Level 1 – AS91026 – 1.1 – 4 Credits – Internal

Apply numeric reasoning in solving problems

Sample question: Holiday

Mike and Huia are planning to travel to England for a holiday. They plan to be in England for three months, and during that time take a three-week bus tour through France and Spain.

Calculate the amount of money Mike and Huia will need to save for their trip and how long it will take them to save it.

Mike and Huia estimate that for each day they are in England, between them they will need on average NZ$250 (to the nearest $50) when they are not on the bus trip.

Huia earns $850 each week. Mike earns $790 each week. Huia is able to save two fifths of her income and Mike 35% of his income to put towards the trip.

The cost for the return air tickets is $2500 each.

The cost of travel insurance is $385 plus GST (15%) for the two of them.

The three-week bus trip costs 2000 Great Britain pounds per person.

Great Britain pounds are used as currency in England.

The Exchange rate is: 1 NZ Dollar = 0.5091 GB Pound (April 2012).

Grade requirements

The requirements for each grade are below.

	Achieved	**Merit**	**Excellence**
Skills	Use three skills correctly to calculate some of the information needed to solve the problem	Use **enough** skills (around 4-5) correctly to calculate **all** of the information needed to solve the problem	
Sequence	*No sequence of steps*	Link the skills together in a logical sequence of steps to solve the problem	
Context	*No context, only calculations*	Relate the calculations to the context, explaining what each calculation means	
Insight	*No insight, only calculations in context and theoretical models*		Show insight. This could include: • understanding the consequences of using estimated or rounded numbers, • rounding calculations appropriate to the context, • realising the likely differences between the theoretical model and reality

Lesson 0.99999…[1] : Operations on integers

Addition and subtraction

You know how to add and subtract numbers. But it can be confusing when negative numbers are involved. You'll recap how those work now. Use algebra tiles to show how it happens.

Show what happens when you take $4 + 3$	Now what about $4 - 3$?

Go negative. What happens when you take $-6 + 4$?	Now what about $-6 - 4$?

Step it up. What happens when you take $5 + (-2)$?	Now what about $5 - (-2)$?

Go for gold. What happens when you take $-4 + (-2)$?	Now what about $-4 - (-2)$?

[1] See appendix C

Multiplication and division

You also know how to multiply and divide numbers. But do you remember having to memorise multiplication facts and tables? A lot of people say it's good to know multiplication tables and I agree with them. But I think there's a more important skill: knowing factors of numbers. For more about this, see appendix A. In the meantime, use algebra tiles to show multiplication and division when negative numbers are involved.

Show what happens when you take 3×3	Now what about $12 \div 6$?

Go negative. What happens when you take -5×3?	Now what about $-12 \div 4$?

Step it up. What happens when you take 5×-3?	Now what about $12 \div -4$?

Go for gold. What happens when you take -2×-3?	Now what about $-10 \div -2$?

Rules

Using these examples, you can make some rules to remind you how operations on integers work. Make as many rules as you would like to.

Lesson 1 questions

Here are some questions for each operation, only do as many as you need to. The first five questions in each set can be done with algebra tiles. The others will require the rules you just wrote. For more questions of each type, see the problems sets in appendix B.

Addition	**Subtraction**	**Multiplication**	**Division**
Q1. $12 + (-5)$	Q16. $-12 - 1$	Q31. 5×-5	Q46. $14 \div 2$
Q2. $-1 + 2$	Q17. $11 - 1$	Q32. -1×1	Q47. $-9 \div -3$
Q3. $4 + (-8)$	Q18. $-2 - (-8)$	Q33. 3×6	Q48. $-16 \div 4$
Q4. $-11 + (-3)$	Q19. $5 - 7$	Q34. -4×-2	Q49. $18 \div -9$
Q5. $11 + (-10)$	Q20. $-8 - (-12)$	Q35. -2×1	Q50. $-20 \div 5$
Q6. $-17 + (-47)$	Q21. $-2 - 18$	Q36. -2×-13	Q51. $-30 \div -3$
Q7. $-31 + 12$	Q22. $30 - (-10)$	Q37. -8×-4	Q52. $-12 \div 6$
Q8. $38 + 3$	Q23. $16 - 31$	Q38. 9×7	Q53. $1 \div -1$
Q9. $42 + (-42)$	Q24. $22 - (-37)$	Q39. -6×-2	Q54. $44 \div 11$
Q10. $31 + 19$	Q25. $-48 - (-1)$	Q40. -11×6	Q55. $30 \div -2$
Q11. $-32 + 49$	Q26. $42 - 43$	Q41. -13×-8	Q56. $0 \div -2$
Q12. $-2 + 17$	Q27. $-11 - 3$	Q42. 8×2	Q57. $130 \div -13$
Q13. $-23 + 28$	Q28. $-29 - (-43)$	Q43. -12×-1	Q58. $48 \div 6$
Q14. $44 + (-29)$	Q29. $14 - (-4)$	Q44. -8×-13	Q59. $-48 \div 12$
Q15. $-26 + 14$	Q30. $-11 - 42$	Q45. 10×12	Q60. $30 \div 5$

Lesson Two: Order of operations

What happens if you combine addition, subtraction, multiplication, and division into the same calculation? Not much, to be honest. But you do need to be aware of which order to do each operation in. Some time in history, a group of mathematicians got together and decided on an order which was great because everyone could agree on the same order. You could choose your own order but until you convince the rest of the mathematical world to switch to yours, you'll have to use BEDMAS.

B	Brackets are used to disrupt the usual order. Use them to do addition or subtraction before a multiplication, for example. Brackets can be nested within each other to create layers of brackets.
E	Exponents AKA powers. This also includes roots because roots are fractional powers.
D M	Multiplication. This also includes division because division is multiplication by a fraction. Example:
A S	Addition. This also includes subtraction because subtraction is addition with a negative number. Example:

Common calculator mistakes

Often the calculator calculates something different to what you expect and sometimes you won't realise and blindly use the result in further calculations. Here are some common calculator mistakes that come from forgetting BEDMAS.

What is -4 squared?

