1. Effects of garbage in quantum circuits

The CNOT gate performs the mapping $(i, j) \to (i, i + j \bmod 2)$ from an input on two bits $(i, j \in \{0, 1\})$ to an output on two bits. It is interesting to note (though not relevant to the problem) that this mapping is not of the form $(z, 0) \to (z, f(z))$ as considered last week, since the function $f(i, j) = i + j \bmod 2$ does not require the whole input to be present in the output for reversibility.
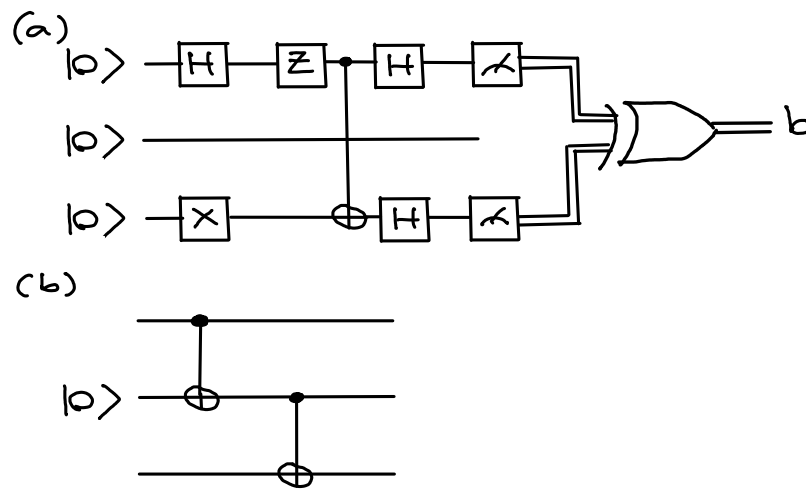

Figure 1: Circuits!

(a) Consider the circuit in Fig. 1 (a). This is composed of both a quantum part, and an irreversible classical part. Determine the value of the output bit b.

(b) Consider the circuit in Fig. 1 (b). Show that this effectively acts as a CNOT between the first and third qubits, but with the additional effect of producing a garbage output on the second.

(c) Replace the CNOT in Fig. 1 (a) with the element in Fig. 1 (b). Show that this changes the behaviour of the output bit b.

(d) Use uncomputation to 'correct' Fig. 1 (b) by removing the garbage.