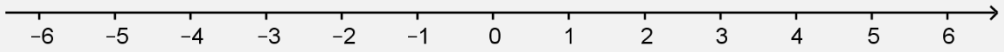
Name: _____

Level 1 – AS91026
Standard 1.1
4 credits – Internal

Apply numeric reasoning in solving problems

Lesson One: Operations on integers

Adding and subtracting positive numbers
Integers are whole numbers including positive and negative numbers and zero. Adding and subtracting integers is like moving right and left along the number line.



The addition $2 + 5$ is like starting at 2 and moving to the right by 5, giving a result of 7. Notice that this is the same as $5 + 2$, that is, starting at 5 and moving to the right by 2. This property that the order doesn't matter is called **commutative.**

The subtraction $6 - 4$ is like starting at 6 and moving to the left by 4, giving a result of 2. Notice that subtraction is **not commutative**. The subtraction $4 - 6$, that is, starting at 4 and moving to the left by 6 gives a different result.

Adding and subtracting negative numbers
Negative numbers can be added and subtracted too. Consider the addition $-3 + 4$. This is like starting at -3 and moving to the right by 4, giving a result of 1. But what about $4 + (-3)$? Because addition is commutative, you know it has to result in 1. This is like starting at 4 and moving to the right by -3, i.e. moving to the left by 3, giving a result of 1.

The subtraction $-2 - 3$ is like starting at -2 and moving left by 3, giving a result of -5.

But what about $1 - (-3)$? This is like starting at 1 and moving to the left by -3, i.e. moving to the right by 3, giving a result of 4.

Calculate the following results.

$12 + (-5)$	$-1 + 2$	$4 + (-8)$	$-11 + (-3)$
_____	_____	_____	_____
$-12 - 1$	$11 - 1$	$-2 - (-8)$	$5 - 7$
_____	_____	_____	_____

Multiplying and dividing negative numbers
When multiplying or dividing numbers:

- If both numbers are positive, the result is positive.
- If one number is positive and the other is negative, the result is negative.
- If both numbers are negative, the result is positive.

Calculate the following results.

5×-5	-4×-2	$14 \div 2$	$-16 \div 4$
_____	_____	_____	_____

Order of operations (BEDMAS)

What happens if you combine addition, subtraction, multiplication, and division into the same calculation? Well, you do the operations one-by-one in order. But you do need to be aware of which order to do each operation in. Some time in history, a group of mathematicians got together and decided on an order which was great because everyone could agree on the same order. You could choose your own order but until you convince the rest of the mathematical world to switch to yours, you'll have to use BEDMAS.

B	Brackets are used to disrupt the usual order. Use them to do addition or subtraction before a multiplication, for example. Brackets can be nested within each other to create layers of brackets.
E	Exponents AKA powers. This also includes roots because roots are fractional powers.
D M	Multiplication. This also includes division because division is multiplication by a fraction.
A S	Addition. This also includes subtraction because subtraction is addition with a negative number.

Common calculator mistakes

Often the calculator calculates something different to what you expect and sometimes you won't realise and blindly use the result in further calculations. Here are some common calculator mistakes that come from forgetting BEDMAS.

What is -4 squared?

[-] [4] [x^2] [EXE]

gives $-4^2 = -16$

[( ] [−] [4] [ )] [x^2] [EXE]

gives $(-4)^2 = 16$

What is $\frac{1+2}{4+5}$?

[1] [+] [2] [÷] [4] [+] [5] [EXE]

gives $1 + \frac{2}{4} + 5 = 6\frac{1}{2}$

[( ] [1] [+] [2] [ )] [÷]

[( ] [4] [+] [5] [ )] [EXE]

gives $\frac{1+2}{4+5} = \frac{3}{9} = \frac{1}{3}$

When you're ready, answer the following questions.

Lesson One questions

Q1. $11 + (-10)$

Q2. $-17 + (-47)$

Q3. $-31 + 12$

Q4. $38 + 3$

Q5. $42 + (-42)$

Q6. $-8 - (-12)$

Q7. $-2 - 18$

Q8. $30 - (-10)$

Q9. $16 - 31$

Q10. $22 - (-37)$

Q11. -2×-13

Q12. -8×-4

Q13. 9×7

Q14. -6×-2

Q15. -11×6

Q16. $-30 \div -3$

Q17. $-12 \div 6$

Q18. $1 \div -1$

Q19. $44 \div 11$

Q20. $30 \div -2$

Q21. $(4 + 15) \times (5 - 10) + 10^2$

Q22. $(6 - 11) \times (12 - 3) + 7^2$

Q23. $4^3 \times 6^2 - 2^4 + 6^2$

Q24. $(-4)^2 + (-3)^3 \div 3 + 7 \times 7$

Q25. $\big((11 - 4) + 6\big) \times 3$

Q26. $((15 + 2) - 11)^2 \times 11^2$

Q27. $(-3)^2 + 14 \div 2 + 3 \times 2$

Q28. $(4^2 + 10^2) \times (12 + 7 \times 7)$

Q29. $\Big(\big(\big((15 - 10) + 3\big) \times 5\big) + 11\Big) \times 5$

Lesson Two: Operations on fractions

Adding and subtracting fractions
You can only add and subtract fractions with the same denominator. When adding or subtracting, add or subtract the numerator but leave the denominator unchanged.

