Eigenface Algorithm for Facial Recognition

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April 18, 2017

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1 Introduction

Eigenfaces is an algorithm for performing facial recognition. In order to achieve this, we need a dataset of images of faces all the same size as well as in approximately the same position in the image.

The algorithm provides a simple and efficient way for performing facial recognition. The images are compressed via dimensionality reduction, and it is the reduced image space in which the faces are represented.

2 Algorithm

1. Assume we have a dataset D, where $|D| = M$ and $I_i \forall i \in \{1, \ldots, M\}$ are the individual images in the set. Further, assume each image I_i is of dimensionality $N\mathtt{x}M$.

2. For each image I_i, reshape it into an image vector Γ_i of dimensionality $(N\mathtt{x}M)\mathtt{x}1$.

3. Calculate the mean face, ψ: $\psi = \frac{1}{M}\sum_{i=1}^{M}\Gamma_i$.

4. Calculate the mean-normalized face, for each image: $\phi_i = \Gamma_i - \psi$.

5. Form a matrix A of the mean-normalized faces, where each column in the matrix is a face: $A = [\phi_1\phi_2 \ldots \phi_M]$. The dimensionality of A will be $(N\mathtt{x}M)\mathtt{x}M$.

6. Instead of calculate the eigenvectors and eigenvalues of covariance matrix AA^T, which is $(N\mathtt{x}M)\mathtt{x}(N\mathtt{x}M)$, we instead calculate them for the matrix A^TA, since this has dimensionality $M\mathtt{x}M$. This is for speedup reasons. It should be noted that the M eigenvalues of A^TA (along with their associated eigenvectors) correspond to the M largest eigenvalues of AA^T (along with their associated eigenvectors).

7. We then reduce the dimensionality of eigenvector matrix V calculate above from $M\mathtt{x}M$ to $M\mathtt{x}K$, where K is the number of principal components we want to retain. These K eigenvectors correspond to the largest K eigenvectors of the matrix.

8. We then calculate the projection matrix u: $u = AV$.

9. Lastly, we find the eigenface subspace Ω: $\Omega = u^TA$.

3 Projecting Points on Eigenface Subspace

1. Consider the test image I. Reshape it into a vector Γ of dimensionality $(N\mathtt{x}M)\mathtt{x}1$.

2. Normalize the vector by subtracting the mean face calculated above to yield $\phi = \Gamma - \psi$.

3. Project the face onto the eigenface subspace by calculating: $\hat{\phi} = u^T \phi$.

4 Predicting Points

Given an image projected onto the eigenface subspace - $\hat{\phi}$, the class prediction can be found using the following formula:

$$prediction = \arg\min_i (\Omega_i - \hat{\phi}_i)^2 \tag{1}$$