Level 1 – AS91027

Standard 1.2

4 Credits – External

Apply algebraic procedures in solving problems

Contents

A note on the content

Any information inside a thin dotted border is key words or concepts being introduced for the first time. A short definition and/or explanation is given.

Any information or questions in a thick dashed border is extension work and is not explicitly required by the standard. However, it is helpful material and should be attempted if possible. At times it will be at a similar level of difficulty to the rest of the work, and at other times it will be harder and require more attention and care.

Any text in a rounded solid border is a summary of the main idea or the "rule". Any text in these borders should be well understood by the end of the unit. These will occur at the end of each lesson, just before the questions.

Homework tracking

As you do homework for this standard, keep track of it all right here. Do enough to keep up with the course work.

Week	Day of the week	Length of time	What did you do?
Term 3, Week 5			
Term 3, Week 6			
Term 3, Week 7			
Term 3, Week 8			

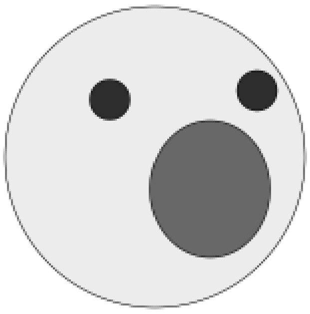
Lesson Zero: Variables

Variable [noun]: An element, feature, or factor that varies or changes.

Variables

Algebra is closely related to number which you explored in the 1.1 standard. Whereas number deals with fixed numbers such as 15, -4, π, and $\sqrt{2}$, algebra deals with variable numbers. Most concepts in number such as operations[1], factors, and simplifying also exist in algebra but with variables instead of numbers.

It would be beneficial for you to be familiar with variables before proceeding to lesson one. One way to familiarise yourself is to start the [Intro to JS: Drawing & Animation]() course on Khan Academy. Completing the first four modules (intro to programming, drawing basics, colouring, and variables) will prepare you for working with variables in this unit.



Lesson Zero: Variables summary

What number does with numbers, algebra does with variables.

[1] Operations include addition, subtraction, multiplication, and division.

Lesson One: Operations on variables

What you will learn:

- How to use the algebra tiles
- Adding and subtracting variables (like terms only)
- Multiplying and dividing variables
- Exponent laws

Constant [noun]:	A quantity that can't change, e.g. 2, 3, 15, -4, π, $\sqrt{2}$. The opposite of a variable.
Term [noun]:	Numbers and variables multiplied together, e.g. $3x$, x^2, xy
Like terms [noun]:	Terms that have the same variables to the same power, e.g. $5x$ and $8x$ are like, $2xy$ and $7yx$ are like, xy and x^2y are unlike, and x and x^2 are unlike.

You're already familiar with the algebra tiles from the 1.1 number workbook. You used them to operate on numbers. Remind yourself how to multiply 3×4 and -5×3 using the algebra tiles.

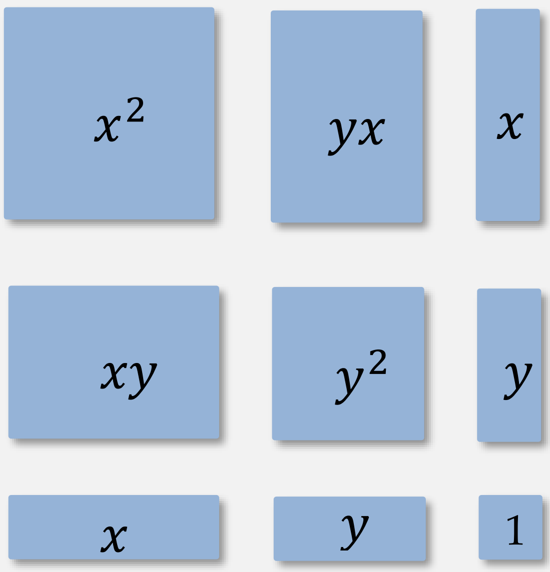
3×4	-5×3

Now you'll use the algebra tiles to operate on variables. I'll restrict you to using only two variables: x and y which gives you the following six terms: x^2, y^2, xy, x, y, and the constant term.

The new algebra tiles

You already know the 1 tile [the constant term]. Now we introduce the x term. It is x long and 1 high (or 1 long and x high). We also introduce the x^2 term with side length x on all sides.

When we add a second variable, y, we get three more terms: y^2, y, and xy. Notice that xy and yx are the same thing. Just like in number, $3 \times 4 = 4 \times 3$.



Addition and subtraction

Each term can only be added or subtracted with like terms. If the shape and size of the algebra tiles are the same, they can be added or subtracted from each other. You can use addition and subtraction to 'simplify' some algebraic expressions.

https://youtu.be/LTm7PeiqAgs

Example: Simplify $3x + 2 + 5 - y + x$. Draw how you use the algebra tiles to do so.

Multiplication and division

Recall what x^2 is short for: $x \times x$

In general, x^n is x multiplied by itself n times, e.g. $x \times x \times x \times x \times \ldots$

We can expand any term into a longer form, e.g.

$$4x^3y^2 = 4 \times x \times x \times x \times y \times y$$

We can use this ability to simplify algebraic expressions containing multiplication and division.

Example: Simplify $x^2 \times x^3 \times 4 \times y \times y^2 \times 2$

$$\begin{aligned} & x^2 \times x^3 \times 4 \times y \times y^2 \times 2 \\ &= x \times x \times x \times x \times x \times 4 \times y \times y \times y \times 2 \\ &= 4 \times 2 \times x \times x \times x \times x \times x \times y \times y \times y \\ &= 8x^5y^3 \end{aligned}$$

Example: Simplify $\frac{48x^3y}{18xy^2}$

$$\begin{aligned}\frac{48x^3y}{18xy^2} &= \frac{48 \times x \times x \times x \times y}{18 \times x \times y \times y} \\ &= \frac{8 \times x \times x}{3 \times y} \\ &= \frac{8x^2}{3y}\end{aligned}$$

Some mathematicians find having so many $\times$'s makes things hard to read, even if they use the fancy curved symbol for x. Therefore, mathematicians prefer to write multiplication with dots as in the example below.

Example: Simplify $x^2 \cdot x^3 \cdot 4 \cdot y \cdot y^2 \cdot 2$

$$\begin{aligned}&x^2 \cdot x^3 \cdot 4 \cdot y \cdot y^2 \\ &= x \cdot x \cdot x \cdot x \cdot x \cdot 4 \cdot y \cdot y \cdot y \cdot 2 \\ &= 4 \cdot 2 \cdot x \cdot x \cdot x \cdot x \cdot x \cdot y \cdot y \cdot y \\ &= 8x^5y^3\end{aligned}$$

Note: Mathematicians write variables in alphabetical order, e.g. $3x^4y^2$, not $3y^2x^4$

https://youtu.be/3-6MmtRPylY

Exponent laws

Notice that when you multiply two exponents with the same base, you add the exponents together, e.g.

$$x^2 \cdot x^3 = x^5$$

Similarly, when you divide two exponents with the same base, you subtract the exponents, e.g.

$$\frac{x^3}{x} = x^2$$

Finally, when you take the exponent of an exponent, you multiply them, e.g.

$$(x^3)^2 = x^6$$

These are known as the exponent laws.

Prime factorisation

Product [noun]:	Two or more numbers multiplied together.
Factor [noun]:	A number that multiplies with another factor to give a certain number, e.g. 4 and 3 are factors of 12.
Factorisation [noun]:	The process of decomposing a number into some of its factors. The verb is 'to factorise'.
Prime factor [noun]:	Factors that are prime numbers.
Prime factorisation [noun]:	The process of decomposing a number into its prime factors.

Every number can be written as a product of prime factors. For example:

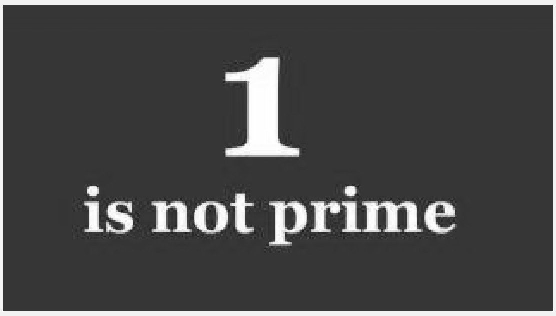
$$\begin{aligned} 36 &= 3 \cdot 12 \\ &= 3 \cdot 3 \cdot 4 \\ &= 3 \cdot 3 \cdot 2 \cdot 2 \\ &= 2^2 \cdot 3^2 \end{aligned}$$

Decomposing numbers into their prime factors is useful when simplifying fractions and finding the greatest common factor (GCF) and the lowest common multiple (LCM). For example:

$$\begin{aligned} \frac{74}{272} &= \frac{2 \cdot 36}{2 \cdot 136} \\ &= \frac{2^2 \cdot 18}{2^2 \cdot 68} \\ &= \frac{2^3 \cdot 9}{2^3 \cdot 34} \\ &= \frac{2^3 \cdot 3^2}{2^4 \cdot 17} \\ &= \frac{3^2}{2 \cdot 17} && \text{Cancelling terms from the numerator and denominator} \\ &= \frac{9}{34} \end{aligned}$$

The fact that each number has a *unique* prime factorisation is the Fundamental Theorem of Arithmetic. For more information on this, watch the video below.