Calculate these results.

$\frac{1}{6}+\frac{2}{3}$ $\frac{2}{8}+\frac{1}{4}$ $\frac{3}{14}+\frac{3}{7}$ $\frac{3}{2}+\frac{1}{4}$

Multiplying and dividing fractions
To multiply fractions, multiply the numerators together and multiply the denominators together. To divide fractions, flip the second fraction upside down and then multiply, e.g.:

$$\frac{2}{3}\times\frac{3}{5}$$
$$=\frac{2\times 3}{3\times 5}$$
$$=\frac{6}{15}$$
$$=\frac{2}{5}$$

$$\frac{3}{4}\div\frac{2}{3}$$
$$=\frac{3}{4}\times\frac{3}{2}$$
$$=\frac{3\times 3}{4\times 2}$$
$$=\frac{9}{8}$$

You should simplify your result if possible.

$\frac{4}{6}\times\frac{4}{12}$ $\frac{2}{4}\times\frac{1}{7}$ $\frac{3}{6}\div\frac{3}{5}$ $\frac{4}{6}\div\frac{6}{14}$

When you're ready, answer the following questions.

Lesson Two questions

Do these questions in class.

Q1. $\frac{4}{11}+\frac{10}{11}$

Q2. $\frac{5}{12}+\frac{3}{4}$

Q3. $\frac{3}{10}+\frac{2}{7}$

Q4. $\frac{1}{12}+\frac{2}{5}$

Q5. $\frac{13}{14}+\frac{14}{13}$

Q6. $\frac{3}{4}-\frac{3}{8}$

Q7. $\frac{8}{11}-\frac{2}{5}$

Q8. $\frac{10}{15}-\frac{2}{3}$

Q9. $\frac{6}{13}-\frac{3}{5}$

Q10. $\frac{2}{7}-\frac{1}{2}$

Q11. $\frac{6}{10}\times\frac{2}{14}$

Q12. $\frac{1}{5}\times\frac{2}{8}$

Q13. $\frac{9}{11}\times\frac{11}{13}$

Q14. $\frac{7}{12}\times\frac{13}{12}$

Q15. $\frac{10}{2}\times\frac{7}{13}$

Q16. $\frac{9}{12}\div\frac{3}{4}$

Q17. $\frac{5}{7}\div\frac{3}{4}$

Q18. $\frac{9}{12}\div\frac{1}{6}$

Q19. $\frac{14}{9}\div\frac{6}{13}$

Q20. $\frac{7}{11}\div\frac{8}{3}$

Lesson Three: Percentages

Converting between fractions, decimals, and percentages

Percentage means "out of 100". This is the same as a fraction with 100 as the denominator. This means that you can easily convert percentages to fractions and likewise, you can easily convert percentages to decimals. For example:

$$68\% = \frac{68}{100} = 0.68$$

Percentage of an amount

You will likely be asked to find the percentage of an amount. For example what is 15% of \$60, what is 40% of 230 people, what is 30% of 120GB? In all of these examples, the amount you're finding is less than the original amount (you could say it's a fraction of the original amount). Because percentages are hard to work with (there's no % button on the calculator), you'll need to convert the percentage into a fraction or a decimal.

Adding or subtracting a percentage of an amount

Usually, you're finding the percentage of an amount because you want to add it to or subtract it from the original amount.

You could add or subtract the percentage amount from the original amount like this example of adding 15% to \$60:

To find 15% of \$60:

$$0.15 \times \$60 = \$9$$

Adding to the original amount:

$$\$60 + \$9 = \$69$$

Alternatively you could multiply the original amount like this:

To add 15% to \$60 :

$$1.15 \times \$60 = \$69$$

Adding GST

GST is goods and services tax. It is a tax applied to anything that is sold in NZ, whether it is a good or a service. Whenever a shop sells something, they pay the GST to the government. GST has been 15% since 2010.

Some companies, especially power companies give prices excluding GST so their products look cheaper. In these cases, you'll need to add 15% to calculate the true cost.

Finding the percentage amount or the original amount

As well as increasing by a percentage, sometimes you'll be given the increased amount and are asked to find either the original amount or the percentage increase. Algebra is a powerful tool for this, specifically: rearranging.

When you're ready, answer the following questions.

Lesson Three questions

Rewrite each fraction/ decimal/percentage as the other two forms.

Q1. 0.1	Q2. 15%	Q3. 25%	Q4. $\frac{1}{3}$	Q5. $\frac{2}{5}$
Q6. 0.65	Q7. $\frac{3}{4}$	Q8. 75%	Q9. 80%	Q10. 0.85

Add GST to each of these amounts.

