

Note: $\text{tr}(\underline{C}) = \text{tr}(\underline{b})$

$$\psi = \frac{\mu}{2} [\text{tr}(\underline{C}) - \dim - 2 \ln(J)] + \lambda \ln^2(J) \quad ; \quad S = 2 \frac{d\psi}{d\underline{C}}, \quad H = 4 \frac{d^2\psi}{d\underline{C} \otimes d\underline{C}}$$

$$\frac{dJ}{d\underline{C}} = \frac{1}{2} J \underline{C}^{-1} \quad \frac{d\underline{C}^{-1}}{d\underline{C}} \Rightarrow \left[\frac{d\underline{C}^{-1}}{d\underline{C}} \right]_{ABCD} = -\frac{1}{2} [C_{AC}^{-1} C_{BD}^{-1} + C_{AD}^{-1} C_{BC}^{-1}]$$

$$\begin{aligned} \frac{d\psi}{d\underline{C}} &= \frac{\mu}{2} \left[\frac{d(\text{tr}(\underline{C}))}{d\underline{C}} - \underline{0} - 2 \cdot \frac{1}{J} \cdot \frac{dJ}{d\underline{C}} \right] + \lambda \cdot 2 \ln(J) \cdot \frac{d(\ln(J))}{d\underline{C}} \\ &= \frac{\mu}{2} \left[\underline{I} - \frac{2}{J} \cdot \frac{dJ}{d\underline{C}} \right] + \lambda \cdot 2 \ln(J) \cdot \frac{1}{J} \cdot \frac{dJ}{d\underline{C}} \\ &= \frac{\mu}{2} \underline{I} - \left[\frac{\mu}{2} \cdot \frac{2}{J} - \lambda \cdot 2 \ln(J) \cdot \frac{1}{J} \right] \frac{dJ}{d\underline{C}} \\ &= \frac{\mu}{2} \underline{I} - \left[\mu - 2\lambda \ln(J) \right] \frac{1}{J} \cdot \frac{1}{2} J \underline{C}^{-1} \\ &= \frac{\mu}{2} \underline{I} - \frac{1}{2} [\mu - 2\lambda \ln(J)] \underline{C}^{-1} \end{aligned}$$

$$\begin{aligned} \tau = F S F^T = \chi(S) &= F (\mu \underline{I} - [\mu - 2\lambda \ln(J)] \underline{C}^{-1}) F^T \\ &= \mu \underline{b} - [\mu - 2\lambda \ln(J)] \underline{I} \end{aligned}$$

$$\begin{aligned} \frac{dz\psi}{d\underline{C} \otimes d\underline{C}} &= -\frac{1}{2} \left[\underline{C}^{-1} \otimes \frac{d}{d\underline{C}} [\mu - 2\lambda \ln(J)] + [\mu - 2\lambda \ln(J)] \frac{d\underline{C}^{-1}}{d\underline{C}} \right] \\ &= -\frac{1}{2} \left[\underline{C}^{-1} \otimes \left[-2\lambda \frac{1}{J} \cdot \frac{dJ}{d\underline{C}} \right] + [\mu - 2\lambda \ln(J)] \frac{d\underline{C}^{-1}}{d\underline{C}} \right] \\ &= -\frac{1}{2} \left[\underline{C}^{-1} \otimes \left[-2\lambda \frac{1}{J} \cdot \frac{1}{2} J \underline{C}^{-1} \right] + [\mu - 2\lambda \ln(J)] \frac{d\underline{C}^{-1}}{d\underline{C}} \right] \\ &= +\frac{\lambda}{2} \underline{C}^{-1} \otimes \underline{C}^{-1} + \frac{1}{2} [\mu - 2\lambda \ln(J)] \left(-\frac{d\underline{C}^{-1}}{d\underline{C}} \right) \end{aligned}$$

$$\chi(\underline{C}^{-1} \otimes \underline{C}^{-1}) = \underline{I} \otimes \underline{I}$$

$$\chi\left(-\frac{d\underline{C}^{-1}}{d\underline{C}}\right) = \underline{S}$$

$$\begin{aligned} J\underline{C} = \chi\left(4 \frac{dz\psi}{d\underline{C} \otimes d\underline{C}}\right) &= 4 \left[\frac{\lambda}{2} \underline{I} \otimes \underline{I} + \frac{1}{2} [\mu - 2\lambda \ln(J)] \underline{S} \right] \\ &= 2\lambda \underline{I} \otimes \underline{I} + 2 [\mu - 2\lambda \ln(J)] \underline{S} \end{aligned}$$

"action" $\Rightarrow J\underline{C} : (\cdot) = 2[\mu - 2\lambda \ln(J)] (\cdot) + 2\lambda \text{tr}(\cdot) \underline{I}$