1 and Prime Numbers – Numberphile



https://youtu.be/IQofiPqhJ_s

Lesson One: Operations on variables summary

Only like terms can be added or subtracted.

Don't write × for multiplication, i.e. $x \times y = xy$. The dot symbol also means multiply, e.g. $x \cdot y = xy$.

The exponent laws are:

$$x^a \cdot x^b = x^{a+b}$$

$$\frac{x^a}{x^b} = x^{a-b}$$

$$(x^a)^b = x^{ab}$$

Addition and subtraction questions

Q1. $5 + 5x + 1 - x$

Q2. $3a - 1 + 7a + 3$

Q3. $-3x - 2x + 1 - 3$

Q4. $k - k + 2k - 2 - 3k + 5$

Q5. $x + 4xy - 4x + 3y - yx$

Q6. $2ab - 3a + ba - 2b + 3$

Q7. $x^2 - 4 + 3x - 2 + x^2$

Q8. $b^2 - b - 4b^2 + 7a - ab$

Q9. $4xy - 5xy + 2x^2 - 3y^2 + y$

Q10. $2t - t^2 - 5t + t$

Multiplication and division questions

Q1. $2y^2 \cdot 3y^3 \cdot x$

Q2. $4k \cdot 2k^2 \cdot k$

Q3. $xy \cdot x^2y \cdot 2$

Q4. $ab \cdot ab^2 \cdot 3$

Q5. $3xy \cdot yx \cdot 4y$

Q6. $a^2b \cdot (ab)^2$

Q7. $0.5x \cdot y^2 \cdot 6x$

Q8. $-t \cdot t^2 \cdot 4t$

Q9. $(3x^3y^2)^2$

Q10. $\frac{1}{3}a^2 \cdot b \cdot ab$

Q11. $\dfrac{6x^2y^2}{2x}$

Q12. $\dfrac{9xy^3}{81x^3y^3}$

Q13. $\dfrac{25x \cdot xy}{15y^2 \cdot x}$

Q14. $\dfrac{(2xy^2)^3}{3x^4y}$

Q15. $\dfrac{(4a^3b^2)^3}{(2ab)^3}$

Q16. $\dfrac{(5tu^2)^2 \cdot 3t^2}{tu^2}$

Lesson Two: Expanding brackets

What you will learn:

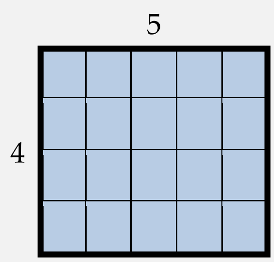
- Why you'd want to use brackets
- How to expand brackets

Introduction to brackets

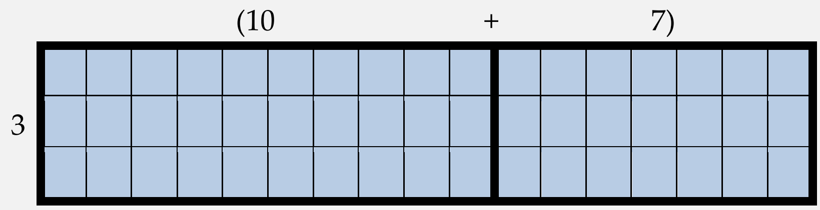
In 1.1 number you encountered brackets in BEDMAS. What does BEDMAS tell you about brackets?

What does the rest of BEDMAS stand for?

As an introduction to why you'd want to use brackets in algebra, I'll give you a situation where you use brackets in number. Imagine you wanted to multiply 4×5. You could make a rectangle with side lengths 4 and 5 out of algebra tiles and count how many small squares are contained in the rectangle.



This is clearly more work than is necessary for a simple multiplication. But it makes more sense if you wanted to multiply 3×17. You could make a rectangle with side lengths 3 and 17. Or, even better, you could split the side with length 17 into $(10 + 7)$. This is how brackets are introduced into the calculation.



Now it's easier to see that:

$$\begin{aligned} 3 \times 17 &= 3(10 + 7) \\ &= 3 \times 10 + 3 \times 7 \\ &= 30 + 21 \\ &= 51 \end{aligned}$$

What if you used some variables as well? How would you multiply 4 and $x + 2$?

$$\begin{aligned} 4(x + 2) &= 4 \times x + 4 \times 2 \\ &= 4x + 8 \end{aligned}$$

Draw how you would use the algebra tiles to multiply $4(x+2)$

Expand: Multiplying two numbers together, e.g. $4(x+2)$. This gets rid of brackets.

<u>Expanding one bracket</u>

When you multiplied $4(x+2)$ you "expanded" it. You can expand when there's more than one variable. For example:

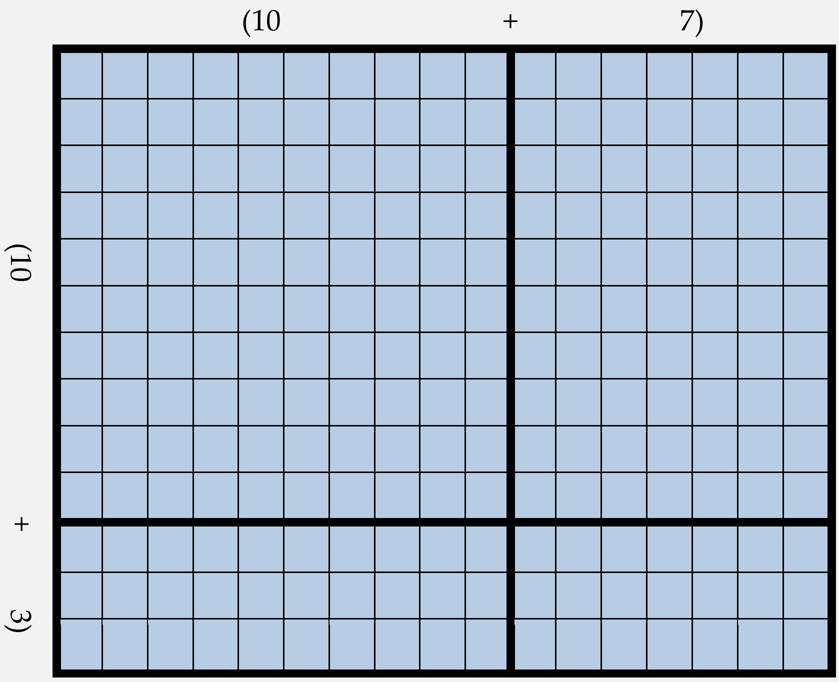
$$x(2y-4) = 2xy - 4x$$

<u>Expanding two brackets</u>

If you have two brackets multiplied together, you need to multiply everything in the first bracket by everything in the second bracket. Usually this means you'll have four multiplications to do.

In number, it looks like this:

$$\begin{aligned}&13 \times 17\\&= (10+3)(10+7)\\&= 10 \times 10 + 10 \times 7 + 3 \times 10 + 3 \times 7\\&= 100 + 70 + 30 + 21\\&= 221\end{aligned}$$



https://youtu.be/pkujj35xwG4

https://youtu.be/qRfV9_WOEhY

Use the algebra tiles to expand $(x+2)(x+3)$

<u>Lesson Two: Expanding brackets summary</u>

Expanding brackets involves multiplying everything in the first bracket by everything in the second bracket. The brackets disappear.

You need to simplify your expansions by combining like terms, e.g.

$$
\begin{aligned}
&(x+2)(x+3)\\
&= x^2+3x+2x+6\\
&= x^2+5x+6
\end{aligned}
$$

Expanding brackets questions

Q1. $3(x + 4)$

Q2. $5(2x - y + z)$

Q3. $2x(6 - x)$

Q4. $-4(x + 2)$

Q5. $-3x(x + 7)$

Q6. $-2x(x - 4)$

Q7. $6x(2y + 3)$

Q8. $-x^2(2x + 3)$

Q9. $(x + 4)(x + 7)$

Q10. $(x + 11)(x - 3)$

Q11. $(y + 2)(6 - y)$

Q12. $(x - 3)(y - 4)$

Q13. $(x + 5)(x - 5)$

Q14. $2(x - 10)(x + 10)$

Q15. $(x + 3)^2$

Q16. $3(x - 4)^2$

Q17. $(3x - 3)(2x + 2)$

Q18. $4(7x - 4) - 3(x + 2)$

Q19. $(x + 1)(2x + 1) + (x - 2)(x + 3)$

Q20. $(6 - x)^2 + (2x + 1)^2$

Lesson Three: Factorising brackets

What you will learn:

- How to factorise brackets using common factors
- How to factorise brackets using grouping
- How to factorise quadratics (two brackets)
- How to factorise quadratics with $a \neq 1$

Factorise:	Splitting a number into its factors. This introduces new brackets where there weren't any before. It is the opposite of expanding.