[-] [4] [x^2] [EXE]

gives $-4^2 = -16$

[(] [−] [4] [)] [x^2] [EXE]

gives $(-4)^2 = 16$

What is $\frac{1+2}{4+5}$?

[1] [+] [2] [÷] [4] [+] [5] [EXE]

gives $1 + \frac{2}{4} + 5 = 6\frac{1}{2}$

[(] [1] [+] [2] [)] [÷]

gives $\frac{1+2}{4+5} = \frac{3}{9} = \frac{1}{3}$

[(] [4] [+] [5] [)] [EXE]

Lesson 2 questions

BEDMAS

Q1. $(4 + 15) \times (5 - 10) + 10^2$

Q2. $(6 - 11) \times (12 - 3) + 7^2$

Q3. $4^3 \times 6^2 - 2^4 + 6^2$

Q4. $10^3 + 3^3 \div 3 + 7 \times 7$

Q5. $4 - 2 \div 2 + 11$

Q6. $14 + 6 \times 2 - 3$

Q7. $3^2 + 14 \div 2 + 3 \times 2$

Q8. $\big((11 - 4) + 6\big) \times 3$

Q9. $((15 + 2) - 11)^2 \times 11^2$

Q10. $(4^2 + 10^2) \times (12 + 7 \times 7)$

Q11. $\Big(\big(\big((15 - 10) + 3\big) \times 5\big) + 11\Big) \times 5$

Q12. $12 \div (3 + 1) \times 15 - 6$

Q13. $7^2 - 9 \times 8 + 1$

Q14. $7 \times 9 + 1 + (10 - 14)$

Q15. $2 - 14 + 14 \times 10$

Q16. $2^5 - 11 - (5 \times -4)$

Q17. $8 - 11 + 4 \times (15 - 15)$

Q18. $8 \times (15 - 14)^2$

Q19. $2 \times (1 - 2) \times 11$

Q20. $10 \times (6 + 14) \div 2$

BEDMAS with implied brackets

Q21. $\dfrac{3\times(7+8)}{1+11\times2}$

Q22. $\dfrac{1-(9+2)^2}{7+13\times2}$

Q23. $\dfrac{(11+7)\div1}{3\times8+11}$

Q24. $\dfrac{(6+14)\times10}{3\times4+8}$

Q25. $\dfrac{15+12\times2}{6-5-3}$

Q26. $\dfrac{10+1\times4}{7-7\times2}$

Q27. $\dfrac{9\times(6+2)}{2\times5}$

Q28. $\dfrac{(3-13)^2}{3\times-4}$

