Computational MRI Coursework 2

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December 2020

1 Problem 1

1.1 Task 1, part 1

In this problem a 1D line of 21 spins is modelled. They are spaced 1mm apart over a distance of 20mm along the x axis. These spins are in a static magnetic field B0 which is aligned in the z direction. The spins are exited via a B1 pulse along the x axis which causes them to flip towards the y direction in 0.32ms. The spins then relax for time $T_s = 5.12ms$. In this time, the spins gradually relax via T1 and T2 relaxation back towards the z direction. In this animation,the spins are viewed in the transverse plane after the B1 flip. Therefore only T2 relaxation is included in the code as it has the exact same visual effect as T1 relaxation would in this case. A T2 time of 10ms was chosen here in order for the signal to visibly decay exponentially over time. As viewed in the transverse plane, all of the spins remain aligned facing the y direction as there are no magnetic gradients.

1.2 Task 1, part 2

Now the spins are affected with a gradient along the x axis of strength -4.6mT/m after the B1 pulse. This causes a change in the precession frequencies of the spins based on their position along the x axis. The reference frame is for the spin in the centre. Therefore this spin remains stationery throughout the duration of the gradient. The spins furthest from the middle experience the greatest difference in field strength compared with the central spin, therefore these ones precess at the greatest speeds. Over the time, the transverse magnetisation gets smaller as the spins become more out of phase.

1.3 Task 1, part 3

In this section we now consider a negative gradient on for $\frac{T_s}{2}$ and a positive gradient on for T_s. The negative gradient causes the spins to de-phase as they did before. But when the gradient is reversed, the spins eventually join up and produce an echo at time 5.44ms. 5.44ms coincides with the positive gradient being on for time $\frac{T_s}{2}$. This means that the frequencies accumulated in the dephasing lobe of the readout are completely cancelled by those in the re-phasing lobe. The echo occurs only for the y component of transverse magnetisation. The x component is always zero due to the two ends of the 1D segment cancelling out. At this time all the spins are aligned and produce a sinc function as a signal. After the echo, the spins become out of phase once again and the signal drops.

1.4 Task 2

Increasing L from 20mm has the effect of increasing the number of oscillations of the signal. This is due to the spins being further from the isocentre, hence they experience stronger magnetic field gradients. This causes the spins to have greater variance in precession frequency, hence more oscillations in the signal.

Increasing G_x has the same effect as increasing L. As we are considering a linear magnetic field gradient in units of $\frac{mT}{m}$, increasing the gradient has the same implication of increasing the distance from the isocentre. Therefore we also see an increase in the number of oscillations.

Increasing N gives a clearer signal. This is due to more data points being recorded.

Increasing T_s causes the echo to occur later. This is due to the dephasing lobe being on for longer. Therefore the rephasing lobe has to be on for longer before the echo occurs. The sampling rate is effective reduced due to $\frac{T_s}{N}$ being greater. This results in lesser clarity in the signal.

1.5 Task 3

Gradients oriented in directions other than the x axis have no effect on the precession frequencies of the spins relative to each other. Therefore making no impact on the signal. For an echo to occur, the gradient must alter the precession frequencies of the spins. As the spins are aligned along the x axis, only gradients along the x axis will adjust the signal.

2 Problem 2

2.1 Task 2, part 1

For the two squares, a total of 9 spins per square was chosen. The squares each had a width of 10mm and were centred at (-5,-5) and (5,5). Each square had a spin at it's centre and there were two spins at the origin (where the squares touched). Computational intensiveness limited the number of spins that could be animated and the highest value of N that was feasible. The distance between the centres of 14.14mm was chosen as this gave a reasonable number of oscillations to be depicted in the signal.

The duration of the phase encoding gradient was chosen to be $\frac{T_s}{2}$ as this is identical to the length of the dephasing lobe of the readout gradient in the first problem. This ensured that the coverage of k space would be equivalent for both x and y.

The step change in magnitude of Δk_y was determined by the following equation (1). Here $\Delta G_y = \frac{2G_x}{256}$, $\tau_y = \frac{T_s}{2}$, γ is the gyro-magnetic ratio of the sample and $G_x = 4.6\frac{mT}{m}$. This gives us a Δk_y value of $0.092\frac{\gamma}{2\pi}\frac{mTms}{m}$.

$$\Delta k_y = \frac{\gamma}{2\pi}\Delta G_y \tau_y \tag{1}$$

With these parameters it is clear that the step change in the gradient for each of the 256 phase encoding steps would be $\frac{2G_x}{256} = 0.036\frac{mT}{m}$.