Q11. $15	Q12. $50	Q13. $65	Q14. $95	Q15. $100
Q16. $125	Q17. $155	Q18. $250	Q19. $310	Q20. $2000

Q21. $20 is increased by 100%. What is the new amount?

Q22. $50 is increased by 50%. What is the new amount

Q23. $50 is increased to $60. What is the percentage increase?

Q24. $70 is decreased to $40. What is the percentage decrease?

Q25. An amount is increased by 30% to $100. What was the original amount?

Q26. An amount is decreased by 45% to $60. What was the original amount?

Lesson Four: Proportions and converting currency

Converting currency
As well as GST, another common problem is giving costs in different currencies. In these cases, you'll need to convert currencies. Algebra, again, is a powerful tool here. Specifically, proportions.

A conversion rate is a type of proportion. For example, let's assume \$1 NZD = \$0.65 USD. How much would a \$100 NZD item be worth in USD?

$$\$100\text{NZD} \times 0.65 = \$65\text{USD}$$

How much would a \$250 NZD item be worth in USD?

$$\$250\text{NZD} \times 0.65 = \$162.50\text{USD}$$

How much would a \$170 USD item be worth in NZD?

$$\$250\text{USD} \div 0.65 = \$384.62\text{NZD}$$

It can be easy to get the conversion the wrong way around if you don't keep your common sense. You could use estimation to make sure your converted in the right direction, i.e. should the NZD amount or USD amount be greater?

When you're ready, answer the following questions.

Lesson Four questions

Q1. Convert \$280 NZD to USD

Q2. Convert \$240 NZD to USD

Q3. Convert \$250 NZD to USD

Q4. Convert \$1 010 NZD to USD

Q5. Convert \$890 NZD to USD

Q6. Convert \$410 NZD to USD

Q7. Convert \$1 350 NZD to USD

Q8. Convert \$1 660 NZD to USD

Q9. Convert \$930 USD to NZD

Q10. Convert \$1 430 USD to NZD

Q11. Convert \$1 040 USD to NZD

Q12. Convert \$1 630 USD to NZD

Lesson Five: Time

Time
Time has particularly odd conversions. Rather than being multiples of 10, there are 60 minutes in an hour, 24 hours in a day, and other unpredictable conversions.

Months are particularly troublesome as there isn't a fixed number of days in a month. If you can avoid months, you should. For example, you rather than dividing a year into 12 months then into 4-ish weeks for each month, you could divide the year into 52 weeks (with one day left over). Or, even better, you could divide a year into 365 days (ignoring leap years).

If you are forced to deal with months, it's a good idea to know how many or how few days it could mean. For example, if a holiday costs $250/day for three months, how many days is that?

It could be as few as:

It could be as many as:

Therefore the cost of the holiday is:

Lesson Five questions
Q1. How many weeks are in a year?

Q2. How many days are in a year?

Q3. How many days are in a week?

Q4. How many hours are in a day?

Q5. How many minutes are in a day?

Q6. How many seconds are in an hour?

Q7. How many days are in a fortnight?

Q8. How many hours are in a fortnight?

Q9. How many seconds are in a week?

Q10. How many hours are in a year?

Lesson Six: Rounding and estimation

When numbers have been rounded before they're given to you, the actual number is within a range. For example, Mike and Huia spending $250/day to the nearest $50 means they could spend anywhere between $225 and $274.99 (really per day.

It is possible to calculate using such a range. For example, the money they spend over three months can be calculated with the above range.

And while we're talking about it, the $250/day is an estimation that Mike and Huia made. Who's to say it can't be more or less than that? How can you account for this in your conclusion?

Calculate the amount that Mike and Huia will spend on the holiday.

Lesson Seven: Income tax

Introduction
Goods and services tax (GST) in an example of tax. Another example is income tax. GST is paid by a consumer when they buy a good or a service from a business. Income tax is paid when a worker is paid some income, e.g. wages or salary.

Your income tax rate depends on how much you earn; the more you earn, the greater the proportion of your income is taxed.

Some examples
The first \$14 000 you earn in a year is taxed at a rate of 10.5%. For example, if you earn \$12 000 in a year then you'll be taxed:

$$\$12\,000 \times 0.105 = \$1\,260$$

If you earn \$20 000 in a year, \$14 000 of it will be taxed at 10.5% and the remaining \$6 000 will be taxed at the higher rate of 17.5%. For example:

$$\$14\,000 \times 0.105 = \$1\,470$$
$$\$6\,000 \times 0.175 = \$1\,050$$
$$\$1\,740 + \$1\,050 = \$2\,790$$

Anyone earning \$20 000 a year pays \$2 790 in income tax so they actually only earn:

$$\$20\,000 - \$2\,790 = \$17\,210$$

The different tax rates
Here are the different tax rates.

Income	Tax rate
\$0 - \$14 000	10.5%
\$14 001 - \$48 000	17.5%
\$48 001 - \$70 000	30%
\$70 000 +	33%

Calculate the income tax and the income remaining after tax on the following yearly incomes:

\$8 000

\$ 40 000

\$55 000

\$90 000