1. $f: \mathbb{R}^2 \to \mathbb{R}^2$

$$f(x,y) = \begin{cases} \dfrac{xy^5 + yx^3}{x^2+y^4}, & (x,y) \neq (0,0) \\ a, & (x,y) = (0,0) \end{cases}$$

a) $$a = \lim_{(x,y)\to(0,0)} \frac{xy^5+yx^3}{x^2+y^4} = \begin{bmatrix} x = \rho\cos\varphi \\ y = \rho\sin\varphi \end{bmatrix} = \lim_{\rho\to 0} \frac{\rho\cos\varphi \cdot \rho^5\sin^5\varphi + \rho\sin\varphi \cdot \rho^3\cos^3\varphi}{\rho^2\cos^2\varphi + \sin^4\varphi \cdot \rho^4} =$$

$$= \lim_{\rho\to 0} \frac{\rho^2\cos\varphi\sin\varphi\,\cancel{(\sin^4\varphi \cdot \rho^4 + \rho^2\cos^2\varphi)}}{\cancel{\rho^2\cos^2\varphi + \rho^4\sin^4\varphi}} =$$

$$= \lim_{\rho\to 0} \overset{\to 0}{\rho^2} \underbrace{\cos\varphi}_{[0,1]} \underbrace{\sin\varphi}_{[0,1]} = \boxed{0} \Rightarrow \boxed{a = 0}$$

b) $$f_x' = \frac{(y^5+3x^2y)(x^2+y^4) - (xy^5+yx^3)(2x)}{(x^2+y^4)^2} =$$

$$= \frac{x^2y^5 + y^9 + \cancel{3}x^4y + \cancel{3}x^2y^5 \cancel{- 2x^2y^5} \cancel{- 2x^4y}}{(x^2+y^4)^2} =$$

$$x^4 + 2x^2y^4 + y^8$$

$$= \frac{2x^2y^5 + y^9 + x^4y}{x^4 + 2x^2y^4 + y^8} = \frac{y\,\cancel{(2x^2y^4 + y^8 + x^4)}}{\cancel{x^4 + 2x^2y^4 + y^8}} = \boxed{y}$$

$$f_y' = \frac{(5y^4x + x^3)(x^2+y^4) - (xy^5+yx^3)(4y^3)}{(x^2+y^4)^2} =$$

$$= \frac{\cancel{5}x^3y^4 + \cancel{5}xy^8 + x^5 + x^3y^4 \cancel{- 4xy^8} \cancel{- 4x^3y^4}}{(x^2+y^4)^2} = \frac{x^5 + 2x^3y^4 + xy^8}{(x^2+y^4)^2} =$$

$$= \frac{x\,\cancel{(x^2+y^4)^2}}{\cancel{(x^2+y^4)^2}} = \boxed{x}$$

$$f_x'(0,0) = \lim_{h\to 0} \frac{f(h,0) - f(0,0)}{h} = \frac{0-0}{h} = \boxed{0}$$

$$f_y'(0,0) = \lim_{h\to 0} \frac{f(0,h) - f(0,0)}{h} = \frac{0-0}{h} = \boxed{0}$$

$L = 0$ ✓

f-ja je dif!

$$L = \lim_{(h,k)\to(0,0)} \frac{f(h,k) - f(0,0) - h f_x'(0,0) - k f_y'(0,0)}{\sqrt{h^2+k^2}} =$$

$$= \lim \frac{\frac{hk^5 + kh^3}{h^2+k^4} - 0 - 0}{\sqrt{h^2+k^2}} = \lim \frac{\frac{hk(k^4+h^2)}{h^2+k^4}}{\sqrt{h^2+k^2}} = \lim \frac{hk}{\sqrt{h^2+k^2}} = \begin{bmatrix} h = \rho\cos\varphi \\ k = \rho\sin\varphi \end{bmatrix} = \lim_{\rho\to 0} \frac{\rho\cos\varphi\,\rho\sin\varphi}{\sqrt{\rho^2\cos^2\varphi + \rho^2\sin^2\varphi}}$$

$$= \frac{\rho^2(\cos\varphi\sin\varphi)}{\rho^2\sqrt{\frac{\cos^2\varphi}{\rho^2} + \sin^4\varphi}} = \frac{\text{ok}}{\infty} = 0$$

$[-1,1]$ $\to 0$ $\to \infty$ $\to \infty$

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(1) $f(x,y) = \begin{cases} \dfrac{y^3}{x^2+y^2} & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$

a) nepr.? Teorema o dva policajca!

$$0 \leq \left|\frac{y^3}{x^2+y^2}\right| \leq \left|\frac{y^3}{y^2}\right| = |y| \xrightarrow{\lim_{(x,y)\to(0,0)}} 0 \checkmark$$

Jeste neprekidna

b) dif.?

$$f'_x(0,0) = \lim_{h\to 0} \frac{f(h,0)-f(0,0)}{h} = \lim_{h\to 0} \frac{\frac{0}{h} - 0}{h} = 0 \checkmark$$

$$f'_y(0,0) = \lim_{h\to 0} \frac{f(0,h)-f(0,0)}{h} = \lim_{h\to 0} \frac{\frac{h^3}{h^2} - 0}{h} = \lim_{h\to 0} \frac{h}{h} = 1$$

$$L = \frac{f(h,k) - f(0,0) - h f'_x(0,0) - k f'_y(0,0)}{\sqrt{h^2+k^2}} = \frac{\frac{k^3}{h^2+k^2} - 0 - 0 - k}{\sqrt{h^2+k^2}} =$$

$$= \frac{\frac{k^3 - k(h^2+k^2)}{h^2+k^2}}{\sqrt{h^2+k^2}} = \frac{\frac{\cancel{k^3} - h^2k - \cancel{k^3}}{h^2+\cancel{k^2}}}{\cancel{\sqrt{h^2+k^2}}} = \frac{-h^2k}{\sqrt{h^2+k^2}(h^2+k^2)}$$

$$= \left[\begin{array}{l} h = \rho\cos\varphi \\ k = \rho\sin\varphi \end{array}\right] = \frac{-\rho^2\cos^2\varphi \cdot \rho\sin\varphi}{(\rho^2+\rho^2)\sqrt{\rho^2\cos^2\varphi + \rho^2\sin^2\varphi}} =$$

$$= \frac{\cancel{\rho^3}\cos^2\varphi\sin\varphi}{\cancel{\rho^2}\cancel{\rho}} = \cos^2\varphi\sin\varphi \neq 0$$

Nije dif!

$$a = \left(\frac{1}{n}, \frac{1}{n}\right) \to \frac{-\frac{1}{n^2} \cdot \frac{1}{n}}{\sqrt{\frac{1}{n^2}+\frac{1}{n^2}} \cdot \left(\frac{1}{n^2}+\frac{1}{n^2}\right)} = \frac{-\frac{1}{\cancel{n^3}}}{\frac{\sqrt{2}}{\cancel{n}} \cdot \frac{2}{\cancel{n^2}}} = \left|-\frac{1}{2\sqrt{2}}\right| \not\to 0$$

↳ Nije dif!

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(2.) $f: \mathbb{R}^2 \to \mathbb{R}^2$

$$f(x,y) = \begin{cases} (x^4+y^4)\cdot \sin\frac{1}{x^4+y^4}, & (x,y)\neq(0,0) \\ 0, & (x,y)=(0,0) \end{cases}$$

a) непр.?

$$\lim_{(x,y)\to(0,0)} \underbrace{(x^4+y^4)}_{\to 0}\cdot \underbrace{\sin\frac{1}{x^4+y^4}}_{\to[-1,1]} = 0 \quad \text{// jeste neprekidna}$$

$$f'_x(0,0) = \lim_{h\to 0}\frac{f(h,0)-f(0,0)}{h} = \lim \frac{h^4\cdot\sin\frac{1}{h^4}-0}{h} = \lim h^3\cdot\sin\frac{1}{h^4} \to 0 \quad //$$

$$f'_y(0,0) = \lim_{h\to 0}\frac{f(0,h)-f(0,0)}{h} = \lim_{h\to 0}\frac{h^4\cdot\sin\frac{1}{h^4}}{h} = \lim h^3\cdot\sin\frac{1}{h^4} \to 0 \quad //$$

$$L = \lim \frac{f(h,k)-f(0,0)-h f'_x(0,0)-k f'_y(0,0)}{\sqrt{h^2+k^2}} = \lim \frac{(h^4+k^4)\cdot\sin\frac{1}{h^4+k^4}}{\sqrt{h^2+k^2}} = \begin{bmatrix} x=\rho\cos\varphi \\ y=\rho\sin\varphi \end{bmatrix} =$$

$$= \lim_{\rho\to 0} \frac{\overbrace{\rho^4}^{\to 0}\overbrace{(\cos^4\varphi+\sin^4\varphi)}^{\to ogr.}\cdot\overbrace{\sin\ldots}^{\to ogr.}}{\rho\,\underbrace{\sqrt{\cos^2\varphi+\sin^2\varphi}}_{\to 1}} = 0 \quad //$$

b) $$f'_x = 4x^3\sin\frac{1}{x^4+y^4} - \cancel{(x^4+y^4)}\cdot\cos\frac{1}{x^4+y^4}\cdot\frac{4x^3}{\cancel{(x^4+y^4)^2}} =$$

$$= 4x^3\sin\frac{1}{x^4+y^4} - \frac{4x^3}{x^4+y^4}\cdot\cos\frac{1}{x^4+y^4} \quad //$$

$$g(x) = f'_x(x,x) = 4x^3\sin\frac{1}{2x^4} - \frac{\cancel{4x^3}^{\,2}}{\cancel{2x^4}_{\,x}}\cdot\cos\frac{1}{2x^4} =$$

$$= 4x^3\cdot\sin\frac{1}{2x^4} - \frac{2}{x}\cdot\cos\frac{1}{2x^4}$$

$$a = \left(\frac{1}{n},\frac{1}{n}\right) \to (0,0)$$

$$\to 4\frac{1}{n^3}\sin\frac{1}{2\cdot\frac{1}{n^4}} - \frac{2}{\frac{1}{n}}\cdot\cos\frac{1}{2\cdot\frac{1}{n^4}} =$$

$$= \underbrace{\left(\frac{4}{n^3}\right)}_{\to 0}\cdot\underbrace{\sin\frac{n^4}{2}}_{\to 0} - \underbrace{(2n)}_{\to \frac{1}{0}=\infty\ \to\infty}\cdot\underbrace{\cos\frac{n^4}{2}}_{\to ogr} \to 0-\infty \to \boxed{-\infty} \neq 0 \quad //$$

Nije neprekidna!

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c) $\lim\limits_{(x,y)\to(0,0)} \underbrace{4x^3}_{\to 0} \cdot \underbrace{\sin\frac{1}{x^4+y^4}}_{\text{ogr.}} - \frac{4x^3}{(x^4+y^4)} \cdot \cos\frac{1}{x^4+y^4} = \begin{bmatrix} x=\rho\cos\varphi \\ y=\rho\sin\varphi \end{bmatrix}$

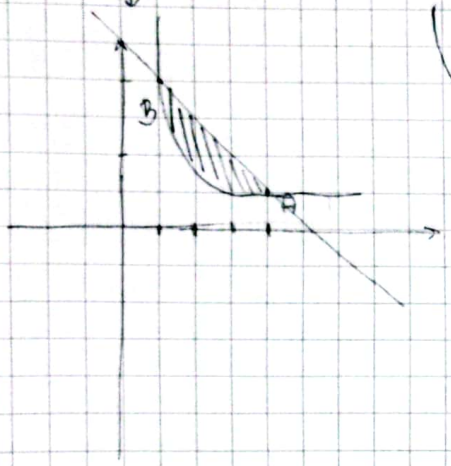
$$= \lim_{\rho\to 0} -\frac{4\cdot\rho^{\cancel{3}}\cdot\overbrace{\cos^3\varphi}^{\text{ogr.}}}{\underbrace{\rho^{\cancel{4}}}_{\to 0}\cdot\underbrace{(\sin^4\varphi+\cos^4\varphi)}_{\text{ogr.}}}\cdot\overbrace{\cos\frac{1}{\dots}}^{\text{ogr.}} = \boxed{-\infty}$$

$\to \infty$

Nije neprekidna u tački $(0,0)$!