Q29. $\dfrac{9\times(13-12)}{(1+1)^3}$

Q30. $\dfrac{12-15\times-3}{12-13\times-1}$

Q31. $\dfrac{(9+3)\div3}{7+8}$

Q32. $\dfrac{12+8-6}{4\times1\times11}$

Q33. $\dfrac{4+6\times7}{15-6-15}$

Q34. $\dfrac{2+1-10}{2\times3}$

Q35. $\dfrac{7\times8}{15}$

Q36. $\dfrac{(4+6)\times3}{(11-3)\times1}$

Q37. $\dfrac{4\times(12-7)}{(8+12)\times2}$

Q38. $\dfrac{9^2-13\times2}{(3+7)\times14}$

Q39. $\dfrac{\big((5+8)\times2\big)-14}{4^2-3^2}$

Q40. $\dfrac{(10^2-11\times8)^2}{\big((11-4)\times4\big)+13}$

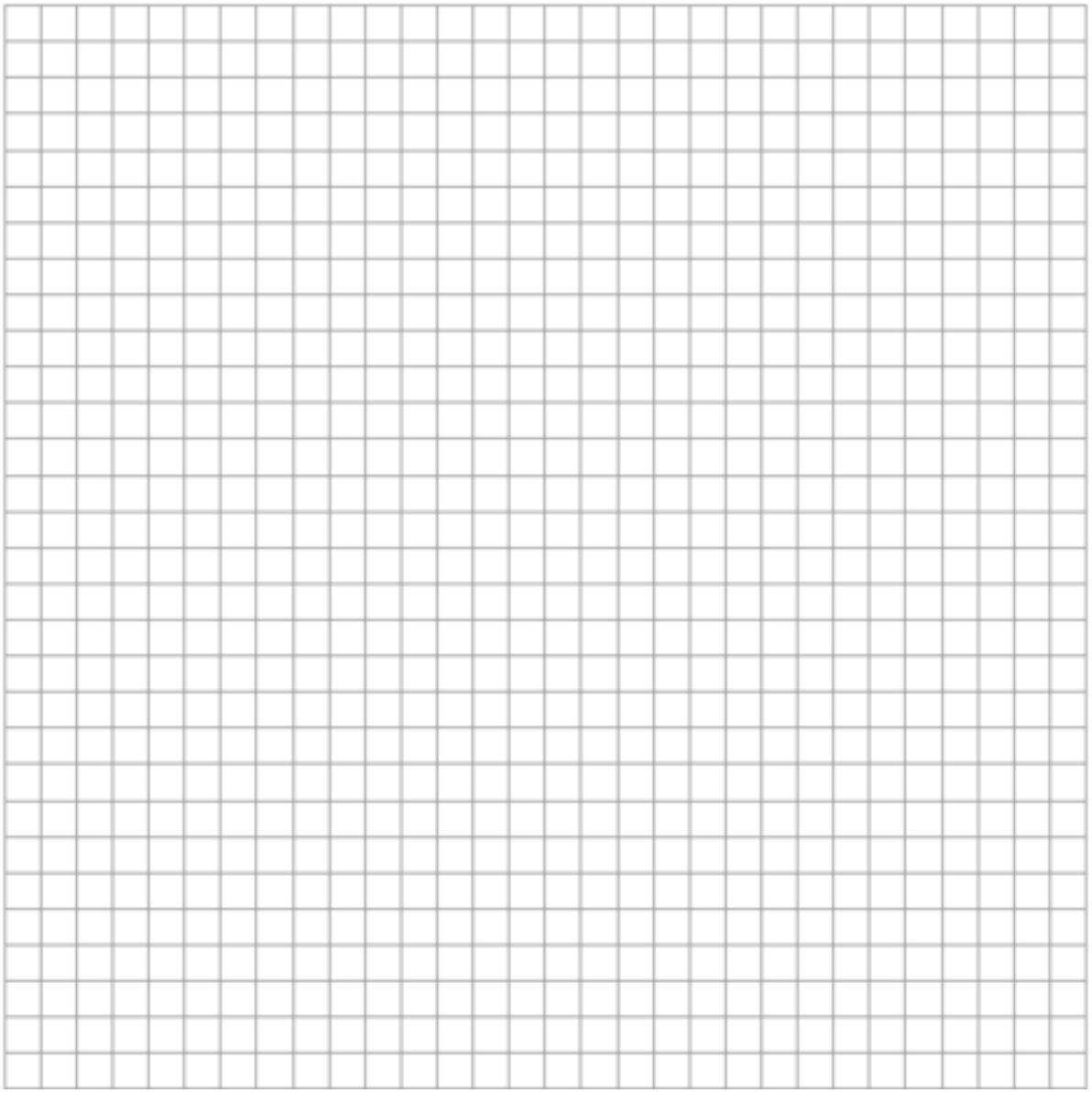
Lesson Three: Operations on fractions

Operations on fractions observe the same order as operations on integers, BEDMAS. But there are a few peculiarities with fractions, one of them being you can only add and subtract fractions with the same denominator. Why is that?

Multiplication

Let's recap integer multiplication. What was -5×3?

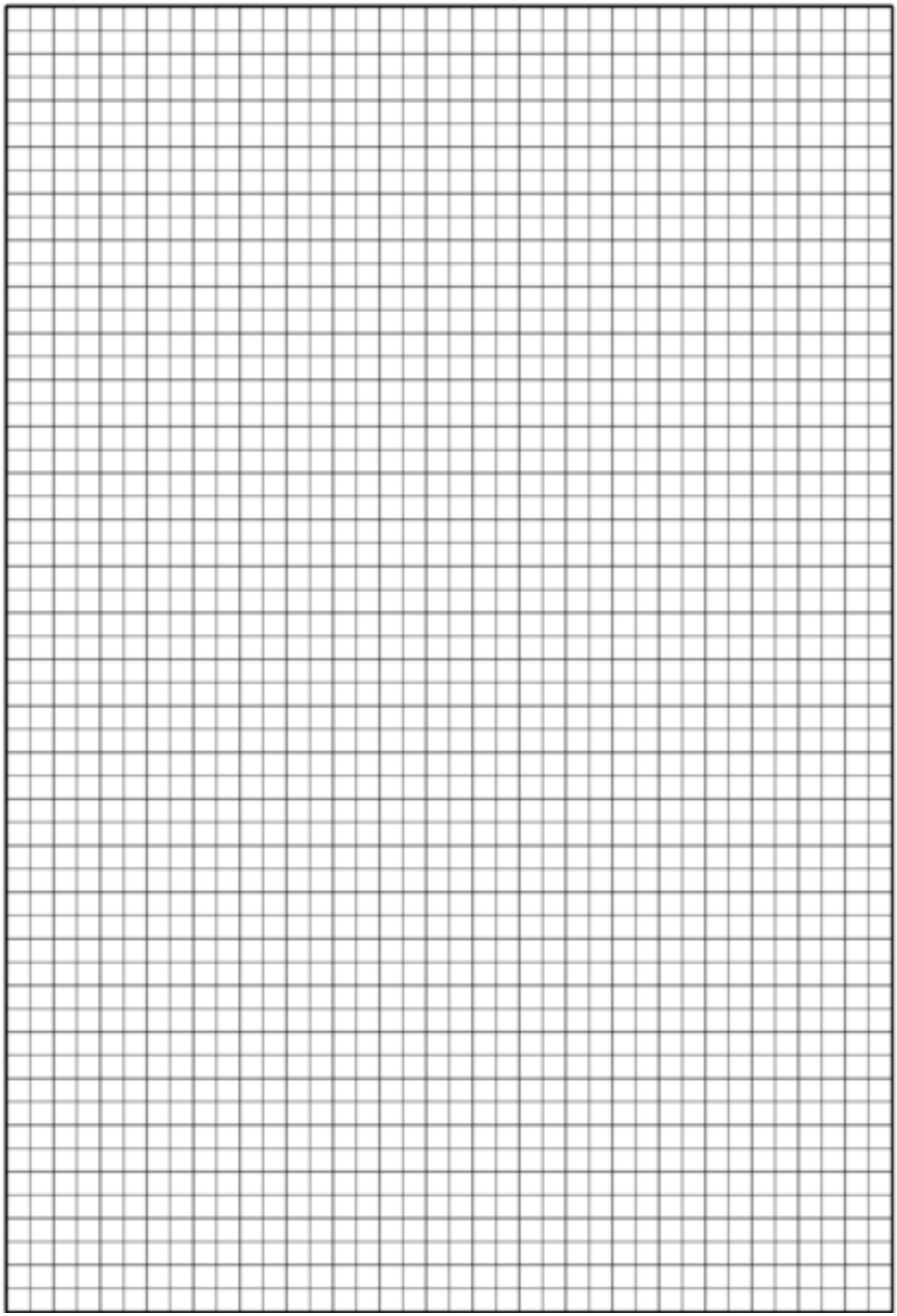
Now let's understand $\frac{2}{3} \times \frac{4}{5}$ and $1\frac{1}{2} \times \frac{3}{7}$



Division

Let's recap integer division. What was $-12 \div 4$?

Now let's understand $\frac{3}{4} \div \frac{2}{3}$ and $1\frac{1}{2} \div \frac{2}{5}$



Lesson 3 questions

Use your newfound (or revised) understanding of operations on fractions to practice on the questions below. Only do as many as you need to. I suggest you first estimate the answer by rounding the fractions.

