Model predictive control for autonomous unmanned helicopters

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Abstract

Purpose – The purpose of this paper is to investigate the feasibility of controlling a small-scale helicopter by using the model predictive control (MPC) method.

Design/methodology/approach – The MPC control synthesis is employed by considering five linear models representing the flight of a small-scale helicopter from hover to high-speed cruise. The internal model principle is employed for the trajectory tracking design.

Findings – It is found that the MPC handles well the transition problems between the models, yields satisfactory tracking control performance and produces a suitable control signal. The performance of the tracking control of the helicopter is considerably influenced by the parameter selection in the states and inputs weighting matrices of the MPC. Simulation results also showed that faster dynamics, coupling problems, input and output constraints and changing linearized multi-inputs multi-outputs dynamics models in the small-scale helicopter can be handled simultaneously by the MPC controller.

Research limitations/implications – The present study is limited for the application of MPC for the control of small-scale helicopters with non-aggressive maneuvers.

Practical implications – The result can be extended to design a full envelope controller for an autonomous small-scale helicopter without the need to resort to a conventional gain scheduling technique.

Originality/value – Helicopter control system designs using MPC with a single either linear or non-linear model have been studied and reported in numerous literatures. The main contribution of the paper is in the application of MPC to handle the control problems of a small-scale helicopter defined as a mathematical model with several different modes during a flight mission.

Keywords Rotorcraft-based UAV, Autonomous control, Small scale helicopter, Model predictive control (MPC), Multiple models, Helicopters, Controllers

Paper type Research paper

Introduction

Autonomous flight control system for small-scale helicopters presents significant challenges due to their highly non-linear, unstable nature of the vehicles and multi-input-multi-output (MIMO) characteristics. Helicopter is a non-linear complex system comprising many modes in its flight trajectories where each mode has different characteristics. The presence of strong cross-coupling and non-linearities makes helicopter difficult to control. Small-scale helicopters, especially, are known to have more reactive response and faster dynamic characteristics compared with the full scale helicopters (Mettler, 2003). This leads to a challenging control design problem as the controller must stabilize the flight and provide good tracking characteristics simultaneously.

A conventional approach to the control synthesis is to linearize the non-linear model in order to have a simple linear model for each mode, then the well-known proportional-integral-derivative (PID) controller or its variant is implemented by trial and error-tuning approach until satisfactory responses are met. This approach has been adapted from long time ago until now and accepted as a proven and simple method to solve the PID control system design in most application areas. The approach does not consider the coupling effects that can degrade and even lead to unstable response. Recently, a MIMO PID controller has been developed and implemented for stabilizing a small-scale helicopter in a hover condition

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(Pradana *et al.*, 2011). In this paper, the parameters of the PID controller are found by using the robust H_∞ – integral back-stepping technique, this method was developed by Joelianto (Joelianto and Tommy, 2003; Joelianto *et al.*, 2008) based on state space representation of the PID controller to exploit the advantage of the state space mathematical of MIMO model of the small-scale helicopter.

Recently, many modern methods have been developed and proposed to reduce drawbacks in the PID controller. One of the promising modern control methods is known as model predictive control (MPC), for example (Maciejowsky, 2002; Camacho and Bordon, 2004). MPC has gained popularity two decades ago in the process control applications with many different names depending on the mathematical model representation used in the control design. The application of MPC in the process control has grown very fast and has been available commercially for advanced process control technology (Takatsu *et al.*, 1998). MPC methodology is appealing to the practitioners as constraints commonly encountered in applications can be explicitly accounted for in the controller. Moreover, it has become the accepted standard to handle complex constrained multivariable problems in the process industries (Qin and Badgwell, 1997). Starting from the current state, the MPC algorithm solves an optimal control problem over finite horizon at each sampling instance. By means of the receding horizon strategy, only the first element of the optimal control sequence is applied to the plant. The optimization process is then repeated at the next sampling time based on new measurements. In its simplest form, MPC employs a linear dynamic model of the plant, linear constraints of inputs, outputs, and state, resulting in a linear program (Bemporad *et al.*, 2002) or quadratic program where efficient and well-tested solvers, such as active set (Fletcher, 1987) or interior point methods (Wright, 1997), are available.

The application of MPC for controlling the up-right position of a rotary double inverted pendulum was successfully demonstrated by Lu *et al.* (2007). The work showed that MPC can handle system with fast dynamics characteristics as well as the slow process dynamics. This opens potential application of MPC beyond advanced process control domain. MPC offers many advantages as the controller deals naturally with multivariable systems, takes into account actuator limitations, and allows operation closer to constraints. It can be used to control great variety of systems (simple to complex, long-time delay and non-minimum phase). The treatment of constraints is theoretically simple so that it can be made in the design step (Qin and Badgwell, 1997). Moreover, the MPC is very practical when the future reference is known a priori such as in robotics and mechatronics. Recently, the potential application of MPC in other domains has been intensively investigated. The application of MPC to control a helicopter was studied by several researchers including Wan and Bogdanov (2001), Singh and Fuller (2001), Kim *et al.* (2002), Castillo *et al.* (2009) and Palunko and Bogdan (2008). The papers used the same fundamental of MPC but it is applied with different concepts and techniques to handle the problems in small-scale helicopters.

This paper is concerned with an implementation of MPC for a small-scale helicopter with multiple mathematical models that represent several modes contained in the helicopter's flying envelope. The main contribution of the paper compared to the previously published papers is in the application of MPC to handle the control problems of a small-scale helicopter defined as a mathematical model with several different modes during a flight mission. The multiple models under consideration were obtained by linearizing the first principle non-linear model in each trim condition of helicopter's flight mode. The models represent the dynamic behavior of a small-scale helicopter in hover and a series of cruise with different flight velocities. In the control design, the design parameters of the MPC are selected appropriately for each model. The performance the MPC in controlling the multiple models is then demonstrated and evaluated by simulation using MATLAB® by combining the models in a certain flight scenario. The conference version of the paper has appeared in Joelianto *et.al.* (2010).

Mathematical models of helicopter

The small-scale helicopter non-linear dynamics model has been developed and reported in numerous literatures. The basic linearized equations of motion for a model helicopter dynamics is derived from the non-linear Newton-Euler equation for a rigid body that has six degrees of freedom to move in space. The equations of motion of the model helicopter are expressed as follows for the fuselage and coupled rotor-fly-bar dynamics (Gavrilets, 2003):

Fuselage

$$\begin{aligned}
&X_{MR}+X_{TR}+X_{fus}=m(\dot{u}-rv+qw)+mg\sin\theta\\
&Y_{MR}+Y_{TR}+Y_{fus}=m(\dot{v}-pw+ru)+mg\sin\varphi\cos\theta\\
&Z_{MR}+Z_{TR}+Z_{fus}=m(-\dot{w}-qu+pv)-mg\cos\varphi\cos\theta\\
&L_{MR}+L_{TR}+L_{fus}=I_{xx}\dot{p}-(I_{yy}-I_{zz})qr\\
&M_{MR}+M_{TR}+M_{fus}=I_{yy}\dot{q}-(I_{zz}-I_{xx})pr\\
&N_{MR}+N_{TR}+N_{fus}=I_{zz}\dot{r}-(I_{xx}-I_{yy})pq\\
&\dot{\varphi}=p+(q\sin\varphi+r\cos\varphi)\tan\theta \quad \dot{\theta}=q\cos\varphi-r\sin\varphi\\
&\dot{\psi}=(q\sin\varphi+r\cos\varphi)\sec\theta
\end{aligned} \tag{1a}$$