1. $I = \iint\limits_D xy\,dx\,dy \qquad D: x\cdot y = 1 \quad x+y=\frac{5}{2}$

$y = \frac{5}{2} - x = p(x)$



$y = \frac{1}{x}$

$\frac{1}{x} = \frac{5}{2} - x \quad /\cdot x$

$1 = \frac{5}{2}x - x^2$

$x^2 - \frac{5}{2}x + 1 = 0 \to x_{1,2} = \frac{1\frac{5}{2} \pm \sqrt{\frac{25}{4}-4}}{2} = \frac{1\frac{5}{2} \pm \sqrt{\frac{9}{4}}}{2} =$

$\frac{1\frac{5}{2} \pm \frac{3}{2}}{2} \to 2 \Rightarrow y_1 = \frac{1}{2}$

$\to \frac{1}{2} \Rightarrow y_1 = 2$

$A(2, \frac{1}{2}) \quad B(\frac{1}{2}, 2)$

presečne tačke

$D = \left\{ (x,y)\in\mathbb{R}^2 \;\middle|\; \frac{1}{2} \le x \le 2,\ \frac{1}{x} \le y \le \frac{5}{2} - x \right\}$

$$\int_{1/2}^{2}\int_{1/x}^{5/2-x} xy\,dx\,dy = \int_{1/2}^{2} x\,\frac{y^2}{2}\Big|_{1/x}^{5/2-x}dx = \int_{1/2}^{2} x\left(\left(\frac{5}{2}-x\right)^2 - \frac{1}{x^2}\right)\cdot\frac{1}{2}dx =$$

$$= \int_{1/2}^{2} x\left(\frac{25}{4} - 10x + x^2 - \frac{1}{x^2}\right)\cdot\frac{1}{2}dx$$

$$= \int_{1/2}^{2} \frac{25}{4}x - 5x^2 + x^3 - \frac{1}{x}\,dx =$$

$$= \left(\frac{25\cdot x^2}{8} - \frac{5x^3}{3} + \frac{x^4}{4} - \ln x\right)\Big|_{1/2}^{2}$$

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$$\frac{1}{2}\left(\frac{25}{4}\,\frac{x^2}{2} - \frac{5}{3}x^3 + \frac{x^4}{4} - \ln x\right)\Big|_{1/2}^{2} =$$

$$= \frac{1}{2}\left(\frac{25\cdot 4}{4\cdot 2} - \frac{5}{3}\,8 + \frac{16}{4} - \ln 2 - \left(\frac{25}{4\cdot 2} - \frac{5}{3\cdot 8} + \frac{1}{4\cdot 16} - \ln\frac{1}{2}\right)\right) =$$

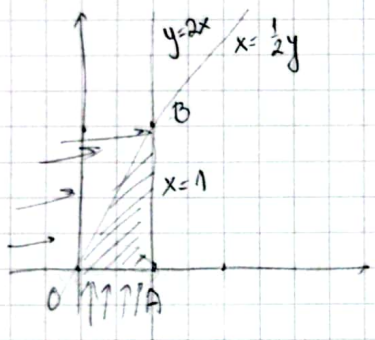
$$= \frac{1}{2}\left(\frac{25}{2} - \frac{40}{3} + \frac{16}{4} - \cancel{\ln 2} - \frac{25}{16} + \frac{5}{24} + \frac{1}{64} + \cancel{\ln 2}\right) =$$

$$= \frac{1}{2}\left(\frac{800 + 256 - 100 + 1}{64} + \frac{-320 + 5}{24}\right) =$$

$$= \frac{1}{2}\left(\frac{955}{64}^{\,|3} - \frac{315}{24}^{\,|8}\right) = \frac{1}{2}\left(\frac{2865 - 2520}{192}\right) = \frac{344}{2\cdot 192} = \frac{172}{192} = \frac{\cancel{86}^{\,43}}{\cancel{96}_{\,43}} = \boxed{\frac{43}{48}}$$

(2) $O(0,0)$ $A(1,0)$ $B(1,2)$

$D = ?$ $\quad I = \iint_D f(x,y)\cdot dx\,dy$



Prvo po y

(I) $y = L_1(x)$, gornja

$$I = \int_0^1 \left(\int_0^{2x} f(x,y)\,dy\right)dx$$

donja

II po x

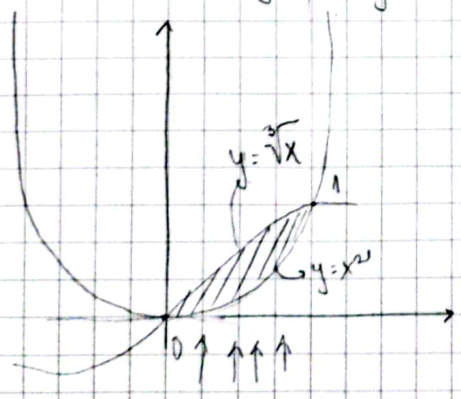
$$I = \int_0^2 \left(\int_{1/2 y}^{1} f(x,y)\,dx\right)dy$$

(3) $P(D) = ?$

D = između $y = x^2$, $x = y^3$

$$\left.\begin{matrix} y = x^2 \\ x = y^3 \end{matrix}\right\} \Rightarrow \begin{matrix} x = x^6 \\ x = 0 \quad \vee \quad x^5 = 1 \end{matrix}$$

$$\begin{matrix} x = 0 & x = 1 \\ y = 0 & y = 1 \end{matrix}$$



$$P(D) = \iint_D dx\,dy =$$

$$= \int_0^1 \left(\int_{x^2}^{\sqrt[3]{x}} dy\right)dx = \int_0^1 \left(\sqrt[3]{x} - x^2\right)dx =$$

$$= \left(\frac{3}{4}x^{4/3} - \frac{x^3}{3}\right)\Big|_0^1 = \frac{3}{4} - \frac{1}{3} - 0 = \frac{5}{12}$$

$$P(D) = \frac{5}{12}$$

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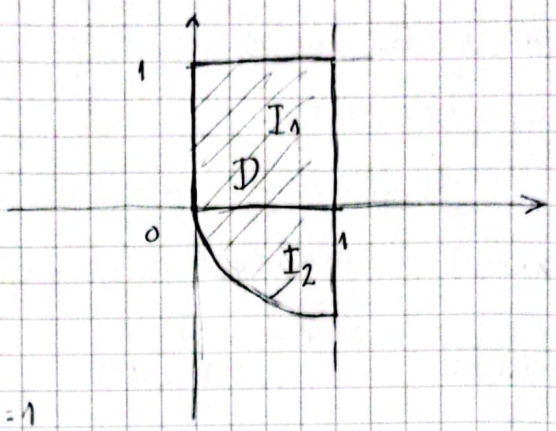
(4) $I = \int_0^1 \left(\int_{-\sqrt{2x-x^2}}^{1} f(x,y)\,dy \right) dx$



D=?

$x \in [0,1] \rightsquigarrow y \in [-\sqrt{2x-x^2}, 1]$

$y = -\sqrt{2x-x^2}$

$\Leftrightarrow y<0 \ \wedge \ y^2 = 2x-x^2$

$\Leftrightarrow y<0 \ \wedge \ y^2+x^2-2x+1=1$

$\Leftrightarrow y<0 \ \wedge \ (x-1)^2+y^2=1 \rightarrow$ krug, center $(1,0)$, polup. $=1$

$y \in [-1,1] \rightarrow$ gore $\boxed{x=1}$

Dole

- $y \in [0,1] \rightarrow x=0$
- $y \in [0,-1] \rightarrow (x-1)^2+y^2=1$

$|x-1| = \sqrt{y^2-1}$

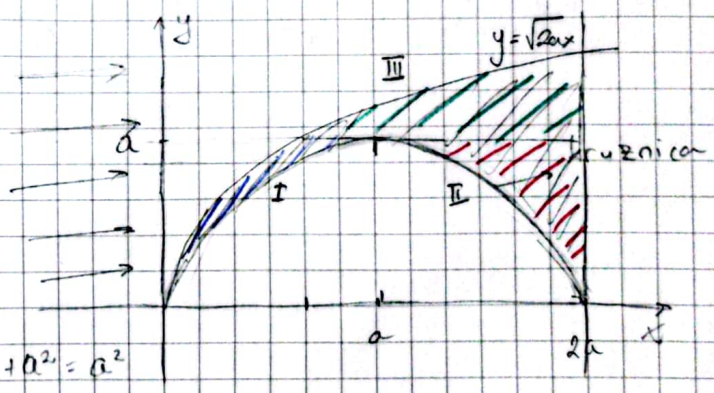
$x \le 1 : |x-1| = 1-x = \sqrt{1-y^2} \rightarrow x = 1-\sqrt{1-y^2}$

Za $y \in [0,1] \rightarrow x \in [0,1]$

Za $y \in [-1,0] \leftrightarrow x \in [1-\sqrt{1-y^2}, 1]$

$I = I_1 + I_2 = \int_0^1 \left(\int_0^1 f(x,y)\,dx \right) dy + \int_{-1}^{0} \left(\int_{1-\sqrt{1-y^2}}^{1} f(x,y)\,dx \right) dy$

(5) $I = \int_0^{2a} \left(\int_{\sqrt{2ax-x^2}}^{\sqrt{2ax}} f(x,y)\,dy \right) dx$



$y = \sqrt{2ax} = \sqrt{2a}\sqrt{x}$

$y = \sqrt{2ax-x^2} \Leftrightarrow y>0$ i $y^2 = 2ax-x^2$

$y^2+x^2-2ax+a^2 = a^2$

$y^2+(x-a)^2 = a \rightarrow$ Krug

da li je $\sqrt{2ax} \ge \sqrt{2ax-x^2} \ /^2$

$\cancel{2ax} \ge \cancel{2ax} - x^2$

$0 \ge -x^2$ ✓

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I $y \in \sim \to x \in \sim$

$y \in [0,a] \to$ donja: $y = \sqrt{2ax} \Rightarrow x = \frac{y^2}{2a}$

gornjo: $(x-a)^2 + y^2 = a^2$

$|x-a| = \sqrt{a^2-y^2}$

$x \in [0,a]$ $\quad a - x = \sqrt{a^2-y^2}$

$x = a - \sqrt{a^2-y^2}$

$\frac{y^2}{2a} \le x \le a - \sqrt{a^2-y^2}$

II

$y \in [0,a] \to$ donja: $x = a + \sqrt{a^2-y^2}$

gornjo: $x = 2a$

$a + \sqrt{a^2-y^2} \le x \le 2a$

III $y \in [a, 2a] \to$ donja $y = \sqrt{2ax} \quad /^2$

$y^2 = 2ax \Rightarrow \boxed{x = \frac{y^2}{2a}}$

gornja $y = 2a$

$\frac{y^2}{2a} \le x \le 2a$

$$I = I_1 + I_2 + I_3 = \int_0^a \left(\int_{\frac{y^2}{2a}}^{a-\sqrt{a^2-y^2}} f(x,y)\,dx \right) dy + \int_a^{2a} \left(\int_{\frac{y^2}{2a}}^{2a} f(x,y)\,dx \right) dy + \int_a^{2a} \left(\int_{a+\sqrt{a^2-y^2}}^{2a} f(x,y)\,dx \right) dy$$

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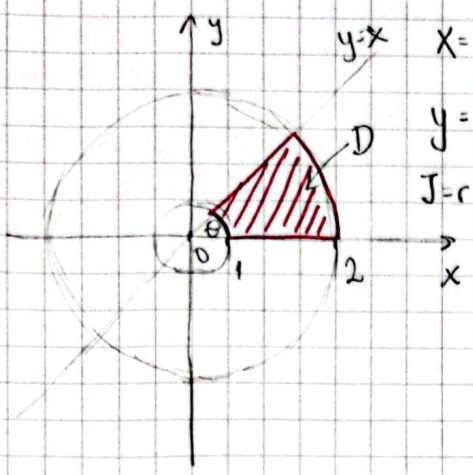
Smena u dvostrukom

(1.) $I = \iint_D \frac{1}{2x^2+y^2}\,dx\,dy \qquad D = \{(x,y)\in\mathbb{R}^2 \mid 1 \le x^2+y^2 \le 4,\ 0 \le y \le x\}$

Prsten formiran pomoću 2 kruga



$x = r\cdot\cos\theta$

$y = r\cdot\sin\theta$

$J = r$

$1 \le x^2+y^2 = r^2 \le 4$

$1 \le r \le 2 \Rightarrow \boxed{r\in[1,2]}$

$\theta \in [0, \pi/4]$

$$I = \int_0^{\pi/4}\left(\int_1^2 \frac{1}{2r^2\cos^2\theta + r^2\sin^2\theta}\cdot r\cdot dr\right)d\theta =$$

$$= \int_0^{\pi/4}\left(\int_1^2 \frac{1}{r^{\cancel{2}}(\cos^2\theta+1)}\cdot \cancel{r}\,dr\right) = \int_0^{\pi/4}\frac{1}{1+\cos^2\theta}\cdot \underbrace{\ln r\Big|_1^2}_{\ln 2 - \ln 1 = \ln 2}\,d\theta =$$

$$= \ln 2\int_0^{\pi/4}\frac{1}{1+\cos^2\theta}\,d\theta \overset{\text{smena}}{=} \ln 2\int_0^1 \frac{1}{1+\frac{1}{1+t^2}}\cdot\frac{dt}{1+t^2} = \ln 2\int_0^1\frac{dt}{2+t^2}$$

$t = \operatorname{tg}\theta \rightsquigarrow \cos^2\theta = \frac{1}{1+t^2}$

$\theta\in[0,\pi/4] \longrightarrow t\in[0,1]$

$dt = \frac{1}{1+t^2}\,dt$

$$= \ln 2\cdot\frac{1}{\sqrt{2}}\cdot \operatorname{arctg}\frac{t}{\sqrt{2}}\Big|_0^1 =$$

$$= \ln\cdot\frac{1}{\sqrt{2}}\cdot\operatorname{arctg}\frac{1}{\sqrt{2}} =$$

Скенирано помоћу ЦамСцаннер-а

Smenu sa trostrukim:

① $V(T)=?$ $\quad T=\{(x,y,z)\in\mathbb{R}^3 \mid \underbrace{x^2+y^2+z^2\le 9}_{\text{sfera}},\ \underbrace{z^2\ge \frac{x^2+y^2}{3}}_{\text{konus}},\ z\ge 0\}$

$\rightarrow r\in[0,3]$

$x=\rho\sin\varphi\cos\theta$

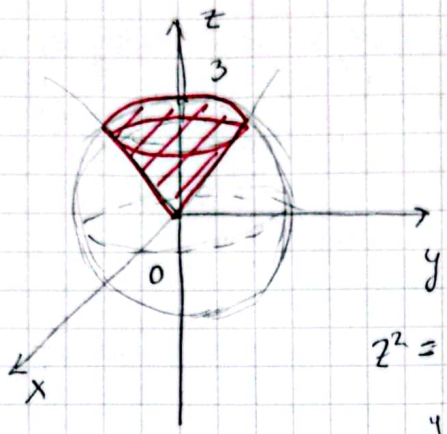
$y=\rho\sin\varphi\sin\theta$

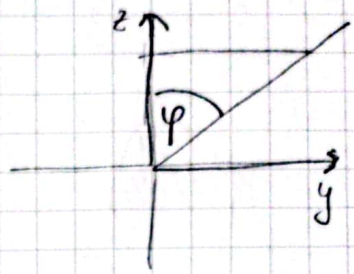
$z=\rho\cos\varphi$

$|J|=\rho^2\sin\varphi$

$\theta\in[-\pi,\pi)$

$\varphi\in[0,?]$





$z^2=\frac{x^2+y^2}{3} \overset{x=0}{=} \frac{y^2}{3}$

$z=\frac{y}{\sqrt{3}} \qquad tg\,\varphi=\frac{y}{z}=\sqrt{3}$

$\varphi=arctg\sqrt{3} \Rightarrow \boxed{\varphi=\pi/3} \rightarrow ?$

$\rightarrow \varphi\in[0,\pi/3]$

$$V(T)=\iiint_T dx\,dy\,dz=\int_{-\pi}^{\pi}\int_0^{\pi/3}\int_0^3 \rho^2\cdot\sin\varphi\,d\rho\,d\varphi\,d\theta$$

$$=\int_{-\pi}^{\pi}\int_0^{\pi/3}\sin\varphi\cdot\frac{\rho^3}{3}\Big|_0^3\,d\varphi\,d\theta$$

$$=\int_{-\pi}^{\pi}\int_0^{\pi/3}9\sin\varphi\,d\varphi\,d\theta$$

$$=\int_{-\pi}^{\pi}9\,(-\cos\varphi)\Big|_0^{\frac{\pi}{3}}=9\pi$$

Скенирано помоћу ЦамСцаннер-а

JUL 2022.