Addition and subtraction	**Multiplication**	**Division**
Q1. $\frac{1}{6}+\frac{2}{3}$	Q16. $\frac{4}{6}\times\frac{4}{12}$	Q31. $\frac{3}{6}\div\frac{3}{5}$
Q2. $\frac{2}{8}+\frac{1}{4}$	Q17. $\frac{2}{4}\times\frac{1}{7}$	Q32. $\frac{6}{6}\div\frac{6}{14}$
Q3. $\frac{3}{14}+\frac{3}{7}$	Q18. $\frac{6}{10}\times\frac{2}{14}$	Q33. $\frac{9}{12}\div\frac{3}{4}$
Q4. $\frac{3}{2}+\frac{1}{4}$	Q19. $\frac{1}{5}\times\frac{2}{8}$	Q34. $\frac{5}{7}\div\frac{3}{4}$
Q5. $\frac{4}{11}+\frac{10}{11}$	Q20. $\frac{9}{11}\times\frac{11}{13}$	Q35. $\frac{9}{12}\div\frac{1}{6}$
Q6. $\frac{5}{12}+\frac{3}{4}$	Q21. $\frac{7}{12}\times\frac{13}{12}$	Q36. $\frac{14}{9}\div\frac{6}{13}$
Q7. $\frac{3}{10}+\frac{2}{7}$	Q22. $\frac{10}{2}\times\frac{7}{13}$	Q37. $\frac{7}{11}\div\frac{8}{3}$
Q8. $\frac{1}{12}+\frac{2}{5}$	Q23. $\frac{7}{15}\times\frac{10}{12}$	Q38. $\frac{1}{9}\div\frac{2}{10}$
Q9. $\frac{13}{14}+\frac{14}{13}$	Q24. $\frac{2}{9}\times\frac{11}{13}$	Q39. $\frac{7}{5}\div\frac{9}{7}$
Q10. $\frac{1}{13}+\frac{1}{12}$	Q25. $\frac{8}{3}\times\frac{4}{11}$	Q40. $\frac{9}{6}\div\frac{7}{3}$
Q11. $\frac{3}{4}-\frac{3}{8}$	Q26. $2\frac{2}{4}\times\frac{12}{14}$	Q41. $2\frac{1}{7}\div\frac{2}{5}$
Q12. $\frac{8}{11}-\frac{2}{5}$	Q27. $1\frac{3}{6}\times\frac{12}{15}$	Q42. $\frac{1}{14}\div 1\frac{1}{2}$
Q13. $\frac{10}{15}-\frac{2}{3}$	Q28. $\frac{6}{10}\times 7\frac{1}{2}$	Q43. $1\frac{3}{12}\div\frac{1}{8}$
Q14. $\frac{6}{13}-\frac{3}{5}$	Q29. $1\frac{4}{10}\times\frac{13}{15}$	Q44. $2\frac{2}{5}\div 4\frac{1}{3}$
Q15. $\frac{2}{7}-\frac{1}{2}$	Q30. $2\frac{2}{3}\times 1\frac{6}{7}$	Q45. $1\frac{4}{7}\div 1\frac{1}{5}$

Lesson Four: Percentages

Converting between fractions, decimals, and percentages
What does percentage mean?

The advantage of this is that you can easily convert percentages to fractions.

And likewise, you can easily convert percentages to decimals.

Going backwards is also possible.

Percentage of an amount
You will likely be asked to find the percentage of an amount. For example what is 15% of $60, what is 40% of 230 people, what is 30% of 120GB? In all of these examples, the amount you're finding is less than the original amount (you could say it's a fraction of the original amount). Because percentages are hard to work with (there's no % button on the calculator), you'll need to convert the percentage into a fraction or a decimal.

Adding or subtracting a percentage of an amount
Usually, you're finding the percentage of an amount because you want to add it to or subtract it from the original amount. For example 15% discount off $60 or a 30% increase in storage capacity from 120GB.

You could simply add or subtract the percentage amount from the original amount like this:

Or you could, even more simply, multiply the original amount by a specific fraction or decimal like this:

Adding GST

GST is goods and services tax. It is a tax applied to anything that is sold in NZ. Whenever a shop sells something, they pay the GST to the government. GST has been 15% since 2010.

Companies, especially power companies (and internal assessments) love giving prices excluding GST so their products look cheaper. You'll get very used to adding 15% very soon.

Finding the percentage amount or the original amount

As well as increasing by a percentage, sometimes you'll be given the increased amount and are asked to find either the original amount or the percentage increase. Algebra is a powerful tool for this, specifically: rearranging.

Questions

Conversions

Rewrite each fraction/ decimal/percentage as the other two forms.

Q1. 0.1

Q2. 15%

Q3. 25%

Q4. $\frac{1}{3}$

Q5. $\frac{2}{5}$

Q6. 0.65

Q7. $\frac{3}{4}$

Q8. 75%

Q9. 80%

Q10. 0.85

GST

Add GST to each of these amounts.

Q11. $15

Q12. $50

Q13. $65

Q14. $95

Q15. $100

Q16. $125

Q17. $155

Q18. $250

Q19. $310

Q20. $2000

Percentage increase or decrease

Q21. $20 is increased by 100%. What is the new amount?

Q22. $50 is increased by 50%. What is the new amount

Q23. $50 is increased to $60. What is the percentage increase?

Q24. $70 is decreased to $40. What is the percentage decrease?

Q25. An amount is increased by 30% to $100. What was the original amount?

Q26. An amount is decreased by 45% to $60. What was the original amount?

Q27. $120 is increased by 40%. What is the new amount?

Q28. $120 is decreased by 40%. What is the new amount?

Lesson Five: Proportions and converting currency

As well as GST, another common problem is giving costs in different currencies. In these cases, you'll need to convert currencies. Algebra, again, is a powerful tool here. Specifically, proportions.

A conversion rate is a type of proportion. For example, let's assume $1 NZD = $0.65 USD. How much would a $100 NZD item be worth in USD?

How much would a $250 NZD item be worth in USD?

How much would a $170 USD item be worth in NZD?

It can be easy to get the conversion the wrong way around if you don't keep your common sense. Proportions are helpful for this. Use proportions to solve the above problems.

