A Foucault's pendulum design

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In this article we explain our design and their performance, of a Foucault pendulum capable of continuous operation with a following system of the oscillation plane. We have built two pendulums, one of 2.835 m length and other of 4.975 m length. The oscillation amplitudes in either case were 1.8 and 1 deg, respectively. In our latitude the precession speed (9.89 deg/h counterclockwise) can be measured within 0.5 and 0.2 deg/h. The arrangement detected some earthquakes of medium intensity that took place as far as 765 km. © *2010 American Institute of Physics.*
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I. INTRODUCTION

Since Jean Bernard Leon Foucault in 1851 hung his 67 m pendulum at the Pantheon in Paris, a significant amount of work has been done around the world to show this elucidating demonstration of the Earth's rotation. Later, Bravais and others who repeated the same experiment observed that after a certain time (typically around 1 h) they launched the pendulum in an oscillation plane, depending on its length, the trajectory of oscillation became elliptical. No explanation was known at that time. It was Heike Kamerlingh-Onnes, in his Ph.D. thesis,[1] working with a rigid pendulum, who first solved the equations of motion, including the asymmetric terms, which give rise to the elliptical movement. The reason was due to the asymmetry of the support system which gives different inertia moment of the pendulum depending on the azimuth of oscillation, thus changing the period of oscillation. Nowadays, the development of classical mechanics in the past century provides the equations of motion for a spherical pendulum.[2] If the trajectory describes an ellipse, there is a precession proportional to the ratio of the area of the ellipse and the square of the pendulum length. This increases or decreases the natural precession due to the Earth's rotation depending on the rotation sense. H. Kamerlingh-Onnes solved this problem, calculating and adding masses to the support of the pendulum in a way that maintains the moment of inertia constant around the azimuth of oscillation, thus eliminating the asymmetries. In more recent days, Crane[3,4] began to build short Foucault pendulums capable of showing the Earth's rotation. To eliminate the elliptical movement that appears after certain time from launch, he made the bob of the pendulum hit a very light copper ring (Charron ring). This limited the oscillation amplitude and at the same time, the strike stopped or at least decreased the perpendicular movement in the oscillating plane. This pendulum had a permanent magnet at the bottom of the bob which is part of a system to keep the amplitude oscillation constant. He built several pendulums which worked satisfactorily. At the University of Guelph (Ontario, Canada) a Foucault pendulum was built following Crane's design and there is a beautiful discussion[5] about the Foucault pendulum mechanics given by Professor B. Nickel from that University.

This fascinating demonstration of Earth's rotation led us to build a short pendulum which allowed us to observe and measure the precession of oscillation plane. In the following lines, we explain our Foucault's pendulum design which allows observation the Earth's rotation and even more.

II. THE APPARATUS

The objective of this experiment was to measure the azimuth of the oscillating plane by keeping the pendulum in continuous oscillation. Several items were revised, beginning with the system to keep the oscillation constant, the method for decreasing the ellipticity, and the measure of the oscillation plane azimuth. Foucault speed of precession is

$$\omega = \lambda \cos \theta,$$

where ω is the speed of Foucault precession, λ is the Earth's angular velocity, and θ is the site latitude. The Centro Atómico Bariloche latitude is $-41°.119$, and this gives a precession speed of 9.869 deg/h.

Figure 1 shows a simple sketch of the pendulum. To keep the amplitude constant, we adopted the same system used by Crane, i.e., a permanent magnet at the bottom of the bob, which is accelerated by an electric coil. At the center of this coil there is another small coil to detect the passage of the bob through the center of oscillation. In our design we accelerate the bob only toward the center for reasons we will explain later. The feedback electronic circuit is shown in Fig. 2. This circuit is a pulsed current source on the excitation coil, which is triggered by a 555 timer in a monostable configuration. The 555 timer is triggered by the detection coil through an amplifier (LM741, BC548, National Semiconductor). The 555 timer works in an inverse situation, that is, when its output is high (pin 3) the current on the excitation coil is zero. So, when the bob reaches the center of equilibrium, and the detection coil triggers the 555, the current in the excitation coil becomes zero. The variable resistance of 100 kΩ on pin 7 determines the time constant, and the mo-