1.

$$f(x) = \begin{cases} \dfrac{x^3 - xy^2}{x^2+y^2}, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

непр?

$$\lim_{(x,y)\to(0,0)} \frac{x^3 - xy^2}{x^2+y^2} = \begin{bmatrix} x = \rho\cos\varphi \\ y = \rho\sin\varphi \end{bmatrix} = \lim_{\rho\to 0} \frac{\rho^3\cos^3\varphi - \rho\cos\varphi\cdot\rho^2\sin^2\varphi}{\rho^2\cos^2\varphi + \rho^2\sin^2\varphi} =$$

$$= \lim_{\rho\to 0} \frac{\rho^{\cancel{3}}(\cos^3\varphi - \cos\varphi\cdot\sin^2\varphi)}{\cancel{\rho^2}\cdot\underbrace{(\cos^2\varphi+\sin^2\varphi)}_{1}} = \lim_{\rho\to 0} \rho\cos\varphi\underbrace{(\cos^2\varphi - \sin^2\varphi)}_{\cos 2\varphi} =$$

$$= \lim_{\rho\to 0} \underbrace{\rho}_{\to 0}\,\underbrace{\cos\varphi}_{\text{ogr.}}\,\underbrace{\cos 2\varphi}_{\text{ogr.}} = 0$$

Jeste непр.!

$$f'_x = \frac{f(h,0) - f(0,0)}{h} = \frac{\frac{\cancel{h^2}}{\cancel{h^2}} - 0}{h} = 1$$

$$f'_y = \frac{f(0,h) - f(0,0)}{h} = \frac{\frac{0}{h^2} - 0}{h} = 0$$

$$L = \lim \frac{f(h,k) - f(0,0) - \frac{h}{\sqrt{h^2+k^2}}\cdot f'_x(0,0) - \frac{k}{\sqrt{h^2+k^2}} f'_y(0,0)}{\sqrt{h^2+k^2}} = \frac{\frac{h^3 - hk^2}{h^2+k^2} - h}{\sqrt{h^2+k^2}} =$$

$$= \frac{\cancel{h^3} - hk^2 - \cancel{h^3} - hk^2}{(h^2+k^2)\sqrt{h^2+k^2}} = \frac{-2hk^2}{\sqrt{(h^2+k^2)^3}}$$

$\vec{v} = (\alpha,\beta) \nparallel (0,0)$

$$\lim_{h\to 0} \frac{f(\alpha h, \beta h) - f(0,0)}{h} =$$

$$= \lim_{h\to 0} \frac{\frac{\alpha^3 h^3 - \alpha h \beta^2 h^2}{\alpha^2 h^2 + \beta^2 h^2} - 0}{h} =$$

$$= \lim_{h\to 0} \frac{\frac{\cancel{h^3}(\alpha^3 - \alpha\beta^2)}{\cancel{h^2}(\alpha^2+\beta^2)}}{h} = \lim_{h\to 0} \frac{\alpha^3 - \alpha\beta^2}{\alpha^2+\beta^2} = \boxed{\frac{\alpha(\alpha^2-\beta^2)}{\alpha^2+\beta^2}}$$

smena

$$= \lim_{\rho\to 0} \frac{-2\rho^3\cos\varphi\sin^2\varphi}{\sqrt{\rho^6(\sin^2\varphi+\cos^2\varphi)^3}} = \lim_{\rho\to 0} \frac{-2\cancel{\rho^3}\cos\varphi\sin^2\varphi}{\cancel{\rho^3}} =$$

$$= \lim_{\rho\to 0} \cos\varphi\sin^2\varphi = \cos\varphi\sin^2\varphi \neq 0$$

↳ Nije dif!

Скенирано помоћу ЦамСцаннер-а

② $f(x,y) = \sin(x)\sin(y)\sin(x+y)$

$\sin y \sin(2x+y)$

$f'_x = \cos x \sin y \sin(x+y) + \sin x \sin y \cos(x+y) = \sin y(\cos x \sin(x+y) + \sin x \cos(x+y))$

$f'_y = \sin x(\cos y \sin(x+y) + \sin y \cos(x+y)) = \sin x \sin(x+2y)$

$\sin y = 0 \quad \vee \quad \sin(2x+y) = 0$

$\hookrightarrow y_1 = 0 \vee y_2 = \pi$

$M_1(0,0)$

$M_2(0,\pi)$

$M_3(0,\pi) \quad M_4(\pi,\pi)$

$y_2 = \pi$:

$\sin x \cdot \sin(x+2\pi) = 0$

$\sin x \cdot \sin x = 0$

$\sin^2 x = 0 \rightarrow x_3 = 0 \quad x_4 = \pi$

$y_1 = 0$: $\sin(x)\sin(x) = \sin^2 x = 0 \Rightarrow x_1 = 0 \vee x_2 = \pi$

$f''_{xy} = \cos y \sin(2x+y) + \sin y \cos(2x+y) = \sin(2x+2y)$

$f''_{xx} = 2\cos(2x+y)\sin y$

$f''_{yy} = 2\sin x \cos(x+2y)$

$\sin(2x+y) = 0$

$2x+y = 0 \Rightarrow \quad 2x+y = \pi$

$\Rightarrow y = -2x \quad y = \pi - 2x$

$y = -2x$:

$\sin x \sin(x+2(-2x)) =$

$= \sin x \cdot \sin(-3x)$

$\underbrace{x = 0 \vee -3x = 0}_{x = 0 \Rightarrow y = 0} \quad \vee \quad x_6 = \pi \quad \vee \quad -3x = \pi \Rightarrow x_7 = \frac{-\pi}{3}$

$\hookrightarrow y = 2\pi \qquad \hookrightarrow y = \frac{2\pi}{3}$

$\hookrightarrow$ vec rešeno

$M_5(\pi, -2\pi) \Rightarrow M_5(0,\pi)$

$M_5(-\pi/3, 2\pi/3)$

$y = \pi - 2x$

$\sin x \cdot \sin(x + 2\pi - 2x) = \sin x \cdot \sin(-2\pi - 3x)$

$2\pi - 3x = 0 \qquad 2\pi - 3x = \pi \qquad M_6(2\pi/3, -\pi/3)$

$x = \frac{2\pi}{3} \qquad 3x = \pi \qquad M_7(\pi/3, \pi/3)$

$x = \pi/3$

$$d^2f(M_6) = \begin{bmatrix} 2\cdot\cos\pi \cdot \sin -\pi/3 & \sin 2\pi/3 \\ \sin 2\pi/3 & 2\cdot\sin(2\pi/3)\cdot\cos 0 \end{bmatrix} = \begin{bmatrix} \sqrt{3} & \sqrt{3}/2 \\ \sqrt{3}/2 & \sqrt{3} \end{bmatrix} = 3 - \frac{3}{4} > 0$$

> 0

$\hookrightarrow$ Lokalni minimum!!

$$d^2f(M_7) = \begin{bmatrix} 2\cdot\cos\pi \cdot \sin(\pi/3) & \sin(4\pi/3) \\ \sin(4\pi/3) & 2\sin(\pi/3)\cos\pi \end{bmatrix} = \begin{bmatrix} -\sqrt{3} & -\sqrt{3}/2 \\ -\sqrt{3}/2 & -\sqrt{3} \end{bmatrix} = 3 - \frac{3}{4} > 0$$

$\hookrightarrow$ Lok. maksimum!!

Скенирано помоћу ЦамСцаннер-а

③ $\iint_D e^{-x^2-y^2} \sin(x^2+y^2)\,dx\,dy \qquad D = \{(x,y) \mid \underbrace{x^2+y^2 \le 1}_{\text{Krug}}\}$

$x = \rho\cos\varphi \rightarrow \rho \in [0,1]$

$y = \rho\sin\varphi \rightarrow \varphi \in [0, 2\pi)$

$J = \rho$

$-\rho^2\cos^2\varphi - \rho^2\sin^2\varphi = -\rho^2$

$\sin(\rho^2\cos^2\varphi + \rho^2\sin^2\varphi) = \sin(\rho^2)$

$$= \int_0^1 \int_0^{2\pi} \rho e^{-\rho} \cdot \sin(\rho^2) \cdot d\varphi \cdot d\rho =$$

$$= \int_0^1 \rho e^{-\rho} \cdot \sin\rho^2 \cdot \varphi\, d\rho \Big|_0^{2\pi} = \int_0^1 \rho e^{-\rho^2} \cdot \sin\rho^2 \cdot 2\pi \cdot d\rho =$$

$$= 2\pi \int_0^1 \rho e^{-\rho^2} \cdot \sin\rho^2 \cdot d\rho =$$

$$= \begin{bmatrix} \rho^2 = t \\ d\rho \cdot 2\rho = dt \\ \rho\, d\rho = \frac{dt}{2} \end{bmatrix} = \pi \int_0^1 e^{-t} \cdot \sin t \cdot dt =$$

$$I = \int_0^1 \frac{\sin t}{e^t}\, dt = \begin{bmatrix} u = \sin t & dv = \frac{1}{e^t} dt \\ du = \cos t\, dt & v = -\frac{1}{e^t} \end{bmatrix} =$$

$$= -\frac{\sin t}{e^t} - \int_0^1 \frac{\cos t\, dt}{e^t} = \begin{bmatrix} u = \cos t & dv = \frac{1}{e^t} \\ du = -\cos t\, dt & v = -\frac{1}{e^t} \end{bmatrix}$$

$$= -\frac{\sin 1}{e} - \frac{\cos 1}{e} + \frac{\cos 0}{e^0} - \int_0^1 \frac{\sin t}{e^t} \cdot dt =$$

$$= -\frac{\sin 1}{e} - \frac{\cos 1}{e} + 1 - I = I$$

$$2I = -\frac{\sin 1}{e} - \frac{\cos 1}{e} + 1$$

$$I = -\frac{\sin 1}{2e} - \frac{\cos 1}{2e} + \frac{1}{2} = \boxed{\frac{e - \sin 1 - \cos 1}{2e}}$$

Скенирано помоћу ЦамСцаннер-а

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(4) $xy' + 4y = x^4y^2, \quad y(2) = -1$

$xy' + 4y = x^4y^2 \;/ : xy^2$

$\frac{y'}{y^2} + \frac{4}{xy} = x^3$

$\frac{1}{y} = z \Rightarrow z' = -\frac{y'}{y^2}$

$\Rightarrow -z' = \frac{y'}{y^2}$

$z + \frac{4}{xz} = -x^3$

$z = e^{-\int -\frac{4}{x}\cdot dx} \cdot \left(c + \int -x^3 \cdot e^{\int -\frac{4}{x} dx} \cdot dx\right) =$

$= e^{4\ln x} \cdot \left(c + \int -x^3 \cdot e^{-4\ln x} dx\right) =$

$= x^4 \cdot \left(c + \int -x^3 \cdot \frac{1}{x^4} dx\right)$

$= x^4 \left(c + \int -\frac{1}{x} dx\right) = \boxed{x^4 \cdot c + x^4 \ln x}$

$\frac{1}{y} = x^4 c - x^4 \ln x$

$\frac{1}{y} = x^4 (c - \ln x)$

$y = \frac{1}{x^4(c - \ln x)}$

$y(2) = -1 \Rightarrow -1 = \frac{1}{16(c - \ln 2)}$

$-16c + 16\ln 2 = 1$

$16\ln 2 - 1 = 16c$

$c = \frac{16\ln 2 - 1}{16} = \boxed{\ln 2 - \frac{1}{16}}$

$y = \frac{1}{x^4\left(\ln 2 - \frac{1}{16} - \ln x\right)} \Rightarrow y = \frac{1}{x^4\left(\ln \frac{2}{x} - \frac{1}{16}\right)}$ //

Скенирано помоћу ЦамСцаннер-а

③ Pov. granice $T = \{(x,y,z) \in \mathbb{R}^3 \mid x^2+y^2 \le 1,\ 3x+2y+z \le 6,\ x+y+z \ge 1\}$

α: $3x+2y+z=6$

β: $x+y+z=1$

$$P(\partial T) = P(S_1) + P(S_2) + P(S_3)$$

S_1:

$z = 6-3x-2y$

$(x,y) \in ?$ $(x,y) \in D$

$D = \{(x,y) \in \mathbb{R}^2 \mid x^2+y^2 \le 1\}$

$$P(S_1) = \iint_D ds = \iint_D \sqrt{1+(z'_x)^2+(z'_y)^2}\, dx\,dy$$

$$= \iint_D \sqrt{1+(-3)^2+(-2)^2}\, dx\,dy = \sqrt{14} \iint_D dx\,dy$$

$$= \sqrt{14} \cdot P(D) = \sqrt{14}\, r^2 \pi = \sqrt{14} \cdot 1^2 \pi = \boxed{\pi\sqrt{14}}$$

S_2: deo β: $x+y+z=1$ $\quad z(x,y) = 1-x-y$

$(x,y) \in D \rightsquigarrow P(S_2) = \sqrt{3}\pi$

S_3: $x=?$ $y=?$ $z=?$

$x = \cos u$ $\quad y = \sin u$ $\rightarrow$ idu po krugu

$u \in [-\pi, \pi]$

$z = v \rightarrow v \in [\underbrace{1-\cos u-\sin u}_{1-x-y}, \underbrace{6-3\cos u-2\sin u}_{6-3x-2y}]$

$a \qquad b$

$r(u,v) = (\cos u, \sin u, v)$ $\quad (u,v) \in D_1$

$r'_u = (-\sin u, \cos u, 0)$ $\quad r'_v = (0,0,1)$

$$|r'_v \times r'_u| = \begin{vmatrix} i & j & k \\ 0 & 0 & 1 \\ -\sin u & \cos u & 0 \end{vmatrix} = (\cos u, \sin u, 0) \qquad \|r'_u \times r'_v\| = \sqrt{\cos^2 u + \sin^2 u + 0} = 1$$

$$P(S_3) = \iint_{S_3} ds = \iint_{D_1} \|r'_u \times r'_v\|\, du\, dv = \iint_{D_1} 1\, du\, dv = \int_{-\pi}^{\pi}\left(\int_a^b 1\, dv\right) du =$$

$$= \int_{-\pi}^{\pi} 6 - 3\cos u - 2\sin u - 1 + \cos u + \sin u =$$

$$= \int_{-\pi}^{\pi} (5 - 2\cos u - \sin u)\, du = \boxed{10\pi}$$

$$P(S) = P(S_1) + P(S_2) + P(S_3) =$$

$$= \pi\sqrt{14} + \pi\sqrt{3} + 10\pi$$

$$= \pi(\sqrt{14} + \sqrt{3} + 10)$$

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Скенирано помоћу ЦамСцаннер-а

Stp. 2017.

① $f(x,y) = \begin{cases} \dfrac{y^3}{x^2+y^2} & , (x,y) \neq (0,0) \\ 0 & , (x,y) = (0,0) \end{cases}$

a) depr.?

$$\lim_{(x,y)\to(0,0)} \frac{y^3}{x^2+y^2} = \begin{bmatrix} x = \rho\cos\varphi \\ y = \rho\sin\varphi \end{bmatrix} = \lim_{\rho\to 0} \frac{\rho^{\cancel{3}}\sin^3\varphi}{\rho^{\cancel{2}}\underbrace{(\cos^2\varphi+\sin^2\varphi)}_{1}} = \lim_{\rho\to 0} \overbrace{\rho}^{\to 0}\underbrace{\sin^3\varphi}_{\text{ograničeno } [-1,1]} \to 0 \checkmark$$

Depr. je u $(x,y) = (0,0)$

b) $f'_x = \dfrac{-y^3\cdot(2x)}{(x^2+y^2)^2}$

$f'_y = \dfrac{3y^2-2y}{(x^2+y^2)^2}$

$$f'_x(0) = \lim_{h\to 0} \frac{f(h,0)-f(0,0)}{h} = \lim_{h\to 0} \frac{\frac{0}{h^2+0}-0}{h} = 0$$

$$f'_y(0) = \lim_{k\to 0} \frac{f(0,k)-f(0,0)}{k} = \lim_{k\to 0} \frac{\frac{k^3}{k^2}-0}{k} = \boxed{1}$$

$$L = \lim_{(h,k)\to(0,0)} \frac{f(h,k)-f(0,0)-\cancel{h}f'_x(0,0)-\cancel{k}f'_y(0,0)}{\sqrt{h^2+k^2}} =$$

$$= \lim_{(h,k)\to(0,0)} \frac{\frac{k^3}{h^2+k^2}-0-0-k}{\sqrt{h^2+k^2}} = \lim_{(h,k)\to(0,0)} \frac{\frac{\cancel{k^3}-kh^2-\cancel{k^3}}{h^2+k^2}}{\sqrt{h^2+k^2}} =$$

$$= \lim_{(h,k)\to(0,0)} \frac{-kh^2}{\sqrt{(h^2+k^2)^3}}$$

$a = \left(\frac{2}{n}, \frac{1}{n}\right) \to (0,0)$

$$\frac{-\frac{2}{n}\cdot\frac{1}{n^2}}{\sqrt{\left(\frac{4}{n^2}+\frac{1}{n^2}\right)^3}} = -\frac{\frac{1}{n^3}}{\sqrt{\frac{125}{n^6}}} = -\frac{\frac{1}{n^3}}{\frac{5\sqrt{5}}{n^3}} = -\frac{1}{5\sqrt{5}}$$

$b = \left(\frac{1}{n}, \frac{2}{n}\right) \to (0,0)$

$$\frac{-\frac{1}{n}\cdot\frac{4}{n^2}}{\sqrt{\left(\frac{1}{n^2}+\frac{4}{n^2}\right)^3}} = \frac{-\frac{4}{n^3}}{\frac{5\sqrt{5}}{n^3}} = -\frac{4}{5\sqrt{5}}$$

$$-\frac{1}{5\sqrt{5}} \neq -\frac{4}{5\sqrt{5}}$$

↳ Nije dif!!

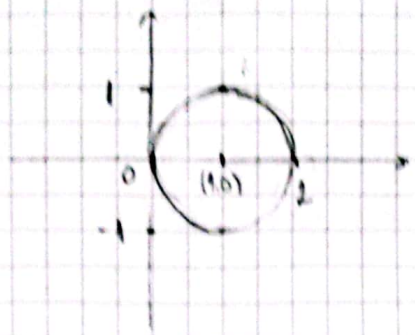
STUDENTSKA UNIJA MATEMATIČARA

Скенирано помоћу ЦамСцаннер-а

(8.) $\iint\limits_D \frac{1}{\sqrt{x^2+y^2}}\,dx\,dy \qquad D=\{(x-1)^2+y^2\le 1\}$

$L_1\ C(1,0),\ r=1$

→ Da smo išli normalnim načinom dobili bismo neki taj integral.



$x=\rho\cos\varphi \qquad (x-1)^2+y^2\le 1$

$y=\rho\sin\varphi \qquad \boxed{x^2+y^2\le 2x}$ → radoviđićemo ih jer se ovako pod korenom sve skrati

$\varphi\in[-\pi,\pi] \qquad \rho\in[0,2\cos\varphi]$

$$J=\det\begin{bmatrix} x'_\rho & x'_\varphi \\ y'_\rho & y'_\varphi\end{bmatrix}=\det\begin{bmatrix}\cos\varphi & -\rho\sin\varphi\\ \sin\varphi & \rho\cos\varphi\end{bmatrix}=\rho\cos^2\varphi+\rho\sin^2\varphi=\rho\underbrace{(\cos^2\varphi+\sin^2\varphi)}_{1}=\rho$$

$$\iint\limits_D\left(\frac{1}{\sqrt{\rho^2\cos^2\varphi+\rho^2\sin^2\varphi}}\cdot\rho\,d\rho\,d\varphi\right)=$$

$$=\iint\limits_D\left(\frac{1}{\rho}\cdot\rho\,d\rho\,d\varphi\right)=$$

$$=\int_{-\pi}^{\pi}\left(\int_0^{2\cos\varphi}d\rho\,d\varphi\right)=\int_{-\pi}^{\pi}\rho\Big|_0^{2\cos\varphi}d\varphi=$$

$$=\int_{-\pi}^{\pi}2\cos\varphi\,d\varphi=$$

$$=2\sin\varphi\Big|_{-\pi}^{\pi}=2\cdot 0-2\cdot 0=0$$

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Скенирано помоћу ЦамСцаннер-а

③ $\iint_S F \cdot dS \quad F(x,y,z) = (yz, xz, xy) \quad S = \{(x,y,z) \in \mathbb{R}^3 \mid z = 9 - x^2 - y^2,\ z = 5\}$

→ Nisam stigao još dotle :)

④ $y' - 3y = e^{2x} \qquad y(0) = 3$

$$y = e^{-\int -3dx}\left(c + \int e^{2x} \cdot e^{\int -3dx} dx\right) =$$

$$= e^{3x}\left(c + \int e^{2x} \cdot e^{-3x} dx\right) =$$

$$= e^{3x}\left(c + \int e^{-x} dx\right) =$$

$$= e^{3x}\left(c - \frac{1}{e^x}\right) = e^{3x}c - e^{2x} \,//$$

$$3 = e^0 c - e^0 \Rightarrow 3 = c - 1 \Rightarrow \boxed{c = 4}$$

$$y = 4e^{3x} - e^{2x} \,//$$

Скенирано помоћу ЦамСцаннер-а

JUN 2 2018.

1) $f: \mathbb{R}^2 \to \mathbb{R}^2$

$$f(x,y) = \frac{y\sin x}{x^2+y^2} + \frac{(x-1)^2 y}{(x-1)^2+y^2}$$

a) 1° $x^2+y^2 \neq 0 \Leftrightarrow (x,y) \neq (0,0)$ $\quad 2^\circ$ $(x-1)^2+y^2 \neq 0 \Leftrightarrow (x,y) \neq (1,0)$ $\Big\} \Rightarrow D = \{(x,y) \in \mathbb{R}^2 \mid (x,y) \neq (0,0) \wedge (x,y) \neq (1,0)\}$ //

b) Svodi se na to da vidimo kako se limesi od (0,0) i (0,1) ponašaju

$$\lim_{(x,y)\to(0,0)} \frac{y\sin x}{x^2+y^2} + \lim_{(x,y)\to(1,0)} \frac{(x-1)^2 y}{(x-1)^2+y^2}$$

ne zanima nas sinus

$$\lim_{(x,y)\to(0,0)} \frac{y}{x^2+y^2} + \lim_{(x,y)\to(1,0)} \frac{(x-1)^2 y}{(x-1)^2+y^2} = \left| \begin{array}{l} x = \rho\cos\varphi \\ y = \rho\sin\varphi \end{array} \right| =$$

$$= \lim_{\rho\to 0} \frac{\sin\varphi}{\rho} + \lim_{\rho\to 0} \frac{\rho^2\cos^2\varphi \cdot \rho\sin\varphi}{\rho^2\cos^2\varphi + \rho^2\sin^2\varphi} =$$

$\to \infty$ $\quad \to 0$ $\quad \to 0$ gr.

znači za (0,0) ne može ništa!

$$= \lim_{\rho\to 0} \frac{\rho^3(\cos^2\varphi \cdot \sin\varphi)}{\rho^2} = 0 \;//$$

za (1,0) možemo da gurnemo do 0! //

$f: \mathbb{R}^2 \to \mathbb{R}^2$

$$f(x,y) = \begin{cases} \dfrac{y\sin x}{x^2+y^2} + \dfrac{(x-1)^2 y}{(x-1)^2+y^2}, & (x,y) \neq (0,0) \wedge (x,y) \neq (1,0) \\ 0, & (x,y) = (1,0) \end{cases}$$

//

Скенирано помоћу ЦамСцаннер-а

c) $f'_x = \dfrac{y\cos x\cdot(x^2+y^2) - y\sin x\cdot 2x}{(x^2+y^2)^2} + \dfrac{(2x-2)y\cdot((x-1)^2+y^2) - (x-1)^2 y\cdot(2x-2)}{((x-1)^2+y^2)^2}$

$\ulcorner (2xy-2y)((x-1)^2+y^2) - (2xy-2y)(x-1)^2 =$

$\dots$

$= (2xy-2y)(\cancel{(x-1)^2} + y^2 - \cancel{(x-1)^2}) =$

$\dots$

$= \dfrac{(2x-2)y^3}{\dots} \lrcorner$

$f'_x = \dfrac{y\cos x\,(x^2+y^2) - y\sin x\cdot 2x}{(x^2+y^2)^2} + \dfrac{y^3(2x-2)}{((x-1)^2+y^2)^2}$ //

$f'_y = \dfrac{\sin x\,(x^2+y^2) - y\sin x\,(2y)}{(x^2+y^2)^2} + \dfrac{(x-1)^2((x-1)^2+y^2) - (x-1)^2\cdot y\cdot 2y}{((x-1)^2+y^2)^2}$

$\ulcorner (x-1)^2((x-1)^2+y) - (x-1)^2\cdot 2y^2 =$

$= (x-1)^2((x-1)^2+y^2-2y^2) \lrcorner$

$f'_y = \dfrac{\sin x\,(x^2-y^2)}{(x^2+y^2)^2} + \dfrac{(x-1)^2((x-1)^2-y^2)}{((x-1)^2+y^2)^2}$

② $f: \mathbb{R}^2 \to \mathbb{R}$

$f(x,y) = xy^2 + x^2$, $D = \{(x,y)\in\mathbb{R}^2 \mid y^2-2 \le x \le 2\}$

min, max?