You could also use estimation to make sure your converted in the right direction, e.g. should the NZD amount or USD amount be greater?

Questions

Q1. Convert $280 NZD to USD

Q2. Convert $240 NZD to USD

Q3. Convert $250 NZD to USD

Q4. Convert $1 010 NZD to USD

Q5. Convert $890 NZD to USD

Q6. Convert $410 NZD to USD

Q7. Convert $1 350 NZD to USD

Q8. Convert $1 660 NZD to USD

Q9. Convert $930 USD to NZD

Q10. Convert $1 430 USD to NZD

Q11. Convert $1 040 USD to NZD

Q12. Convert $1 630 USD to NZD

Q13. Convert $1 170 USD to NZD

Q14. Convert $920 USD to NZD

Q15. Convert $350 USD to NZD

Q16. Convert $1 560 USD to NZD

Lesson Six: Time

Another common problem is having to work with different measurements for time, e.g. the holiday costs $250/day for three months. How many days is that?

It could be as few as:
It could be as many as:

Which one will you choose for your calculations?

Recall how to convert between:
Years
Months
Fortnights
Weeks
Days
Hours
Minutes
Seconds

Lesson Seven: Rounding and estimation

When numbers have been rounded before they're given to you, the actual number is within a range. For example, Mike and Huia spending $250/day to the nearest $50 means they could spend anywhere between $225 and $274.99 per day.

It is possible to calculate using such a range. For example, the money they spend over three months (minus three weeks) can be calculated with the above range.

And while we're talking about it, the $250/day is an estimation that Mike and Huia made. Who's to say it can't be more or less than that? How can you account for this in your calculations?

Practice internal

The Big Day Out is (was) a three-day music festival. At the Big Day Out, audience members can purchase "green days". The organisers will use all the income from these to buy carbon credits. These carbon credits will be used to purchase trees to offset the carbon emissions from the event.

This activity requires you to determine the minimum number of trees that could be purchased using carbon credits from the Big Day Out.

An estimate for the size of the audience at the Big Day Out was 19,000 (to the nearest 1000).

Big Day Out attendees can choose to purchase 0, 1, 2, or 3 green days when they buy their tickets. 23% of the attendees purchased green days.

Of those who purchased green days, one quarter purchased three days, one third purchased two days and the rest purchased one day.

One green day costs NZ$1.34.

A carbon credit costs US$18.

Exchange rate: US$1= NZ$1.2262 (April 2016).

The organisers are able to buy 10 trees with every 3 carbon credits.

Appendix A: Factors of numbers

What is a factor?

What are the factors of 24?

What are some strategies to quickly find factors?

Questions

Write the factors of each number

Q1. 48	Q11. 140	Q21. 143	Q31. 77
Q2. 13	Q12. 104	Q22. 94	Q32. 108
Q3. 59	Q13. 115	Q23. 97	Q33. 83
Q4. 32	Q14. 124	Q24. 60	Q34. 66
Q5. 104	Q15. 74	Q25. 85	Q35. 130
Q6. 84	Q16. 119	Q26. 128	Q36. 18
Q7. 8	Q17. 78	Q27. 117	Q37. 93
Q8. 95	Q18. 137	Q28. 23	Q38. 68
Q9. 79	Q19. 16	Q29. 118	Q39. 134
Q10. 67	Q20. 112	Q30. 29	Q40. 17

Appendix B: Problem sets

Here are some large problem sets if you would like lots of questions to practice. They are grouped into types so you can practice similar problems and are in the same order as the lessons.

Operations on integers: addition

Q1. $-29 + 47$	Q11. $8 + 25$	Q21. $15 + 20$	Q31. $33 + 34$
Q2. $-23 + 49$	Q12. $4 + 27$	Q22. $-18 + (-1)$	Q32. $11 + (-4)$
Q3. $21 + 10$	Q13. $23 + (-10)$	Q23. $48 + (-7)$	Q33. $45 + 33$
Q4. $-34 + (-17)$	Q14. $47 + 1$	Q24. $1 + (-6)$	Q34. $39 + 6$
Q5. $40 + (-41)$	Q15. $-40 + (-5)$	Q25. $41 + 46$	Q35. $-37 + 24$
Q6. $-25 + (-10)$	Q16. $-29 + 30$	Q26. $14 + (-10)$	Q36. $37 + (-48)$
Q7. $16 + (-47)$	Q17. $-21 + 46$	Q27. $3 + 32$	Q37. $0 + 43$
Q8. $38 + 43$	Q18. $1 + (-19)$	Q28. $37 + (-5)$	Q38. $-6 + 18$
Q9. $-4 + 15$	Q19. $14 + (-9)$	Q29. $14 + (-36)$	Q39. $20 + 7$
Q10. $28 + 2$	Q20. $33 + 26$	Q30. $-33 + 44$	Q40. $-11 + 7$

Operations on integers: subtraction

Q1. $42 - (-23)$	Q11. $19 - 5$	Q21. $29 - 0$	Q31. $19 - (-28)$
Q2. $46 - 32$	Q12. $-21 - 41$	Q22. $-1 - 43$	Q32. $-18 - (-27)$
Q3. $-38 - 37$	Q13. $29 - 4$	Q23. $-21 + 24$	Q33. $-10 - (-26)$
Q4. $-49 - (-49)$	Q14. $2 - 7$	Q24. $47 - 49$	Q34. $-49 - (-10)$
Q5. $-45 - (-42)$	Q15. $-17 - (-48)$	Q25. $-33 - (-7)$	Q35. $-17 - 9$
Q6. $2 - (-40)$	Q16. $14 - (-35)$	Q26. $42 - (-16)$	Q36. $32 - 7$
Q7. $17 - 26$	Q17. $4 - 21$	Q27. $20 - 6$	Q37. $-5 - 40$
Q8. $40 - (-16)$	Q18. $21 - 38$	Q28. $22 - 48$	Q38. $3 - (-36)$
Q9. $37 - 0$	Q19. $49 - 26$	Q29. $-43 - (-8)$	Q39. $30 - 15$
Q10. $19 - (-3)$	Q20. $47 - 14$	Q30. $-18 - (-13)$	Q40. $38 - (-15)$