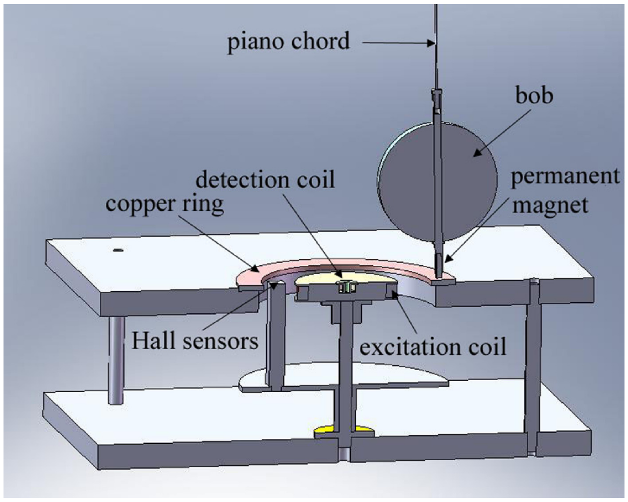

FIG. 1. (Color online) Simple sketch of the pendulum arrangement.

ment when the excitation coil is energized again. There is a test point for the personal computer (PC), branched on the 555 output (pin 3). This is used to synchronize the pendulum with the PC. In series with the excitation coil there is another test point used to measure the height and width of the current pulse.

Actually, the trajectory of the pendulum always describes an ellipse, larger or smaller depending on the anisotropy of the suspension system. To keep the ellipticity small we use an electromagnetic brake. When the permanent magnet at the bottom of the bob is arriving at the end of the maximum amplitude, there is a copper surface which interacts with the magnet and through Foucault currents it reduces the perpendicular speed of the bob as well as the final speed of oscillation. This does not stop the perpendicular movement of the bob but it does not allow it to increase up to intolerable values, which change the natural precession of the azimuth in large values.

To measure the azimuth of oscillation we use the permanent magnet at the bottom of the bob and a pair of Hall sensors, separated by 3 mm and mounted near the end of the maximum amplitude of oscillation. An electric signal is obtained from the output of an instrumentation amplifier which takes the difference of the Hall sensors signals (see inset of Fig. 3). The Hall sensors were mounted at the top of a tower, which also supports the Hall sensors amplifier circuit, and is placed on an aluminum disk whose angular position was controlled by a stepper motor through a mechanical reduction. The angle and height of this tower can be regulated to obtain the best output from the Hall sensors. When the permanent magnet is over the pair of Hall sensors and we move the Hall sensors we obtain the signal plotted in Fig. 3. There is a zero value of the voltage when the magnet is symmetrically aligned between the two sensors. When the magnet moves toward the right there is a negative voltage which increases over approximately 2°. At angles greater than 15°, the voltage drops to zero. Moving the magnet to the left, the opposite value of the voltage is obtained. There is a region where the voltage changes very quickly from negative to positive values and this is where we operate the pendulum. With the aid of an acquisition card in a computer and the stepper motor, we keep the position of the Hall sensors at zero voltage, thus following the plane oscillation of the pendulum. Counting the number of steps necessary to correct the position of the Hall sensors we obtain the angle. In our case, each step of the motor equals $5 \times 10^{-4\circ}$. The feedback sys-

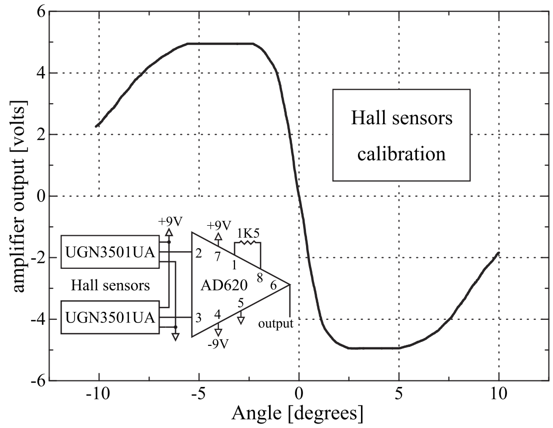

FIG. 3. Hall sensors calibration and amplifier circuit.