Rotor-fly-bar

$$\begin{aligned}
\tau_e\dot{a}_{1s}&=-a_{1s}+\frac{\partial a_{1s}}{\partial\mu_{MR}}\frac{\mu_a}{(\Omega R)_{MR}}+\frac{\partial a_{1s}}{\partial\mu_{MR}}\frac{w_a}{(\Omega R)_{MR}}+\cdots\\
&\quad-\tau_e q+A_{\delta_{Long}}\delta_{Long}\\
\tau_e\dot{b}_{1s}&=-b_{1s}-\frac{\partial b_{1s}}{\partial\mu_{MR}}\frac{v_a}{(\Omega R)_{MR}}-\tau_e p+B_{\delta_{Long}}\delta_{Long}
\end{aligned} \tag{1b}$$

Forces in *x, y, z* directions are denoted as *X, Y, Z*, respectively, and moments for roll, pitch and yaw are designated as *L, M, N*, respectively. The subscripts *MR, TR, fus* denote contribution from main rotor, tail rotor, and fuselage, respectively. The above set of equations can be linearized around trim conditions to obtain a series of linear models for the purpose of control design.

The linearization process was determined by first selecting the trim conditions in the non-linear model that corresponds to the hover and a series of cruise with different speeds. The small perturbation is applied around trim conditions and aerodynamic forces are expanded in Taylor's series leading to stability derivative model structure given as the following matrix equation:

$$
\begin{bmatrix} \dot{u} \\ \dot{w} \\ \dot{q} \\ \dot{\theta} \\ \dot{a}_{ls} \\ \dot{v} \\ \dot{p} \\ \dot{r} \\ \dot{\varphi} \\ \dot{b}_{ls} \end{bmatrix}
=
\begin{bmatrix}
X_u & X_w & 0 & X_\theta & X_{a_{ls}} & 0 & 0 & 0 & 0 & X_{b_{ls}} \\
Z_u & Z_w & Z_q & Z_\theta & Z_{a_{ls}} & Z_v & 0 & 0 & Z_\varphi & Z_{b_{ls}} \\
M_u & M_w & M_q & 0 & M_{a_{ls}} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \Theta_q & 0 & 0 & 0 & 0 & \Theta_r & 0 & 0 \\
\frac{1}{\tau_e}A_u & 0 & \frac{1}{\tau_e}A_q & 0 & \frac{1}{\tau_e}A_{a_{ls}} & 0 & 0 & 0 & 0 & 0 \\
Y_u & Y_w & 0 & Y_\theta & 0 & Y_v & Y_p & Y_r & Y_\varphi & Y_{b_{ls}} \\
L_u & L_w & L_q & 0 & 0 & L_v & 0 & L_r & 0 & L_{b_{ls}} \\
N_u & N_w & N_q & 0 & 0 & N_v & 0 & N_r & 0 & 0 \\
0 & 0 & \Phi_q & 0 & 0 & 0 & \Phi_p & \Phi_r & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{1}{\tau_e}B_v & \frac{1}{\tau_e}B_p & 0 & 0 & \frac{1}{\tau_e}B_{b_{ls}}
\end{bmatrix}
$$

$$
\times \begin{bmatrix} u \\ w \\ q \\ \theta \\ a_{ls} \\ v \\ p \\ r \\ \varphi \\ b_{ls} \end{bmatrix}
+
\begin{bmatrix}
X_{\delta_{Coll}} & 0 & 0 & 0 \\
Z_{\delta_{Coll}} & 0 & 0 & 0 \\
M_{\delta_{Coll}} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\frac{1}{\tau_e}A_{\delta_{Coll}} & \frac{1}{\tau_e}A_{\delta_{Long}} & 0 & 0 \\
Y_{\delta_{Coll}} & 0 & Y_{\delta_{Ped}} & 0 \\
L_{\delta_{Coll}} & 0 & L_{\delta_{Ped}} & 0 \\
N_{\delta_{Coll}} & 0 & N_{\delta_{Ped}} & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{1}{\tau_e}B_{\delta_{Lat}}
\end{bmatrix}
$$

$$
\times \begin{bmatrix} \delta_{Coll} \\ \delta_{Long} \\ \delta_{Ped} \\ \delta_{Lat} \end{bmatrix} \tag{2a}
$$

$$
\begin{bmatrix} u \\ w \\ q \end{bmatrix}
=
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix} u \\ w \\ q \\ \theta \\ a_{1s} \\ v \\ p \\ r \\ \varphi \\ b_{1s} \end{bmatrix} \tag{2b}
$$

The linear model is then represented in a matrix form known as the state-space model representation:

$$
\begin{aligned}
\dot{x}(t) &= Ax(t) + Bu(t) \\
y(t) &= Cx(t)
\end{aligned} \tag{3}
$$

where x denotes the state vector in R^n and u is the input vector in R^m. The system matrix A contains the stability derivatives with dimension $n \times n$; the input matrix B contains the input derivatives with dimension $n \times m$. The matrix C represents the output matrix with the appropriate dimension depends on the number of helicopter's outputs and states. The model parameters are identified for hover and a number of cruise flight conditions which will then be used to elaborate the performance of MPC in an unmanned aerial vehicle (UAV) application.

MPC design

Constrained MPC can be formulated in a linear, discrete-time, state-space model of the plant, in the following expression:

$$
\begin{aligned}
x(k+1) &= A_d x(k) + B_d u(k) \\
y(k) &= C_d x(k) \quad z(k) = C_z x(k)
\end{aligned} \tag{4}
$$

where:

$$
A_d = e^{AT_s} \quad B_d = \int_0^{T_s} e^{A\sigma} B d\sigma \quad C_d = C
$$

where $y(k)$, $z(k)$, $u(k)$, and $x(k)$ represent system outputs, controlled outputs, inputs, and states, respectively, for each mode at time instants $t = kT_s,\ k = 0, 1, 2, \ldots$ where T_s denotes the sampling time interval used to discretize the state-space model equation (3). In this paper, it is further assumed that the system outputs equal to controlled outputs $C_z = C_d$. The predicted outputs are obtained by iterating the model for each mode:

$$
\begin{aligned}
\hat{y}(k+i|k) &= C_d \hat{x}(k+i|k) \\
&= C_d A_d^i x(k) + \sum_{j=1}^{i} C_d A_d^{j-1} B_d \hat{u}(k+i-j|k)
\end{aligned} \tag{5a}
$$

By collecting the predicted outputs into a vector, then the predicted outputs are given in a vector-matrix form:

$$
\begin{bmatrix} \hat{y}(k+1) \\ \hat{y}(k+2) \\ \vdots \\ \hat{y}(k+H_p) \end{bmatrix}
=
\begin{bmatrix} C_d A_d \\ C_d A_d^2 \\ \vdots \\ C_d A_d^{H_p} \end{bmatrix} x(k)
+
\begin{bmatrix}
C_d B_d & 0 & \cdots & 0 \\
C_d A_d B_d & C_d B_d & & \vdots \\
\vdots & \vdots & \ddots & \vdots \\
C_d A_d^{H_p-1} B_d & C_d A_d^{H_p-2} B_d & \cdots & C_d B_d
\end{bmatrix}
\times
\begin{bmatrix} \hat{u}(k) \\ \hat{u}(k+1) \\ \vdots \\ \hat{u}(k+H_p-1) \end{bmatrix} \tag{5b}
$$