1° int D

$f'_x = y^2+2x \to 2x = -y^2 \to y^2 = 0 \to \boxed{y=0}$

$f'_y = 2xy \to 2xy = 0 \to \boxed{x=0}$

$M_1(0,0)$ //

$f(M_1) = 0$ //

2° dD

$\varphi(x,y) = y^2-2-x$

$F(x,y,\lambda) = xy^2+x^2 + \lambda(y^2-2-x)$

$F'_x = y^2+2x-\lambda = 0$

$F'_y = 2xy + 2\lambda y = 0$

$F'_\lambda = y^2-2-x = 0$

$x = (y^2-2)$

$\lambda = y^2+2x = y^2+2(y^2-2)$

$\Rightarrow 2(y^2-2)\cdot y + 2y\cdot(y^2+2(y^2-2)) =$

$= 2y(y^2-2) + 2y^3 + 4y(y^2-2) =$

$= 6y(y^2-2) + 2y^3$

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$= 2y\,(3(y^2-2)+2y^2) = 0$

$\Rightarrow 2y\,(3y^2-2+2y^2) = 0$

$\Rightarrow 2y\,(5y^2-2) = 0$

$y = 0 \;\vee\; y = \pm\sqrt{\frac{2}{5}}$

3° tačka

$x = 2$

$y^2 - 2 = 2$

$y^2 = 4$

$y = \pm 2$

$M_6(2,2)$

$M_7(2,-2)$

$\boxed{\min = M_1}$

$\boxed{\max = M_7}$

$y = 0 \Rightarrow x = -2$ $\quad M_2(-2,0)$

$y = \pm\sqrt{\frac{2}{5}} \Rightarrow x = \frac{2}{5} - 2 = -\frac{8}{5}$ $\quad M_3(-8/5, +\sqrt{2/5})$

$M_4(-8/5, -\sqrt{2/5})$

$x = 2$

$f(2,y) = 2y^2 + 4 = g(y)$

$g'(y) = 4y = 0 \Rightarrow y = 0$

$M_5(2,0)$

$f(M_2) = 4$

$f(M_3) = -\frac{8}{5}\cdot\frac{2}{5} + \frac{64}{25} =$

$= \frac{64-16}{25} = \frac{48}{25} = 1{,}92 = f(M_4)$

$f(M_5) = 4$

$f(M_6) = 2\cdot 4 + 4 = 12 = f(M_7)$

$\{0;\ 1{,}92;\ 4,\ 12\}$

$\hookrightarrow$ min $\quad \hookrightarrow$ max

(3) $\iint\limits_D xy\cdot dx\cdot dy \qquad D = \{(x,y)\in\mathbb{R}^2 \mid 9x^2 + 16y^2 \le 1,\ y \le x\}$

$x = \frac{1}{3}\rho\cdot\cos\varphi$

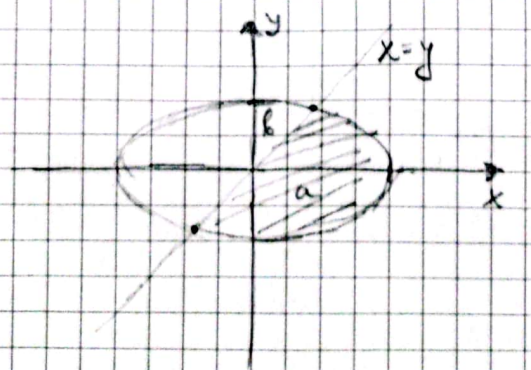
$y = \frac{1}{4}\rho\cdot\sin\varphi$

$9x^2 + 16y^2 = 1$

$\frac{x^2}{\frac{1}{9}} + \frac{y^2}{\frac{1}{16}} = 1 \qquad a^2 = \frac{1}{9} \Rightarrow a = \pm\frac{1}{3} = \frac{1}{3}$

$b^2 = \frac{1}{16} \Rightarrow b = \pm\frac{1}{4} = \frac{1}{4}$

$\begin{vmatrix} \frac{1}{3}\cos\varphi & -\frac{1}{3}\rho\sin\varphi \\ \frac{1}{4}\sin\varphi & \frac{1}{4}\rho\cos\varphi \end{vmatrix} = \frac{1}{12}\rho\cos^2\varphi + \frac{1}{12}\rho\sin^2\varphi = \boxed{\frac{\rho}{12}} \qquad \rho\in[0,1]$



$\left.\begin{array}{l} -\frac{3\pi}{4} \longrightarrow -\frac{\sqrt{2}}{2} \\ \frac{\pi}{4} \longrightarrow \frac{\sqrt{2}}{2} \end{array}\right\} \Rightarrow \varphi \in \left[-\frac{3\pi}{4}, \frac{\pi}{4}\right]$

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$$\iint_D \frac{1}{3}\rho\cos\varphi \cdot \frac{1}{4}\rho^2\sin\varphi \cdot \frac{1}{12}\rho \, d\rho \, d\varphi =$$

$$= \int_0^1 \left(\int_{-\frac{3\pi}{4}}^{\pi/4} \frac{1}{12}\rho^2 \cdot \frac{1}{12}\rho \cdot \cos\varphi \sin\varphi \cdot d\varphi \cdot d\rho \right) =$$

$$= \int_0^1 \frac{1}{144} \cdot \rho^3 \left(\int_{-\frac{3\pi}{4}}^{\frac{\pi}{4}} \cos\varphi \cdot \sin\varphi \, d\varphi \cdot d\rho \right) = \begin{bmatrix} t = \cos\varphi \\ dt = -\sin\varphi \cdot d\varphi \end{bmatrix} =$$

$\rightarrow t \in [-\sqrt{2}/2, \sqrt{2}/2]$

$$= \int_0^1 \frac{1}{144} \cdot \rho^3 \left(\int_{-\frac{\sqrt{2}}{2}}^{\sqrt{2}/2} t \cdot \cancel{\sin\varphi} \cdot \frac{dt}{\cancel{\sin\varphi}} \cdot d\rho \right) =$$

$$= \int_0^1 \frac{1}{144} \cdot \rho^3 \left(\int_{-\frac{\sqrt{2}}{2}}^{\frac{\sqrt{2}}{2}} t \cdot dt \right) d\rho =$$

$$= \int_0^1 \frac{1}{144} \cdot \rho^3 \left. \frac{t^2}{2} \right|_{-\frac{\sqrt{2}}{2}}^{\frac{\sqrt{2}}{2}} \cdot d\rho = \int_0^1 \frac{1}{144} \rho^3 \left(\frac{2}{8} - \frac{2}{8} \right) d\rho =$$

$$= \int_0^1 \frac{1}{144} \rho^3 \cdot 0 \, d\rho = \int_0^1 0 \, d\rho = \boxed{0}$$

Скенирано помоћу ЦамСцаннер-а

JUN 1 2020.

① $f: \mathbb{R}^3 \to \mathbb{R}^3$

ND $f(x,y,z) = x^2 + 3y^2 - 25$

min? max? $\quad A = \{(x,y,z) \in \mathbb{R}^3 \mid x^2 + 4y^2 + z^2 = 36\}$

$f'_x = 2x \qquad x = 0$

$f'_y = 6y \qquad y = 0$ $\Big\} \Rightarrow z^2 = 36 \Rightarrow z = \pm 36$

$f'_z = 0$ $\qquad M_1(0,0,6)$ $\quad M_2(0,0,-6)$

ΔD $\quad \varphi(x,y,z) = 36 - x^2 - 4y^2 - z^2$

$F(x,y,z,\lambda) = x^2 + 3y^2 - 25 - \lambda(36 - x^2 - 4y^2 - z^2)$

$F'_x = 2x - 2\lambda x \Rightarrow 2x(1-\lambda) = 0$

$F'_y = 6y - 8y\lambda \Rightarrow 2y(3 - 4\lambda) = 0$ $\Big\} \Rightarrow \lambda = 1 \vee x = 0$; $\lambda = 3/4 \vee y = 0$ $\quad M_3(0,0,0)$

$F'_z = 2z\lambda \Rightarrow 2z\lambda = 0 \Rightarrow z = 0$

$F'_\lambda = 36 - x^2 - 4y^2 - z^2$

tačka: ? $\qquad f(M_1) = -25 = f(M_2) = f(M_3)$ → samo tačka ekstremuma

② $f(x,y) = \begin{cases} (x^4+y^4)\sin\frac{1}{x^4+y^4}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$

a) neprek.?

$$\lim_{(x,y)\to(0,0)} (x^4+y^4)\sin\frac{1}{x^4+y^4} = \begin{bmatrix} x = \rho\cos\varphi \\ y = \rho\sin\varphi \end{bmatrix} = \lim_{\rho\to 0} \underbrace{\rho^4}_{\to 0}(\cos^4\varphi + \sin^4\varphi)\cdot \underbrace{\sin\frac{1}{\rho^4(\cos^4\varphi+\sin^4\varphi)}}_{\text{ogr.}}$$

$= 0$ ✓ jeste neprek.

df?

$$f'_x = \lim_{h\to 0}\frac{f(h,0) - f(0,0)}{h} = \lim_{h\to 0}\frac{h^4\cdot\sin\frac{1}{h^4}}{h} = \lim_{h\to 0} h^3 = 0 = f'_y$$

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$$L = \lim_{(h,k)\to(0,0)} \frac{f(h,k) - f(0,0) - h f'_x(0,0) - k f'_y(0,0)}{\sqrt{h^2+k^2}} =$$

$$= \lim_{(h,k)\to(0,0)} \frac{(h^4+k^4)\cdot \sin\frac{1}{h^4+k^4}}{\sqrt{h^2+k^2}} = \begin{bmatrix} x = \rho\cos\varphi \\ y = \rho\sin\varphi \end{bmatrix} =$$

$$= \lim_{\rho\to 0} \frac{\rho^4(\cos^4\varphi + \sin^4\varphi)\cdot \sin\frac{1}{\rho^4(\cos^4\varphi+\sin^4\varphi)}}{\sqrt{\rho^2\underbrace{(\cos^2\varphi+\sin^2\varphi)}_{1}}} =$$

$\to 0$ $\to$ ograničeno

$$= \lim_{\rho\to 0} \rho^3 = 0 \checkmark \quad \text{Jeste dif!!}$$

b) $g: \mathbb{R}\to\mathbb{R}$

$g(x) = f'_x(x,x)$

$$f'_x(x,y) = 4x^3\cdot \sin\frac{1}{x^4+y^4} + (\cancel{x^4+y^4})\cdot \cos\frac{1}{x^4+y^4}\cdot \frac{4x^3}{((x^4+y^4))^2}$$

$$= 4x^3\left(\sin\frac{1}{x^4+y^4} + \cos\frac{1}{x^4+y^4}\left(-\frac{4x^3}{x^4+y^4}\right)\right)$$

$$g(x) = f'_x(x,x) \Rightarrow g(x) = 4x^3\left(\sin\frac{1}{2x^4} - \frac{\cos\frac{1}{2x^4}}{x}\right)$$

npr. ??

$$\lim_{x\to 0} g(x) = \lim_{x\to 0} \underbrace{4x^3\sin\frac{1}{2x^4}}_{\to 0} - \underbrace{\frac{\cos\frac{1}{2x^4}}{x}}_{\to\infty} \to \text{Ne teži ka } 0!!$$

c) $$f'_x = 4x^3\left(\sin\frac{1}{x^4+y^4} + \cos\frac{1}{x^4+y^4}\left(-\frac{4x^3}{x^4+y^4}\right)\right) = \begin{bmatrix} x = \rho\cos\varphi \\ y = \rho\sin\varphi \end{bmatrix}$$

$$= -\lim_{\rho\to 0} \frac{4\cos^7\varphi\cdot\cos\frac{1}{\rho^4(\ldots)}}{\underbrace{\rho(\ldots)}_{\to 0}} = \infty \to \text{ne teži ka nuli}$$

$\to$ ograničeno

$\to\infty$

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③ $\iiint_T (4+z)\,dx\,dy\,dz$

$$T=\left\{(x,y,z)\in\mathbb{R}^3 \mid (x-1)^2+(y-2)^2+(z-3)^2\le 16 \ \wedge\ z\ge 3\right\}$$

$$x = 1+\rho\cos\theta\cos\varphi \qquad \rho\in[0,4]$$
$$y = 2+\rho\sin\theta\cos\varphi \qquad \theta\in[0,2\pi]$$
$$z = 3+\rho\cos\varphi \qquad \varphi\in[0,\pi/2]$$

$$J=\begin{vmatrix} \cos\theta\cos\varphi & -\rho\sin\theta\cos\varphi & -\rho\cos\theta\sin\varphi \\ \sin\theta\cos\varphi & \rho\cos\theta\cos\varphi & -\rho\sin\theta\sin\varphi \\ \sin\varphi & 0 & \rho\cos\varphi \end{vmatrix}$$

$$=\rho^2\cos\varphi$$

$$\int_0^4\int_0^{\pi/2}\int_0^{2\pi}(7+\rho\cos\varphi)\,\rho^2\cos\varphi\,d\theta\,d\varphi\,d\rho=$$

$$=\iiint_T 7\rho^2\cos\varphi+\rho^3\cos^2\varphi\,d\theta\,d\varphi\,d\rho=$$

$$=\iint_0^4\int_0^{\pi/2} 14\pi\cos\varphi\,\rho^2+2\pi\rho^3\cos^2\varphi\,d\varphi\,d\rho=$$

$$=\int_0^4 \rho^2 14\pi\sin\varphi\Big|_0^{\pi/2}+2\pi\rho^3\frac{\sin(2x)+x}{2}\Big|_0^{\pi/2}d\rho=$$

$$=\int_0^4 \rho^2 14\pi+\underbrace{2\pi\rho^3\frac{1+\frac{\pi}{2}}{2}}_{\rho^3\left(\pi+\frac{\pi^2}{2}\right)}d\rho=\frac{\rho^3}{3}14\pi\Big|_0^4+\frac{\rho^4}{4}\left(\pi+\frac{\pi^2}{2}\right)\Big|_0^4=$$

$$=\frac{64\cdot 14}{3}\pi+\frac{\cancel{256}^{\,64}}{\cancel{4}}\left(\pi+\frac{\pi^2}{2}\right)=$$

$$=64\cdot\frac{17}{3}\pi+32\pi^2$$

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4. γ $[0,2]^3$ $x+y+z=5$ iz $(200, 200, 0)$

$$\oint_\gamma (e^x+2y-z)\,dx + (e^{2y}+4x-z)\,dy + (e^{3x}+3y)\,dz$$

$$\nabla \times f = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ P & Q & R \end{vmatrix} = (R'_y - Q'_z,\ P'_z - R'_x,\ Q'_x - P'_y)$$

$$\left.\begin{matrix} R'_y = 3 & P'_z = -1 & Q'_x = 4 \\ Q'_z = -1 & R'_x = 3e^{3x} & P'_y = 2 \end{matrix}\right\} \Rightarrow \nabla \times f = (9, -1-3e^{3x}, 2)$$

$x+y+z=5$

$x-y=2 \Rightarrow z=1$

$x \in [1,2]$

$y+z=5-x \Rightarrow z \in [4-y, 3-y]$

$z = 5-x-y$

$r'_x = (1,0,-1)$

$r'_y = (0,1,-1)$

$$r'_x \times r'_y = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 0 & -1 \\ 0 & 1 & -1 \end{vmatrix} = (1,1,1)$$

$$\iint_D (9, -1-3e^{3x}, 2) \circ (1,1,1)\,dx\,dy =$$

$$= \iint_D 9-1-3e^{3x}+2\,dx\,dy = \iint_D 6-3e^{3x}\,dx\,dy = ?$$

Скенирано помоћу ЦамСцаннер-а

Sep 1. 2021.

