

# exercise11-2

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## 1 Exercise Sheet 11 (Bonus)- MLP

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```
In [36]: # initialize numpy enviroment
%matplotlib notebook
import numpy as np
from numpy import square
import matplotlib.pyplot as plt
```

```
In [2]: # Code for visualizing MLP structure
```

```
def draw_neural_net(ax, left, right, bottom, top, layer_sizes):
    '''
        Draw a neural network cartoon using matplotilb.

    :usage:
        >>> fig = plt.figure(figsize=(12, 12))
        >>> draw_neural_net(fig.gca(), .1, .9, .1, .9, [4, 7, 2])

    :parameters:
        - ax : matplotlib.axes.AxesSubplot
            The axes on which to plot the cartoon (get e.g. by plt.gca())
        - left : float
            The center of the leftmost node(s) will be placed here
        - right : float
            The center of the rightmost node(s) will be placed here
        - bottom : float
            The center of the bottommost node(s) will be placed here
        - top : float
            The center of the topmost node(s) will be placed here
        - layer_sizes : list of int
            List of layer sizes, including input and output dimensionality
    '''
    n_layers = len(layer_sizes)
    v_spacing = (top - bottom)/float(max(layer_sizes))
    h_spacing = (right - left)/float(len(layer_sizes) - 1)
    # Nodes
```

```

for n, layer_size in enumerate(layer_sizes):
    layer_top = v_spacing*(layer_size - 1)/2. + (top + bottom)/2.
    for m in range(layer_size):
        circle = plt.Circle((n*h_spacing + left, layer_top - m*v_spacing), v_spacing/2.,
                             color='w', ec='k', zorder=4)
        ax.add_artist(circle)
# Edges
for n, (layer_size_a, layer_size_b) in enumerate(zip(layer_sizes[:-1], layer_sizes)):
    layer_top_a = v_spacing*(layer_size_a - 1)/2. + (top + bottom)/2.
    layer_top_b = v_spacing*(layer_size_b - 1)/2. + (top + bottom)/2.
    for m in range(layer_size_a):
        for o in range(layer_size_b):
            line = plt.Line2D([n*h_spacing + left, (n + 1)*h_spacing + left],
                              [layer_top_a - m*v_spacing, layer_top_b - o*v_spacing])
            ax.add_artist(line)

```

## 1.1 Structure of the Multiple Layer Perceptron

We use the classic structure of Multiple Layer Perceptron, with

1. Input Layer: 2 input neurons and 1 bias unit
2. One Hidden Layer: 2 hidden neurons with 1 bias Unit
3. Output Layer: 1 output neurons

### 1.1.1 Bias unit

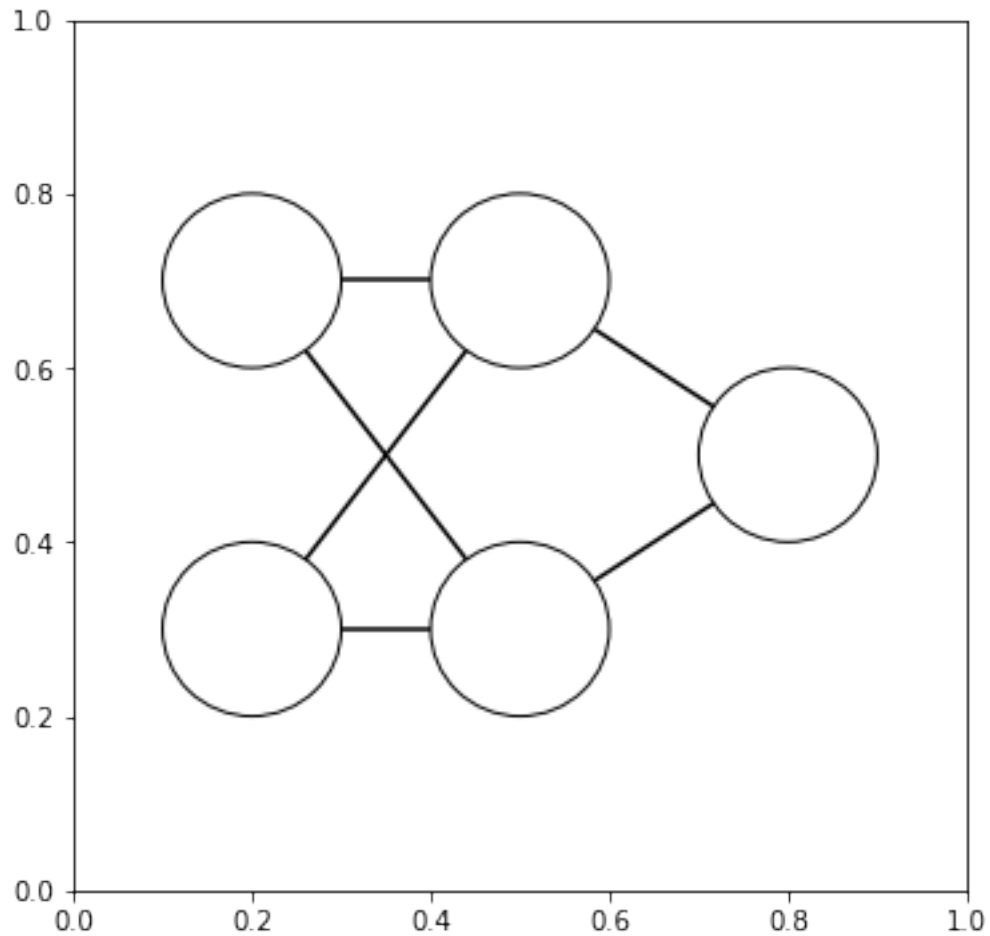
Bias unit guarantees that it is a affine combination of previous layer inside of the neuron, increasing the model flexibility of each neurons.

The following is the visualization of the MLP ( note that bias unit is emitted)