The input prediction $\hat{u}(k+i)$ can be expressed in term of input increment $\Delta\hat{u}(k+i)$ where $\Delta\hat{u}(k+i) = \hat{u}(k+i) - \hat{u}(k+i-1)$. The predicted inputs now become:

$$\begin{bmatrix} \hat{u}(k) \\ \hat{u}(k+1) \\ \vdots \\ \hat{u}(k+H_p-1) \end{bmatrix} = \begin{bmatrix} u(k-1) \\ u(k-1) \\ \vdots \\ u(k-1) \end{bmatrix} + \begin{bmatrix} I & 0 & \cdots & 0 \\ I & I & & \vdots \\ \vdots & & \ddots & \vdots \\ I & I & \cdots & I \end{bmatrix} \begin{bmatrix} \Delta\hat{u}(k) \\ \Delta\hat{u}(k+1) \\ \vdots \\ \Delta\hat{u}(k+H_p-1) \end{bmatrix} \tag{6}$$

It is assumed that the input only changes at time instants $k, k+1, \ldots, k+H_u-1$ such that for $H_u \leq i \leq H_p-1, \hat{u}(k+i) = \hat{u}(k+H_u-1)$.

The predicted outputs now can be written as follow:

$$\begin{bmatrix} \hat{y}(k+1) \\ \vdots \\ \hat{y}(k+H_u) \\ \hat{y}(k+H_u+1) \\ \vdots \\ \hat{y}(k+H_p) \end{bmatrix} = \begin{bmatrix} C_dA_d \\ \vdots \\ C_dA_d^{H_u} \\ C_dA_d^{H_u+1} \\ \vdots \\ C_dA_d^{H_p} \end{bmatrix} x(k) + \begin{bmatrix} C_dB_d \\ \vdots \\ \sum_{i=0}^{H_u-1} C_dA_d^iB_d \\ \sum_{i=0}^{H_u} C_dA_d^iB_d \\ \vdots \\ \sum_{i=0}^{H_p} C_dA_d^iB_d \end{bmatrix} u(k-1)$$
$$+ \begin{bmatrix} C_dB_d & \ldots & 0 \\ C_dA_dB_d + C_dB_d & & 0 \\ \vdots & & \vdots \\ \sum_{i=0}^{H_u-1} C_dA_d^iB_d & \cdots & C_dB_d \\ \sum_{i=0}^{H_u} C_dA_d^iB_d & \ddots & C_dA_dB_d + C_dB_d \\ \vdots & & \vdots \\ \sum_{i=0}^{H_p} C_dA_d^iB_d & \cdots & \sum_{i=0}^{H_p-H_u} C_dA_d^iB_d \end{bmatrix} \begin{bmatrix} \Delta\hat{u}(k) \\ \Delta\hat{u}(k+1) \\ \vdots \\ \Delta\hat{u}(k+H_u-1) \end{bmatrix} \tag{7}$$

Therefore, the predicted outputs can be written in more compact equation:

$$\hat{\Upsilon} = \underbrace{\Phi x(k) + \Gamma u(k-1)}_{\text{past}} + \underbrace{G\hat{U}}_{\text{future}} \tag{8}$$

If the measured disturbance is brought to the calculation, equation (4) becomes:

$$x(k+1) = A_d x(k) + B_d u(k) + B_{dm} d(k) \tag{9}$$

where B_{dm} is a measured disturbance matrix and $d(k)$ is the measured disturbance. In MPC design, it is more convenient to express the model with an increment input and one possibility (Maciejowsky, 2002), is given as follows:

$$\begin{bmatrix} x(k+1) \\ u(k) \end{bmatrix} = \begin{bmatrix} A_d & B_d \\ 0 & I \end{bmatrix} \begin{bmatrix} x(k) \\ u(k-1) \end{bmatrix} + \begin{bmatrix} B_d \\ I \end{bmatrix} \Delta u(k) + \begin{bmatrix} B_{dm} \\ 0 \end{bmatrix} d(k) \tag{10a}$$
$$y(k) = \begin{bmatrix} C_d & 0 \end{bmatrix} \begin{bmatrix} x(k) \\ u(k-1) \end{bmatrix}$$

The state-space model can be written in more compact form with the discrete-time representation as subscript:

$$\begin{aligned} x_{k+1} &= Ax_k + B\Delta u_k + B_m d_k \\ y_k &= Cx_k \end{aligned} \tag{10b}$$

where:

$$x_{k+1} \equiv \begin{bmatrix} x(k+1) \\ u(k) \end{bmatrix}, \quad \Delta u_k \equiv \Delta u(k), \quad d_k \equiv d(k).$$

The predicted outputs are then given by:

$$\begin{bmatrix} \hat{y}_{k+1} \\ \hat{y}_{k+2} \\ \vdots \\ \hat{y}_{k+Hp} \end{bmatrix} = \begin{bmatrix} CA \\ CA^2 \\ \vdots \\ CA^{Hp} \end{bmatrix} x_k + \begin{bmatrix} CB & 0 & \cdots & 0 \\ CAB & CB & & \vdots \\ \vdots & & \ddots & \vdots \\ CA^{Hp-1}B & CA^{Hp-2}B & \cdots & CA^{Hp-Hu}B \end{bmatrix} \times \begin{bmatrix} \Delta\hat{u}_k \\ \Delta\hat{u}_{k+1} \\ \vdots \\ \Delta\hat{u}_{k+Hu-1} \end{bmatrix} + \begin{bmatrix} CB_m & 0 & \cdots & 0 \\ CAB_m & CB_m & & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ CA^{Hp-1}B_m & CA^{Hp-2}B_m & \cdots & CB_m \end{bmatrix} \times \begin{bmatrix} \hat{d}_k \\ \hat{d}_{k+1} \\ \vdots \\ \hat{d}_{k+Hp-1} \end{bmatrix} \tag{11a}$$

This equation can be written in more compact equation:

$$\hat{Y}_k = \underbrace{\Phi x_k}_{\text{past}} + \underbrace{\Theta \hat{U}_k + \Xi \hat{D}_k}_{\text{future}} \tag{11b}$$

where $\hat{Y}_k = [\ \hat{y}_{k+1} \hat{y}_{k+2} \ldots \hat{y}_{k+H_p}]^T$, $\Delta\hat{U}_k = [\Delta\hat{u}_k \quad \Delta\hat{u}_{k+1} \ldots \Delta\hat{u}_{k+H_u-1}]^T$, and $\hat{D}_k = [\hat{d}_k \quad \hat{d}_{k+1} \quad \ldots \hat{d}_{k+H_p-1}]^T$.