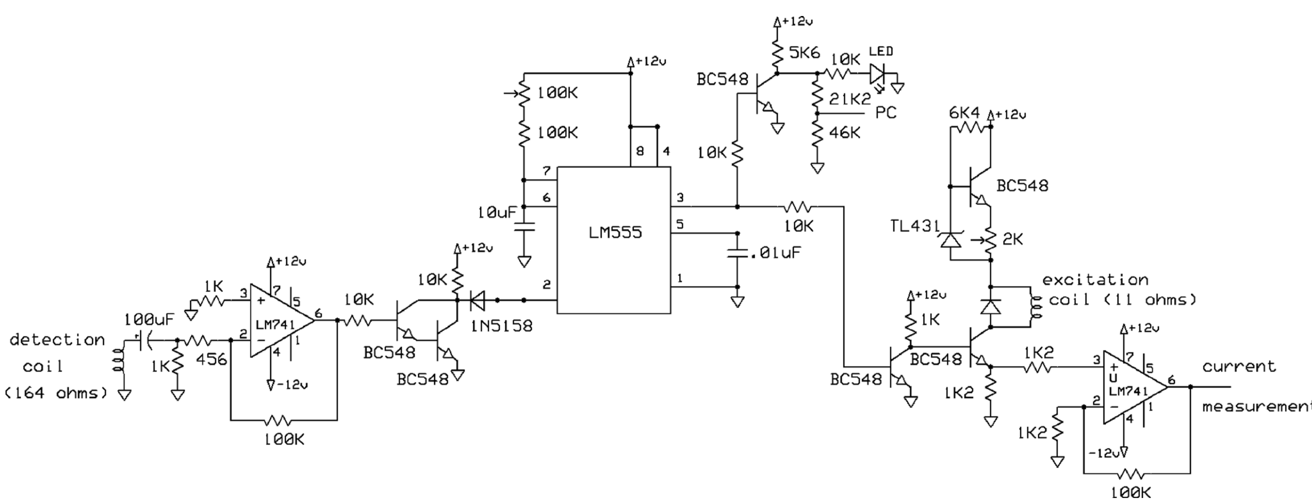

FIG. 2. Schematic of the feedback circuit.

tem for keeping constant the amplitude is analogical (Fig. 2) and it works continuously. To follow the plane of oscillation it is necessary to read the output of the Hall sensors with a 12 bit interface card (Advantech model PCL-711B) connected to a PC. The out of equilibrium voltage of the Hall sensors is then used to send a calculated number of pulses to the stepper motor to correct their position. The measurement cycle made by the PC, beginning at the rest position is: during the first quarter cycle of the pendulum, and a bit more, until the voltage of the Hall sensors is measured twice (going outward and returning toward the equilibrium position). The bob is then accelerated toward the center until the pendulum arrives at the rest position. Then the excitation coil current is set to zero, and during this state, the stepper motor corrects the position of the Hall sensors. In the last quarter the bob is again accelerated toward the rest position. The support system adopted was the conventional "mandrel" fastener of the suspension wire.

We have tested the system with two pendulums: the first of 2835 mm (period=3.38 s) and the second of 4970 mm (period=4.48 s), the lead bob is spherical and weighs 12.5 kg (used in both pendulums) and the suspension wire was steel wire SAE 1070 (ASTM 227 class 2) of 0.92 mm diameter and also a chord piano wire Diamond Brand (USA) of the same diameter. The amplitude of oscillation always used was 86.4 mm (depending on the copper ring size and the feedback current). The measured Q of the long pendulum is approximately 5540 (calculated from the logarithmic decrement). We cannot calculate the energy that we provide to the pendulum to keep the amplitude constant, but from the measured Q and the total gravitational potential energy, we can calculate that we have to provide nearly 1.6×10^{-5} J each pendulum cycle.

