
Quantum Information
Sheet 5

2020

Hints: 15/04/2020

Hand-in: 22/04/2020

Solutions: 29/04/2020

There will be no penalty for late hand-in, but handing in on-time will allow the TAs to address your problems.

1. Teleportation

Consider an arbitrary state of the form

$$|\psi\rangle = a|0\rangle + b|1\rangle.$$

The qubit in this state belongs to Alice. She and Bob also have one qubit each of a Bell state

$$|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle).$$

The total state can then be written as

$$|\psi\rangle = a|0\rangle \otimes |\Phi^+\rangle + b|1\rangle \otimes |\Phi^+\rangle.$$

Here the two qubits on the left (one as part of the Bell pair) belong to Alice. The one on the right belongs to Bob.

- (a) Rewrite the state such that Alice's qubits are expressed in the Bell basis, and Bob's qubit is expressed in the z basis.
- (b) Find the resulting state for Bob's qubit for each possible outcome of a Bell measurement on Alice's qubits.
- (c) Find the single qubit rotation required by Bob in each case, to rotate his state to $|\psi\rangle$. Show that this depends only on the knowledge of Alice's result, and requires no knowledge of $|\psi\rangle$.
- (d) Repeat all the above, but with Alice and Bob instead sharing two qubits in the $|\Psi^-\rangle$ state.