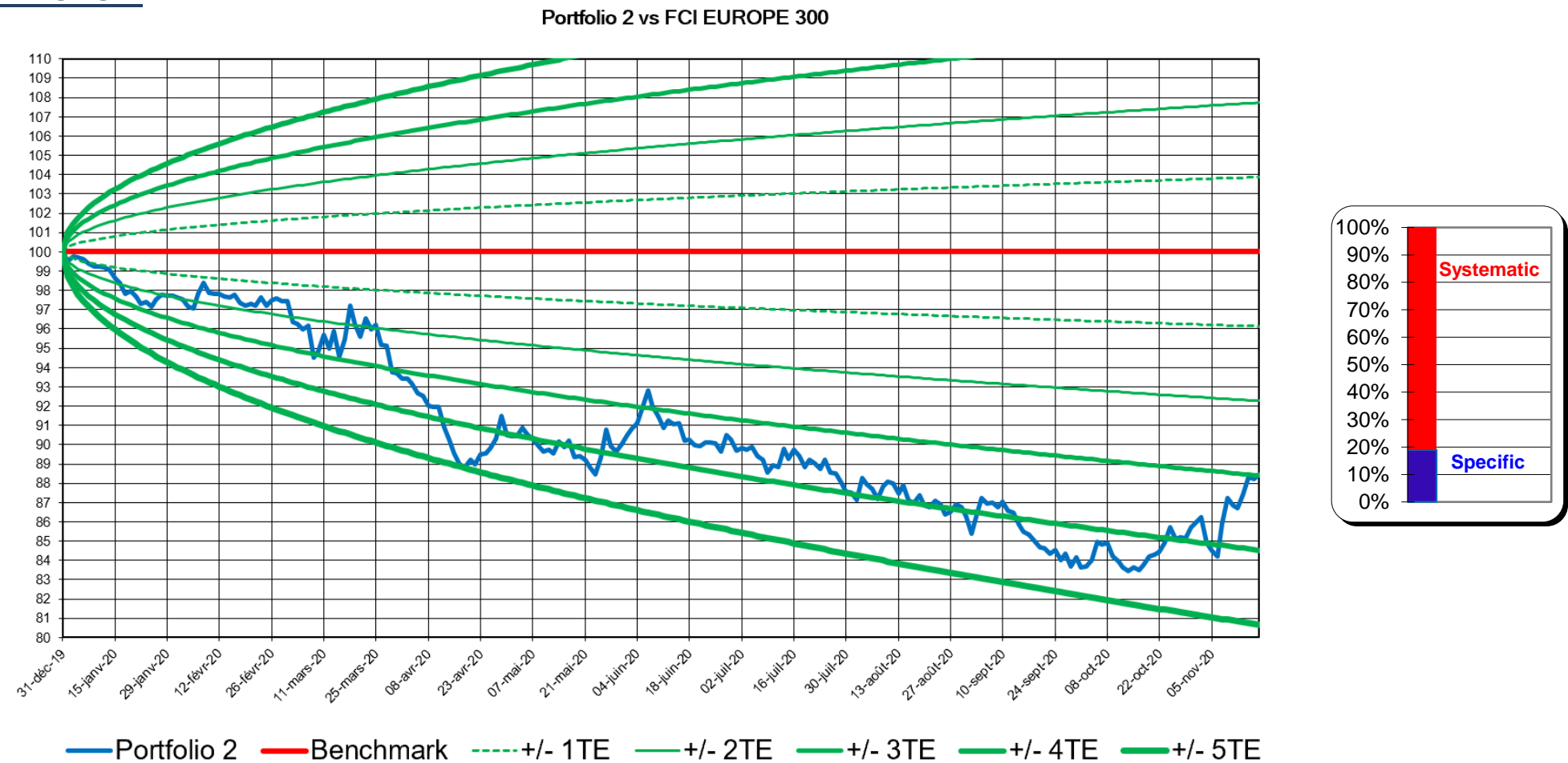
PORTFOLIO 2: RELATIVE RISK AS OF 2019-12-31

