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Neural Network-Based Distributed Attitude Coordination Control for Spacecraft Formation Flying With Input Saturation

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***Abstract*— This brief considers the attitude coordination control problem for spacecraft formation flying when only a subset of the group members has access to the common reference attitude. A quaternion-based distributed attitude coordination control scheme is proposed with consideration of the input saturation and with the aid of the sliding-mode observer, separation principle theorem, Chebyshev neural networks, smooth projection algorithm, and robust control technique. Using graph theory and a Lyapunov-based approach, it is shown that the distributed controller can guarantee the attitude of all spacecraft to converge to a common time-varying reference attitude when the reference attitude is available only to a portion of the group of spacecraft. Numerical simulations are presented to demonstrate the performance of the proposed distributed controller.**

***Index Terms*— Attitude coordination control, Chebyshev neural networks, control input saturation, quaternion, spacecraft formation flying.**

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I. Introduction

Attitude coordination control for spacecraft formation flying (SFF) has received significant attention in recent years. In general, there exist several parameterizations to represent the orientation angles, i.e., the three-parameter representations (e.g., the Euler angles, Gibbs vector, Cayley–Rodrigues parameters, and modified Rodrigues parameters) and the four-parameter representations (e.g., unit quaternion). Using the modified Rodrigues parameters for attitude representations, the problem of attitude coordination has been studied in [1]–[7]. However, the three-parameter representations always exhibit singularity, i.e., the Jacobian matrix in the spacecraft kinematics is singular for some orientations, and it is well known that the four-parameter representations (e.g., unit quaternion) are considered for global representation of orientation angles without singularities.

Using unit quaternion for attitude representation, two distributed formation control strategies for maintaining attitude alignment among a group of spacecraft were proposed in [8]. In [9], a decentralized variable structure controller was presented for attitude coordination control of multiple spacecraft in the presence of model uncertainties, external disturbances, and intercommunication time delays. Based on the state-dependent Riccati equation technique, a decentralized attitude coordinated control algorithm for SFF was proposed in [10]. In these works [8]–[10], the common reference attitude was assumed to be a constant. The problem of quaternion-based attitude synchronization for a group of spacecraft to a common time-varying reference attitude was studied in [11]–[14]. However, the common time-varying reference attitude was assumed to be available to each agent in the group. In practice, it is more realistic that a common time-varying reference attitude is available only to a subset of the group members, and it is highly desirable to develop a distributed control law that can force a group of spacecraft to a common time-varying reference attitude even when only a subset of the team members has access to the common reference attitude.

In the current literature, attitude coordination control that can be robust against both structured and unstructured uncertainties has not received much attention. In [14], a quaternion-based decentralized adaptive sliding-mode control law was proposed for attitude coordination control of SFF in the presence of model uncertainties and external disturbances. However, the common time-varying reference attitude was assumed to be available to each spacecraft in the formation, and the problem of control input saturation was not considered. Universal function approximations such as neural networks (NNs) have been used in the robust control of nonlinear uncertain systems [15]–[19], due to the learning and adaptive abilities of NNs. It is worthwhile mentioning that the NN-based approaches for multiagent systems developed in [17]–[19] cannot be directly applied to spacecraft attitude dynamics because of the inherent nonlinearity in quaternion kinematics. Recently, a pinning impulsive control strategy was proposed for the synchronization of stochastic dynamical networks with nonlinear coupling in [20].

Chebyshev neural network (CNN) is a functional link network whose input is generated by use of a subset of Chebyshev polynomials. CNN has been shown to be capable of approximating any continuous functions over a compact set to arbitrary accuracy [21], [22], which makes CNN ideal for attitude coordination control of SFF in the presence of both structured and unstructured uncertainties. CNNs have been applied for attitude control of a single spacecraft [23], [24]. On the other hand, there always exists control input saturation in practical systems because the actuators of a control system can only provide a limited actuation. Moreover, saturation of the control input influences significantly the performance of adaptive and learning control systems, and therefore, the issue of control input saturation should be considered in designing an adaptive and learning control system. In [25], an adaptive-learning-based control scheme was proposed for preventing the presence of input saturation from inhibiting the learning capabilities and memory of an online approximator in feedback control systems.