Factorising brackets (with common factors)

Factorising is the opposite of expanding. It takes you from $4x + 8$ to $4(x + 2)$. How do you do that? The clue is in the name. You need to look for factors that are common to both terms in the bracket. Recall what factors are:

Both $4x$ and 8 have a factor of 4 in common so you can divide both of them by 4. Put brackets around these divided numbers and put the factor in front of the bracket. For example:

$$\begin{aligned} 4x + 8 &= 4\left(\frac{4x}{4} + \frac{8}{4}\right) \\ &= 4(x + 2) \end{aligned}$$

Factorise these expressions:

$6x + 21$ $\qquad$ $5x^2 + 30x$ $\qquad$ $24 - 4x$ $\qquad$ $x(x + 2) + 3(x + 2)$

What if there are no common factors across all terms? Can you still factorise? Yes! There are actually four different methods you can use to factorise, depending on the situation. You've seen the first one: common factors. The second is called grouping.

https://youtu.be/341wp4eT0rY

Grouping

If there are no common factors across all terms, it may be possible that there are pairs of terms that have common factors. For example, in

$$x^2 + 5x + 3x + 15$$

there is no common factor. But if you look at pairs of terms in isolation, you can see that $x^2 + 5x$ has a common factor of x and that $3x + 15$ has a common factor of 3.

If you factorise both of these pairs in isolation, you get:

$$x(x + 5) + 3(x + 5)$$

Now, both of these terms have a common factor of $(x + 5)$ so you can factorise again to get:

$$(x + 5)(x + 3)$$

Grouping helps you factorise in situations where the solution exists but you can't reach it just by using common factors.

Quadratics

If there are no common factors across all terms and there are no groupings or pairs you can factorise in isolation, you could use the third tool for factorising: quadratics. For example, how could you factorise:

$$x^2 + 8x + 15$$

Notice that it's the same as the question you just looked at because:

$$x^2 + 8x + 15 = x^2 + 5x + 3x + 15$$

So you know the factorisation will be $(x + 5)(x + 3)$, but how could you factorise this properly? Notice that the numbers in the factorisation [5 and 3] add to 8 and multiply to 15 which are the numbers in the second and third terms of the question. This is a reliable shortcut and always works as long as there isn't a number in front of the x^2 term.

For example, to factorise $x^2 + 4x - 21$, you need two numbers that multiply to -21 and add to 4. The only two numbers that do this are 7 and -3. Therefore, the factorisation is:

$$(x + 7)(x - 3)$$

https://youtu.be/YGoYvkfpTSI

Quadratics with multiple x^2 terms

If there is a multiplier [AKA coefficient] in front of the x^2 term then you can't use the quadratic shortcut you just learned. Hopefully, you can factorise a common factor to remove the coefficient. For example:

$$\begin{aligned}&3x^2 + 15x + 18\\&= 3(x^2 + 5x + 6)\\&= 3(x + 2)(x + 3)\end{aligned}$$

If you can't, you'll need to split the middle term [the x term] into two pieces. But you'll need to do it in a very precise way. To work out the precise split that you need to make, multiply the first and last terms. For example:

$$\begin{gathered}2x^2 + 11x + 12\\2x^2 \cdot 12 = 24x^2\end{gathered}$$

Now you need two numbers that multiply to make what you just calculated [$24x^2$] and sum to the x term [$11x$]. The factors of $24x^2$ are $\{x, 24x\}$, $\{2x, 12x\}$, $\{3x, 8x\}$, and $\{4x, 6x\}$. The only factors of $24x^2$ that add to $11x$ are $3x$ and $8x$. This is the split you need to make.

$$\begin{aligned}&2x^2 + 11x + 12\\&= 2x^2 + 3x + 8x + 12\end{aligned}$$

Now, there are still no common factors but there are pairs of terms that you can factorise separately.

$$\begin{aligned}&2x^2 + 3x + 8x + 12\\&= x(2x + 3) + 4(2x + 3)\\&= (2x + 3)(x + 4)\end{aligned}$$

And you're done! You can now factorise quite complicated expressions using one of the four methods.

https://youtu.be/cCUgnulHazM

Why four different methods?

You might be wondering why you only need one method of expanding but four methods of factorising. Here's a secret: there is actually only *one* method of factorising.

That one method was mentioned at the very beginning of the lesson in the definition. Copy the first sentence of the definition of factorising below:

How do you split a number into its factors? Let's look at number first, before moving on to algebra. If I asked you for the factors of 12 I'm actually asking you which rectangles can you make with 12 squares? Try this with the algebra tiles and draw the rectangles below.

So the factors of 12 are: $\{1, 12\}, \{2, 6\}, \{3, 4\}$. You could also include $\{4, 3\}, \{6, 2\}, \{12, 1\}$ but they're double-ups so you can omit them.

Do the same process for these algebraic expressions and draw how you used the algebra tiles to factorise them.

$4x + 2$	$3x^2 + 6x + 2x + 4$	$x^2 + 6x + 8$	$2x^2 + 9x + 4$

Lesson Three: Factorising brackets summary

Factorising is the opposite of expanding and involves splitting an expression into its factors. It introduces brackets that weren't originally there.

It is worthwhile expanding your answer to check you get back to the original question.

Methods	Use when…	For example…
Common factors	There is a common factor across all terms	$4x + 2 = 2(2x + 1)$
Grouping	There is no common factor but the terms can be grouped into pairs that can be factorised	$3x^2 + 6x + 2x + 4$ $= 3x(x + 2) + 2(x + 2)$ $= (x + 2)(3x + 2)$
Quadratics	There is no common factor and there are no groups of pairs	$x^2 + 6x + 8$ $= (x + 2)(x + 4)$
Quadratics with multiple x^2 terms	There is no common factor, no groups of pairs, are multiple x^2 terms, and you cannot divide by a common factor to eliminate the multiple x^2 term	$2x^2 + 9x + 4$ $= 2x^2 + 8x + x + 4$ $= 2x(x + 4) + (x + 4)$ $= (2x + 1)(x + 4)$

Factorising brackets questions

Q1. $14x + 21$

Q2. $5xy - 15x$

Q3. $6x^2 + 3x$

Q4. $30 - 4x$

Q5. $7x(x + 2) - 4(x + 2)$

Q6. $-4x(x - 1) - 2(x - 1)$

Q7. $4x^2 + 8xy + 5x + 10y$

Q8. $3x + 3y + 2xy + 2x^2$

Q9. $6a + 12b + 4ax + 8bx$

Q10. $ax + bx + ay + by$

Q11. $x^2 + 5x + 6$

Q12. $x^2 - 5x + 6$

Q13. $x^2 + 8x + 16$

Q14. $x^2 + 16x + 64$

Q15. $x^2 - 16$

Q16. $x^2 - 64$

Q17. $2x^2 + 16x + 30$

Q18. $5x^2 - 4x - 1$

Q19. $2x^2 - 7x + 6$

Q20. $4x^2 - 20x + 25$

Lesson Three-point-five: Rational expressions

Rational:	Able to be expressed as a ratio, in other words, as a fraction.