```

In [3]: mlp_fig = plt.figure(figsize=(6, 6))
        draw_neural_net(mlp_fig.gca(), .2, .8, .1, .9, [2, 2, 1])

```



## 1.2 Implementation of the MLP

Firstly, let's define the necessary activation function  $\sigma$  and the quadratic loss function

```
In [4]: def sigmoid(x):  
        """Sigmoid Function  
        Input: A vector in any dimension  
        Output: Applying Sigmoid function elementwise  
        """  
        exp_part = np.exp(-x)  
        return 1 / (1 + exp_part)  
  
def sigmoid_grad(x):  
    """Gradient of the sigmoid function  
    Input: A vector in any dimension  
    Output: Calculate Gradient of Sigmoid function elementwise  
    """  
    sig = sigmoid(x)
```

```

    return np.multiply(sig, 1 - sig)

def loss(y_pred, y):
    diff = y_pred - y
    np.sum(np.square(diff))

def mean_square_error(y_pred, y):
    m, n = y_pred.shape
    l = loss(y_pred, y)
    return l / m / n

```

The Gradient of the Sigmoid function is calculated using the following formula:

$$\sigma'(x) = \sigma(x) \cdot (1 - \sigma(x))$$

Given two weight matrix  $W_1, W_2$ , representing the weight from the input layer to the hidden layer and from the hidden layer to the output layer, including the bias term, we can conclude that  $W_1 \in \mathbb{R}^{2 \times 3}$  and  $W_2 \in \mathbb{R}^{1 \times 3}$ .

### 1.3 Forward Propagation

Our Entire Neural Network can be represent as a single function:

$$N : \mathbb{R}^2 \rightarrow \mathbb{R}$$

Such function will: 1. Associate the input  $X \in \mathbb{R}^2$  with a bias term, forming  $X_{biased}$  2. Calculate the offline combination

$$z_2 = W_1 \cdot X_{biased}$$

, where  $z_2 \in \mathbb{R}^2$  3. Apply the Sigmoid function, obtaining the value of the neurons

$$a_2 = \sigma(z_2)$$

, where  $a_2 \in \mathbb{R}^2$  4. Associate  $a_2$  with a bias term 5. Calculate the affine combination and return

$$z_3 = W_2 \cdot a_2$$

, where  $z_3 \in \mathbb{R}$  6. Calculate  $a_3 = \sigma(z_3)$ . Note that in the lecture note, we do not apply sigmoid function for the last layer. However, we applied sigmoid here because we want the result to be in the interval  $[0, 1]$  instead of  $\mathbb{R}$

```

In [5]: def neural_net(X, W1, W2):
        """Implementation of the MLP
        Input: takes input data X in 2 dimension, 2 weight matrix
        """

        # bias term
        X_b = np.ones((3,1))
        X_b[1:] = X
        a_1 = X_b

```

```

# calculate z_2
z_2 = np.matmul(W1, a_1)

# calculate a_2
a_2 = sigmoid(z_2)

# associate with bias
b_2 = np.ones((3, 1))
b_2[1:] = a_2
a_2 = b_2

# calculate z_3
z_3 = np.matmul(W2, a_2)

# calculate a_3
a_3 = sigmoid(z_3)

return a_3[0][0]

```

#### 1.4 Calculate the gradient with respect to $W_1$ and $W_2$ using Backpropagation

We are using backpropagation to train  $W^{(1)}$  and  $W^{(2)}$

First, perform a full forward propagation of the neural network to obtain  $a_1, a_2, a_3, z_2, z_3$

Second, calculate

$$\delta^{(3)} = a_k^{(3)} - y_k$$

$$\delta^{(2)} = W_2' \delta^{(3)} \cdot \sigma'(z^{(2)})$$

Finally, calculate the gradient at layer  $l$  using

$$\frac{\partial}{\partial W_{ij}^{(l)}} = \delta^{(l+1)} \cdot (a^{(l)})' \quad (1)$$