Due to the inherent nonlinearity in quaternion kinematics, the problem of attitude coordination control of SFF using quaternion-based attitude representation is challenging, especially when the common time-varying reference attitude is available only to a subset of the group members. To the best of our knowledge, this problem is still unresolved even when the spacecraft attitude dynamics are exactly known. Therefore, this brief attempts to solve the above-mentioned problem. It is assumed that the topology of the information flow between spacecraft is described by an undirected connected graph. A decentralized sliding-mode observer is proposed to obtain an accurate estimation of the common time-varying reference attitude in finite time. The separation principle theorem is trivial in this case, which implies that the observer can be designed separately from the controller. By the use of CNNs, graph theory, smooth projection algorithm, and robust control technique, a distributed attitude coordination control scheme is proposed for SFF when the common time-varying reference attitude is available only to a subset of group members. Furthermore, a modified adaptive law is proposed such that the presence of actuator limits causes neither the instability in the closed-loop system nor "unlearning" in the online approximation process. The stability of the overall closed-loop system is proved by a Lyapunov-based approach. In contrast to the existing works on quaternion-based attitude coordination control [11]–[14], the main contribution of this brief is the development of a CNN-based attitude coordination control law for SFF without requiring all spacecraft to have access to the virtual leader (or the leader).

II. Preliminaries

A. Spacecraft Attitude Kinematics and Dynamics

Consider a group of n spacecraft modeled as rigid bodies. The equations of motions for the ith$(i = 1, 2, \ldots, n)$ spacecraft are given by [26]

$$J_i\dot{\omega}_i = -\omega_i^{\times} J_i\omega_i + \tau_i + \vartheta_i \tag{1}$$

$$\dot{q}_i = \frac{1}{2}\begin{pmatrix} q_{i4}I_3 + \bar{q}_i^{\times} \\ -\bar{q}_i^T \end{pmatrix}\omega_i \equiv \frac{1}{2}M(q_i)\omega_i \tag{2}$$

where $\omega_i \in R^3$ is the angular velocity of the ith spacecraft with respect to an inertial frame I and expressed in the body frame B_i, $J_i \in R^{3\times3}$ is the inertia matrix of the ith spacecraft, $\tau_i \in R^3$ is the torque control of the ith spacecraft, $\vartheta_i \in R^3$ is the bounded external disturbance, $I_3 \in R^{3\times3}$ is the identity matrix, and the matrix $x^{\times} \in R^{3\times3}$ represents a skew-symmetric matrix. The unit quaternion q_i is defined as $q_i = (\bar{q}_i^T, q_{i4})^T = (q_{i1}, q_{i2}, q_{i3}, q_{i4})^T \in R^4$, where $\bar{q}_i = (q_{i1}, q_{i2}, q_{i3})^T = \bar{e}_i \sin(\theta_i/2)$ represents the vector part of the unit quaternion, and $q_{i4} = \cos(\theta_i/2)$ denotes the scalar part of the unit quaternion. In this notation, $\bar{e}_i \in R^3$ is the Euler axis, and $\theta_i \in R$ is the Euler angle. The elements of a unit quaternion q_i are subject to the constraint $q_i^T q_i = 1$, and the unit quaternion q_i and $-q_i$ represent the same attitude. The inverse of a unit quaternion q_i is given by $q_i^{-1} = (-\bar{q}_i{}^T, q_{i4})^T$.

The quaternion multiplication of two unit quaternion q_i and q_j, namely, $q_i \odot q_j$, is defined as

$$q_i \odot q_j = \begin{pmatrix} q_{i4}\bar{q}_j + q_{j4}\bar{q}_i + \bar{q}_i^{\times}\bar{q}_j \\ q_{i4}q_{j4} - \bar{q}_i^T\bar{q}_j \end{pmatrix} \tag{3}$$

which is also a unit quaternion.

In the practical applications, the control torque $\tau_i(i = 1, 2, \ldots, n)$ is subject to some constraints because of the actuation limits. Assume that the control torque $\tau_i = (\tau_{i1}, \tau_{i2}, \tau_{i3})^T$ is constrained by $\tau_m \le \tau_{ij}(t) \le \tau_M$, $j = 1, 2, 3$, where τ_m and τ_M are known constants, respectively.