Rational expressions

Rational expressions are those that are, or can be, expressed as a fraction. These are examples of rational expressions:

$$\frac{x}{y} \qquad \frac{x(x+3)}{(x+3)} \qquad \frac{(x+4)(x-2)}{(x+4)} \qquad \frac{x^2+2x-8}{(x+4)}$$

If a factor occurs in both the numerator [the top number] and the denominator [the bottom number] then you can simplify it by cancelling out both of the factors. For example:

$$\frac{x(x+3)}{7(x+3)} = \frac{x}{7} \qquad \frac{4x(x-2)}{8(x-2)} = \frac{4x}{8} = \frac{4x}{4 \cdot 2} = \frac{x}{2}$$

You will almost certainly need to factorise the numerator and/or the denominator before you can simplify. For example:

$$\begin{aligned} &\frac{x^2+2x-8}{x+4} \\ &= \frac{(x+4)(x-2)}{x+4} \\ &= (x-2) \end{aligned}$$

https://youtu.be/JjkIEXFuAOA

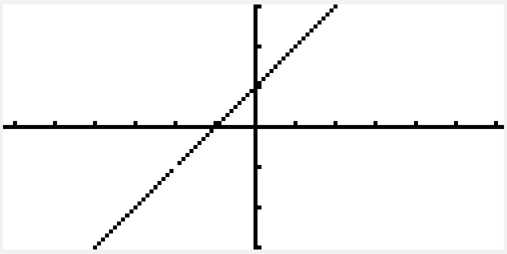
Holes in graphs

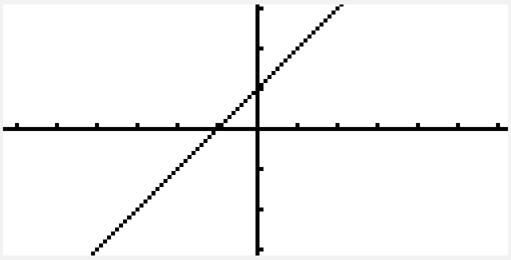
The graphs of:

$$y = \frac{(x+2)(x+1)}{(x+2)} \qquad \text{and} \qquad y = x + 1$$

are almost undistinguishable from one another. Why do you think they'd be so similar?

They only have one *tiny* difference. Here are the plots of both graphs below:





What is the difference and where does it occur?

Why did it occur at that particular coordinate?

We say that the graph is "not defined" or that it "does not exist" for $x = -2$.

Lesson Three-point-five: Rational expressions summary

Rational expressions require you to simplify fractions that include numbers and variables.

If a factor occurs in the numerator AND the denominator, you can simplify the expression by cancelling them out.

You'll need to factorise the numerator or the denominator (or both) before you can simplify.

Rational expressions questions

Simplify these rational expressions:

Q1.
$$\frac{2x(x+4)}{x}$$

Q2.
$$\frac{8x(x+2)}{2}$$

Q3.
$$\frac{(x-1)(x-5)}{(x-1)}$$

Q4.
$$\frac{(x+4)(x-4)}{(x-4)}$$

Q5.
$$\frac{x^2+7x+10}{x+2}$$

Q6.
$$\frac{x^2+9x-22}{x+11}$$

Q7.
$$\frac{2x^2+8x+6}{x+3}$$

Q8.
$$\frac{2x^2-7x+6}{x-2}$$

Q9.
$$\frac{3x^2-8x+4}{x^2+2x-8}$$

Q10.
$$\frac{4x^2-20x+25}{2x^2-x-10}$$

Lesson Four: Linear equations and inequations

What you will learn:

- The true meaning of =
- How to keep equations equal
- How to solve equations
- What inverse operations are
- How to solve inequations

Equation:	Able to be expressed as a ratio, in other words, as a fraction.

The true meaning of =

A common misconception of the equal sign is that it means "the answer follows". But it means exactly what it says, that two things are *equal.*

Consider the bus driver riddle:
"You are driving a bus. At your first stop, you pick up 29 people. On your second stop, 18 people get off, and 10 new people arrive. At your next stop, 3 people get off, and 13 people come on. On your fourth stop 4 of the people get off, then 17 new people get on. What is the bus driver's name?

If you wanted to calculate the number of passengers on the bus, you could show working like this:

$$29 - 18 = 11 + 10 = 21 - 3 = 18 + 13 = 31 - 4 = 27 + 17 = 44$$

You might read the working to say: "29 people are on the bus and 18 get off which is 11. Then 10 people get on and 11 + 10 is 21. Then 3 people get off, 21 – 3 is 18. 13 people get on and 18 + 13 is 31, and so on…"

But the equals sign means that thing are equal. What you're actually saying is:

$$\begin{array}{c} 29 - 18 = 11 + 10 = 21 - 3 = 18 + 13 = 31 - 4 = 27 + 17 = 44 \\ 11 \quad = \quad 21 \quad = \quad 18 \quad = \quad 31 \quad = \quad 27 \quad = \quad 44 \quad = 44 \end{array}$$

which is obviously not true. This is a common misconception among maths students. Rewrite the working so that it makes sense:

You've just used the equal sign correctly to mean "equal". Another example of using it correctly is I could say that:

$$my\ level\ of\ awesome = your\ level\ of\ awesome = high$$

which, in other words, says "my level of awesome is the same as yours and they're both high". Or simply "you and I are both awesome".

Here's a final example of the equal sign being used correctly to mean "equal":

$$25 = 25 = 25 = 25$$

Keeping equations equal

Equations are descriptions of two or more things that are equal to each other. For example:

$$4 \times 8 = 16 \times 2$$

Or similarly:

$$3 + 8 = 15 - 4$$

Very often, you'll want to change one side of the equation. For example, you could add 1 to the right hand side (RHS) of the equation like this:

$$3 + 8 \neq 15 - 4 + 1$$

Notice how the equal sign turned into a "not equal" sign because it is no longer an equation. What you are looking at is an *inequation*.

There is a rule for keeping both sides of an equation equal. What is it?

____

____

Solving equations

Solve:	Calculate the value of the variable, e.g. "solve for x", means find the value of x in that particular situation. It requires you to isolate the variable on one side of the equation.

When you are asked to solve an equation, you need to find the value of the variable in that particular situation. For example, I have a bag full of (insert an arbitrary noun here) and I want to share them with all my friends. I have only seven friends... I give them all out and each of my friends get eight. How many (arbitrary noun)s were in my bag?

Find the answer before continuing. Write it here: ______

You could solve for the starting number of items in this case quite easily. You probably didn't need to use algebra. If you were to use algebra, you could call the starting number of items x and say that:

$$\frac{x}{7} = 8$$

What does solving an equation require you to do?

And how could you do that?

Inverse operations

Inverse operations "undo" other operations. For example, subtraction is the inverse operation of addition and vice versa.

What are the other inverse operations?

Operation	Inverse operation
Addition	
Multiplication	
Square	
Cube	

What is the inverse operation of subtraction?

What is the inverse operation of division?

https://youtu.be/IHNvnQq4OYg

Inequations

It is possible to solve inequations the same way. There are examples of inequations:

$3x + 2 < 11$ $\qquad$ $\frac{x}{4} \leq 20$ $\qquad$ $x + 5 > -2$ $\qquad$ $2x - 1 \geq 20$

They are solved exactly the same as equations except that if you multiply of divide by a negative number, you flip the direction of the inequality.

https://youtu.be/KAj1sLtlRrs

Lesson Four: Linear equations and inequations summary

Equations are balanced and need to stay balanced. What you do to one side, you have to do to the other.

Use inverse operations to isolate the variable on one side of the equation to solve it.

If you multiply or divide an inequation by a negative number, flip the direction of the inequality.

Lesson Four: Linear equations and inequations questions

Solve the following equations and inequations for x

Q1.

$$5x + 18 = 33$$

Q2.

$$20 = 3x + 14$$

Q3.

$$2(x - 3) = 8$$

Q4.

$$5x + 7 = x - 2$$

Q5.

$$\frac{x}{3} + 7 = 11$$

Q6.

$$\frac{x - 5}{2} = 6$$

Q7.

$$5x + 18 < -17$$

Q8.

$$9 + 2x \geq -19$$

Q9.

$$\frac{x}{5} + 1 < -3$$

Q10.

$$\frac{2 - 3x}{2} \leq -5$$

Q11.

$$\frac{20}{x} = 4$$

Q12.

$$\frac{12}{x} = \frac{3}{4}$$

Q13.

$$\frac{3x}{5} = \frac{x + 4}{3}$$

Q14.

$$\frac{2x + 5}{x - 1} = \frac{-1}{3}$$

Q15.

$$\frac{4x - 5}{2} = \frac{3}{4}$$

Q16.

$$\frac{x}{5} + \frac{2x}{3} = 6$$

Q17.

$$8 - \frac{x}{3} > 6$$

Q18.

$$\frac{-3x + 1}{7} > 5$$

Q19.

$$-8x + \frac{3x}{2} \leq 5$$

Q20.

$$\frac{10x}{x} + \frac{12}{x} = 14$$

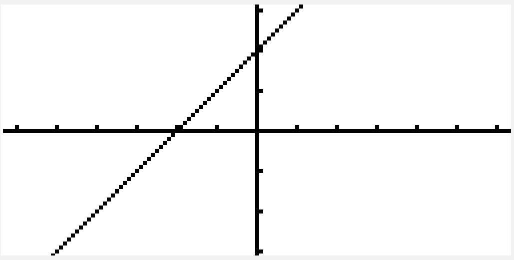
Lesson Five: Quadratic equations

What you will learn:

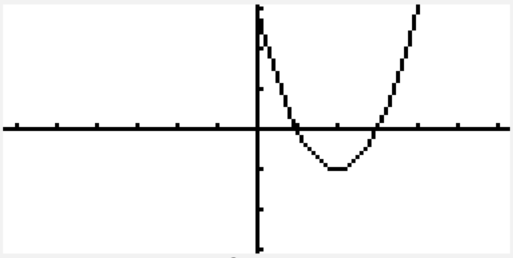
- The number of solutions a quadratic can have
- How to solve quadratic equations

Introduction to quadratic equations

In the previous lesson you were solving linear equations which required you to find the one (and only one) value of x that balances the equation. If you had chosen any other value for x, the equation would be unbalanced and unsolved. Quadratics equations are different from linear equations. Here's an example of each:



$y = x + 2$



$y = x^2 - 4x + 3$

Try solving the linear equation above for when $y = 0$

$$x + 2 = 0$$
$$x =$$

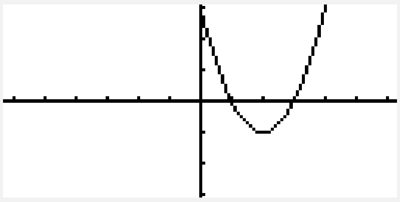
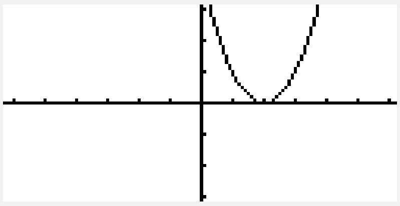
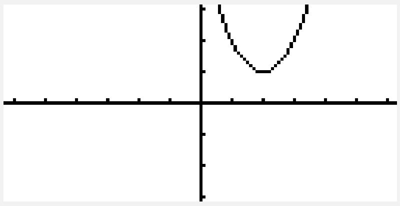
Try solving the quadratic equation above for when $y = 0$ by substituting different values of x

x	$x^2 - 4x + 3$
-1	
0	
1	
2	
3	
4	

With this particular quadratic, there are two values of x that balance the equation so there are two solutions.

Number of solutions

Quadratics can have either 0, 1, or 2 solutions.

Two solutions	One solution	No solutions
		

Solving quadratic equations

It's easiest to solve quadratic equations in factorised form. For example, to solve:

$$x^2 - 4x + 3 = 0$$

put it in factorised form:

$$(x - 1)(x - 3) = 0$$

You want the left hand side (LHS) of the equation to equal the RHS, that is, to equal zero. The LHS has two numbers multiplied together and zero multiplied by anything is zero. So either:

$$x - 1 = 0$$
or
$$x - 3 = 0$$

In other words:

$$x = 1 \; or \; x = \; 3$$

You may need to rearrange the equation so that the right hand side (RHS) is zero. For example:

$$\begin{aligned} x^2 &= -4x - 4 \\ x^2 + 4x + 4 &= 0 \\ (x + 2)^2 &= 0 \\ x &= -2 \end{aligned}$$

https://youtu.be/6rahURA-2GU

Lesson Five: Quadratic equations summary

Quadratic equations can have 0, 1, or 2 solutions.

To solve a quadratic equation, have the RHS of the equation equal zero and factorise the quadratic. The solutions can then be easily calculated.

Lesson Five: Quadratic equations questions

Solve the following quadratic equations

Q1.
$$(x-4)(x-6)=0$$

Q2.
$$(x+5)(x-8)=0$$

Q3.
$$x(x-2)=0$$

Q4.
$$x(x+3)=0$$

Q5.
$$(2x+2)(x-3)=0$$

Q6.
$$6(x-1)(3x+2)=0$$

Q7.
$$(x-3)^2=0$$

Q8.
$$(x+7)^2=0$$

Q9.
$$x^2+x-12=0$$

Q10.
$$x^2-13x+12=0$$

Q11.
$$x^2+8x+12=0$$

Q12.
$$x^2-3x+2=0$$

Q13.
$$x^2-64=0$$

Q14.
$$x^2=x+30$$

Q15.
$$4x^2+2x-12=0$$

Q16.
$$9x^2=16$$

Q17.
$$7x^2+x=0$$

Q18.
$$5x^2=24x+5$$

Q19.
$$x=\frac{2x+3}{x}$$

Q20.
$$x=\frac{1}{x}$$

Lesson Six: Simultaneous equations

What you will learn:

- How to visualise simultaneous equations
- How to solve simultaneous equations using graphs
- How to solve simultaneous equations using algebra

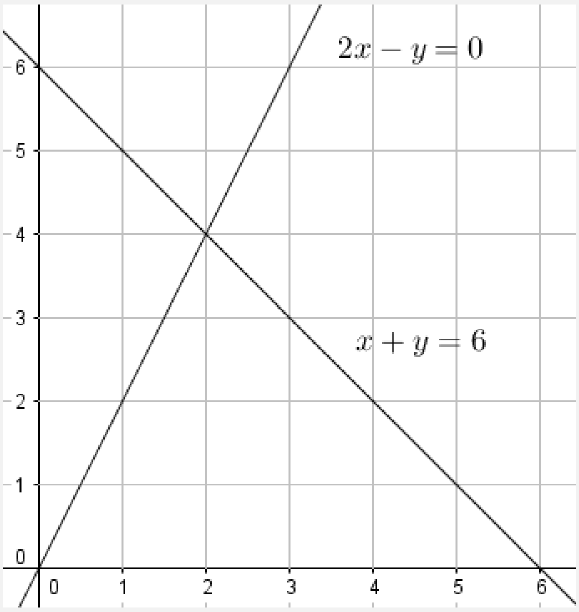
Introduction to simultaneous equations

Up until now, you've been solving one equation at a time with one variable. It is possible to solve two equations at the same time, that is to say, *simultaneously*. They also have two variables to solve rather than one.

Perhaps it's best not to think about it too much and instead, consider how to visualise them.

Visualising simultaneous equations

Take two equations: $2x - y = 0$ and $x + y = 6$



The point at which they intersect is the solution to both equations. In this case, $x = 2$ and $y = 4$

Confirm this by substituting $x = 2$ and $y = 4$ into both equations.

There are many methods to solve simultaneous equations. I'll show you two: algebra and graphs.

https://youtu.be/QPQUCgZ7u80

Solving simultaneous equations with graphs

I'll solve the earlier example using graphs. I have the equations $x + y = 6$ and $2x - y = 0$ and I want to calculate the intersection of the two.

I'd rearrange the equations into the form $y = ax + b$

$$\begin{aligned} x + y &= 6 \\ y &= -x + 6 \end{aligned}$$

$$\begin{aligned} 2x - y &= 0 \\ 2x &= y \\ y &= 2x \end{aligned}$$

Now I can set both equations equal to each other:

$$-x + 6 = 2x$$

and solve for the x variable:

$$\begin{aligned} -x + 6 &= 2x \\ 6 &= 3x \\ x &= 2 \end{aligned}$$

Now I can substitute $x = 2$ into either of the equations (it doesn't matter which one) to solve for the y variable:

$$\begin{aligned} x + y &= 6 \\ 2 + y &= 6 \\ y &= 4 \end{aligned}$$

$$\begin{aligned} 2x - y &= 0 \\ 2 \times 2 - y &= 0 \\ y &= 4 \end{aligned}$$

Solving simultaneous equations with algebra

I'll solve the earlier example again, but this time, using algebra. I need to rearrange the equations into the form $ax + by = c$ (which they already are).

$$x + y = 6$$

$$2x - y = 0$$

Now I can write them on top of each other like this:

$$\begin{aligned} x + y &= 6 \\ 2x - y &= 0 \end{aligned}$$

I want to eliminate one of the variables. I could eliminate y by adding the two equations together:

$$\begin{aligned} &(x + y = 6) \\ &\underline{+(2x - y = 0)} \\ &3x \qquad = 6 \end{aligned}$$

which I can now solve for $x = 2$. Now I can substitute this into either of the equations to solve for y just like before.

Solving simultaneous equations with a matrix

Unfortunately, no one can be told what the Matrix is. You have to see it for yourself.

