## Quantum Information

2020

Sheet 5

Hints: 01/04/2020

Hand-in: 08/04/2020

Solutions: 15/04/2020

There will be no penalty for late hand-in, but handing in on-time will allow the TAs to address your problems.

## 1. Single Qubit Clifford Gates

If a gate U is a Clifford gate, the following property holds

$$UPU^{\dagger} \sim \pm P'$$
.

Here both P and P' are Pauli operators, and the  $\sim$  denotes equality up to a factor of  $\pm 1$  or  $\pm i$ 

- (a) Show that the Pauli operators themselves are Clifford gates.
- (b) Show that H, S and  $S^{\dagger}$  are Clifford gates.
- (c) Show that  $T = S^{1/2}$  is not a Clifford gate.

## 2. Two Qubit Clifford Gates

For more than one qubit, Clifford gates map between tensor products of Pauli operators. For two qubits

$$U(P \otimes Q) U^{\dagger} \sim \pm (P' \otimes Q').$$

Here P, P' Q and Q' are all Pauli operators.

- (a) Show that the controlled-NOT is a Clifford gate.
- (b) Show that the controlled-Hadamard is not a Clifford gate.

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