Quantum Information 2020

Sheet 7

Hints: 15/04/2020 Hand-in: 22/04/2020 Solutions: 29/04/2020

There will be no penalty for late hand-in, but handing in on-time will allow the TAs to address your problems.

1. Teleportation

Consider an arbitrary state of the form

$$|\psi\rangle = a\,|0\rangle + b\,|1\rangle\,.$$

The qubit in this state belongs to Alice. She and Bob also have one qubit each of a Bell state

$$\left|\Phi^+\right\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right).$$

The total state can then be written as

$$|\psi\rangle = a\,|0\rangle \otimes \left|\Phi^+\right\rangle + b\,|1\rangle \otimes \left|\Phi^+\right\rangle.$$

Here the two qubits on the left (one as part of the Bell pair) belong to Alice. The one on the right belongs to Bob.

(a) Rewrite the state such that Alice's qubits are expressed in the Bell basis, and Bob's qubit is expressed in the z basis.

(b) Find the resulting state for Bob's qubit for each possible outcome of a Bell measurement on Alice's qubits.

(c) Find the single qubit rotation required by Bob in each case, to rotate his state to $|\psi\rangle$. Show that this depends only on the knowledge of Alice's result, and requires no knowledge of $|\psi\rangle$.

(d) Repeat all the above, but with Alice and Bob instead sharing two qubits in the $|\Psi^-\rangle$ state.

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