-Morpheus

A matrix is an array of numbers like these:

$$\begin{bmatrix} 1 & 4 \\ 7 & 2 \end{bmatrix} \qquad \begin{bmatrix} -3 & 2 \\ 1 & 2 \\ 0 & 2 \end{bmatrix} \qquad \begin{bmatrix} 0 & 4 & 1 \\ 1 & 5 & 2 \end{bmatrix} \qquad \begin{bmatrix} 4 & 1 \end{bmatrix} \qquad \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Matrices are useful for storing large amounts of data in a single object. For example, I can store the coefficients of the two equations in a matrix. In other words:

$$\begin{aligned} x + y &= 6 \\ 2x - y &= 0 \end{aligned}$$

becomes:

$$\begin{bmatrix} 1 & 1 & 6 \\ 2 & -1 & 0 \end{bmatrix}$$

I want to perform row operations on the matrix to reduce it to the form:

$$\begin{bmatrix} 1 & 0 & x \\ 0 & 1 & y \end{bmatrix}$$

I can start by adding row 2 to row 1:

$$\begin{bmatrix} 3 & 0 & 6 \\ 2 & -1 & 0 \end{bmatrix} \begin{matrix} \rho_1 + \rho_2 \\ \end{matrix}$$

Then divide row 1 by 3:

$$\begin{bmatrix} 1 & 0 & 2 \\ 2 & -1 & 0 \end{bmatrix} \begin{matrix} \rho_1 \div 3 \\ \end{matrix}$$

Now I can subtract two lots of row 1 from row 2:

$$\begin{bmatrix} 1 & 0 & 2 \\ 0 & -1 & -4 \end{bmatrix} \begin{matrix} \\ \rho_2 - 2\rho_1 \end{matrix}$$

And finally I can multiply row 2 by -1

$$\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 4 \end{bmatrix} \begin{matrix} \\ \rho_2 \times -1 \end{matrix}$$

This matrix corresponds to the solutions $x = 2$ and $y = 4$. A quick note about the symbol ρ. It's the Greek letter "rho" and is pronounced the same as "row". It's a little mathematical pun.

Lesson Six: Simultaneous equations summary

Simultaneous equations have two variables and are solved at the same time.

You can solve them using graphs or algebra (or matrices).

Lesson Six: Simultaneous equations questions

Q1.
$$x = y - 3$$
$$2x - 3y = 4$$

Q2.
$$3x + y = 7$$
$$x - y = 5$$

Q3.
$$2y - 2x = 8$$
$$y + 2x = 10$$

Q4.
$$6x + 2y = 6$$
$$x - 2y = 8$$

Q5.
$$3x + 4y = 1$$
$$2x - 3y = -5$$

Q6.
$$y = 2x + 7$$
$$y = x + 4$$

Q7.
$$x = y$$
$$3x - 5y = 8$$

Q8.
$$8x + 7y = 42$$
$$2y - 2x = -3$$

Q9.
$$2x + 5y = -9$$
$$3x - 4y = 21$$

Q10.
$$x + y = 7$$
$$2x - 3y = 4$$

Q11.
$$2y + x = 3$$
$$y - 2x = -6$$

Q12.
$$y - 3x = 3$$
$$2y = x + 6$$

Q13.
$$3x = 4 + 2y$$
$$y = 5 - 2x$$

Q14.
$$5x = 30 - 2y$$
$$3x - y = 7$$

Q15.
$$4x + 2y = 10$$
$$x - 3y = 6$$

Q16.
$$x = y - 4$$
$$4x - 2y + 8 = 0$$

Q17.
$$3x + 2y = 12$$
$$4x - 3y = -1$$

Q18.
$$2x + 3y + 5 = 0$$
$$x - 2y + 6 = 0$$

Q19.
$$5x - 6y = -13$$
$$y = 8x - 5$$

Q20.
$$3y - 5x = 1$$
$$4y + x = 9$$

Weekly exercise 1

1. Define variable.

2. Which of these terms are like and which are unlike?

$16x^2, -3x^2$	$4x, 7y$	$-x, x$	xy, yx	x^2y, xy^2
$15, 5y$	$2y^2, 3y^2$	$24, 7$	$2x, -x$	$-5x, -24x$

3. Expand $(x-5)(x+3)$

4. Simplify fully $(2a^4)^3$

5. Solve $m^3 = 64$

6. Solve $3^n = 81$

7. Sam has gone on holiday and forgotten to return a reference book to the library. He is fined \$3 for the first week he is late returning the bool. He is fined 2 times as much if he is 2 weeks late, 4 times the original fine if he is 3 weeks late, and so on.

   The formula used to calculate the total fine, T, that Sam has to pay is $T = 3 \times 2^{(n-1)}$, where w is the number of weeks he is late in returning the book. How many weeks late did he return the book if the fine was \$192? *You must show use of the formula.*

8.
   a. Jamie and Pippa go to a fun park for a day out. They go on different rides. Jamie's day out costs 1.5 times as much as Pippa's. Write an equation for the **total cost** of the day out in terms of P (the cost of Pippa's day out).

   b. Zack joins his two friends for the day out. His day out costs \$5 more than Pippa's. The total cost of the day out for the three of them is \$75. Use algebra to find the cost of Zack's day out.

Weekly exercise 2

1. Factorise $x^2 - 2x - 63$

2. Solve $(m + 6)(m - 2) = 0$

3. Factorise the expression below and write it in its simplest form:
$$8ab^2 - 3a^2b + 4a^2b$$

4. Write as a single fraction:
$$x - \frac{x-2}{5}$$

5. Solve $9 - 4x = 1$

6. Solve $4(2n - 3) = 3(n - 11)$

7. Solve $6x - 3 \geq 8x + 9$

8.
   a. Sharee wants to know how much money Tama has saved for the holidays. Tama says he has saved **at least** \$18, plus ¾ of the amount Sharee has saved. Write an equation for the amount, T, Tama has saved in terms of the amount, S, Sharee has saved.

   b. Sharee has saved \$72. Find the amount Tama has saved and show how it relates to Sharee's amount. *You must justify your statement using algebra and show that you have used your expression from part 1.*

9. For what values of n will h be negative when $h = n^2 - 7n + 10$?

10. Write an expression for f in terms of u and v.
$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

11. Using the same formula as above, write an expression for v in terms of u and f.

Weekly exercise 3

1. Sophia and Rewa are being paid to tidy the grounds of their neighbours' house. Sophia, being the elder of the two, is paid \$17 an hour, and Rewa is paid \$13 an hour. Altogether they earned a total of \$176.

    a. If Sophia worked for s hours and Rewa worked for r hours, write an equation showing the above information.

    b. Altogether they worked a total of 12 hours. Use algebra to show how many more hours Rewa worked than Sophia.

2. A cylinder and a cone have the same height. They volume of the cylinder is half the volume of the cone. The volume of the cylinder is $\pi r^2 h$ and the volume of the cone is $\frac{1}{3}\pi R^2 h$. Describe the relationship between the radius of the cone and the radius of the cylinder. *You must show algebraic working and then describe the relationship in words.*

3. Solve $x^2 + 5x - 14 = 0$

4. Jamie and Pippa go to a fun park for a day out. They go on different rides. Jamie's day out costs 1.5 times as much as Pippa's.
    a. Write an equation for the **total cost** of the day out in terms of P (the cost of Pippa's day out).

    b. Zack joins his two friends for the day out. His day out costs \$5 more than Pippa's. The total cost of the day out for the three of them is \$75. Use algebra to find the cost of Zack's day out.

5. Simplify:
$$\frac{r^2 - 1}{r^2 + r}$$

6. An n-sided polygon had D diagonal, where $D = \frac{n}{2}(n-3)$. Use the formula to find how many sides the polygon has, if there are 20 diagonals.

7. For the sequence of numbers where $T = n^2 - n + a$, show that for any value of n, if $n = a$, then T will never be a prime number. Assume $n > 1$.

Appendix A: Answers

Lesson One: Addition and subtraction questions

Q1.
$5 + 5x + 1 - x$
$= 4x + 6$

Q2.
$3a - 1 + 7a + 3$
$= 10a + 2$

Q3.
$-3x - 2x + 1 - 3$
$= -5x - 2$

Q4.
$k - k + 2k - 2 - 3k + 5$
$= -k + 3$

Q5.
$x + 4xy - 4x + 3y - yx$
$= 3xy - 3x + 3y$

Q6.
$2ab - 3a + ba - 2b + 3$
$= 3ab - 3a - 2b + 3$

Q7.
$x^2 - 4 + 3x - 2 + x^2$
$= 2x^2 + 3x - 6$

Q8.
$b^2 - b - 4b^2 + 7a - ab$
$= -3b^2 + 7a - b - ab$

Q9.
$4xy - 5xy + 2x^2 - 3y^2 + y$
$= 2x^2 - 3y^2 - xy + y$

Q10.
$2t - t^2 - 5t + t$
$= -t^2 - 2t$

Lesson One: Multiplication and division questions

Q1.
$2y^2 \cdot 3y^3 \cdot x$
$= 6xy^5$

Q2.
$4k \cdot 2k^2 \cdot k$
$= 8k^4$

Q3.
$xy \cdot x^2y \cdot 2$
$= 2x^3y^2$

Q4.
$ab \cdot ab^2 \cdot 3$
$= 3a^2b^3$

Q5.
$3xy \cdot yx \cdot 4y$
$= 12x^2y^3$

Q6.
$a^2b \cdot (ab)^2$
$= a^2b \cdot a^2b^2$
$= a^4b^3$

Q7.
$0.5x \cdot y^2 \cdot 6x$
$= 3x^2y^2$

Q8.
$-t \cdot t^2 \cdot 4t$
$= -4t^4$

Q9.
$(3x^3y^2)^2$
$= 9x^6y^4$

Q10.
$\frac{1}{3}a^2 \cdot b \cdot ab$
$= \frac{1}{3}a^3b^2$

Q11.
$\frac{6x^2y^2}{2x}$
$= 3xy^2$

Q12.
$\frac{9xy^3}{81x^3y^3}$
$= \frac{1}{9x^2}$

Q13.