(1.) $f(x,y) = \begin{cases} \dfrac{xy^2}{x^4+2y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$

a) Непр?

$$\lim_{(x,y)\to(0,0)} \frac{xy^2}{x^4+2y^2} = \begin{bmatrix} x = \rho\cos\varphi \\ y = \rho\sin\varphi \end{bmatrix} = \lim_{\rho\to 0} \frac{\rho\cos\varphi \cdot \rho^2\sin^2\varphi}{(\rho^4\cos^4\varphi + 2\rho^2\sin^2\varphi)} =$$

$$= \lim_{\rho\to 0} \frac{\rho^3 \cos\varphi \sin^2\varphi}{\rho^3 \underbrace{(\rho\cos^4\varphi)}_{\to 0} \underbrace{\left(\frac{2\sin^2\varphi}{\rho}\right)}_{\to\infty}} = 0$$

ogr $\to$ $\frac{ogr}{\infty} \to 0$

Непр!!

$$0 \le \left|\frac{xy^2}{x^4+2y^2}\right| \le \frac{|x|}{2} \cdot \underbrace{\frac{2|y^2|}{|x^4+2y^2|}}_{\le 1} \le \frac{|x|}{2} \xrightarrow[(x,y)\to(0,0)]{} 0$$

b) $$f'_x = \frac{y^2(x^4+2y^2) - xy^2(4x^3)}{(x^4+2y^2)^2} = \frac{x^4y^2 + 2y^4 - 4x^4y^2}{(x^4+2y^2)^2} =$$

$$= \frac{y^2(2y^2 - 3x^4)}{(x^4+2y^2)^2}$$

$$f'_y = \frac{2xy(x^4+2y^2) - (xy^2)(4y)}{(x^4+2y^2)^2} = \left[\, 2x^5y + \cancel{4xy^3} - \cancel{4xy^3} = \boxed{2x^5y} \,\right]$$

c) dif?

$$L = \lim_{(h,k)\to(0,0)} \frac{f(h,k) - f(0,0) - hf'_x(0,0) - kf'_y(0,0)}{\sqrt{h^2+k^2}}$$

$$f'_x = \lim_{h\to 0} \frac{f(h,0) - f(0,0)}{h} = \lim_{h\to 0} \frac{0-0}{h} = 0$$

$$f'_y = \lim_{h\to 0} \frac{f(0,h) - f(0,0)}{h} = \lim_{h\to 0} \frac{0-0}{h} = 0$$

$$= \lim_{(h,k)\to(0,0)} \frac{\frac{hk^2}{h^4+2k^2}}{\sqrt{h^2+k^2}} = \begin{bmatrix} h = \rho\cos\varphi \\ k = \rho\sin\varphi \end{bmatrix} =$$

$$= \lim_{\rho\to 0} \frac{\overbrace{\rho^3 \cdots}^{\to 0}}{\rho(\rho^4\cos^4\varphi + 2\sin^2\varphi)} =$$

$$= \frac{1}{2\sin^2\varphi} \to \text{nije dif.}$$

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$a = \left(\frac{1}{n}, \frac{1}{n}\right) \to (0,0)$

$$\frac{\frac{1}{n} \cdot \frac{1}{n^2}}{\left(\frac{1}{n^4} + \frac{2}{n^2}\right) \cdot \sqrt{\frac{1}{n^2} + \frac{1}{n^2}}} = \frac{\frac{1}{n^3}}{\frac{1+2n^2}{n^4} \cdot \frac{\sqrt{2}}{n}} = \frac{\frac{1}{n^3}}{\frac{\sqrt{2}(1+2n^2)}{n^5}} = \frac{n^2}{\sqrt{2}n^3 + 2\sqrt{2}n^5} = \frac{1}{2\sqrt{2}} \not\to 0 \,//$$

$\downarrow$ 0

g) $(x^4 + 2y^2) \cdot \left(x \cdot f'_x(x,y) + \frac{3y}{2} f'_y(x,y)\right) = 2y^2 f(x,y)$

$$(x^4 + 2y^2) \cdot \left(x \cdot \frac{y^2(2y^2 - 3x^4)}{(x^4+2y^2)^2} + \frac{3y}{2} \cdot \frac{2x^5 y}{(x^4+2y^2)^2}\right) =$$

$$= \frac{4xy^4 - \cancel{6x^5y^2} + \cancel{6x^5y^2}}{2(x^4+2y^2)^2} = \frac{4xy^4}{2(x^4+2y^2)^2} \perp$$

$$\cancel{(x^4+2y^2)} \cdot \frac{\cancel{4}xy^4}{\cancel{2}(x^4+2y^2)^{\cancel{2}}} = \frac{2xy^4}{(x^4+2y^2)} \quad // \checkmark$$

Jesu jednaki ! :)

$$2y^2 \cdot f(x,y) = 2y^2 \cdot \frac{xy^2}{x^4+2y^2} = \frac{2xy^4}{(x^4+2y^2)} \checkmark$$

(2) $F(x,y,z) = (xz + 2zy,\ y + \sin z,\ e^{\operatorname{arctg}(xy)} - 2)$

$\iint_S F \cdot dS$ $\quad S \to$ unutrašnja strana

$T = \{(x,y,z) \in \mathbb{R}^3 \mid x^2+y^2+z^2 \leq 1,\ z+y-1 \geq 0\}$

Primenićemo Gausa :)

$S = \partial T$, $T \subseteq \mathbb{R}^3$, F neprek. dif!, $F(P,Q,R)$

$$\iint_S F \cdot dS = \iiint_T F' \, dx\,dy\,dz$$

$\hookrightarrow$ Radimo sa spoljne strane površi, ali pošto je u zadatku rečeno da gledamo sa unutrašnjosti, dodaćemo minus na naš novodobijeni integral!

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$$\iint_S F\cdot ds = -\iiint_T F'\,dx\,dy\,dz = -\iiint_T z\,dx\,dy\,dz$$

$$\ulcorner F' = \frac{\partial F}{\partial x} + \frac{\partial F}{\partial y} + \frac{\partial F}{\partial z} = z + 1 - 1 = z \lrcorner$$

Na osnovu T vidimo $\rightarrow$ $\sqrt{1-x^2-y^2} \geq z \geq 1-y$

$$= -\iint_D \left(\int_{1-y}^{1-\sqrt{x^2+y^2}} z\,dz \right) = -\iint_D \left(\frac{1-x^2+y^2}{2} - \frac{1-2y+y^2}{2} \right) =$$

$$= -\iint_D \left(\frac{\cancel{1}-x^2-y^2-\cancel{1}+2y-y^2}{2} \right) = -\iint_D \frac{-x^2-2y^2+2y}{2}$$

$\ulcorner$ D je projekcija na xy-ravan $\rightarrow$ $\sqrt{1-x^2-y^2} = 1-y /^2$

$$x^2+2y^2-2y=0$$

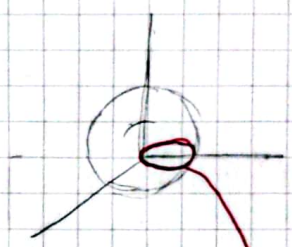
$$x^2+2\left(y^2-y+\frac{1}{4}-\frac{1}{4}\right)=0$$

$$x^2+2\left(\left(y-\frac{1}{2}\right)^2-\frac{1}{4}\right)=0$$

$$x^2+2\left(y-\frac{1}{2}\right)^2-\frac{1}{2}=0$$

$$x^2+2\left(y-\frac{1}{2}\right)^2=\frac{1}{2}$$

$$x^2+\frac{\left(y-\frac{1}{2}\right)^2}{\left(\frac{1}{\sqrt{2}}\right)^2}=\left(\frac{1}{\sqrt{2}}\right)^2$$



$x = \rho\cos\varphi$

$y-\frac{1}{2} = \frac{1}{\sqrt{2}}\rho\sin\varphi$

$r \in \left[0, \frac{1}{\sqrt{2}}\right]$ $\varphi \in [0,\pi]$

$\llcorner$ Zbog projekcije kruga $\lrcorner$

$$|J| = \begin{vmatrix} \cos\varphi & -\rho\sin\varphi \\ \frac{1}{\sqrt{2}}\sin\varphi & \frac{1}{\sqrt{2}}\rho\cos\varphi \end{vmatrix} =$$

$$= \rho\frac{1}{\sqrt{2}}\cos^2\varphi + \rho\frac{1}{\sqrt{2}}\sin^2\varphi =$$

$$= \rho\frac{1}{\sqrt{2}}\overbrace{(\sin^2\varphi+\cos^2\varphi)}^{1} =$$

$$= \boxed{\rho\frac{1}{\sqrt{2}}} \lrcorner$$

$$\frac{1}{2}\iint_D x^2+2y^2+2y\,dx\,dy = \frac{1}{2}\iint_D \left(x^2+2\left(y-\frac{1}{2}\right)^2-\frac{1}{2}\right)dx\,dy =$$

$$= \frac{1}{2}\int_0^{\pi}\int_0^{\frac{1}{\sqrt{2}}} \left(\rho^2\cos^2\varphi + \cancel{2}\cdot\frac{1}{\cancel{2}}\cdot\rho^2\sin^2\varphi - \frac{1}{2}\right)\cdot\rho\frac{1}{\sqrt{2}}$$

$$= \frac{1}{2}\int_0^{\pi}\int_0^{\frac{1}{\sqrt{2}}} \left(\rho^2-\frac{1}{2}\right)\rho\cdot\frac{1}{\sqrt{2}} = \frac{1}{2\sqrt{2}}\int_0^{\pi}\int_0^{\frac{1}{\sqrt{2}}} \rho^3-\frac{1}{2}\rho\,d\rho\,d\varphi =$$

$$= \frac{1}{2\sqrt{2}}\int_0^{\pi} \frac{\rho^4}{4}-\frac{1}{2}\cdot\frac{\rho^2}{2}\Big|_0^{\frac{1}{\sqrt{2}}} =$$

$$= \frac{1}{2\sqrt{2}}\int_0^{\pi}\left(\frac{1}{16}-\frac{1}{2}\right) = \frac{1}{2\sqrt{2}}\int_0^{\pi} -\frac{7}{8}\,d\varphi$$

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$\frac{1}{2\sqrt{2}} \cdot \frac{-7}{16} \cdot \theta \Big|_0^{\pi} = \frac{-7\pi}{32\sqrt{2}} - 0 = \frac{-7\pi}{32\sqrt{2}}$

(3.) $y' = \frac{2x+y}{2x}, \quad x>0$

a) $y=?$ $\quad y' = 1 + \frac{y}{2x}$

$y' - \frac{y}{2x} = 1$

$y = e^{-\int -\frac{1}{2x}dx}\left(c + \int 1 \cdot e^{\int -\frac{1}{2x}dx} dx\right) =$

$= e^{\frac{1}{2}\int \frac{1}{x}dx}\left(c + \int 1 \cdot e^{-\frac{1}{2}\int \frac{1}{x}dx} dx\right) =$

$= e^{\frac{1}{2}\ln x}\left(c + \int e^{-\frac{1}{2}\ln x} dx\right) =$

$= x^{\frac{1}{2}}\left(c + \int \frac{1}{\sqrt{x}} dx\right) =$

$= \sqrt{x}\left(c + 2\sqrt{x}\right) = \boxed{c\sqrt{x} + 2x}$

b) $y(1) = 1 \Rightarrow \quad 1 = c\sqrt{1} + 2 \cdot 1$

$1 = c + 2$

$c = 1 - 2 = -1 \Rightarrow \boxed{y = -\sqrt{x} + 2x}$

(4.) $f: \mathbb{R}^2 \to \mathbb{R}$

$f(x,y) = x^2 + 2y^2 - 2xy - 5y + x$

a) lok. ekstr.

