Quantum Information
Exercise 3

1. Mutually unbiased bases

Show that the X, Y and Z bases are all unbiased with respect to each other: each state for one basis results in a completely random result for the other two bases.

2. Shifting certainty

The state of a single qubit is characterized by three numbers, $\langle \sigma^x \rangle$, $\langle \sigma^y \rangle$ and $\langle \sigma^z \rangle$, defined as

$$\langle \sigma^\alpha \rangle = p_0^\alpha - p_1^\alpha$$

Calculate these for the following states and verify that $\langle \sigma^x \rangle^2 + \langle \sigma^y \rangle^2 + \langle \sigma^z \rangle^2$ remains constant.

a)

$$|\,\psi\rangle = \cos(\theta)\,|\,0\rangle + \sin\theta\,|\,1\rangle$$

b)

$$|\,\psi\rangle = \frac{|\,0\rangle + e^{i\theta}\,|\,1\rangle}{\sqrt{2}}$$

2. A useful matrix

Find a matrix σ^z such that

$$\langle \sigma^z \rangle = \langle \psi\,|\,\sigma^z\,|\,\psi\rangle \qquad \forall\,|\,\psi\rangle$$