Tracking error :	4.00
-Systematic	3.61
-Specific	1.72
Specif/Syst ratio :	19/81
Systematic Beta	0.95
Systematic correlation	94.40%
Tracking at Risk (68%)	6.63
Tracking at Risk (96%)	19.37



ACTIVE RISK MEASUREMENT IN DIFFERENT VOLATILITY REGIMES

PORTFOLIO 2



APT TRACKING ERROR & TRACKING AT RISK

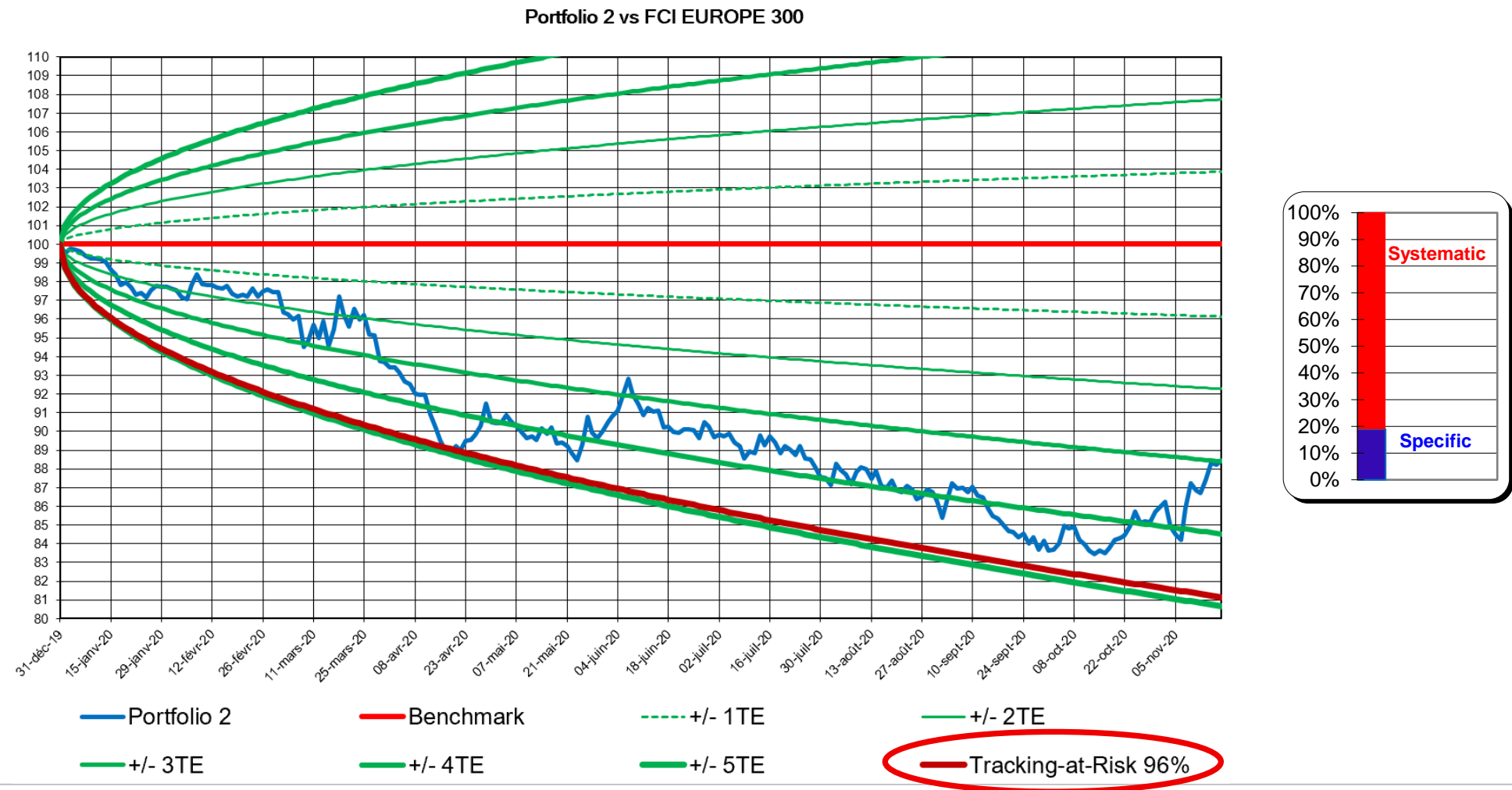
Lower Bound	Upper Bound	Probability
-1	1	68.2689492%
-2	2	95.4499736%
-3	3	99.7300204%
-4	4	99.9936658%
-5	5	99.9999427%
-6	6	99.9999998%