Operations on integers: multiplication

Q1. -8×-12	Q11. 12×10	Q21. 2×-8	Q31. -15×-13
Q2. -11×-2	Q12. -13×5	Q22. -5×-11	Q32. -2×-7
Q3. 13×13	Q13. 13×14	Q23. 5×-1	Q33. -2×-11
Q4. -2×-11	Q14. -13×8	Q24. -9×-8	Q34. 9×-7
Q5. 11×8	Q15. -8×13	Q25. 12×15	Q35. -8×9
Q6. -11×6	Q16. 14×8	Q26. -8×8	Q36. -10×5
Q7. -14×8	Q17. -10×15	Q27. 10×-10	Q37. 11×6
Q8. 3×6	Q18. -14×-10	Q28. 9×14	Q38. 6×-1
Q9. -2×-10	Q19. 2×10	Q29. -15×15	Q39. -5×-10
Q10. 0×3	Q20. -15×-15	Q30. -6×5	Q40. -12×-12

Operations on integers: division

Q1. $18 \div 2$	Q11. $-150 \div -10$	Q21. $-16 \div -8$	Q31. $-45 \div 3$
Q2. $-12 \div -4$	Q12. $-11 \div -1$	Q22. $12 \div -2$	Q32. $8 \div -1$
Q3. $35 \div 5$	Q13. $14 \div 2$	Q23. $-140 \div 7$	Q33. $-108 \div 9$
Q4. $143 \div 13$	Q14. $-54 \div 6$	Q24. $11 \div 1$	Q34. $54 \div -9$
Q5. $0 \div -5$	Q15. $-49 \div -7$	Q25. $-21 \div -3$	Q35. $33 \div -3$
Q6. $13 \div -13$	Q16. $-10 \div 5$	Q26. $-18 \div 3$	Q36. $24 \div 12$
Q7. $26 \div 13$	Q17. $64 \div 8$	Q27. $-60 \div 10$	Q37. $90 \div 15$
Q8. $2 \div -1$	Q18. $-117 \div 13$	Q28. $-24 \div -2$	Q38. $-40 \div 8$
Q9. $-72 \div 8$	Q19. $-80 \div 10$	Q29. $44 \div 4$	Q39. $12 \div -1$
Q10. $36 \div 6$	Q20. $-143 \div 13$	Q30. $14 \div -7$	Q40. $-21 \div -7$

BEDMAS

Q1. $5 \times 12 + 6 \div 3$	Q11. $13 - 2 \times 12$	Q21. $(8 - 6)^4 - 1 \times 15$
Q2. $\big((6 + 7) \times 2\big) - 14$	Q12. $(4 + 1)^2 + 14 \times -2$	Q22. $(1 + 11) \div 6 \times 3^2$
Q3. $(11 + 9) \times 11 \div 4$	Q13. $7 + 7 \times 9$	Q23. $(1 - 7)^2 + 11 \times 4$
Q4. $10 + 2^3 \times 13$	Q14. $(12 - 12) \times 8 + 4$	Q24. $9 \times 4 - 13 + 7$
Q5. $(13 - 8)^2 - 4 \times 9$	Q15. $2 \times 8 - 4 \times 3$	Q25. $1 \times 5 + 14 \times 4$
Q6. $10 \times (4 - 4) + 12 \times 11$	Q16. $(2 \times 10)^2 - 11 \times 7^2$	Q26. $(12 - 12)^2 - 2 \times 11$
Q7. $(4 - 14)^2 + 1 \times 12 - 4$	Q17. $(15 \div 5 \times 2)^2 - 7 \times 2$	Q27. $(10 + 14) \times 2 - 6^2$
Q8. $(13 + 6) \times 2 - 10^2$	Q18. $4 + 15 \times 1 - 9$	Q28. $12 \times 11 - 15 \times 10$
Q9. $(1 - 9) \times 3^2$	Q19. $((5 + 9) \div 2)^2$	Q29. $8 \times 12 + 14 \div 2$
Q10. $12 - 6 + 3 \times 10 - 5$	Q20. $3 - 1 \times 5 + 2^3$	Q30. 4×8^2

BEDMAS with implied brackets

Q1. $\frac{(7+11)\div 2}{(1+14)^2}$	Q8. $\frac{7+6\times 2}{1\times 10+15}$	Q15. $\frac{12-6\times 2}{13+11\times 2}$
Q2. $\frac{11-12\times -2}{11+7\times 2}$	Q9. $\frac{(10-3)^2-14}{2+15\times 14}$	Q16. $\frac{(3-4)^4}{12\times 3\div 4}$
Q3. $\frac{14+14-15}{12\div 2^2}$	Q10. $\frac{(10+1)\times 11}{15\times 10}$	Q17. $\frac{7+11\times 7}{9\times 10-11}$
Q4. $\frac{(6+2)^2}{9-1\times 3}$	Q11. $\frac{(14-2)^2}{10\times(11-15)}$	Q18. $\frac{5+(14-5)^2}{8+(14-12)^2}$
Q5. $\frac{(13-9)\times 15}{4\times 3\times 5}$	Q12. $\frac{(6-9)^2}{(9-6)^2}$	Q19. $\frac{4-1+6\times 12}{10^3-12\times 7}$
Q6. $\frac{9\times 3^2}{(15-8)\times 10}$	Q13. $\frac{14\div 1+6}{11\times 12\div 3}$	Q20. $\frac{(1+15)\times -2}{(7-3)^2\times -1}$
Q7. $\frac{13+14+15}{7^2}$	Q14. $\frac{11^2-3\times 4}{6\times 3+15}$	