```

In [6]: def neural_net_train(X, W1, W2, Y):
        """Implementation of the MLP
        Input: takes input data X in 2 dimension, 2 weight matrix, y
        Output:
        """

        # bias term
        X_b = np.ones((3,1))
        X_b[1:] = X
        a_1 = X_b

        # calculate z_2
        z_2 = np.matmul(W1, a_1)

        # calculate a_2
        a_2 = sigmoid(z_2)

```

```

# associate with bias
b_2 = np.ones((3, 1))
b_2[1:] = a_2
a_2 = b_2

# calculate z_3
z_3 = np.matmul(W2, a_2)

a_3 = sigmoid(z_3)
# forward propagation end
# -----

# start of backpropagation

# delta 3 in 1 x 1
delta_3 = a_3 - Y

# delta 2 in 2 x 1
delta_2 = np.multiply(np.matmul(W2.T, delta_3)[1:], sigmoid_grad(z_2))

# we don't have delta 1
grad_w_1 = np.matmul(delta_2, a_1.T)

grad_w_2 = np.matmul(delta_3, a_2.T)
# grad_w_2 = np.matmul(delta_3.T, a_2)

return a_3, grad_w_1, grad_w_2# grad_w_2

```

## 1.5 Dealing with Multiple training data at the same time

The gradient formula is averaged. Namely

$$\frac{\partial}{\partial W_{ij}^{(l)}} = \frac{1}{m} \sum_{i=1}^m \delta_{(i)}^{(l+1)} \cdot (a_{(i)}^{(l)})'$$

```

In [7]: # accumulate the gradient result
def batch_neural_net_train(X, W1, W2, Y):
    # X contains m training examples as row vectors
    # X is m x 3
    m, _ = X.shape

    grad_w_1_acc = np.zeros((2,3))
    grad_w_2_acc = np.zeros((1,3))
    pred_all = np.zeros((m, 1))
    for i in range(m):
        pred, grad_w_1, grad_w_2 = neural_net_train(np.asmatrix(X[i]).T, W1, W2, np.as
        grad_w_1_acc = grad_w_1_acc + grad_w_1

```

```

grad_w_2_acc = grad_w_2_acc - grad_w_2
pred_all[i] = pred[0][0]
error = np.sum(np.square(pred_all - Y)) / m
return pred_all, grad_w_1_acc / m, grad_w_2_acc / m, error

```

## 1.6 Gradient Decent

We optimize our weight using gradient decent. We update the weight using the following formula:

$$W_{ij}^{(l)} = W_{ij}^{(l)} - \alpha * \frac{\partial}{\partial W_{ij}^{(l)}}$$

```

In [8]: def train(X, Y, step = 2000, alpha = 4.0):
        # initiate w_1, w_2
        w_1 = np.random.randn(2,3)
        w_2 = np.random.randn(1, 3)

        # train
        for i in range(step):
            pred, grad_w_1, grad_w_2, err = batch_neural_net_train(X, w_1, w_2, Y)
            w_1_new = w_1 + alpha * grad_w_1
            w_2_new = w_2 + alpha * grad_w_2
            w_1 = w_1_new
            w_2 = w_2_new
            print('step: {} - error: {}'.format(i, err))
        return w_1, w_2

```

We explicitly print the training error, and after the experiment, we find the optimal  $\alpha = 4.0$  and  $step = 2000$

```

In [9]: # build up a training data
        X = np.asmatrix([[1,1], [1,0], [0,1], [0, 0]])
        Y = np.asmatrix([[0], [1], [1], [0]])

        # train the neural network
        w_1, w_2 = train(X, Y)

```

```

step: 0 - error: 0.3633636945095289
step: 1 - error: 0.26172880815262456
step: 2 - error: 0.2533828480173811
step: 3 - error: 0.25070177852172537
step: 4 - error: 0.24988666935103154
step: 5 - error: 0.24955784725618854
step: 6 - error: 0.24936947115359331
step: 7 - error: 0.24921940356510075
step: 8 - error: 0.24907749784066913
step: 9 - error: 0.24893517119266312
step: 10 - error: 0.24878919721553783
step: 11 - error: 0.24863798039983362