APT TRACKING ERROR & TRACKING AT RISK

Below # STD	Probability
1	15.8655254%
2	2.2750132%
3	0.1349898%
4	0.0031671%
5	0.0000287%
6	0.0000001%

ACTIVE RISK MEASUREMENT IN DIFFERENT VOLATILITY REGIMES

PORTFOLIO 2



ACTIVE RISK MEASUREMENT IN DIFFERENT VOLATILITY REGIMES

GFC 2008 :: ILLUSTRATION WITH A MAC PORTFOLIO

Period & Benchmark:

Period : 2007-06-31 to 2009-02-28

Portfolio : Balanced portfolio

Reference Benchmark: FCI Europe 300

3 – Portfolio 3:

- *Flexible Allocation*
- *APT Ex-Ante TE = 4%*
- *High Active **Systematic** Risk*

APT TRACKING ERROR & TRACKING AT RISK

PORTFOLIO 3: RELATIVE RISK AS OF 2007-06-30

	LIA	Bench
EuroMTS 5-7	17%	30%
CAC MidSmall	28%	5%
CAC 40	43%	30%
Eonia	12%	35%
	100%	100%

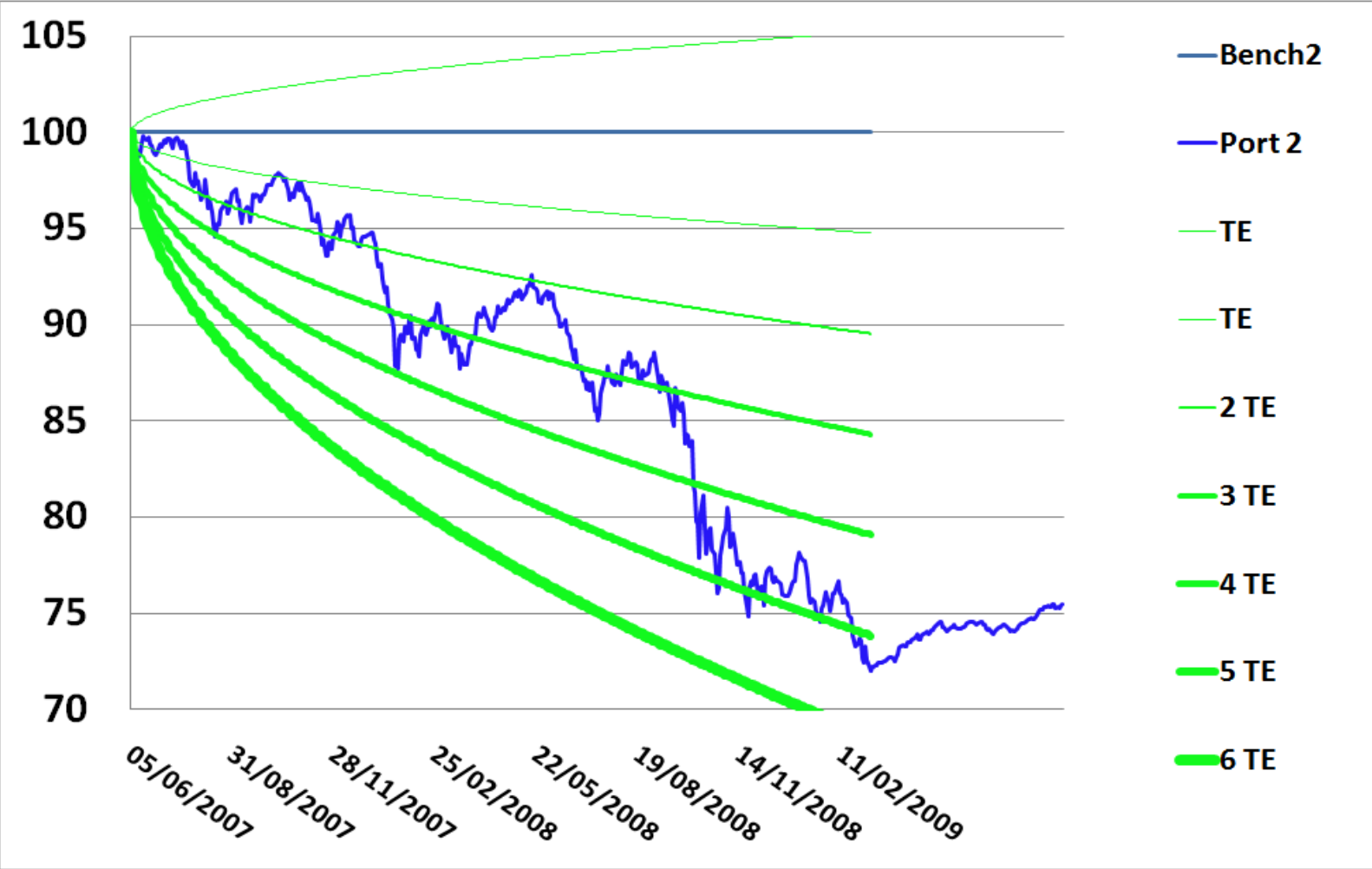
TE (ex ante)

3,87

TaR 68 %, 1Y (ex ante)