III. RESULTS

When first mounting the pendulum it is necessary to align its rest position with the center of the excitation and detection coils. The second step is to calibrate the Hall sensors, i.e., how many motor steps changes the output voltage of the instrumentation amplifier by 1 V, in the region of maximum change. We called this calibration constant K_H. This calibration together with the mechanical reduction of the stepper motor, i.e., number of steps per degree of the azimuth (K_M=1509.96 motor steps per degree), allow us to calculate the correction due to the movement of the oscillation plane (Foucault precession). By re-establishing the zero output voltage to the Hall sensors, the movement of the plane of oscillation is followed. To calculate the number of correction motor steps we used

$$ms = K_H \times K_M \times V_{\mathrm{OE}}.$$

V_{OE}: out of equilibrium voltage of the Hall sensors.

What we normally obtain as V_{OE} versus time is shown in Fig. 4. There are two peaks of voltage: the first when the bob moves outward from the center and the second when it comes back to the center of the coils. V_{OE} is determined adding the amplitudes of both peaks with their signs. The sign of the first peak gives the rotation sense of the ellipse described by the bob and, consequently, if it is increasing or decreasing the Foucault precession. In Fig. 4 it is also shown the current pulses on the excitation coil when the bob moves toward the equilibrium position. The mean amplitude of the peaks gives the amplitude of the ellipse and allows us to correct the total measured precession. The correction due to the developed ellipse can be calculated with (Ref. 2, page 342) as

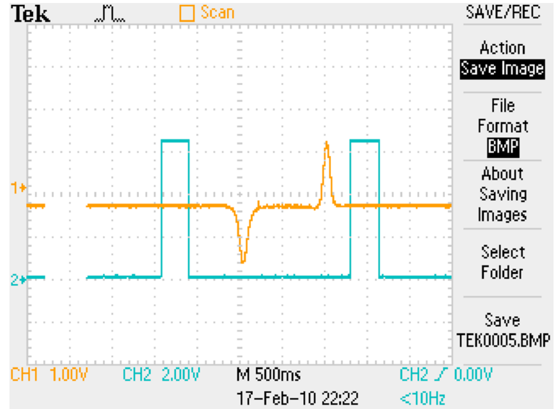

FIG. 4. (Color online) Pulse current on the excitation coil and Hall sensors signal.

$$w_s = -\pi \times d_s \times K_H \times V_d/(K_M \times 180)\ \ [\mathrm{mm}],$$

$$r_m = w_s/\{2 \times \sqrt{[1-(d_s/d_a)^2]}\}\ \ [\mathrm{mm}],$$

$$\mathrm{area} = \pi \times d_a \times r_m\ \ [\mathrm{mm}^2],$$

$$E_p = 0.75 \times \mathrm{area}/(L^2 \times T) \times (180/\pi)\ \ [\mathrm{deg/h}],$$

where

d_a: amplitude of oscillation [mm].

d_s: distance of the Hall sensors to the center of the coils [mm].

L: length of the pendulum [mm].

T: calculated period of the pendulum [h].

V_d: voltage difference of the two peaks [V].

w_s: width of the ellipse at the Hall sensors position [mm].

r_m: minor radius of the ellipse [mm].

area: surface of the ellipse [mm^2].

E_p: calculated ellipse precession [deg/h].

For the short pendulum, the largest ellipse minor radius was 1.4 mm and its maximum precession speed 2.8 deg/h. The mean value of the Foucault precession speed was 10.05 ± 0.55 deg/h (fitted in a five day measurement). The first mounting was assembled with steel wire SAE 1070 (ASTM 227 class 2) and in a second step, to see the influence of the suspension wire, we have changed it for a chord piano wire, Diamond Brand (USA) of the same diameter. The developed ellipse was smaller (minor radius <0.6 mm) and its precession speed decreased to a maximum of 0.85 deg/h.