$\dfrac{25x \cdot xy}{15y^2 \cdot x}$

$= \dfrac{5x}{3y}$

Q14.

$\dfrac{(2xy^2)^3}{3x^4y}$

$= \dfrac{8x^3y^6}{3x^4y}$

$= \dfrac{8y^5}{3x}$

Q15.

$\dfrac{(4a^3b^2)^3}{(2ab)^3}$

$= \dfrac{64a^9b^6}{8a^3b^3}$

$= 8a^6b^3$

Q16.

$\dfrac{(5tu^2)^2 \cdot 3t^2}{tu^2}$

$= \dfrac{25t^2u^4 \cdot 3t^2}{tu^2}$

$= \dfrac{75t^4u^4}{tu^2}$

$= 75t^3u^2$

Lesson Two: Expanding brackets questions

Q1.
$3(x+4)$
$= 3x + 12$

Q2.
$5(2x - y + z)$
$= 10x - 5y + 5z$

Q3.
$2x(6 - x)$
$= 12x - 2x^2$

Q4.
$-4(x+2)$
$= -4x - 8$

Q5.
$-3x(x+7)$
$= -3x^2 - 21x$

Q6.
$-2x(x-4)$
$= -2x^2 + 8$

Q7.
$6x(2y+3)$
$= 12xy + 18x$

Q8.
$-x^2(2x+3)$
$= -2x^3 - 3x^2$

Q9.
$(x+4)(x+7)$
$= x^2 + 7x + 4x + 28$
$= x^2 + 11x + 28$

Q10.
$(x+11)(x-3)$
$= x^2 - 3x + 11x - 33$
$= x^2 + 8x - 33$

Q11.
$(y+2)(6-y)$
$= 6y - y^2 + 18 - 2y$
$= -y^2 + 4y + 18$

Q12.
$(x-3)(y-4)$
$= xy - 4x - 3y + 12$

Q13.
$(x+5)(x-5)$
$= x^2 - 5x + 5x - 25$
$= x^2 - 25$

Q14.
$2(x-10)(x+10)$
$= 2(x^2 - 100)$
$= 2x^2 - 200$

Q15.
$(x+3)^2$
$= (x+3)(x+3)$
$= x^2 + 6x + 9$

Q16.
$3(x-4)^2$
$= 3(x-4)(x-4)$
$= 3(x^2 - 8x + 16)$
$= 3x^2 - 24x + 48$

Q17.
$(3x-3)(2x+2)$
$= 6x^2 + 6x - 6x - 6$
$= 6x^2 - 6$

Q18.
$4(7x-4) - 3(x+2)$
$= 28x - 16 - (3x+6)$
$= 28x - 16 - 3x - 6$
$= 25x - 22$

Q19.

$(x+1)(2x+1)+(x-2)(x+3)$
$=(2x^2+x+2x+1)+(x^2+3x-2x-6)$
$=3x^2+4x-5$

Q20.

$(6-x)^2+(2x+1)^2$
$=(6-x)(6-x)+(2x+1)(2x+1)$
$=36-12x+x^2+4x^2+4x+1$
$=5x^2-8x+37$

Lesson Three: Factorising brackets questions

Q1.
$14x+21$
$=7(2x+3)$

Q2.
$5xy-15x$
$=5x(y-3)$

Q3.
$6x^2+3x$
$=3x(2x+1)$

Q4.
$30-4x$
$=4(15-x)$

Q5.
$7x(x+2)-4(x+2)$
$=(x+2)(7x-4)$

Q6.
$-4x(x-1)-2(x-1)$
$=(x-1)(-4x-2)$

Q7.
$4x^2+8xy+5x+10y$
$=4x(x+2y)+5(x+2y)$
$=(x+2y)(4x+5)$

Q8.
$3x+3y+2xy+2x^2$
$=3(x+y)+2x(y+x)$
$=(x+y)(2x+3)$

Q9.
$6a+12b+4ax+8bx$
$=6(a+2b)+4x(a+2b)$
$=(a+2b)(6+4x)$

Q10.
$ax+bx+ay+by$
$=x(a+b)+y(a+b)$
$=(a+b)(x+y)$

Q11.
x^2+5x+6

Factors of 6: $\{1,6\}$, $\{2,3\}$, $\{-1,-6\}$, or $\{-2,-3\}$

$=(x+2)(x+3)$

Q12.
x^2-5x+6

Factors of 6: $\{1,6\}$, $\{2,3\}$, $\{-1,-6\}$, or $\{-2,-3\}$

$=(x-2)(x-3)$

Q13.
$x^2+8x+16$
$=(x+4)(x+4)$

Q14.
$x^2+16x+64$
$=(x+8)(x+8)$

Q15.
x^2-16
$=(x+4)(x-4)$

Q16.
x^2-64
$=(x+8)(x-8)$

Q17.
$2x^2+16x+30$
$=2(x^2+8x+15)$
$=2(x+5)(x+3)$

Q18.
$5x^2-4x-1$
$=5x^2-5x+x-1$
$=5x(x-1)+(x-1)$
$=(x-1)(5x+1)$

Q19.
$2x^2-7x+6$
$=2x^2-3x-4x+6$
$=x(2x-3)-2(2x-3)$
$=(2x-3)(x-2)$

Q20.
$4x^2-20x+25$
$=4x^2-10x-10x+25$
$=2x(2x-5)-5(2x-5)$
$=(2x-5)(2x-5)$
$=(2x-5)^2$

Lesson Three-point-five: Rational expressions questions

Q1.

$\frac{2x(x+4)}{x}$

$= 2(x+4)$

Q2.

$\frac{8x(x+2)}{2}$

$= 4x(x+2)$

Q3.

$\frac{(x-1)(x-5)}{(x-1)}$

$= x-5$

Q4.

$\frac{(x+4)(x-4)}{(x-4)}$

$= x+4$

Q5.

$\frac{x^2+7x+10}{x+2}$

$= \frac{(x+2)(x+5)}{x+2}$

$= x+5$

Q6.

$\frac{x^2+9x-22}{x+11}$

$= \frac{(x+11)(x-2)}{x+11}$

$= x-2$

Q7.

$\frac{2x^2+8x+6}{x+3}$

$= \frac{2(x^2+4x+3)}{x+3}$

$= \frac{2(x+3)(x+1)}{x+3}$

$= 2(x+1)$

$= 2x+2$

Q8.

$\frac{2x^2-7x+6}{x-2}$

$= \frac{2x^2-3x-4x+6}{x-2}$

$= \frac{x(2x-3)-2(2x-3)}{x-2}$

$= \frac{(2x-3)(x-2)}{x-2}$

$= 2x-3$

Q9.

$\frac{3x^2-8x+4}{x^2+2x-8}$

$= \frac{3x^2-6x-2x+4}{(x+4)(x-2)}$

$= \frac{3x(x-2)-2(x-2)}{(x+4)(x-2)}$

$= \frac{(3x-2)(x-2)}{(x+4)(x-2)}$

$= \frac{3x-2}{x+4}$

Q10.