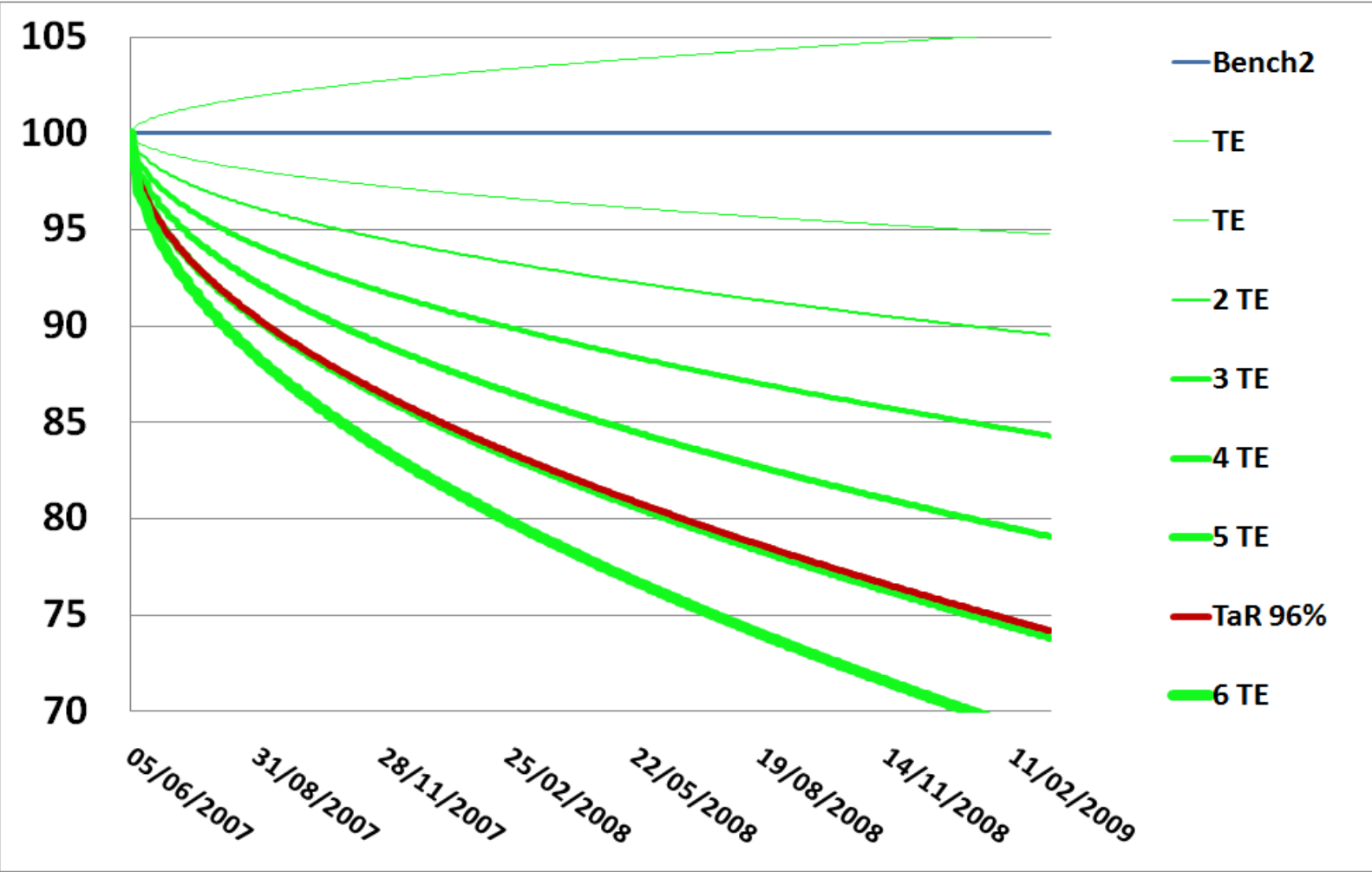
8,20

APT TRACKING ERROR & TRACKING AT RISK

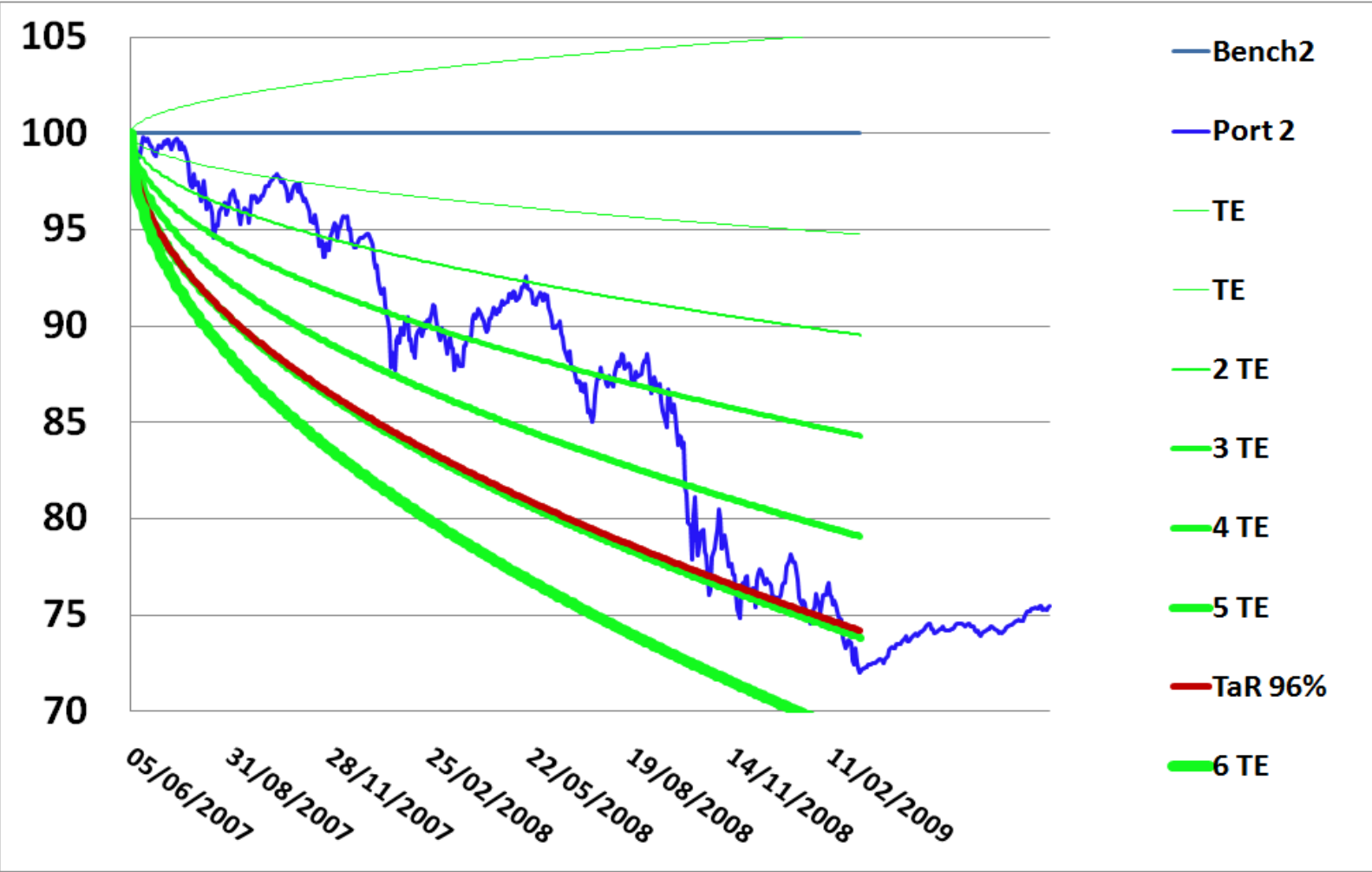


Gaussian Analysis
=
« This should never have occurred »

APT TRACKING ERROR & TRACKING AT RISK



APT TRACKING ERROR & TRACKING AT RISK



Relative Risk measures

Tracking Error

TE

Tracking at Risk

TaR

« *There was still 2% of chance to go lower* »

CONTENTS



I. Risk Models

1. Variance Covariance Matrix
2. Single Factor Model
3. Prespecified Multi Factor Model
4. APT Statistical Multi Factor Model