Operations on fractions: addition and subtraction

Q1. $\frac{3}{9}+\frac{2}{18}$	Q11. $\frac{4}{6}+\frac{6}{3}$	Q 21. $\frac{11}{12}-\frac{5}{6}$	Q31. $\frac{11}{14}-\frac{11}{9}$
Q2. $\frac{4}{7}+\frac{4}{14}$	Q12. $\frac{3}{12}+\frac{3}{8}$	Q22. $\frac{10}{11}-\frac{2}{22}$	Q32. $\frac{3}{11}-\frac{14}{2}$
Q3. $\frac{3}{9}+\frac{2}{9}$	Q13. $\frac{1}{3}+\frac{2}{4}$	Q23. $\frac{1}{7}-\frac{4}{14}$	Q33. $\frac{15}{7}-\frac{2}{9}$
Q4. $\frac{1}{4}+\frac{7}{8}$	Q14. $\frac{1}{5}+\frac{7}{14}$	Q24. $\frac{7}{8}-\frac{1}{2}$	Q34. $\frac{3}{5}-\frac{12}{9}$
Q5. $\frac{4}{14}+\frac{5}{7}$	Q15. $\frac{6}{11}+\frac{2}{5}$	Q25. $\frac{9}{12}-\frac{3}{4}$	Q35. $\frac{13}{4}-\frac{10}{11}$
Q6. $\frac{8}{14}+\frac{11}{28}$	Q16. $1\frac{2}{3}+1\frac{1}{9}$	Q26. $\frac{2}{13}-\frac{1}{26}$	Q36. $1\frac{1}{7}-\frac{13}{14}$
Q7. $\frac{9}{12}+\frac{3}{4}$	Q17. $1\frac{4}{11}+1\frac{7}{8}$	Q27. $\frac{2}{4}-\frac{5}{16}$	Q37. $2\frac{2}{5}-3$
Q8. $\frac{7}{10}+\frac{15}{20}$	Q18. $2\frac{3}{4}+2\frac{3}{6}$	Q28. $\frac{11}{12}-\frac{10}{24}$	Q38. $1\frac{4}{8}-1\frac{1}{11}$
Q9. $\frac{2}{9}+\frac{5}{8}$	Q19. $1\frac{3}{12}+1\frac{2}{3}$	Q29. $\frac{7}{10}-\frac{65}{100}$	Q39. $1\frac{4}{7}-3\frac{1}{4}$
Q10. $\frac{5}{10}+\frac{3}{7}$	Q20. $1\frac{5}{8}+1\frac{6}{8}$	Q30. $\frac{2}{8}-\frac{1}{16}$	Q40. $2-1\frac{1}{3}$

Operations on fractions: multiplication and division

Q1. $\frac{3}{12}\times\frac{3}{9}$	Q11. $\frac{13}{8}\times\frac{6}{10}$	Q21. $\frac{3}{12}\div\frac{2}{10}$	Q31. $\frac{12}{6}\div\frac{14}{15}$
Q2. $\frac{5}{15}\times\frac{2}{12}$	Q12. $\frac{3}{10}\times\frac{13}{2}$	Q22. $\frac{10}{10}\div\frac{4}{7}$	Q32. $\frac{2}{6}\div\frac{7}{2}$
Q3. $\frac{5}{7}\times\frac{4}{11}$	Q13. $\frac{8}{15}\times\frac{12}{12}$	Q23. $\frac{3}{14}\div\frac{2}{5}$	Q33. $\frac{11}{3}\div\frac{5}{10}$
Q4. $\frac{3}{8}\times\frac{4}{14}$	Q14. $\frac{1}{11}\times\frac{6}{4}$	Q24. $\frac{3}{15}\div\frac{14}{15}$	Q34. $\frac{9}{8}\div\frac{13}{10}$
Q5. $\frac{1}{9}\times\frac{4}{14}$	Q15. $\frac{2}{2}\times\frac{11}{13}$	Q25. $\frac{5}{15}\div\frac{2}{7}$	Q35. $\frac{14}{7}\div\frac{12}{6}$
Q6. $\frac{2}{10}\times\frac{2}{8}$	Q16. $1\times1\frac{1}{2}$	Q26. $\frac{9}{14}\div\frac{4}{5}$	Q36. $1\frac{5}{8}\div\frac{8}{2}$
Q7. $\frac{4}{12}\times\frac{1}{15}$	Q17. $1\frac{5}{9}\times\frac{8}{10}$	Q27. $\frac{9}{15}\div\frac{13}{14}$	Q37. $2\frac{2}{4}\div\frac{3}{5}$
Q8. $\frac{4}{11}\times\frac{4}{4}$	Q18. $3\frac{3}{4}\times1\frac{1}{7}$	Q28. $\frac{7}{12}\div\frac{5}{10}$	Q38. $2\frac{1}{2}\div2\frac{2}{5}$
Q9. $\frac{5}{13}\times\frac{1}{10}$	Q19. $3\frac{1}{2}\times2$	Q29. $\frac{4}{11}\div\frac{10}{10}$	Q39. $1\frac{1}{4}\div2\frac{3}{6}$
Q10. $\frac{1}{13}\times\frac{8}{9}$	Q20. $2\frac{2}{3}\times\frac{6}{6}$	Q30. $\frac{3}{10}\div\frac{1}{10}$	Q40. $\frac{4}{4}\div1\frac{1}{10}$

Percentages: conversions

For each question, show the other two forms of the number, e.g. if you are given a percentage, rewrite it as both a fraction and a decimal.

Q1. 7%	Q11. $\frac{40}{100}$	Q21. 0.91
Q2. 58%	Q12. $\frac{31}{100}$	Q22. 0.88
Q3. 36%	Q13. $\frac{37}{100}$	Q23. 0.5
Q4. 74%	Q14. $\frac{14}{100}$	Q24. 0.04
Q5. 79%	Q15. $\frac{8}{100}$	Q25. 0.35
Q6. 31%	Q16. $\frac{41}{100}$	Q26. 0.87
Q7. 95%	Q17. $\frac{11}{100}$	Q27. 0.66
Q8. 80%	Q18. $\frac{40}{100}$	Q28. 0.27
Q9. 71%	Q19. $\frac{10}{100}$	Q29. 0.13
Q10. 7%	Q20. $\frac{33}{100}$	Q30. 0.72

