Quantum Information 2020
Sheet 4
Hints: 25/03/2020 Hand-in: 01/04/2020 Solutions: 08/04/2020

There will be no penalty for late hand-in, but handing in on-time will allow the TAs to address your problems.

1. Alternative Pauli Basis States

There are an infinite number of possible single qubit states. From a theoretical standpoint, the one we choose to label $|\,0\rangle$ is arbitrary. So let's consider the following alternative.

$$|\,\bar{0}\rangle = \cos(\theta)\,|\,0\rangle + \sin(\theta)\,|\,1\rangle$$

(a) For this $|\,\bar{0}\rangle$, find a corresponding orthogonal state $|\,\bar{1}\rangle$.

(b) For this basis $|\,\bar{0}\rangle$ and $|\,\bar{1}\rangle$, find mutually unbiased basis states $|\,\bar{+}\rangle$ and $|\,\bar{-}\rangle$.

(c) Now find the basis states $|\,\bar{\circlearrowright}\rangle$ and $|\,\bar{\circlearrowleft}\rangle$ for a third mutually unbiased basis.

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2. Eigenstates of the Pauli Matrices

Note: Sometimes Pauli matrices are written as X, Y, and Z, and sometimes as σ_x, σ_y and σ_z. For the most part, the convention is an arbitrary choice. Once you've used them enough, you'll hardly notice the difference (to the great annoyance of your students)

The Pauli matrices are defined

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

(a) Show that each squares to the identity matrix.

$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

(b) Find the eigenvalues and eigenvectors of each.

(c) The Hadamard matrix is expressed

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}.$$

Show that this also squares to the identity, and has the same eigenvalues of the Pauli matrices.

(d) Write the Hadamard as a sum of Pauli matrices.

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