Suppose that there exists a virtual leader, labeled as spacecraft 0, and its attitude is given by q_0, which is the common time-varying reference attitude for the whole group. The assumptions with respect to the common reference attitude q_0 and external disturbances are stated as follows.

Assumption 1: The first three derivatives of the reference attitude q_0 (i.e., $\dot{q}_0, \ddot{q}_0, \dddot{q}_0$) are assumed to be bounded for all time.

Assumption 2: The external disturbance $\vartheta_i(i = 1, 2, \ldots, n)$ is assumed to be bounded such that $||\vartheta_i\| \le \vartheta_{Mi}$, where ϑ_{Mi} is a positive constant.

The station-keeping attitude error of the ith spacecraft, namely $q_{i0}(i = 1, 2, \ldots, n)$, which is the attitude error of the ith spacecraft with respect to the common reference attitude q_0, is defined as $q_{i0} = q_0^{-1} \odot q_i$. The corresponding rotation matrix $C_{i0} = C(q_{i0}) \in R^{3\times3}$ is given by

$$C(q_{i0}) = \left(q_{i0,4}^2 - \bar{q}_{i0}^T\bar{q}_{i0}\right) I_3 + 2\bar{q}_{i0}\bar{q}_{i0}^T - 2q_{i0,4}\bar{q}_{i0}^{\times}. \tag{4}$$

Note that $\|C_{i0}\| = 1$ and $\dot{C}_{i0} = -\omega_{i0}^{\times}C_{i0}$, where $\omega_{i0} = \omega_i - C_{i0}\omega_0$ is the relative angular velocity, and $\omega_0 \in R^3$ is the desired angular velocity.

The formation-keeping attitude error is the attitude error of an individual spacecraft with respect to the other spacecraft in the formation. The attitude error between two unit quaternion q_i and q_j is defined by the quaternion multiplication as $q_{ij} = q_j^{-1} \odot q_i$. The relative angular velocity between ω_i and ω_j is defined as $\omega_{ij} = \omega_i - C_{ij}\omega_j$, where $C_{ij} = C(q_{ij})$ denotes the corresponding rotation matrix. It is easy to obtain that $\omega_{ij} = \omega_{i0} - C_{ij}\omega_{j0}$, $C_{ij} = C_{ji}^T$, and $C_{ij} = C_{i0}C_{j0}^T$.

B. Graph Theory

The topology of the information flow between spacecraft (follower) is described by a weighted undirected connected graph $G = (\Upsilon, E, A)$, where $\Upsilon = \{r_1, r_2, \ldots, r_n\}$ is the set of nodes, $E \subseteq \Upsilon \times \Upsilon$ is the set of edges, and $A = (a_{ij}) \in R^{n\times n}$ is the weighted adjacency matrix of graph G with nonnegative elements. Node $r_i (i = 1, 2, \ldots, n)$ represents the ith spacecraft, and an edge in G is denoted by an unordered pair (r_i, r_j). $(r_i, r_j) \in E$ if and only if there is an information exchange between the ith spacecraft and the jth spacecraft, i.e., $(r_i, r_j) \in E \Leftrightarrow (r_j, r_i) \in E$. The adjacency element a_{ij} denotes the communication quality between the ith spacecraft and the jth spacecraft, i.e., $(r_i, r_j) \in E \Leftrightarrow a_{ij} > 0$. It is assumed that $a_{ij} = a_{ji}$ and $a_{ii} = 0$ in this brief; that is, the weighted adjacency matrix A is a symmetric matrix.

Let $D = \text{diag}\{d_1, d_2, \ldots, d_n\}$ represent the degree matrix of the weighted graph G, whose diagonal element is defined by $d_i = \sum_{j=1}^{n} a_{ij} (i = 1, 2, \ldots, n)$. Then the Laplacian matrix L of the weighted graph G is given by $L = D - A$, which is a symmetric matrix. Next, another graph $\bar{G}$ associated with the system consisting of one leader and n follower spacecraft is considered. Let $B = \text{diag}\{a_{10}, a_{20}, \ldots, a_{n0}\}$ be the leader adjacency matrix associated with $\bar{G}$, where the constant $a_{i0} \geq 0 (i = 1, 2, \ldots, n)$ is strictly positive if the ith spacecraft has access to the leader spacecraft; otherwise $a_{i0} = 0$. For $\bar{G}$, if there is a path in $\bar{G}$ from the node r_0 (leader spacecraft) to every node r_i (follower spacecraft i), then $\bar{G}$ is called a connected graph.