Firstly, the phase encoding gradient was turned on for time $\frac{T_s}{2}$ at a magnitude of $-4.6\frac{mT}{m}$ in the y direction. This had the effect of giving a positive My signal with a normalised amplitude of greater than 0.7 with T2 effects included. This is due to the regular arrangement of spins in the square meaning that the phases regularly aligned in the positive y direction. Next, the phase encoding gradient was turned off and the frequency encoding gradient turned on in the x direction with the ADC on also. This caused the signal to drop to much smaller values due to a low signal amplitude at the time of the gradient switch.

After a total of 8 seconds, the gradient echo occurred due to the dephasing and rephasing readout lobes cancelling. The signal at this time is shown in figure 1. Interestingly the signal here is very similar as that in figure 2.The signal has effectively been flipped in the time domain with the trough of the signal on the right of the highest peak in figure 1, but on the left of the highest peak in figure 2. This is the same case with phase encoding gradients of -2.3 and +2.3 in figures 3 and 4 respectively where the signals evolve oppositely to each other with time. This could have interesting consequences when these signals come to be fourier transformed.

Another notable point is that there is no clear gradient echo after 8 seconds. Instead, the same form of signal seems to continue before and after this time. This is likely due to the regular arrangement of spins modelled and would likely not be the case with a more random arrangement, as we wouldn't get such regular constructive interference of spins.

In the bottom right corner of the videos, the trajectory through k_x and k_y is mapped out. The phase encoding gradient causes the trajectory to move along the y axis with $k_x = 0$. When the readout gradient is turned on, the trajectory moves first negative in the x direction, and then to the positive x. The echo occurs as the trajectory passes across the y axis. The following phase encoding gradients of (-4.6,-2.3,0,2.3 and 4.6)mT/m were chosen as these captured the same extent of k space as the frequency encoding gradient, but also included intermediate values and the origin.

2.2 Task 2, part 2

Next the phase and frequency encoding gradients were allowed to overlap with one another. Figure 5 shows the stage with $G_y = 4.6$mT/m. The signal here is identical to that of figure 2. This shows that having the phase and frequency encoding gradients on at the same time has no effect on the signal. It just results in the acquisition taking less time overall, as the echo occurs at 5.44ms rather than 8ms. This means that overlapping gradients are definitely a useful concept. Instead of the trajectory through k space being in the x and y directions solely, the trajectory now moves along both x and y with differing amounts as shown in the bottom right hand plot of figure 6.

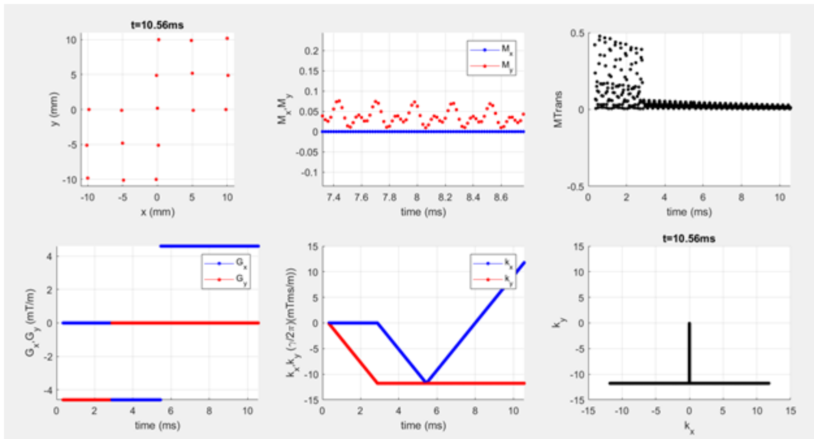

Figure 1: From problem 2 task 1 with $G_y = -4.6mT/m$.

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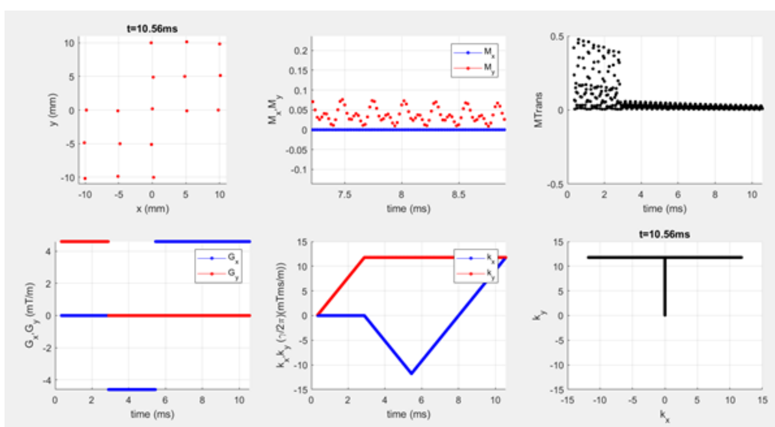

Figure 2: From problem 2 task 1 with $G_y = 4.6mT/m$.

