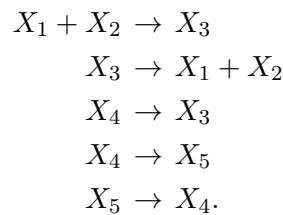


QUIZ #3

Name: _____

1. Consider a Markovian reaction network comprised of the following five species and reactions:



The system is initialized with $X_1(0) = X_2(0) = X_3(0) = X_4(0) = 0$ and $X_5(0) = 1$. In this case, the master equation can be written as

$$\frac{d\mathbf{p}(t)}{dt} = \mathbf{P}\mathbf{p}(t),$$

where the matrix \mathbf{P} is given by

$$\mathbf{P} = \begin{bmatrix} -\kappa_1 & \kappa_2 & 0 & 0 \\ \kappa_1 & -\kappa_2 & \kappa_3 & 0 \\ 0 & 0 & -(\kappa_3 + \kappa_4) & \kappa_5 \\ 0 & 0 & \kappa_4 & -\kappa_5 \end{bmatrix},$$

where κ_1 is the specific probability rate constant of the first reaction, κ_2 is the specific probability rate constant of the second reaction, etc.

- (a) What is the state-space of this system?
- (b) Is the network irreducible, completely reducible, or incompletely reducible?
- (c) Which states are persistent and which are transient?
- (d) If $\kappa_3 = 0$, what happens to the system?
- (e) In the case of (d), what is the probability distribution $\bar{\mathbf{p}}$ of the population process at steady-state?

2. Let $\bar{p}(\tilde{\mathbf{x}})$ be the steady-state distribution of the population density process in a Markovian reaction network of size Ω .
 - (a) Define the potential energy function $V(\tilde{\mathbf{x}}; \Omega) \geq 0$.
 - (b) What is the form of $\bar{p}(\tilde{\mathbf{x}})$ in terms of $V(\tilde{\mathbf{x}}; \Omega)$?
 - (c) If $V_0(\tilde{\mathbf{x}}) = \lim_{\Omega \rightarrow \infty} V(\tilde{\mathbf{x}}; \Omega)$, what is the relationship between the minima of $V_0(\tilde{\mathbf{x}})$ and the stable macroscopic states of the Markovian reaction network at steady-state?
 - (d) What is *Keizer's paradox*?
 - (e) What is a noise-induced mode and how these modes may be generated in a complex network?

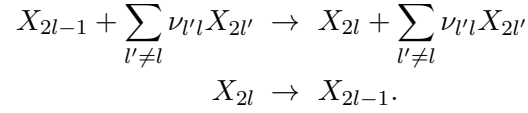
3. In a Markovian reaction network, two thermodynamic balance equations are given by

$$\frac{dS(t)}{dt} = \sigma(t) - h(t) \quad \text{and} \quad \frac{dF(t)}{dt} = f(t) - \sigma(t),$$

where $S(t)$ is the entropy of the system at time t and $F(t)$ is the Helmholtz free energy at time t .

- (a) Define $S(t)$.
- (b) Define $F(t)$.
- (c) What are the thermodynamic roles of $\sigma(t)$, $h(t)$ and $f(t)$?
- (d) How are these quantities related at steady-state?
- (e) How are these quantities related if the system reaches thermodynamic equilibrium at steady-state? What happens to the system in this case?

4. The neural network model discussed in class is comprised of L neurons and is characterized by the reactions:



- (a) What do X_{2l-1} , X_{2l} denote? What does ν_{ij} measure and what is the meaning of its values?
- (b) What do these reactions model?
- (c) The propensity function of the previous reactions are given by

$$x_{2l-1} [\phi_l(\mathbf{x}) > 0] \tanh(\phi_l(\mathbf{x})) \quad \text{and} \quad \gamma_l x_{2l},$$

where $[]$ is the Iverson bracket. What is the form and meaning of $\phi_l(\mathbf{x})$ and γ_l ?

- (d) The pressure $\bar{P}(\Omega)$ and bulk modulus $\bar{B}(\Omega)$ in a neural network with size parameter Ω at steady-state are given by

$$\bar{P}(\Omega) = -\frac{\partial \bar{H}(\Omega)}{\partial \Omega} \quad \text{and} \quad \bar{B}(\Omega) = -\Omega \frac{\partial \bar{P}(\Omega)}{\partial \Omega},$$

where $\bar{H}(\Omega)$ is the Helmholtz free energy. How can these quantities be used in order to quantify robustness and critical behavior in the network? What does “robustness” mean in this case?