II. The need for Statistical Multi Factor Models

1. Risk attribution
2. Risk measurement

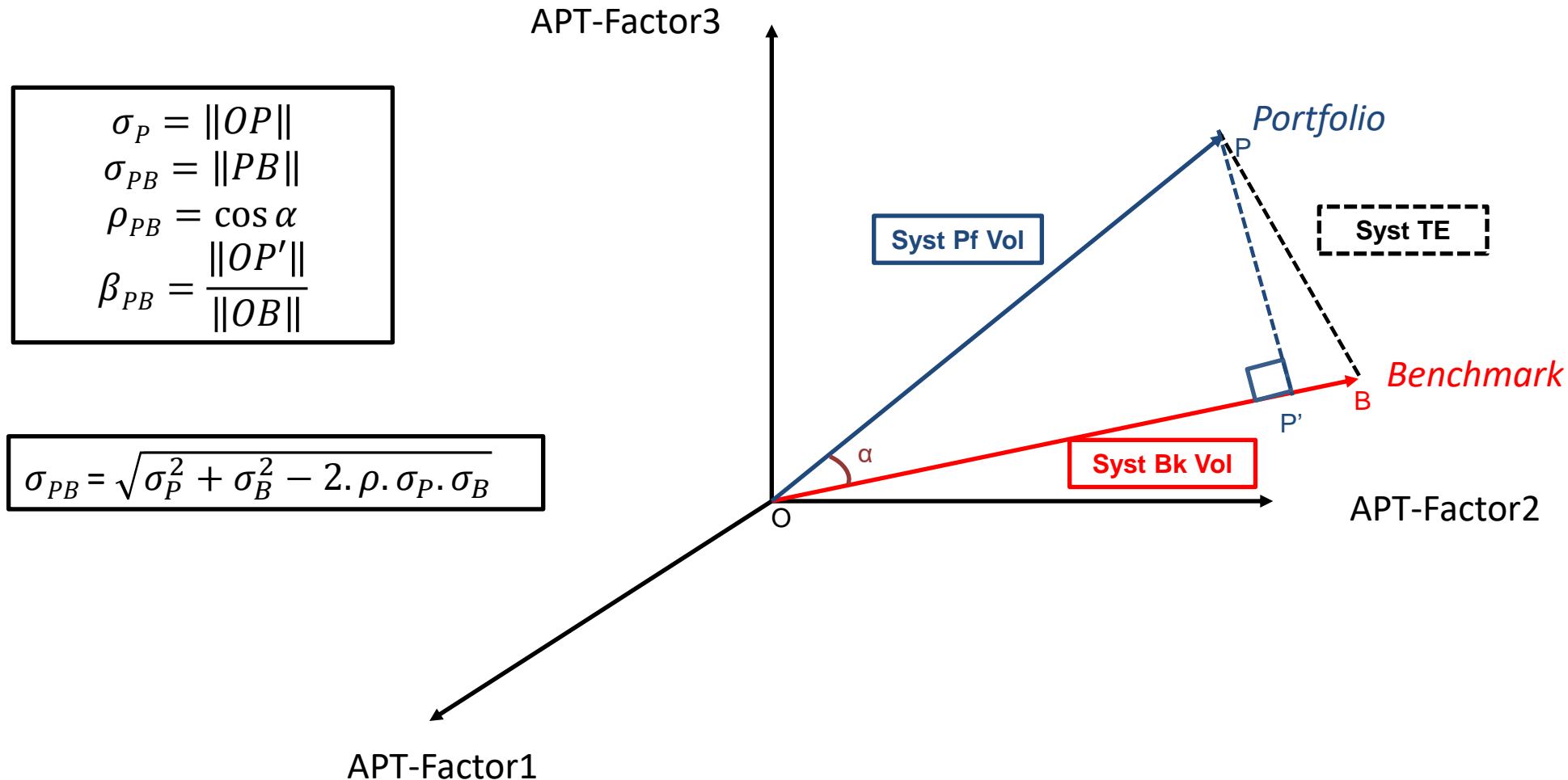
III. Risk Indicators

1. Top Level Risk Indicators
2. Position Based Risk Indicators

IV. Risk Analysis

RELATIONSHIP BETWEEN RISK INDICATORS

Relationship between Risk Indicators



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APT EX-ANTE MARGINAL RISK

Definition :

Marginal Risk is a measure of the sensitivity of the volatility of a portfolio to a change in the weight of a specific security. *Marginal Risk* is defined as the partial derivative of the portfolio's volatility with respect to the weight of the security.

$$\partial_s [\sigma_p] = \frac{\partial \sigma_p}{\partial w_s}$$

$\partial_s [\sigma_p]$ = Marginal Risk of security s in portfolio P

σ_p = Ex-ante volatility of portfolio P

w_s = Portfolio weight in security s

Source: APT Analytics Guide

APT EX-ANTE MARGINAL RISK

$$\sigma_p^{(1)} = \sigma_p + \delta\omega_s \partial_s [\sigma_p]$$

Where:

- $\sigma_p^{(1)}$ = Estimated new forecast volatility in annual portfolio return
- σ_p = Forecast volatility of annual portfolio return
- $\delta\omega_s$ = Change to portfolio weight in security s
- $\partial_s [\sigma_p]$ = Marginal Risk of security s

Other formula

$$\partial_s [\sigma_p] = \rho_{P,S} \cdot \sigma_s$$

Source: APT Analytics Guide

RISK MANAGEMENT – MARGINAL CONTRIBUTION TO VOLATILITY

Preferences :

Loomis Sayles US Growth

Absolute Risk

Position	Port. Weight	Marginal Volatility	Standalone Volatility
1 SHOPIFY SUBD.VTG.SHS.1	2.61%	47.27%	67.09%
2 BLOCK A	1.08%	44.82%	63.15%
3 NVIDIA	7.84%	41.87%	53.77%
4 TESLA	8.71%	37.70%	59.28%
5 NETFLIX	4.90%	30.64%	50.88%
6 META PLATFORMS A	7.32%	29.12%	47.46%
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19 ALPHABET 'A'	6.21%	18.72%	28.62%
20 NIKE 'B'	0.92%	17.07%	31.87%

	Port.
Total Ex-ante Volatility	24.12%

RISK MANAGEMENT – MARGINAL CONTRIBUTION TO VOLATILITY

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Absolute Risk

	Port.
Total Ex-ante Volatility	24.12%

Simulation

	Port.
Total Ex-ante Volatility	23.94%

Δ Total Ex-ante Volatility	-0.18%
----------------------------	--------

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Simulation

	Port.
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Δ Total Ex-ante Volatility	-0.18%
----------------------------	--------

Δ Expected return	+0.15%
-------------------	--------

RISK MANAGEMENT – MARGINAL CONTRIBUTION TO VOLATILITY

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Total Ex-ante Volatility	24.12%

Simulation

	Port.
Total Ex-ante Volatility	23.94%

Δ Total Ex-ante Volatility	-0.18%
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Δ Expected return	+0.15%
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RISK MANAGEMENT – MARGINAL CONTRIBUTION TO VOLATILITY

Preferences :

Loomis Sayles US Growth

Absolute Risk

Position	Port. Weight	Marginal Volatility	Standalone Volatility	Contribution to Volatility	Contribution to Vol (%)
1 SHOPIFY SUBD.VTG.SHS.!	2.61%	47.27%	67.09%	1.24%	5.12%
2 BLOCK A	1.08%	44.82%	63.15%	0.49%	2.01%
3 NVIDIA	7.84%	41.87%	53.77%	3.28%	13.60%
4 TESLA	8.71%	37.70%	59.28%	3.28%	13.62%
5 NETFLIX	4.90%	30.64%	50.88%	1.50%	6.22%
6 META PLATFORMS A	7.32%	29.12%	47.46%	2.13%	8.84%
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10 PAYPAL HOLDINGS	0.99%	25.54%	40.07%	0.25%	1.04%
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RISK ANALYSIS

Fund	Benchmark	Total Volatility	% Systematic Vol	% Specific Vol	Bmk Volatility	TE	% Systematic TE	% Specific TE	Syst Correlation	Syst Beta
Vaughan Nelson U.S. Select Equity	S&P 500	21.4%	95%	5%	17.9%	5.7%	49%	51%	0.993	1.16
Harris Associates U.S. Value Equity	S&P 500 Value	17.9%	98%	2%	14.7%	7.3%	77%	23%	0.939	1.14
Loomis Sayles US Growth Equity	S&P 500 Growth	23.3%	92%	8%	21.9%	5.5%	31%	69%	0.991	1.04
BNP Paribas Sust US Value Mlt-Fctr Eq	S&P 500 Value	12.9%	97%	3%	14.7%	4.5%	56%	44%	0.979	0.85