The general aim of MPC is to minimize the cost quadratic function:

$$\mathcal{J} = \sum_{i=0}^{Hp} \|y_{k+i} - w_{k+i}\|_Q^2 + \sum_{i=1}^{Hu} \|\Delta u_{k+i-1}\|_R^2 \tag{12a}$$

where w is the reference, H_p and H_u are the maximum prediction horizon and the control horizon, respectively, and the weighting matrices Q and R are defined by:

$$Q = \begin{bmatrix} Q(1) & 0 & \cdots & 0 \\ 0 & Q(2) & & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & Q(H_p) \end{bmatrix}$$
$$R = \begin{bmatrix} R(1) & 0 & \cdots & 0 \\ 0 & R(2) & & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & R(H_u-1) \end{bmatrix} \tag{12b}$$

The cost function can then be simplified in a matrix form as:

$$\mathcal{J}_{MPC} = \left\|\hat{Y} - W\right\|_Q^2 - \left\|\Delta\hat{U}\right\|_R^2 \tag{13}$$

Define the error prediction as:

$$\varepsilon(k) = \Gamma(k) - \Phi x(k) - \Xi D(k) \tag{14}$$

where:

$$\Gamma(k) = \begin{bmatrix} r(k+1) \\ \vdots \\ r(k+H_p) \end{bmatrix},$$

$$r(k+i) = w(k+i) - \in(k+i) = w(k+i) - e^{-iT_s/T_{ref}} \in(k)$$

and:

$$\in(k) = w(k) - y(k),$$

T_{ref} denotes the response speed of the reference trajectory that approaches the set-point exponentially. The cost function in the MPC problem becomes a quadratic programming (QP) problem as follows:

$$\begin{aligned} \mathcal{J}_{MPC} &= \left\|\Theta\Delta\hat{U} - \varepsilon\right\|_Q^2 + \left\|\Delta\hat{U}\right\|_R^2 \\ \mathcal{J}_{MPC} &= \varepsilon^T Q\varepsilon - 2\Delta\hat{U}^T\Theta^T Q\varepsilon + \Delta\hat{U}^T[\Theta^T Q\Theta + R]\Delta\hat{U} \\ \mathcal{J}_{MPC} &= constant - \Delta\hat{U}^T G + \Delta\hat{U}^T H\Delta\hat{U} \end{aligned} \tag{15}$$

where $G = 2\Theta^T Q\varepsilon$ and $H = \Theta^T Q\Theta + R$. The minimization of the cost function $\mathcal{J}_{MPC}$ is equivalent to the minimization of the objective function $\Delta\hat{U}^T H\Delta\hat{U} - \Delta\hat{U}^T G$ which is a quadratic function. In this equation, the matrix H is the Hessian matrix, therefore the function can be changed to $\Delta\hat{U}^T H\Delta\hat{U} - G^T\Delta\hat{U}$.

The cost function subjects to linear inequality in the inputs, input increments, and outputs as follow:

$$\begin{aligned} &u_{\min} \le u_{k+i} \le u_{\max}, \quad i = 0, 1, \ldots, H_u - 1 \\ &\Delta u_{\min} \le \Delta u_{k+i} \le \Delta u_{\max}, \quad i = 0, 1, \ldots, H_u - 1 \\ &y_{\min} \le y_{k+i} \le y_{\max}, \quad i = 1, 2, \ldots, H_p \end{aligned} \tag{16}$$

All inequalities in the equation (16) can be arranged together into a single set vector-matrix equation as follows:

$$\Omega\Delta\hat{U} \le \beta + Fu(k-1) + Mx(k) \text{ or } \Omega\Delta\hat{U} \le \omega \tag{17}$$

where:

$$\Delta\hat{U} = \begin{bmatrix} \Delta\hat{u}_k \\ \Delta\hat{u}_{k+1} \\ \vdots \\ \Delta\hat{u}_{k+N_u-1} \end{bmatrix}$$

and:

$$\omega = \beta + Fu(k-1) + Mx(k).$$

Consider the set of upper bound input increment constraints:

$$\Delta\hat{u}_{k+i} \le \Delta u_{\max}, \quad i = 0, 1, \ldots, H_u - 1 \tag{18}$$

The constraints can be written in the general form by setting:

$$\Omega \leftarrow \begin{bmatrix} I & & & \\ & I & & \\ & & \ddots & \\ & & & I \end{bmatrix}, \quad F \leftarrow 0, \quad M \leftarrow 0,$$
$$\beta \leftarrow \begin{bmatrix} \Delta u_{\max} \\ \Delta u_{\max} \\ \vdots \\ \Delta u_{\max} \end{bmatrix} \tag{19}$$

Similarly, the lower bound constraints are set as:

$$\Omega \leftarrow - \begin{bmatrix} I & & & \\ & I & & \\ & & \ddots & \\ & & & I \end{bmatrix}, \quad F \leftarrow 0, \quad M \leftarrow 0,$$
$$\beta \leftarrow - \begin{bmatrix} \Delta u_{\min} \\ \Delta u_{\min} \\ \vdots \\ \Delta u_{\min} \end{bmatrix} \tag{20}$$

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For the set of upper bound input contraints are set:

$$\Omega \leftarrow \begin{bmatrix} I & & & \\ I & I & & \\ \vdots & & \ddots & \\ I & \cdots & \cdots & I \end{bmatrix}, \quad F \leftarrow -\begin{bmatrix} I \\ I \\ \vdots \\ I \end{bmatrix}, \quad M \leftarrow 0,$$
$$\beta \leftarrow \begin{bmatrix} u_{\max} \\ u_{\max} \\ \vdots \\ u_{\max} \end{bmatrix} \tag{21}$$

The set of lower bound input contraints are set:

$$\Omega \leftarrow -\begin{bmatrix} I & & & \\ I & I & & \\ \vdots & & \ddots & \\ I & \cdots & \cdots & I \end{bmatrix}, \quad F \leftarrow \begin{bmatrix} I \\ I \\ \vdots \\ I \end{bmatrix}, \quad M \leftarrow 0,$$
$$\beta \leftarrow -\begin{bmatrix} u_{\min} \\ u_{\min} \\ \vdots \\ u_{\min} \end{bmatrix} \tag{22}$$

Note that the output predictions are given by:

$$\begin{bmatrix} \hat{y}_{k+1} \\ \hat{y}_{k+2} \\ \vdots \\ \hat{y}_{k+H_p} \end{bmatrix} = \Phi x_k + \Theta \Delta \hat{U}_k \tag{23}$$

The set of upper bound output contraints are set:

$$\Omega \leftarrow \Theta, \quad F \leftarrow 0, \quad M \leftarrow -\Phi, \quad \beta \leftarrow \begin{bmatrix} y_{\max} \\ \vdots \\ y_{\max} \end{bmatrix} \tag{24}$$

The set of lower bound output contraints are set:

$$\Omega \leftarrow -\Theta, \quad F \leftarrow 0, \quad M \leftarrow \Phi, \quad \beta \leftarrow -\begin{bmatrix} y_{\min} \\ \vdots \\ y_{\min} \end{bmatrix} \tag{25}$$

Note that the form of the output predictions will depend on the state-space model. The inequality matrix equation (17) needs to be computed once in the MPC control design and it will be used for each optimization process since constraints are constant. Two popular methods used to solve QP problems are active set (Fletcher, 1987) and interior point (Wright, 1997) methods.