$\left.\begin{array}{l} f'_x = 2x - 2y + 1 = 0 \\ f'_y = 4y - 2x - 5 = 0 \end{array}\right\} + \Rightarrow 2y - 4 = 0$

$\boxed{y = 2}$

$2x - 4 + 1 = 0$

$2x - 3 = 0$

$\boxed{x = 3/2}$

$M_1(3/2, 2)$

$f''_{xx} = 2 \quad f''_{xy} = f''_{yx} = -2 \quad f''_{yy} = 4$

$\left.\begin{array}{l} \begin{vmatrix} 2 & -2 \\ -2 & 4 \end{vmatrix} = 8 - 4 = \boxed{4} > 0 \\ A_1 > 0 \end{array}\right\} \Rightarrow$ lok. minimum

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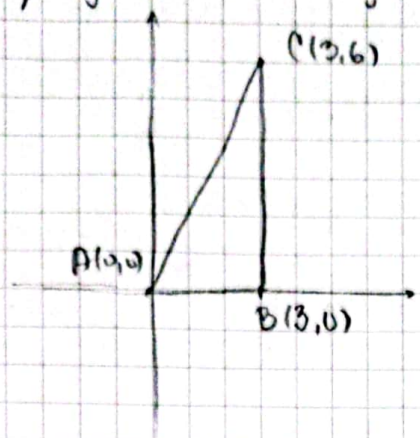
b) $\operatorname{int} D$ iz a) $M_1(\frac{3}{2}, 2)$

$D = \{(x,y) \in \mathbb{R}^2 \mid x \leq 3,\ y \geq 0,\ y \leq 2x\}$



$f(M_1) = \frac{9}{4} + 8 - 2\cdot 3 - 5\cdot 2 + \frac{3}{2} =$

$= \frac{9}{4} + \underbrace{8 - 6 - 10}_{-8} + \frac{3}{2} =$

$= \boxed{-\frac{17}{4}} = -4{,}25$

$f(A) = \boxed{0}$ $\quad f(B) = 2\cdot 9 - 5\cdot 3 = 18 - 15 = \boxed{3}$

$f(C) = 9 + 2\cdot 36 - 2\cdot 3\cdot 6 - 5\cdot 6 + 3 = \boxed{18}$

$AB: (t, 0)$, $\boxed{0 < t < 3}$

$g(t) = f(t,0) = t^2 + t \to g'(t) = 2t + 1 = 0 \Rightarrow t = -\frac{1}{2}$

$(-\frac{1}{2}, 0) \notin AB$

$BC: (3, t)$ $\quad 0 < t < 6$

$g(t) = f(3,t) = 9 + 2t^2 - 2\cdot 3\cdot t - 5t + 3 =$

$= 12 + 2t^2 - 6t - 5t = 12 + 2t^2 - 11t$

$g'(t) = 4t - 11 = 0 \Rightarrow \boxed{t = \frac{11}{4}}$ $\quad (3, \frac{11}{4}) \in BC$ ✓ $\quad f(3, \frac{11}{4}) = -3{,}125$

$AC\ (t, 2t)$ $\quad 0 < t < 3$

$g(t) = f(t, 2t) = t^2 + 2\cdot 4t^2 - 2t\cdot 2t - 5\cdot 2t + t =$

$= 9t^2 - 4t^2 + 10t + t = 5t^2 - 9t$

$g'(t) = 10t - 9 \Rightarrow \boxed{t = \frac{9}{10}} \in (0, 3)$

$(\frac{9}{10}, \frac{9}{5})$

$f(\frac{9}{10}, \frac{9}{5}) = -4{,}05$

$\{-4{,}25;\ -4{,}05;\ -3{,}125;\ 0;\ 3;\ 18\}$

min, max

c) $f(D) = [f_{min}, f_{max}] = [-4{,}25;\ 18]$

$f(M_1),\ f(C)$

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JUL 2022.

(2)

(1.) $f(x) = \begin{cases} \dfrac{x^3 - xy^2}{x^2+y^2} & , x \neq 0 \\ 0 & , x = 0 \end{cases}$

nepr.?

$$\lim_{(x,y)\to(0,0)} \frac{x^3 - xy^2}{x^2+y^2} = \begin{bmatrix} x = \rho\cos\varphi \\ y = \rho\sin\varphi \end{bmatrix} = \lim_{\rho\to 0} \frac{\rho^3\cos^3\varphi - \rho^3\sin^2\varphi\cos\varphi}{\rho^2\cos^2\varphi + \rho^2\sin^2\varphi} = \lim_{\rho\to 0} \frac{\overset{\to 0}{\rho^3}\overset{\to \text{ogr.}}{(\cos^3\varphi - \sin^2\varphi\cos\varphi)}}{\rho^2\underbrace{(\cos^2\varphi + \sin^2\varphi)}_{1}}$$

$\to 0$ ✓ Nepr. je na $x=0$

dif. u svakom pravcu?

$v = (\alpha, \beta) \neq (0,0)$

$$\lim_{h\to 0} \frac{f(\alpha h, \beta h) - f(0,0)}{h} = \lim_{h\to 0} \frac{\frac{\alpha^3 h^3 - \alpha h \cdot \beta^2 \cdot h^2}{h^2(\alpha^2+\beta^2)} - 0}{h} =$$

$$= \lim_{h\to 0} \frac{h^3(\alpha^3 - \alpha\beta^2)}{h^3(\alpha^2+\beta^2)} = \left[\frac{\alpha^3 - \alpha\beta^2}{\alpha^2+\beta^2}\right]$$ Jeste dif. u svim pravcima

$$f'_x = \lim_{h\to 0} \frac{f(h,0) - f(0,0)}{h} = \lim_{h\to 0} \frac{\frac{h^3}{h^2}}{\frac{h}{1}} = \boxed{1}$$

$$f'_y = \lim_{h\to 0} \frac{f(0,h) - f(0,0)}{h} = \lim_{h\to 0} \frac{0}{h} = \boxed{0}$$

$$L = \lim_{(h,k)\to(0,0)} \frac{f(h,k) - f(0,0) - h f'_x(0,0) - k f'_y(0,0)}{\sqrt{h^2+k^2}} = \lim \frac{\frac{h^3 - hk^2}{h^2+k^2} - h}{\sqrt{h^2+k^2}}$$

$$= \lim \frac{\cancel{h^3} - hk^2 - \cancel{h^3} - hk^2}{(h^2+k^2)\sqrt{h^2+k^2}} = \lim \frac{-2hk^2}{\sqrt{(h^2+k^2)^3}}$$

$a = \left(\frac{1}{n}, \frac{1}{n}\right) \to (0,0)$

$$\frac{-2\cdot\frac{1}{n}\cdot\frac{1}{n^2}}{\sqrt{\left(\frac{1}{n^2}+\frac{1}{n^2}\right)^3}} = \frac{-2\cdot\frac{1}{n^3}}{\sqrt{\frac{8}{n^6}}} = \frac{-2\cdot\frac{1}{n^3}}{\frac{2\sqrt{2}}{n^3}} = \frac{-1}{\sqrt{2}} \not\to 0$$

Nije dif!!

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② Lok. ekstr. $f(x,y) = \sin(x)\sin(y)\sin(x+y)$

$f'_x = \cos x \cdot \sin y \cdot \sin(x+y) + \sin(x)\cdot\sin(y)\cdot\cos(x+y)\cdot 1$

$f'_y = \cos y \cdot \sin x \cdot \sin(x+y) + \sin x \sin y \cos(x+y)\cdot 1$

$\cos x \sin y \sin(x+y) - \cos y \sin x \sin(x+y) =$

$= \sin(x+y)\cdot(\cos x \sin y - \cos y \sin x) =$

$= -\sin(x+y)\cdot\sin(x-y)$

$\sin(x+y) = 0$

$x+y = 0 \Rightarrow x = -y$

$x+y = \pi$

$\sin(-2y) = 0$

$-2y = 0 \quad \vee \quad -2y = \pi$

$y = 0 \quad \vee \quad y = -\pi/2$

$\sin(\pi - 2y) = 0 \Rightarrow \sin(-2y) = 0$

$\Rightarrow y = 0 \quad \vee \quad y = -\pi/2$

$M_1(0,0)$

$M_2(\pi,0)$

$M_3(\pi/2, \pi/2)$

$M_4(\pi/2, -\pi/2)$

Hiši me sve da radim preveše zadataka :) !

$f''_{xx} = 2\sin y \cos(2x+y)$

$f''_{yy} = 2\sin x \cos(x+2y)$

$f''_{xy} = f''_{yx} = \sin(2x+2y)$

$f'' \quad 2\sin y \cos(2x+y) = 0$

$\sin y = 0$

$y = 0 \quad \Rightarrow \quad \cos(2x) = 0$

$y = \pi \qquad x = \frac{\pi}{2} \quad \vee \quad x = \frac{3\pi}{2}$

$\cos(2x+\pi) = 0$

$2x + \pi = \frac{\pi}{2} \quad \vee \quad 2x + \pi = \frac{3\pi}{2}$

$2x = -\frac{\pi}{2} \qquad 2x = \frac{\pi}{2}$

$x = -\frac{\pi}{4} \qquad x = \frac{\pi}{4}$

$M_5(\frac{\pi}{2}, 0)$

$M_6(\frac{3\pi}{2}, 0)$

$M_7(-\frac{\pi}{4}, \pi)$

$M_8(\frac{\pi}{4}, \pi)$

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Скенирано помоћу ЦамСцаннер-а

(3) $\int_D e^{-x^2-y^2} \sin(x^2+y^2)\,dx\,dy \qquad D=\{(x,y) \mid \underbrace{x^2+y^2\le 1}_{\text{krug}}\}$

$C(0,0)$

$r=1$

$x=\rho\cos\varphi \qquad \rho\in[0,1]$

$y=\rho\sin\varphi \qquad \varphi\in[0,2\pi)$

$J=\rho$

$$\int_0^1\int_0^{2\pi} \rho e^{-\rho^2\cos^2\varphi-\rho^2\sin^2\varphi}\cdot\sin(\rho^2\cos^2\varphi+\rho^2\sin^2\varphi)\,d\varphi\,d\rho$$

$$=\int_0^1\int_0^{2\pi}\rho e^{-\rho^2}\cdot\sin\rho^2\cdot d\varphi\,d\rho=$$

$$=\int_0^1 \rho e^{-\rho^2}\cdot\sin\rho^2\,\varphi\Big|_0^{2\pi} = 2\pi\int_0^1 e^{-\rho^2}\cdot\sin\rho^2 = \begin{bmatrix}\rho^2=t\\ 2\rho\,d\rho=dt\\ \rho\,d\rho=\frac{dt}{2}\end{bmatrix}=$$

$$=\pi\int_0^1 e^{-t}\cdot\sin t\,dt = \begin{bmatrix}u=\sin t & dv=e^{-t}dt\\ du=\cos t & v=-e^{-t}\end{bmatrix}$$

$$I=-\frac{\sin t}{e^t}\Big|_0^1+\int_0^1\frac{\cos t}{e^t}dt=-\frac{\sin 1}{e}-\frac{\cos t}{e^t}\Big|_0^1-\underbrace{\int\frac{\sin t}{e^t}dt}_{I}=$$

$$=-\frac{\sin 1}{e}-\frac{\cos 1}{e}+1-I=I$$

$$2I=\frac{e-\sin 1-\cos 1}{e}$$

$$I=\frac{e-\sin 1-\cos 1}{2e}$$

$$\pi I=\pi\cdot\frac{e-\sin 1-\cos 1}{2e}$$

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Скенирано помоћу ЦамСцаннер-а

(4.) $xy' + 4y = x^4y^2 \,/\, \cdot xy^2 \quad y(2) = -1$

$$\frac{y'}{y^2} + y \cdot \frac{4}{xy} = x^3 \qquad z = \frac{1}{y} \qquad z' = \frac{-y'}{y^2}$$

$$z' - 4z \cdot \frac{1}{x} = -x^3$$

$$z = e^{-\int -\frac{4}{x}dx}\left(c + \int -x^3 \cdot e^{\int -\frac{4}{x}dx}\, dx\right) =$$

$$= e^{4\ln x}\left(c + \int -x^3 \cdot e^{-4\ln x}\, dx\right) =$$

$$= x^4\left(c + \int -x^{\cancel{3}} \cdot \frac{1}{x^{\cancel{4}}}\, dx\right) =$$

$$= x^4(c - \ln x)$$

$$\frac{1}{y} = x^4(c - \ln x)$$

$$y = \frac{1}{x^4(c - \ln x)} \qquad \to y = \frac{1}{x^4\left(\ln\frac{2}{x} - \frac{1}{16}\right)} =$$

$$-1 = \frac{1}{16(c - \ln 2)} \Rightarrow -16(c - \ln 2) = 1$$

$$c - \ln 2 = \frac{-1}{16}$$

$$c = \ln 2 - \frac{1}{16}$$

Скенирано помоћу ЦамСцаннер-а

① JUN 2022.