III. Distributed Attitude Coordination Controller Design

In this section, we consider the problem of distributed attitude coordination control for SFF in the presence of both structured and unstructured uncertainties as well as in the presence of the control input constraint. We first propose a decentralized sliding-mode observer for each spacecraft to obtain an accurate estimation of the common reference attitude q_0 in finite time, and then present a distributed attitude coordination control scheme for SFF when the common reference attitude is available only to a subset of the team members.

A. Sliding-Mode Observer

We assume that there exist n observers each of which is embedded in an individual spacecraft, and propose a decentralized sliding-mode observer as follows.

Lemma 1: If the graph $\bar{G}$ is connected and the dynamic equation for $p_i (i = 1, 2, \ldots, n)$ is given as

$$\dot{p}_i = -\beta_1 \left(\sum_{j=1}^{n} a_{ij}(p_i - p_j) + a_{i0}(p_i - q_0) \right) - \beta_2 \text{sign} \left(\sum_{j=1}^{n} a_{ij}(p_i - p_j) + a_{i0}(p_i - q_0) \right) \quad (5)$$

where $p_i \in R^4$ is the ith spacecraft's estimate of q_0, $\beta_1 > 0$ and $\beta_2 > \|\dot{q}_0\|$ are some constants, respectively, and $\text{sign}(\cdot)$ denotes the signum function, then p_i converges to q_0 in finite time.

Proof: Let $e_{p_i} = p_i - q_0 (i = 1, 2, \ldots, n)$, and we have

$$\dot{e}_{p_i} = -\beta_1 \left(\sum_{j=1}^{n} l_{ij} e_{p_j} + a_{i0} e_{p_i} \right) - \beta_2 \text{sign} \left(\sum_{j=1}^{n} l_{ij} e_{p_j} + a_{i0} e_{p_i} \right) - \dot{q}_0 \quad (6)$$

where l_{ij} represents the element of the Laplacian matrix L.

In order to prove that p_i converges to q_0 in finite time, we consider the following Lyapunov function:

$$V = \frac{1}{2} e_p^T M_1 e_p \quad (7)$$

where $e_p = (e_{p_1}^T, e_{p_2}^T, \ldots, e_{p_n}^T)^T \in R^{4n}$, $M_1 = (L + B) \otimes I_4 \in R^{4n\times 4n}$, $\otimes$ denotes the Kronecker product, and $I_4 \in R^{4\times 4}$ is the identity matrix. By [27, Lemma 3], we obtain that the matrix M_1 is positive definite.

The time derivative of the Lyapunov function (7) along (6) results in

$$\begin{aligned} \dot{V} &= -\beta_1 e_p^T M_1 M_1 e_p - \beta_2 e_p^T M_1 \text{sign}\left(M_1 e_p\right) - e_p^T M_1 Q_0 \\ &\leq -\beta_1 \lambda_{\min}^2(M_1) e_p^T e_p - (\beta_2 - \|\dot{q}_0\|) \lambda_{\min}(M_1) \|e_p\| \\ &\leq -\rho_1 V - \rho_2 V^{\frac{1}{2}} \end{aligned} \quad (8)$$

where $\rho_1 = 2\beta_1 \lambda_{\min}^2(M_1)/\lambda_{\max}(M_1)$, $\rho_2 = (\beta_2 - \|\dot{q}_0\|)\lambda_{\min}(M_1)\sqrt{2/\lambda_{\max}(M_1)}$, and $Q_0 = (\dot{q}_0^T, \ldots, \dot{q}_0^T)^T \in R^{4n}$. Therefore, we conclude that e_p converges to zero in finite time [28], which in turn implies that p_i converges to q_0 in finite time. ■

The main idea behind the introduction of a decentralized sliding-mode observer is to provide an accurate estimation of the common reference attitude q_0. Lemma 1 states that $p_i (i = 1, 2, \ldots, n)$ converges to q_0 in finite time and, hence, the separation principle is automatically satisfied [29], which implies that we can design the observer and the controller separately; that is, p_i is used for q_0 to generate the station-keeping attitude error q_{i0} in designing the control law τ_i.