Multiple models helicopter with MPC

Helicopter control system designs using MPC with a single either linear or non-linear models have been studied including one reported in Wan and Bogdanov (2001), Singh and Fuller (2001), Kim *et al.* (2002), Castillo *et al.* (2009) and Palunko and Bogdan (2008). Robust hybrid control system design using linear impulsive differential equations for a small-scale helicopter with two models was studied by Sutarto *et al.* (2006). The paper considers the tracking control system using MPC for multiple models of a small-scale helicopter using five different linear models represent hover, acceleration and flying up conditions in one sequence of flying.

Note that the state-space model in equation (2) represents a more general structure than the parameterized state-space model proposed by Mettler (2003). The values of the parameters in the state-space model for the five flight conditions are given in Tables I-VIII.

The solution of the state space representation of the models gives information of helicopter position in the body coordinates. However, it is difficult to have physical interpretation of helicopter position in the body coordinate. Hence it is necessary to transform the body coordinate to local horizon coordinate system. The transformation between the two coordinates is carried out by using the following matrix transformation appeared in Budiyono and Wibowo (2009):

$$T_I = \begin{bmatrix} \cos\theta\cos\psi & \cos\theta\sin x & -\sin\theta \\ \sin\phi\sin\theta\cos\psi - \cos\phi\sin\psi & \sin\phi\sin\theta\sin\psi + \cos\phi\cos\psi & \sin\phi\cos\theta \\ \cos\phi\sin\theta\cos\psi + \sin\phi\sin\psi & \cos\phi\sin\theta\sin\psi - \sin\phi\cos\psi & \cos\phi\cos\theta \end{bmatrix} \tag{26}$$

The transformation matrix T_I relates the positions and the velocities in the two different coordinates as expressed in the following equations:

$$\begin{aligned} \begin{bmatrix} N & E & A \end{bmatrix}^T &= T_I \begin{bmatrix} x & y & z \end{bmatrix}^T \\ \begin{bmatrix} V_x & V_y & V_z \end{bmatrix}^T &= T_I \begin{bmatrix} u & v & w \end{bmatrix}^T \end{aligned} \tag{27}$$

where u, v, w are velocities in the body coordinate and V_x, V_y, V_z are in the local coordinate system. In the small-scale helicopter control problem, the design of the MPC is aimed to control the helicopter is such a way that a certain flight profile from hover to an accelerating cruise can be achieved by the helicopter. The introduction of the flight profile is, in this case, important for two different reasons. First, the desired flight trajectory of the helicopter cannot be defined arbitrarily as it is constrained by the stability and the speed of the helicopter. Second, the MPC controller needs a reference signal in order to dictate the helicopter properly. In this case, the flight profile acts as a virtual flight path that is used as the reference signal in the MPC design.

It is generally known that the tracking control system designs can yield a perfect tracking (the error between the trajectory and the output of the controlled system is 0) if the control system design satisfies the condition called internal model principle (IMP). The IMP requirement states that the perfect tracking will be achieved if the modes of the trajectories are also the modes in the open loop tracking controller. If that is not the case, the perfect tracking can still be achieved by augmenting a finite dimensional system in the open loop of the tracking system. The detail theory and conditions can be found in the

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Table I *X*-axes force derivates

	Speed				
Parameters *X*	**U_0 = 0 m/s**	**U_0 = 4 m/s**	**U_0 = 8 m/s**	**U_0 = 12 m/s**	**U_0 = 16 m/s**
X_u	−0.0133	−0.0555	−0.1607	−0.1647	−0.2218
X_w	0.0001	0.0085	0.0164	0.0175	0.0162
X_q	0.0000	−0.0010	0.0010	0.0010	−9.6915
X_θ	−9.8100	−9.8090	−9.8028	−9.7738	−9.6915
$X_{a_{ls}}$	−24.2699	−11.4711	−11.5216	−11.5100	−11.4057
$X_{\delta_{Coll}}$	−0.2817	−0.6189	−0.0507	0.36569	0.52661

Table II *Y*-axes force derivates

	Speed				
Parameters *Y*	**U_0 = 0 m/s**	**U_0 = 4 m/s**	**U_0 = 8 m/s**	**U_0 = 12 m/s**	**U_0 = 16 m/s**
Y_u	−0.0166	−0.0118	−0.0209	−0.0182	−0.0136
Y_w	−0.0170	0.0058	0.0081	0.0091	0.0094
Y_θ	0.0011	0.0098	0.0231	0.0479	0.0834
Y_v	0.0700	−0.1362	−0.1944	−0.2680	−0.3425
Y_p	0.0000	0.0010	−0.0010	−0.0010	0.0010
Y_r	0.0018	−3.9911	−7.9849	−11.9799	−15.9745
Y_ϕ	9.7804	9.7854	9.7842	9.7580	9.6769
$Y_{b_{ls}}$	24.2692	11.4708	11.5214	11.5100	11.4057
$Y_{\delta_{Coll}}$	1.47798	0.84158	0.76276	0.76978	0.78488
$Y_{\delta_{Ped}}$	−154.71	−4.596	−4.3577	−4.5072	−4.9054

Table III *Z*-axes force derivates

	Speed				
Parameters *Z*	**U_0 = 0 m/s**	**U_0 = 4 m/s**	**U_0 = 8 m/s**	**U_0 = 12 m/s**	**U_0 = 16 m/s**
Z_u	0.0397	−0.4699	−0.4182	−0.2500	−0.1568
Z_w	0.0965	−0.9969	−1.3972	−1.6309	−1.7309
Z_q	0.0000	3.9983	7.9964	11.9792	15.9740
Z_θ	0.0142	0.1419	0.3749	0.8403	1.5180
$Z_{a_{ls}}$	0.0346	0.0010	−0.0438	0.8403	−0.0547
Z_v	0.0397	0.0000	0.0000	0.0000	0.0000
Z_ϕ	−0.7609	−0.6792	−0.6035	−0.5558	−0.5318
$Z_{b_{ls}}$	0.1817	0.0769	0.0680	0.0633	0.0593
$Z_{\delta_{Coll}}$	−197.54	−122.96	−126.53	−139.51	−151.88

Table IV Roll moment derivates

	Speed				
Parameters *L*	**U_0 = 0 m/s**	**U_0 = 4 m/s**	**U_0 = 8 m/s**	**U_0 = 12 m/s**	**U_0 = 16 m/s**
L_u	−0.0622	−0.0198	−0.0573	−0.0550	−0.0425
L_w	−0.0667	0.0662	0.0849	0.0941	0.0965
L_q	−0.0001	−0.0032	−0.0025	−0.0011	−0.0005
L_v	0.3576	−0.1715	−0.2316	−0.2942	−0.3470
L_r	0.0066	0.0322	0.0547	0.0727	0.0925
$L_{b_{ls}}$	556.7226	420.4654	421.0045	420.8826	419.7725
$L_{\delta_{Coll}}$	15.7386	8.96576	8.12507	8.26462	8.42221
$L_{\delta_{Ped}}$	−560.7	−16.657	−15.793	−16.336	−17.779

Table V Pitch moment derivates

			Speed		
Parameters M	$U_0 = 0$ m/s	$U_0 = 4$ m/s	$U_0 = 8$ m/s	$U_0 = 12$ m/s	$U_0 = 16$ m/s
M_u	−0.0004	0.1148	0.2214	0.4004	0.5083
M_w	−0.0010	−0.0231	−0.0047	0.6571	1.9197
M_q	−0.0002	−0.0292	−0.0611	−0.3553	−0.4441
$M_{a_{ls}}$	295.5075	223.1810	223.4664	223.4012	222.8119
$M_{\delta_{Coll}}$	1.59099	0.02908	−2.849	−19.174	−46.921