$\frac{4x^2-20x+25}{2x^2-x-10}$

$= \frac{4x^2-10x-10x+25}{2x^2-5x+4x-10}$

$= \frac{2x(2x-5)-5(2x-5)}{x(2x-5)+2(2x-5)}$

$= \frac{(2x-5)(2x-5)}{(2x-5)(x+2)}$

$= \frac{2x-5}{x+2}$

Lesson Four: Linear equations and inequations questions

Q1.

$$\begin{aligned} 5x + 18 &= 33 \\ 5x &= 33 - 18 \\ 5x &= 15 \\ x &= 3 \end{aligned}$$

Q2.

$$\begin{aligned} 20 &= 3x + 14 \\ 6 &= 3x \\ 2 &= x \\ x &= 2 \end{aligned}$$

Q3.

$$\begin{aligned} 2(x - 3) &= 8 \\ x - 3 &= 4 \\ x &= 7 \end{aligned}$$

Q4.

$$\begin{aligned} 5x + 7 &= x - 2 \\ 4x &= -9 \\ x &= -\frac{9}{4} \end{aligned}$$

Q5.

$$\begin{aligned} \frac{x}{3} + 7 &= 11 \\ \frac{x}{3} &= 4 \\ x &= 12 \end{aligned}$$

Q6.

$$\begin{aligned} \frac{x - 5}{2} &= 6 \\ x - 5 &= 12 \\ x &= 17 \end{aligned}$$

Q7.

$$\begin{aligned} 5x + 18 &< -17 \\ 5x &< -35 \\ x &< -7 \end{aligned}$$

Q8.

$$\begin{aligned} 9 + 2x &\geq -19 \\ 2x &\geq -28 \\ x &\geq -14 \end{aligned}$$

Q9.

$$\begin{aligned} \frac{x}{5} + 1 &< -3 \\ \frac{x}{5} &< -4 \\ x &< -20 \end{aligned}$$

Q10.

$$\begin{aligned} \frac{2 - 3x}{2} &\leq -5 \\ 2 - 3x &\leq -10 \\ -3x &\leq -12 \\ x &\geq 4 \end{aligned}$$

Q11.

$$\begin{aligned} \frac{20}{x} &= 4 \\ 20 &= 4x \\ x &= 5 \end{aligned}$$

Q12.

$$\begin{aligned} \frac{12}{x} &= \frac{3}{4} \\ 48 &= 3x \\ x &= 16 \end{aligned}$$

Q13.

$$\begin{aligned} \frac{3x}{5} &= \frac{x + 4}{3} \\ 9x &= 5x + 20 \\ 4x &= 20 \\ x &= 5 \end{aligned}$$

Q14.

$$\begin{aligned} \frac{2x + 5}{x - 1} &= \frac{-1}{3} \\ 6x + 15 &= -x + 1 \\ 7x &= -14 \\ x &= -2 \end{aligned}$$

Q15.

$$\begin{aligned} \frac{4x - 5}{2} &= \frac{3}{4} \\ 16x - 20 &= 6 \\ 16x &= 26 \\ x &= \frac{13}{8} \end{aligned}$$

Q16.

$$\begin{aligned} \frac{x}{5} + \frac{2x}{3} &= 6 \\ \frac{3x + 10x}{15} &= 6 \\ \frac{13x}{15} &= 6 \\ 13x &= 90 \\ x &= \frac{90}{13} \end{aligned}$$

Q17.

$$\begin{aligned} 8 - \frac{x}{3} &> 6 \\ -\frac{x}{3} &> -2 \\ x &< 6 \end{aligned}$$

Q18.

$$\begin{aligned} \frac{-3x + 1}{7} &> 5 \\ -3x + 2 &> 35 \\ -3x &> 33 \\ x &< -11 \end{aligned}$$

Q19.

Q20.

$$-8x + \frac{3x}{2} \le 5$$
$$-16x + 3x \le 10$$
$$-13x \le 10$$
$$x \ge -\frac{10}{13}$$

$$\frac{10x}{x} + \frac{12}{x} = 14$$
$$10x + 12 = 14x$$
$$12 = 4x$$
$$x = 3$$

Lesson Five: Quadratic equations questions

Q1.
$(x-4)(x-6) = 0$

$x = 4 \text{ or } x = 6$

Q2.
$(x+5)(x-8) = 0$

$x = -5 \text{ or } x = 8$

Q3.
$x(x-2) = 0$

$x = 0 \text{ or } x = 2$

Q4.
$x(x+3) = 0$

$x = -3 \text{ or } x = 0$

Q5.
$(2x+2)(x-3) = 0$

$x = -1 \text{ or } x = 3$

Q6.
$6(x-1)(3x+2) = 0$

$x = -\frac{2}{3} \text{ or } x = 1$

Q7.
$(x-3)^2 = 0$

$x = 3$

Q8.
$(x+7)^2 = 0$

$x = -7$

Q9.
$x^2 + x - 12 = 0$
$(x+4)(x-3) = 0$

$x = -4 \text{ or } x = 3$

Q10.
$x^2 - 13x + 12 = 0$
$(x-12)(x-1) = 0$

$x = 1 \text{ or } x = 12$

Q11.
$x^2 + 8x + 12 = 0$
$(x+6)(x+2) = 0$

$x = -6 \text{ or } x = -2$

Q12.
$x^2 - 3x + 2 = 0$
$(x-2)(x-1) = 0$

$x = 1 \text{ or } x = 2$

Q13.
$x^2 - 64 = 0$
$(x-8)(x+8) = 0$

$x = -8 \text{ or } x = 8$
$x = \pm 8$

Q14.
$x^2 = x + 30$
$x^2 - x - 30 = 0$
$(x-6)(x+5) = 0$

$x = -5 \text{ or } x = 6$

Q15.
$4x^2 + 2x - 12 = 0$
$2x^2 + x - 6 = 0$
$2x^2 + 4x - 3x - 6 = 0$
$2x(x+2) - 3(x+2) = 0$
$(x+2)(2x-3) = 0$

$x = -2 \text{ or } x = \frac{3}{2}$

Q16.

$$9x^2 = 16$$
$$9x^2 - 16 = 0$$
$$9x^2 + 12x - 12x - 16 = 0$$
$$3x(3x + 4) - 4(3x + 4) = 0$$
$$(3x + 4)(3x - 4) = 0$$

$x = \pm\frac{4}{3}$

Q17.

$7x^2 + x = 0$

$x(7x + 1) = 0$

$x = -\frac{1}{7}$ or $x = 0$

Q18.

$5x^2 = 24x + 5$

$5x^2 - 24x - 5 = 0$

$5x^2 - 25x + x - 5 = 0$

$5x(x - 5) + (x - 5) = 0$

$(x - 5)(5x + 1) = 0$

$x = -\frac{1}{5}$ or $x = 5$

Q19.

$x = \frac{2x + 3}{x}$

$x^2 = 2x + 3$

$x^2 - 2x - 3 = 0$

$(x - 3)(x + 1) = 0$

$x = -1$ or $x = 3$

Q20.

$x = \frac{1}{x}$

$x^2 = 1$

$x = \pm\sqrt{1}$

$x = \pm 1$

Lesson Six: Simultaneous equations questions

Only answers will be shown. You may use any valid method to solve these.

Q1.

$x = y - 3$

$2x - 3y = 4$

$x = -13$ and $y = -10$

Q2.

$3x + y = 7$

$x - y = 5$

$x = 3$ and $y = -2$

Q3.

$2y - 2x = 8$

$y + 2x = 10$

$x = 2$ and $y = 6$

Q4.

$6x + 2y = 6$

$x - 2y = 8$

$x = 2$ and $y = -3$

Q5.

$3x + 4y = 1$

$2x - 3y = -5$

$x = -1$ and $y = 1$

Q6.

$y = 2x + 7$

$y = x + 4$

$x = -3$ and $y = 1$

Q7.

$x = y$

$3x - 5y = 8$

$x = -4$ and $y = -4$

Q8.

$8x + 7y = 42$

$2y - 2x = -3$

$x = 3.5$ and $y = 2$

Q9.

$2x + 5y = -9$

$3x - 4y = 21$

$x = 3$ and $y = -3$

Q10.

$x + y = 7$

$2x - 3y = 4$

$x = 5$ and $y = 2$

Q11.

$2y + x = 3$

$y - 2x = -6$

$x = 3$ and $y = 0$

Q12.

$y - 3x = 3$

$2y = x + 6$

$x = 0$ and $y = 3$

Q13.
$3x = 4 + 2y$
$y = 5 - 2x$

$x = 2$ and $y = 1$

Q14.
$5x = 30 - 2y$
$3x - y = 7$

$x = 4$ and $y = 5$

Q15.
$4x + 2y = 10$
$x - 3y = 6$

$x = 3$ and $y = -1$

Q16.
$x = y - 4$
$4x - 2y + 8 = 0$

$x = 0$ and $y = 4$

Q17.
$3x + 2y = 12$
$4x - 3y = -1$

$x = 2$ and $y = 3$

Q18.
$2x + 3y + 5 = 0$
$x - 2y + 6 = 0$

$x = -4$ and $y = 1$

Q19.
$5x - 6y = -13$
$y = 8x - 5$

$x = 1$ and $y = 3$

Q20.
$3y - 5x = 1$
$4y + x = 9$

$x = 1$ and $y = 2$

Appendix B: Summaries

I recommend you write the summaries for each lesson in this appendix so you have them in one place. It makes for really good revision too.

Lesson Zero: Variables summary

Lesson One: Operations on variables summary

:

Lesson Two: Expanding brackets summary

Lesson Three: Factorising brackets summary

Methods	Use when…	For example…

Lesson Three-point-five: Rational expressions summary

Lesson Four: Linear equations and inequations summary

Lesson Five: Quadratic equations summary

Lesson Six: Simultaneous equations summary