In [4], a distributed sliding-mode observer was proposed for attitude containment control of multiple rigid bodies under an undirected follower communication topology. In [30], first-order and second-order decentralized sliding mode estimators were proposed for formation tracking of multiple autonomous vehicles under a directed communication graph. The difference between the observer (5) and that in [4] lies in the term $-\beta_1(\sum_{j=1}^{n} a_{ij}(p_i - p_j) + a_{i0}(p_i - q_0))$, which is used to provide a fast convergence rate [31].

B. Controller Design

We construct an auxiliary error $\alpha_i (i = 1, 2, \ldots, n) \in R^3$ for the ith spacecraft as

$$\alpha_i = \omega_i - \omega_{di} \quad (9)$$

where $\omega_{di} \in R^3$ is defined by

$$\omega_{di} = \frac{\sum_{j=0}^{n} a_{ij} C_{ij} \omega_j - k_1 a_{i0} \bar{q}_{i0} - \sum_{j=1}^{n} k_1 a_{ij} (\bar{q}_{i0} - C_{ij} \bar{q}_{j0})}{\sum_{j=0}^{n} a_{ij}} \tag{10}$$

with k_1 being a positive constant. In (10), p_i is used for q_0 to generate the station-keeping attitude error q_{i0}, i.e., $q_{i0} = p_i^{-1} \odot q_i$.

Using (1) and (9), the dynamic equation for $\alpha_i (i = 1, 2, \ldots, n) \in R^3$ is obtained by (1) as

$$J_i \dot{\alpha}_i = -\omega_i^{\times} J_i \omega_i - J_i \dot{\omega}_{di} + \tau_i + \vartheta_i. \tag{11}$$

Equation (11) can be re-expressed as

$$\begin{aligned} \dot{\alpha}_i &= (I_3 - J_i)\dot{\alpha}_i - \omega_i^{\times} J_i \omega_i - J_i \dot{\omega}_{di} + \tau_i + \vartheta_i \\ &= F_i(X_i) + \tau_i + \vartheta_i \end{aligned} \tag{12}$$

where $F_i(X_i) = (I_3 - J_i)\dot{\alpha}_i - \omega_i^{\times} J_i \omega_i - J_i \dot{\omega}_{di}$, and $X_i = (q_i^T, \omega_i^T, \dot{\omega}_i^T, \dot{\omega}_{di}^T)^T \in R^{13}$. Based on the universal approximation property of CNN, the unknown function $F_i (i = 1, 2, \ldots, n)$ can be approximated by [32]

$$F_i(X_i) = W_i^* \xi_i(X_i) + \varepsilon_i \tag{13}$$

where $W_i^* \in R^{3 \times N_1}$, with $N_1 = 13N_2 + 1$ and N_2 being the order of Chebyshev polynomials, is the optimal weight matrix of the CNN; $\varepsilon_i \in R^3$ is the CNN approximation error; and $\xi_i(X_i) \in R^{N_1}$ represents the Chebyshev polynomial basis function. The Chebyshev polynomial basis function $\xi_i(X_i)$ is obtained by use of Chebyshev polynomials as

$$\begin{aligned} \xi_i(X_i) = \big(&1, T_1(X_{i,1}), \ldots, T_{N_2}(X_{i,1}), \ldots, T_1(X_{i,13}), \\ &\ldots, T_{N_2}(X_{i,13})\big)^T \end{aligned} \tag{14}$$

where $T_k(X_{i,j}) (k = 1, 2, \ldots, N_2, j = 1, 2, \ldots, 13)$ represent Chebyshev polynomials, which are a set of orthogonal polynomials derived from the solution of the Chebyshev differential equation and obtained by using the so-called two-term recursive formula as

$$T_{k+1}(X_{i,j}) = 2X_{i,j} T_k(X_{i,j}) - T_{k-1}(X_{i,j}) \tag{15}$$

where $T_0(X_{i,j}) = 1$, and $T_1(X_{i,j})$ has several definitions, such as $X_{i,j}, 2X_{i,j}, 2X_{i,j} - 1$, and $2X_{i,j} + 1$. In this brief, $T_1(X_{i,j})$ is considered to be $X_{i,j}$.