$$f(x,y) = \begin{cases} \dfrac{xy^5 + yx^3}{x^2+y^4}, & (x,y) \neq (0,0) \\ a, & (x,y) = (0,0) \end{cases}$$

a) $$\lim_{(x,y)\to(0,0)} \frac{xy\cancel{(y^4+x^2)}}{\cancel{x^2+y^4}} = \lim_{(x,y)\to(0,0)} xy = \begin{bmatrix} x = \rho\cos\varphi \\ y = \rho\sin\varphi \end{bmatrix} =$$

$$= \lim_{\rho\to 0} \rho^2 \cos\varphi \sin\varphi = 0 \quad \boxed{a=0}$$

b) $$f'_x = \frac{(y^5+3x^2y)(x^2+y^4) - (xy^5+yx^3)(2x)}{(x^2+y^4)^2}$$

$$= \frac{x^2y^5 + \cancel{2}x^4y + y^9 + \cancel{2}x^2y^5 - \cancel{2x^2y^5} - 2x^4y}{(x^2+y^4)^2} = \frac{y^9 + 2x^2y^5 + x^4y}{(x^2+y^4)^2} =$$

$$= \frac{y^9 + 2x^2y^5 + x^4y}{x^4+2x^2y^4+y^8} = \frac{y\cancel{(y^8+2x^2y^4+x^4)}}{\cancel{x^4+2x^2y^4+y^8}} = y$$

$$f'_y = \frac{(5xy^4+x^3)(x^2+y^4) - (xy^5+yx^3)(4y^3)}{x^4+2x^2y^4+y^8}$$

$$= \frac{5x^3y^4 + \cancel{5}xy^8 + x^5 + x^3y^4 - \cancel{4xy^8} - \cancel{4y^4x^3}}{x^4+2x^2y^4+y^8} = \frac{2x^3y^4 + xy^8 + x^5}{x^4+2x^2y^4+y^8} =$$

$$= \frac{x\cancel{(x^4+2x^2y^4+y^8)}}{\cancel{x^4+2x^2y^4+y^8}} = x$$

$$f'_x(0,0) = \lim_{h\to 0} \frac{f(h,0)-f(0,0)}{h} = \frac{0-0}{h} = 0$$

$$L = \frac{f(h,k) - f(0,0) - hf'_x(0,0) - kf'_y(0,0)}{\sqrt{h^2+k^2}}$$

$$= \frac{\frac{hk^5+kh^3}{h^2+k^4} - 0}{\sqrt{h^2+k^2}} =$$

$$= \frac{\frac{hk(k^4+h^2)}{h^2+k^4}}{\sqrt{h^2+k^2}} = \frac{hk}{\sqrt{h^2+k^2}}$$

$$f'_y(0,0) = \lim_{k\to 0} \frac{f(0,k)-f(0,0)}{k} = \lim_{k\to 0} \frac{0-0}{0} = 0$$

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$$\begin{bmatrix} x = \rho\cos\varphi \\ y = \rho\sin\varphi \end{bmatrix} = \lim_{\rho\to 0} \frac{\rho^2\cos\varphi\sin\varphi}{\rho\sqrt{\cos^2\varphi+\sin^2\varphi}} = 0$$ Jesre diff!

(2.) $f(x,y) = x^2+y^2-4x+2y+4xy+9$

min, max:?

$f(M_1) = 23,56$ $\quad f(M_4) = 13,61$

$f(M_2) = 29$ $\quad f(A) = 17$

$f(M_3) = 32,89$ $\quad f(B) = 49 \to$ max!

$f(C) = 9 \to$ min!

1° int D

$f'_x = 2x-4+4y = 0$

$f'_y = 2y+2+4x = 0 \quad /:2$

$y+1+2x = 0$

$3y-5=0$

$y = 5/3 \quad 2x = -5/3 - 1 = -8/3$

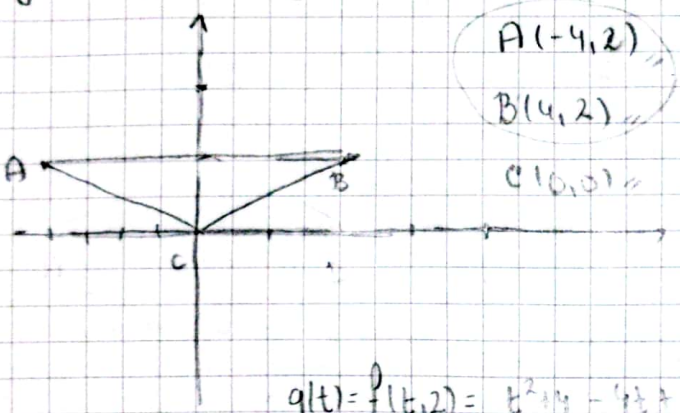
$x = -4/3$

$M_1(-4/3, 5/3)$

2° ∂D

$y=2x,\ y=-2x,\ y=2$

3°



$A(-4,2)$

$B(4,2)$

$C(0,0)$

$AB \in (t,2) \quad t\in[-4,4]$

$AC \in (-2t,t) \quad t\in[0,2]$

$BC \in (2t,t) \quad t\in[0,2]$

$g(t) = f(t,2) = t^2+4-4t+3+8t+9$

$= t^2+4t+28$

$g'(t) = 2t+4 \to \boxed{t=-2}$ $\quad M_2(2,2) \in AB$ ✓

$g(t) = f(-2t,t) = 4t^2+t^2+8t+2t-8t^2+9 =$

$= -3t^2+10t$ $\quad M_3\left(-\frac{10}{3}, \frac{5}{6}\right) \in AC$ ✓

$g'(t) = -6t+10 \to t = \frac{5}{3}$

$g(t) = f(2t,t) = 4t^2+t^2-8t+2t+8t^2+9 =$

$= 13t^2-6t \quad g'(t) = 26t-6 = 0 \quad t = \frac{6}{26} = \frac{3}{13}$

$M_4 \in \left(\frac{6}{13}, \frac{3}{13}\right) \in BC$ ✓

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(4.) $y''' - 2y'' + y' - 2y = e^x + e^{3x}$???

(3.) $f(x,y) = \left(e^{\sin x + \cos x} - y^3 + x^3 y,\ \frac{x^4}{4} + x^3 + \log(y^3 + y^2)\right)$

a, b - poluosc poz. elipse $\parallel x, y$, centrirana u $(0,0)$

$\oint_C F \cdot dr$ $\quad P(x,y) = e^{\sin x + \cos x} - y^3 + x^3 y$

$Q(x,y) = \frac{x^4}{4} + x^3 + \log(y^3 + y^2)$

$P'_y = -3y^2 + x^3$

$Q'_x = x^3 + 3x^2$

$J = abr$

$\iint_D (\cancel{x^3} - 3x^2 + 3y^2 - \cancel{x^3})\, dx\, dy =$ $\quad \rho \in [0,1]$ $\quad \theta \in [0, 2\pi]$

$= \iint_D (3y^2 - 3x^2)\, dx\, dy = \begin{bmatrix} x = a\rho\cos\theta \\ y = b\rho\sin\theta \\ J = abr \end{bmatrix}$

$= \iint_D 3(y^2 - x^2)\, dx\, dy = \iint_D 3(b^2\rho^2\sin^2\theta - a^2\rho^2\cos^2\theta) \cdot abr$

$= \frac{-r^2((b^2+a^2)\cos\theta\sin\theta) + (a^2-b^2)\theta)}{2}\Big|_0^{2\pi} = \frac{2\pi(a^2-b^2)}{2}$

$-r^2\pi(a^2-b^2)$

$\int_0^1 -r^2\pi(a^2-b^2) = -\frac{r^3}{3}\pi(a^2-b^2)\Big|_0^1 =$

$= -\frac{\pi}{3}(a^2-b^2)$

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Скенирано помоћу ЦамСцаннер-а

SEP 1 2022

(1.) $f: \mathbb{R}^2 \to \mathbb{R}^2$

$$f(x,y) = \begin{cases} y-x^2, & y \geq x^2 \\ 0, & -x^2 \leq y < x^2 \\ y+x^2, & y < -x^2 \end{cases}$$

a) $\lim\limits_{y \to x^2} y - x^2 = 0$ //

$\lim\limits_{y \to -x^2} y + x^2 = 0$ //

b) dif $f(0,0) \in \mathbb{R}^2$?

$$f'_x = \lim_{h \to 0^+} \frac{f(h,0) - f(0,0)}{h} = \lim_{h \to 0^+} \frac{-h^2}{h} = 0 \text{ //} = \lim_{h \to 0^-} \text{ //}$$

$$f'_y = \lim_{h \to 0^+} \frac{f(0,h) - f(0,0)}{h} = \lim_{h \to 0^-} \frac{h}{h} = 1 \text{ //} = \lim_{h \to 0^-} \text{ //}$$

$$\alpha = \lim_{(h,k) \to (0,0)} \frac{f(h,k) - f(0,0) - h f'_x(0,0) - k f'_y(0,0)}{\sqrt{h^2+k^2}} = \lim_{(h,k) \to (0,0)} \frac{k - h^2 - k}{(h^2+k^2)\sqrt{h^2+k^2}} = \left[\begin{array}{l} x = \rho\cos\varphi \\ y = \rho\sin\varphi \end{array}\right] = \lim_{\rho \to 0} \frac{-\rho^2 \cos^2\varphi}{\rho} = 0 \text{ //}$$

$\alpha = 0 \Rightarrow$ f-ja je dif.

(2.) $S = \{(x,y) \in \mathbb{R}^2 \mid x^2 = 4y\}$ $(3,0)$ //

$y = \frac{x^2}{4}$

$d(P,S) = \sqrt{(x-3)^2 + y^2}$

$f(x,y) = (x-3)^2 + y^2$

$\varphi(x,y) = x^2 - 4y$

$F(x,y,\lambda) = (x-3)^2 + y^2 + \lambda(x^2 - 4y)$

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$F'_x = 2x - 6 + 2x\lambda = 0$

$F'_y = 2y - 4\lambda = 0 \longrightarrow y = 2\lambda$

$F'_\lambda = x^2 - 4y = 0 \longrightarrow x^2 - 8\lambda = 0$

$\lambda = \frac{x^2}{8}$

$B = (2,1)$

$2x - 6 + \frac{x^3}{4} = 0 \;/ : 2$

$|AB| = \sqrt{1+1} = \boxed{\sqrt{2}}$

$x - 3 + \frac{x^3}{8} = 0 \;/ 8$

$x_1 = 2 \Rightarrow y = \frac{x^2}{4} = \boxed{1}$

$x^3 + 8x - 24 = 0$

$(x^3 + 8x - 24) : (x-2) = x^2 + 2x + 12 = 0$

$x_{2,3} = \frac{-2 \pm \sqrt{4-12}}{2} = \nexists$

I

3. $a > b > 0$

$\int_C (y^2+z^2)dx + (x^2+z^2)dy + (x^2+y^2)dz$

$C = \{ x^2+y^2+z^2 = 2ax \,;\, z > 0 \,;\, x^2+y^2 = 2bx \mid (x,y,z) \in \mathbb{R}^3 \}$

pozitivno sa $(b,0,2022)$

$F(x,y,z) = (\underbrace{y^2+z^2}_{P}, \underbrace{x^2+z^2}_{Q}, \underbrace{x^2+y^2}_{R})$

$z^2 = 2x(a-b)$

$z = \sqrt{2x(a-b)}$

$P'_z = 2z \quad Q'_x = 2x \quad R'_x = 2x$

$P'_y = 2y \quad Q'_z = 2z \quad R'_y = 2y$

S. o.n $= (1,1,1)$

$r'_x \times r'_y = (-z'_x, -z'_y, 1) = \left(\underbrace{-\sqrt{\frac{a-b}{x}}}_{<0}, 0, 1\right)$

$x > 0$

Loš smjer usaglašene orijent!

$\int_C F \cdot dr = \iint_S (R'_y - Q'_z, P'_z - R'_x, Q'_x - P'_y)\, d\vec{S} =$

$= \iint_S (2(y-z), 2(z-x), 2(x-y)) \cdot d\vec{S}$

$z'_x = \sqrt{2(a-b)} \cdot \frac{1}{2\sqrt{x}} = \sqrt{\frac{a-b}{x}}$

...

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II $z = \sqrt{2x(a-b)}$

$x = b + a\cos\varphi$

$y = b\sin\varphi$

$z = \sqrt{2(b+a\cos\varphi)(a-b)}$ $\quad \Big[\, ab - b^2 + a^2\cos\varphi - ab\cos\varphi =$

$= ab(1-\cos\varphi) + a^2\cos\varphi - b^2 \,\Big]$

$z = \sqrt{2(ab - b^2 + a^2\cos\varphi - ab\cos\varphi)}$

$r(\varphi) = \left(b + a\cos\varphi,\ b\sin\varphi,\ +\underbrace{\sqrt{2(ab-b^2+a^2\cos\varphi - ab\cos\varphi)}}_{u}\right)$

$r'(\varphi) = \left(-a\sin\varphi,\ b\cos\varphi,\ \frac{1}{u}\cdot(-a^2\sin\varphi + ab\sin\varphi)\right)$

$F(r(\varphi)) = (\ldots)$ $\qquad \varphi \in [0, 2\pi]$

$\iint F \cdot dr = \iint F(r(\varphi)) \cdot r'(\varphi)$

4. $y' = x^2 \frac{\ln(x) - \ln(y)}{y^2} = x^2 \frac{\ln(x/y)}{y^2}$

$y' = \frac{1}{\frac{y^2}{x^2}}\left(-\ln\left(\frac{y}{x}\right)\right)$ $\quad z = \frac{y}{x} \quad y = zx$

$z' = \frac{y'}{x} - \frac{y}{x^2}$ $\qquad \frac{y'}{x} = z' + \frac{y}{x^2} \,/\cdot x$

$y' = z'x + \frac{zx}{x}$

$y' = z'x + z$

$z'x + z = -\frac{\ln z}{z^2}$

$z' = -\frac{\ln z - z^0}{x}$

$\frac{1}{x} = \frac{z' z^2}{-\ln z - z^3}$

$\int \frac{z^2\, dz}{-\ln z + z^3} = \ln x + C$

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II $y' = \frac{x^2}{y^2} - \ln\left(\frac{x}{y}\right)$

$z = \frac{x}{y}$ $\quad z' = \frac{y'}{y} \Rightarrow y' = z'y = \cancel{z'z x}\ z' \cdot \frac{x}{z}$

$\cancel{y = xz}\ y = \frac{x}{z}$ $\quad z' \cdot \frac{x}{z} = z^2 - \ln(z) \ / \cdot \frac{z}{x}$

$z' = \frac{z^3}{x} - \frac{z \ln(z)}{x}$??

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