Table VI Yaw moment derivates

			Speed		
Parameters N	$U_0 = 0$ m/s	$U_0 = 4$ m/s	$U_0 = 8$ m/s	$U_0 = 12$ m/s	$U_0 = 16$ m/s
N_u	0.4332	0.3998	0.6233	0.5232	0.3888
N_w	0.4334	0.0149	−0.0058	−0.0121	−0.0114
N_q	0.0004	0.0234	0.0187	0.0078	0.0033
N_v	−2.6429	1.2565	1.6970	2.1556	2.5425
N_r	−0.0480	−0.2356	−0.4007	−0.5330	−0.6780
$N_{\delta_{Coll}}$	2.28×10^{-06}	0.02247	0.01681	0.26198	0.24962
$N_{\delta_{Ped}}$	4,108.23	122.048	115.718	119.69	130.264

Table VII Angle derivates

			Speed		
Parameters Θ,Φ	$U_0 = 0$ m/s	$U_0 = 4$ m/s	$U_0 = 8$ m/s	$U_0 = 12$ m/s	$U_0 = 16$ m/s
Θ_q	0.9970	0.9976	0.9981	0.9984	0.9985
Θ_r	−0.0776	−0.0692	−0.0616	−0.0569	−0.0549
Φ_q	−0.0001	−0.0010	−0.0024	−0.0049	−0.0086
Φ_p	1.0000	1.0000	1.0000	1.0000	1.0000
Φ_r	−0.0014	−0.0145	−0.0382	−0.0860	−0.1566

Table VIII Flapping derivates

			Speed		
Parameters () $\times 1/\tau_e$	$U_0 = 0$ m/s	$U_0 = 4$ m/s	$U_0 = 8$ m/s	$U_0 = 12$ m/s	$U_0 = 16$ m/s
A_u	0.0079	0.0030	0.0029	0.0027	0.0025
A_q	−1.0000	−1.0000	−1.0000	−1.0000	−1.0000
$A_{a_{ls}}$	−8.3500	−8.3500	−8.3500	−8.3500	−8.3500
$A_{\delta_{Coll}}$	−0.0003	0.11362	0.22925	0.35564	0.48714
$A_{\delta_{LOng}}$	35.07	35.07	35.07	35.07	35.07
B_v	0.0079	0.0029	0.0026	0.0024	0.0023
B_p	−1.0000	−1.0000	−1.0000	−1.0000	−1.0000
$B_{b_{ls}}$	−8.3500	−8.3500	−8.3500	−8.3500	−8.3500
$B_{\delta_{Lat}}$	35.07	35.07	35.07	35.07	35.07

references Francis and Wonham (1976), Joelianto (2000) or Joelianto and Williamson (2009).

As the aim of the paper is to investigate the application of the MPC for controlling the small-scale helicopter with multiple linearized models, the IMP requirement can be simplified by finding the flight profile that corresponds to each model. Each flight profile of the multiple models is then merged into one flight profile that will serve as the known reference signal of the MPC. In addition, the MPC needs reference signal prediction in order to compute the current control signal. If the future reference signal is not available, there will be problem in the control signal computation. To solve this problem, in the control signal computation, the last model is extended to be the sixth model to generate the future flight profile. However, the control computation is considered only for the first five models. The algorithm of the MPC design is shown in Figure 1. In the first correction, four parameters of MPC (H_p, H_u, Q, R) are tuned to obtain the required performances. If, after several iterations, the performances cannot be achieved, the parameters of the sampling time T_s and the speed respose are then adjusted T_{ref}. The process is then repeated again by tuning the parameters (H_p, H_u, Q, R).

The simulation results are shown in Figures 2-5. Figure 2 shows the tracking performance of the small helicopter using MPC with the six models where the sixth model is similar to the fifth model. The black line is the flight profile generated by the multiple models of the helicopter with respect to its initial condition in each mode with the constants control signal (u) and under a smoothing between the model transition. The color line denotes the output of the helicopter with the control signal is computed by the MPC with certain parameter selection by using trial error method. Figure 3 shows the required responses correspond to the required flight profile generated for the five multiple models by cropping the response of the sixth model in Figure 2. It can be seen that deviations occurred gradually and it becomes significant at the end of the flight segment. The required control signal is shown in Figure 4. The control signal is small which is expected in order that helicopter move smoothly along the desired trajectory. However, it can be seen that at each model transition, the control signal jumps from one value to another. In practice, it will be appropriate if the jump can be reduced in order to decrease sudden movement of the helicopter. Figure 5 shows the desired flight trajectory for the MPC (red line) and the actual flight (blue line) in the local coordinate system.

The MPC parameters are shown in Tables IX and X.

The existence of a slight deviation between targeted and actual trajectory indicates that the present result is not perfect. However, the application of the MPC for the control of a small-scale helicopter with multiple models opens up a new horizon demonstrating that the MPC can deal with switching dynamics of the helicopter and maintain the overall stability. The cost function produced by using the parameters given in Tables IX and X is $1.5980 \times 10^{+007}$. The deviation can be reduced by fine tuning the appropriate parameters of the MPC. At this moment, parameter selection of the MPC is considered as the main problem since there are many combinations that are difficult to find by trial error method.

Conclusion

This paper concerned with the application of model predictive control known as MPC for a small-scale helicopter with five multiple linearized models. As the MPC requires future reference signal, a flight profile is generated from the models that serves as the reference trajectory for the controlled helicopter with MPC. The results indicate that the approach leads to a good tracking performance even without the parameters optimization in the MPC design. Moreover, the MPC can handle the switching dynamics problem well shown by stable responses with sufficient tracking performance. The results demonstrate that MPC gives a new alternative control method in achieving good control for the small-scale helicopter when the models are available.

Further work

The results open many opportunities for further work particularly in the area of control for small-scale helicopters and in the area of unmanned autonomous vehicles or systems (UAV or UAS) in general. First, the integration of the IMP in the MPC control design enables the implementation of difficult and complex flight trajectory for the small-scale helicopter. Second, in real flight, the helicopter