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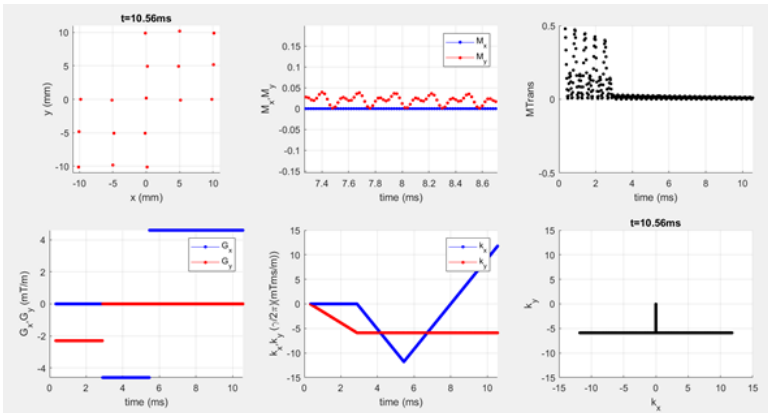

Figure 3: From problem 2 task 1 with $G_y = -2.3mT/m$.

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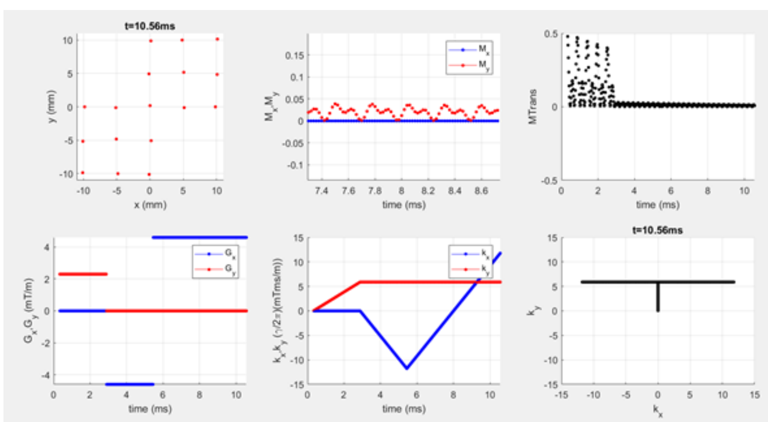

Figure 4: From problem 2 task 1 with $G_y = 2.3mT/m$.

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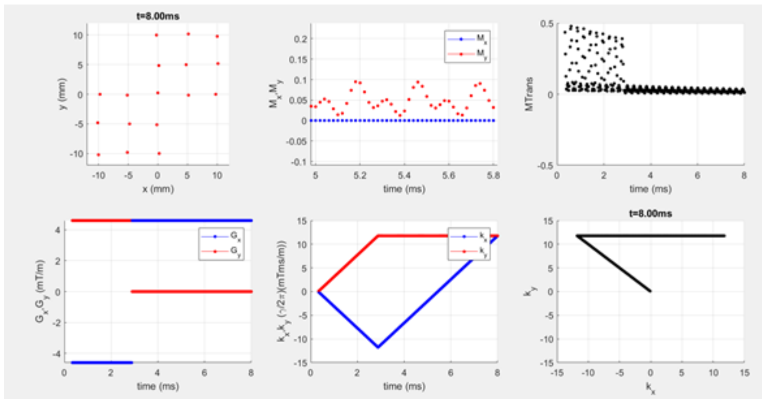

Figure 5: From problem 2 task 2 with $G_y = 4.6mT/m$.

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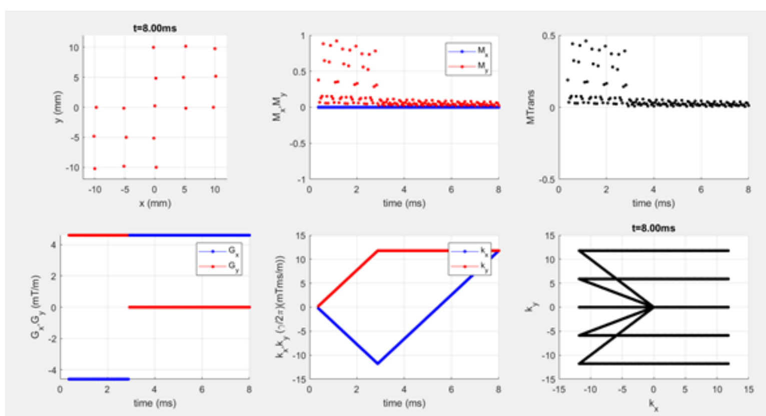

Figure 6: From problem 2 task 2 with all lines of k space covered.