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step: 12 - error: 0.24848032041076992  
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step: 1978 - error: 1.6148477347668138e-06  
step: 1979 - error: 1.61308599185117e-06



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step: 1983 - error: 1.6060676586119122e-06
step: 1984 - error: 1.6043202039578174e-06
step: 1985 - error: 1.6025755884445943e-06
step: 1986 - error: 1.6008338059403327e-06
step: 1987 - error: 1.59909485032996e-06
step: 1988 - error: 1.5973587155142043e-06
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step: 1996 - error: 1.5835704591459987e-06
step: 1997 - error: 1.5818594397305153e-06
step: 1998 - error: 1.580151181015909e-06
step: 1999 - error: 1.578445677081284e-06

```

The prediction is just performing the forward propagation algorithms.

```

In [10]: def predict(X, w_1, w_2):
          m, _ = X.shape
          pred = np.zeros((m, 1))
          for i in range(m):
              if neural_net(np.asmatrix(X[i]).T, w_1, w_2) > 0.5:
                  pred[i][0] = 1
              else:
                  pred[i][0] = 0
          return pred

```

```

In [11]: predict(X, w_1, w_2)

```

```

Out[11]: array([[0.],
                [1.],
                [1.],
                [0.]])

```

## 2 Bonus Question

We use the same neural network architecture. However, we give a different training set for the neural network.

## 2.1 Understanding *Sign* function

$$\text{Sign}(x) = \begin{cases} -1 & x < 0 \\ 0 & x = 0 \\ 1 & x > 0 \end{cases} \quad (2)$$

However, we can safely ignore the second case when  $x = 0$ , since the propability for such a case is 0.

### 2.1.1 Mapping the data into interval $[0, 1]$

We simply define  $S(\text{Sign}(x)) = \frac{\text{Sign}(x)+1}{2}$ . To recover, we have  $S^{-1}(x) = 2x - 1$

```
In [12]: def sign(x):
         if x > 0:
             return 1
         elif x == 0:
             return 0
         return -1

         def S(x):
             return (sign(x) + 1) / 2

         def S_inv(x):
             return 2 * x - 1
```

## 2.2 Initiate the data set

We initiate the dataset using a standard normal detribution. Let's take  $m = 4000$

```
In [13]: Xs = np.random.randn(4000, 2)
         Ys = []
         for data in Xs:
             Ys.append(S(data[0] * data[1]))
         Ys = np.asmatrix(Ys).T
```

## 2.3 Train the dataset

We use the same function for the training.

```
In [14]: M_1, M_2 = train(Xs, Ys, 500, 2.0)
```

```
step: 0 - error: 0.26187646415196025
step: 1 - error: 0.25486890930593475
step: 2 - error: 0.253977777982671055
step: 3 - error: 0.2532320007945011
step: 4 - error: 0.25254475027069334
step: 5 - error: 0.25191054337526547
step: 6 - error: 0.25132419724409055
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```

```
In [15]: Ys_pred = predict(Xs, M_1, M_2)
```

```
In [18]: # calculatethe the error ratio
         np.sum(np.square(Ys_pred - Ys)) / 4000
```

```
Out[18]: 0.24725
```

## 2.4 Error Analysis

24.725% of error is acceptable for such a simple neural network structure. Now we construct a function using the neural net and plot the function together with the original function

```
In [41]: def original_fct(x, y):
         return S(x * y)
         def approximate_fct(x, y):
             if neural_net(np.asmatrix([x, y]).T, M_1, M_2) > 0.5:
                 return 1
             return 0
```

```
In [42]: from mpl_toolkits.mplot3d import axes3d, Axes3D #<-- Note the capitalization!
         fig = plt.figure()
         ax = ax = Axes3D(fig)

         x = y = np.arange(-1.0, 1.0, 0.05)
         X, Y = np.meshgrid(x, y)
         z1s = np.array([original_fct(x,y) for x,y in zip(np.ravel(X), np.ravel(Y))])
         Z1 = z1s.reshape(X.shape)
         z2s = np.array([approximate_fct(x,y) for x,y in zip(np.ravel(X), np.ravel(Y))])
         Z2 = z2s.reshape(X.shape)

         ax.plot_surface(X, Y, Z1)
         ax.plot_surface(X, Y, Z2)
         ax.set_xlabel('X')
         ax.set_ylabel('Y')
```

```
ax.set_zlabel('Value')
```

```
plt.show()
```

```
<IPython.core.display.Javascript object>
```

```
<IPython.core.display.HTML object>
```