Figure 1 Model predictive control design flowchart

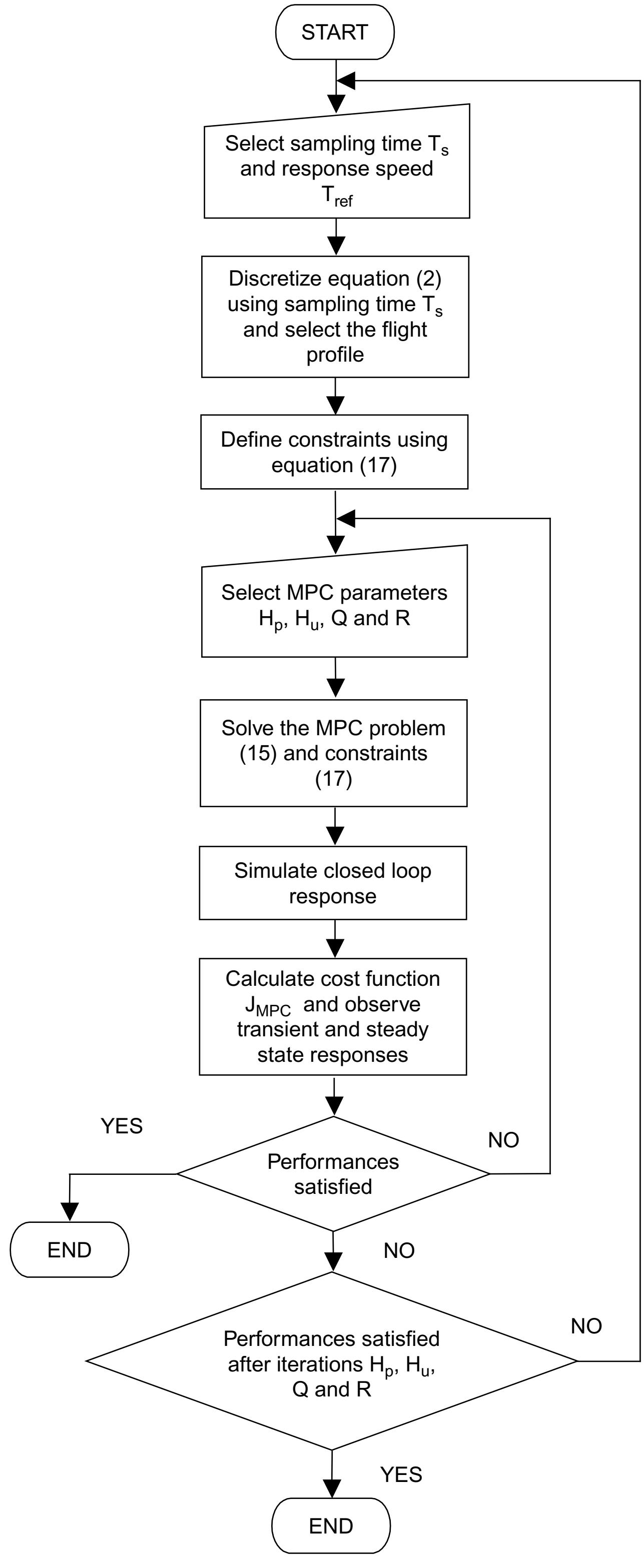

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Figure 2 Simulation results using six models