Percentages: Adding GST

Add GST to these amounts.		These prices include GST. Find the price excluding GST.	
Q1. \$67	Q9. \$21	Q17. \$38	Q25. \$164
Q2. \$283	Q10. \$157	Q18. \$176	Q26. \$26
Q3. \$135	Q11. \$265	Q19. \$77	Q27. \$146
Q4. \$87	Q12. \$285	Q20. \$172	Q28. \$103
Q5. \$215	Q13. \$234	Q21. \$27	Q29. \$232
Q6. \$211	Q14. \$20	Q22. \$245	Q30. \$225
Q7. \$209	Q15. \$254	Q23. \$226	Q31. \$20
Q8. \$142	Q16. \$119	Q24. \$57	Q32. \$135

Percentages: increase or decrease by a percentage

Q1. 360 has been increased by 6%. What is the new amount?	Q16. 660 has been decreased by 2%. What is the new amount?
Q2. 250 has been increased by 91%. What is the new amount?	Q17. 730 has been decreased by 78%. What is the new amount?
Q3. 680 has been increased by 20%. What is the new amount?	Q18. 180 has been decreased by 78%. What is the new amount?
Q4. 590 has been increased by 93%. What is the new amount?	Q19. 560 has been decreased by 56%. What is the new amount?
Q5. 610 has been increased by 64%. What is the new amount	Q20. 540 has been decreased by 39%. What is the new amount?
Q6. 320 has been increased to 660. What is the percentage increase?	Q21. 750 has been decreased to 480. What is the percentage decrease?
Q7. 200 has been increased to 660. What is the percentage increase?	Q22. 960 has been decreased to 620. What is the percentage decrease?
Q8. 240 has been increased to 740. What is the percentage increase?	Q23. 490 has been decreased to 340. What is the percentage decrease?
Q9. 380 has been increased to 890. What is the percentage increase?	Q24. 760 has been decreased to 550. What is the percentage decrease?
Q10. 350 has been increased to 550. What is the percentage increase?	Q25. 920 has been decreased to 130. What is the percentage decrease?
Q11. An amount has been increased by 80% to 490. What was the original amount?	Q26. An amount has been decreased by 23% to 610. What was the original amount?
Q12. An amount has been increased by 20% to 420. What was the original amount?	Q27. An amount has been decreased by 38% to 870. What was the original amount?
Q13. An amount has been increased by 62% to 880. What was the original amount?	Q28. An amount has been decreased by 93% to 920. What was the original amount?
Q14. An amount has been increased by 56% to 820. What was the original amount?	Q29. An amount has been decreased by 47% to 890. What was the original amount?
Q15. An amount has been increased by 52% to 510. What was the original amount?	Q30. An amount has been decreased by 33% to 870. What was the original amount?

Converting currency

Q1. Convert $655 NZD to USD	Q16. Convert $1 010 USD to NZD
Q2. Convert $75 NZD to USD	Q17. Convert $120 USD to NZD
Q3. Convert $1 035 NZD to USD	Q18. Convert $125 USD to NZD
Q4. Convert $1 065 NZD to USD	Q19. Convert $895 USD to NZD
Q5. Convert $1 110 NZD to USD	Q20. Convert $445 USD to NZD
Q6. Convert $535 NZD to USD	Q21. Convert $960 USD to NZD
Q7. Convert $510 NZD to USD	Q22. Convert $95 USD to NZD
Q8. Convert $690 NZD to USD	Q23. Convert $1 215 USD to NZD
Q9. Convert $340 NZD to USD	Q24. Convert $1 380 USD to NZD
Q10. Convert $1 375 NZD to USD	Q25. Convert $255 USD to NZD
Q11. Convert $305 NZD to USD	Q26. Convert $1 045 USD to NZD
Q12. Convert $620 NZD to USD	Q27. Convert $615 USD to NZD
Q13. Convert $425 NZD to USD	Q28. Convert $1 385 USD to NZD
Q14. Convert $1 020 NZD to USD	Q29. Convert $1 110 USD to NZD
Q15. Convert $1 425 NZD to USD	Q30. Convert $25 USD to NZD

Appendix C: Decimal representations

Numbers have decimal representations. For example, the decimal representation of $\frac{2}{5}$ is 0.4. Likewise, the decimal representation of 1 is 1.

But 1 has another decimal representation.

0.9… is "zero with an infinite number of 9s after the decimal point" and is also a decimal representation of one.

In other words, they are exactly the same.

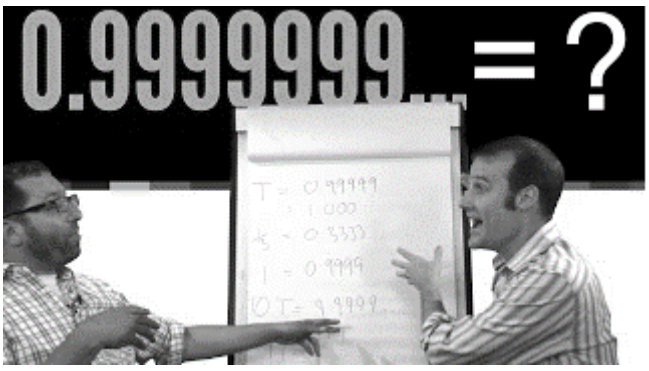
But how does that work? 0.9… must be less than 1 because, well, it just looks less. Like, there's no units in 0.9… but there's one unit in 1.

There are several ways of showing that, for algebra (as a branch of maths) to work, we need to accept that every integer has two decimal representations.[2]

Argument from philosophy: If two numbers are different, then you can fit another number between them, such as their average. But what number could you possibly fit between 0.999... and 1.000...?

Argument from number: $1/3 = 0.333...$ in decimal form. And $1/3 + 1/3 + 1/3 = 3(1/3) = 1$. Reasonably then, 0.333... + 0.333... + 0.333... = 3(0.333...)should also equal 1. But 3(0.333...) = 0.999.... Then 0.999... must equal 1.

Argument from algebra: You can also show this with algebra. To see this argument, watch the video below.



https://youtu.be/rT1sIVqonE8?t=2m18s

[2] All of these methods are from http://www.purplemath.com/modules/howcan1.htm