Quiz 3 Ruirong Huang PCNS

1.

(a) state space: $\left\{ X_1 = \begin{pmatrix}1\\1\\0\\0\\0\end{pmatrix} \quad X_2 = \begin{pmatrix}0\\0\\1\\0\\0\end{pmatrix} \quad X_3 = \begin{pmatrix}0\\0\\0\\1\\0\end{pmatrix} \quad X_4 = \begin{pmatrix}0\\0\\0\\0\\1\end{pmatrix} \right\}$

(b) incompletely reducible

(c) states in P_1 are persistent (X_1, X_2)

states in P_2 are transient (X_3, X_4)

(d) the it becomes a completely reducible network, and state is P_1 / P_2 can only be reached from states in P_1 / P_2, respectively.

In this case, as the initial state is X_4, X_1 and X_2 can never be reached.

(e) it depends on initial condition generally.

In this case, the initial case is X_4, so $\bar{P}_1 = \bar{P}_2 = 0$, $\cancel{X_3}$ $\bar{P}_3 + \bar{P}_4 = 1$

2. (a)

$$V(\tilde{x};\Omega) \triangleq -\frac{1}{\Omega}\ln \frac{\bar{P}_{\tilde{X}}(\tilde{x};\Omega)}{\bar{P}_{\tilde{X}}(\tilde{x}_*;\Omega)} \geq 0$$

(b) from (a), $\bar{P}_{\tilde{X}}(\tilde{x};\Omega) = \dfrac{1}{\sum_u \exp(-\Omega V(u;\Omega))} \exp(-\Omega V(\tilde{x};\Omega))$

(c) as $\Omega \to \infty$, minima of $V_0(\tilde{x})$ is ground state and macroscopic mode of the network.

(d) Keizer's paradox :

$$\underbrace{\lim_{\Omega\to\infty}\lim_{t\to\infty} P_{\tilde{X}}(\tilde{x};t,\Omega)}_{\text{ME}} \neq \underbrace{\lim_{t\to\infty}\lim_{\Omega\to\infty} P_{\tilde{X}}(\tilde{x};t,\Omega)}_{\text{Macro}}$$

when there're more than one global / local minimum of V_0.

(e) when Ω is not large enough, minimizing V may not minimize V_0, so some of the modes will be "polluted" by this "noise", which means they're biased from real macroscopic modes when Ω is large enough.

3.

(a) $S(t) = -\sum_{X} P_X(x;t) \ln P_X(x;t)$

(b) $F(t) = U(t) - S(t)$ where $U(t) = \sum_{X} E(x) P_X(x;t)$

$= \sum_{X} P_X(x;t) \ln\left(\frac{P_X(x;t)}{\bar{P}_X(x)}\right)$

(c) $\sigma(t)$ is the entropy production rate

$h(t)$ is the heat dissipation rate

$f(t)$ is the motive power

(d) $\sigma(t) = h(t) = f(t) \geq 0$

(e) $\sigma(t) = h(t) = f(t) = 0$ the system "dies" at thermodynamic equilibrium

4.

(a) X_{2l-1} means quiet ~~neurons~~ neuron (l)

X_{2l} means active neuron (l)

W_{ij} means the weight of snaptic between neuron i and neuron j

(b)

first reaction: a quiet neuron is activated by other neurons and becomes active

second reaction: an active neuron becomes quiet.

(C) $\phi_l(x)$ is net overall synaptic input to neuron l

r_l is the degration rate of neuron l (from active to quiet)

(d) as indicated by the formulas,

$\bar{P}(\Omega)$ means the changing rate of H (free Helmholtz energy) according to the change of Ω (size)

$\bar{B}(\Omega)$ means the changing rate of pressure according to size, which indicates the robustness of network.

If $\bar{B}(\Omega)$ is small, we say that the network is robust (the pressure change is small)