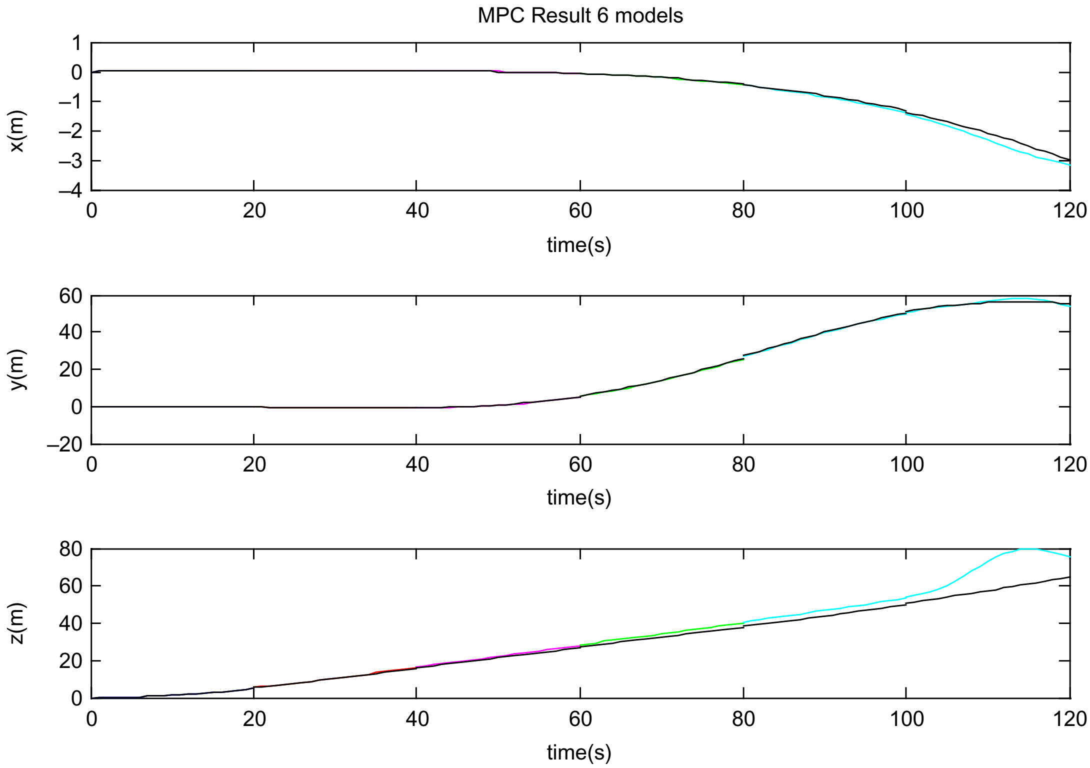

Figure 3 Simulation results for five models

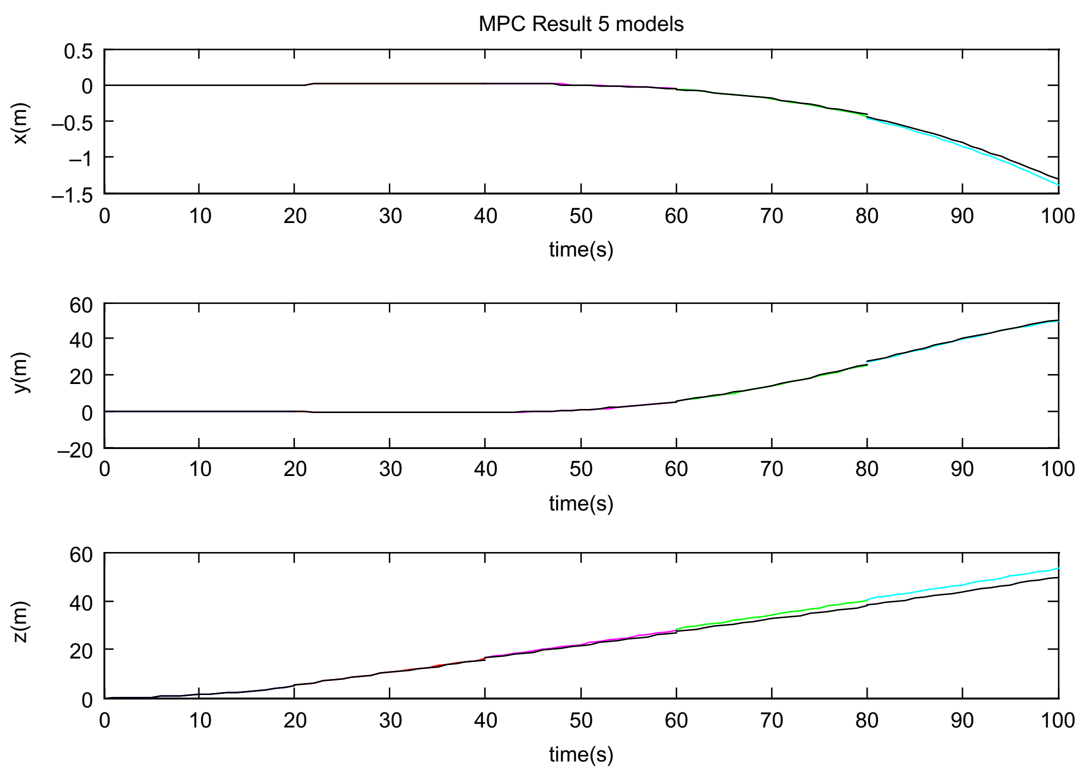

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Figure 4 Control signal

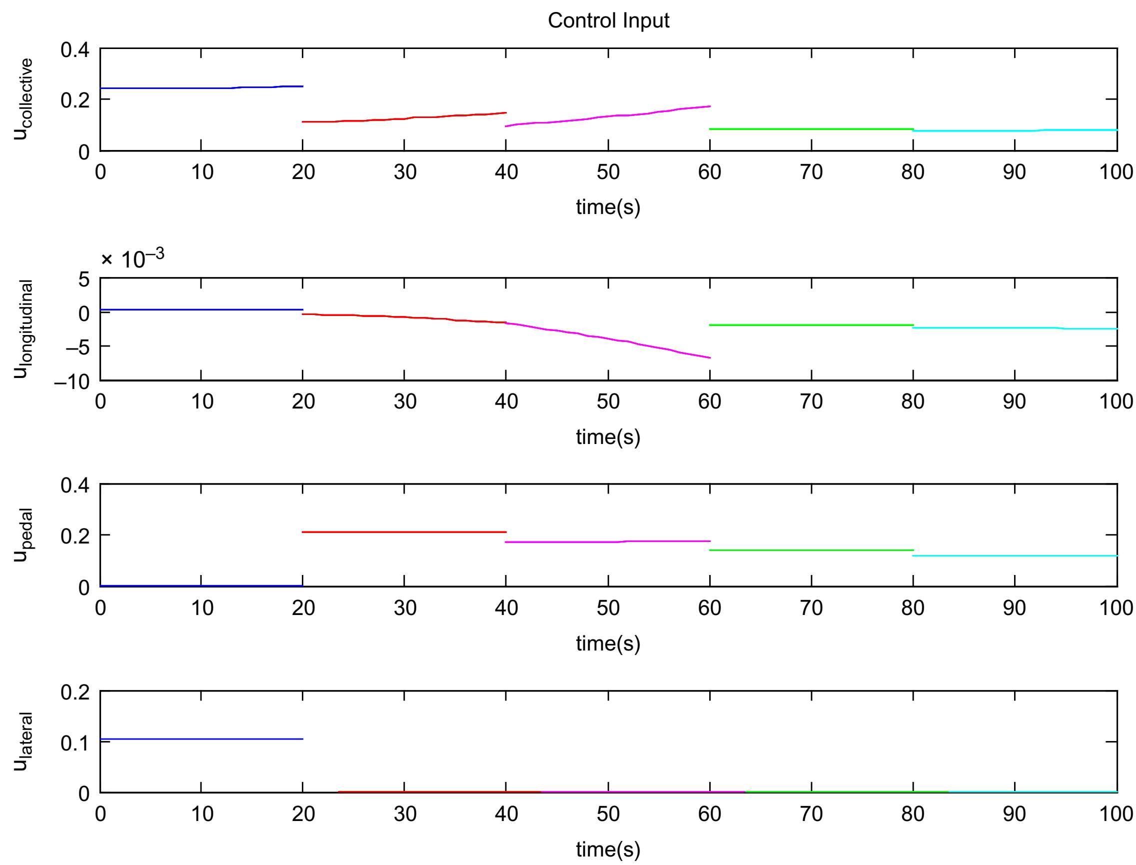
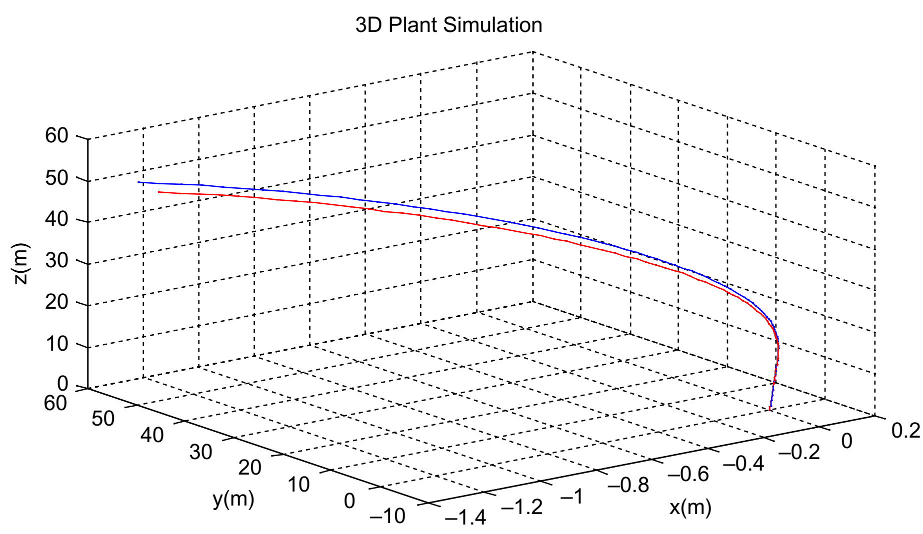


Figure 5 Trajectory in *X-Y-Z* coordinates



will experience wind disturbance and other type of disturbances therefore it will be of interest to investigate the application of robust MPC. There are numerous variants of robust MPC as reported in Joelianto *et al.* (2009a, b). Third, the selection of parameters of the MPC is difficult when the controlled system is complex and has a higher dimension. In this case, an optimization approach can be useful such as genetic algorithm based solver (Joelianto and Hernawan, 2009). Fourth, as multiple models inherently bring the requirement for switching strategy, a variant of MPC with switching design that lead to hybrid control with MPC can be developed. The extension of the hybrid MPC development can simultaneously cover the issue of safety verification and validation which ensure that the flight of the helicopter safe throughout its entire mission.

Table IX Output and input weighting matrices

	Q	R
Hover	0.000000015 *diag(0.1, 0.1, 0.1, 0.1)	0.00000015 *diag(100, 100, 100, 100)
U_0 = 4 m/s	0.000000015 *diag(0.1, 0.1, 0.1, 0.1)	0.00000015 *diag(100, 100, 100, 100)
U_0 = 8 m/s	0.000000015 *diag(0.1, 0.1, 0.1, 0.1)	0.00000015 *diag(100, 100, 100, 100)
U_0 = 12 m/s	0.000000015 *diag(0.0001, 0.0001, 0.0001, 0.0001)	0.00000015 *diag(100, 100, 100, 100)
U_0 = 16 m/s	0.000000015 *diag(0.001, 0.001, 0.001, 0.001)	0.00000015 *diag(100, 100, 100, 100)
U_0 = 16 m/s	0.000000015 *diag(0.001, 0.001, 0.001, 0.001)	0.00000015 *diag(1, 1, 1, 1)

Table X Parameters horizon and constraints

Parameters	Value
Minimum horizon prediction (N_1)	1
Maximum horizon prediction (N_2)	5
Control horizon (N_u)	3
Output constraints	$y_{max} = [0.2;2;15;0]$; $y_{min} = [-0.2; -2;0;0]$;

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