In the long pendulum, the suspension wire was also the chord piano. In this case the amplitude of oscillation is 1°. In Figs. 5 and 6 we show some measurements. The minor radius of the ellipse is less than 0.3 mm (less than the 2835

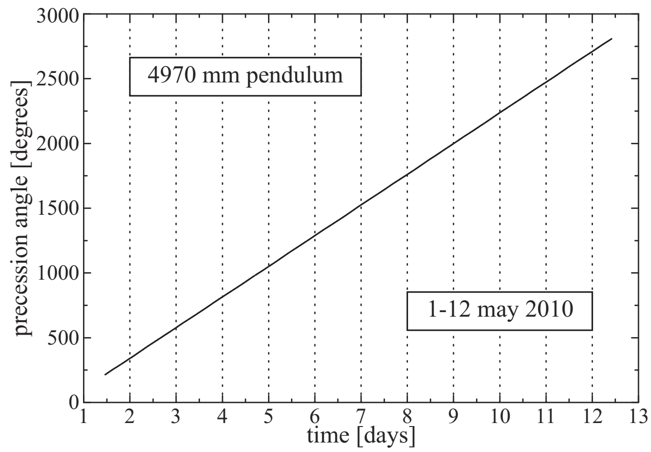

FIG. 5. Total precession angle of 4975 mm pendulum.

mm pendulum). This ellipse radius represents a precession speed of less than 0.1 deg/h. The mean value of the Foucault precession speed was 9.88 ± 0.22 deg/h and the calculation of the derivative of the angle versus time was taken with the last 200 periods. The bob increases its height by 0.75 mm which equals 0.0919 J for the maximum potential gravity energy. The pendulum was covered with a box, 2 m height, made of wood and glass to prevent air movements. Some earthquakes were detected when they were near San Carlos de Bariloche, had a moderate intensity and the plane of oscillation was in a certain position against the seismic wave. In Fig. 6 two such earthquakes are shown. Number 1 is the earthquake that took place in Libertador O'Higgins, Chili on 2 May 2010 14:52:43 h Universal Time at −34°.210 lat., −71°.86 long., intensity 5.9 (765 km from Bariloche). Number 2 is the earthquake which took place in front of Bio-Bio (Chili) on the 3 May 2010 23:09:45 h UT at −38°.11 lat., −73°.65 long. intensity 6.4 (388 km from Bariloche). The earthquake of the 27 February 2010 in Chili (intensity 8.8) put the pendulum completely out of synchronism and the following system lost the plane of oscillation. Some remarks: (a) the pendulum takes several hours to reach a steady state because after launching it with a burned thread, it has a small chaotic behavior that is only noticeable by the following system (that is, the bob does not describe a regular ellipse), (b) the Hall sensors orientation must be regulated carefully so as to minimize the step found in the ellipse speed precession when it changes the rotation sense.

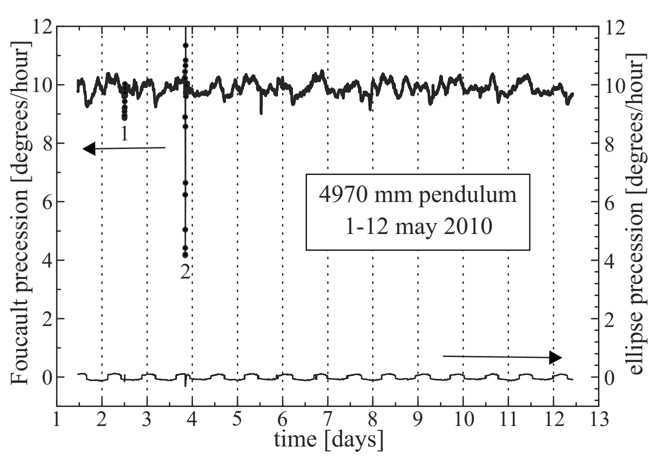

FIG. 6. Corrected Foucault precession speed and ellipse precession speed.

IV. CONCLUSIONS

We have examined two Foucault pendulums which can follow the angular precession of the oscillation plane due to earth rotation with less than 0.5 deg/h and 0.2 deg/h of uncertainty and also detected near earthquakes in certain conditions. The additional issues that were added to a conventional Foucault pendulum, the copper ring and the following system, worked satisfactorily. They allowed us to keep the ellipse in rather small values of its minor axe thus decreasing the correction of the total precession measured. The mechanical reduction of the following system must be adapted to the length of each pendulum because the number of motor steps necessary to correct the position of the oscillation plane gives an idea of the "noise" generated from the integer mode of operation of the stepper motor. Analyzing by Fourier transform several measurements, we found only the periodicity of one turn of the oscillation plane and their harmonics. We did not find any periodicity related to the passage of the Moon (25 h).

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