Remark 2: Note that the unknown function $F_i (i = 1, 2, \ldots, n)$ is a function of the variables q_i, ω_i, $\dot{\omega}_i$, and $\dot{\omega}_{di}$, which implies that angular accelerations are required. However, angular accelerations may not be available for measurements. The high-order sliding-mode observers [33] can be applied to obtain accurate estimation of $\dot{\omega}_i$ and $\dot{\omega}_{di}$ in finite time. Moreover, because the partial information is sufficient to estimate the unknown function F_i, only the variables q_i and ω_i are used as the inputs of the CNN to implement the controller in practice. Note that if p_i converges to q_0 and all attitude and their derivative (i.e., q_i and $\dot{q}_i$) converge to the desired trajectories (i.e., q_0 and $\dot{q}_0$), then $\omega_{ij} = 0$, $C_{ij} = I_3$, $\bar{q}_{i0} = 0$, $\alpha_i = 0$, and $\dot{\omega}_{di}$ converges to $\dot{\omega}_0$, which in turn implies that the uncertain function F_i converges to the *desired nonlinear function* F_{di} given as

$$F_{di} = -\omega_0^{\times} J_i \omega_0 - J_i \dot{\omega}_0 \tag{16}$$

which further implies that the CNN used in the controller approximates the unknown desired nonlinear function F_{di} during the steady state.

The following assumptions are stated for the stability analysis of the overall closed-loop system.

Assumption 3: The optimal weight matrix $W_i^* (i = 1, 2, \ldots, n)$ is assumed to lie in a known bounded set Ω_{Wi} defined by

$$W_i^* \in \Omega_{Wi} = \left\{ W_i^* : W_{i\min} \le W_{i,jk}^* \le W_{i\max} \right\} \tag{17}$$

where $j = 1, 2, 3$, $k = 1, 2, \ldots, N_1$, and $W_{i\min}$ and $W_{i\max}$ are known constants.

Assumption 4: The CNN approximation error $\varepsilon_i (i = 1, 2, \ldots, n)$ is bounded such that $\|\varepsilon_i\| \le \varepsilon_{Mi}$, where ε_{Mi} is a positive constant.

In order to guarantee that the estimated CNN weight matrices remain within the known bounded set, the *smooth projection* algorithm [34], [35] is used to update the CNN weight matrices. Let the estimated weight matrix for $W_i^* (i = 1, 2, \ldots, n)$ be W_i, and define a smooth projection of W_i as

$$\pi_i(W_i) = W_{\pi i} = (\pi_{i,jk}(W_{i,jk})) \tag{18}$$

where $j = 1, 2, 3$, $k = 1, 2, \ldots, N_1$, and the projection operator $\pi_{i,jk}$: $R \to R$ is a real-valued smooth nondecreasing function defined by [34]

$$\begin{aligned} &\pi_{i,jk}(W_{i,jk}) = W_{i,jk}, \forall W_{i,jk} \in [W_{i\min}, W_{i\max}] \\ &\pi_{i,jk}(W_{i,jk}) \in [W_{i\min} - \varepsilon_{Wi}, W_{i\max} + \varepsilon_{Wi}], \quad \forall W_{i,jk} \in R \end{aligned} \tag{19}$$

where ε_{Wi} is a small positive constant.

The control law with consideration of control input saturation for the ith spacecraft, $\tau_i (i = 1, 2, \ldots, n)$, is given by

$$\tau_i = \mathrm{sat}(\tau_{i0}, \tau_m, \tau_M), \ \tau_{i0} = -W_{\pi i} \xi_i - k_2 \alpha_i - \psi_i \tag{20}$$

$$\dot{W}_i = \delta_i (\alpha_i - \chi_i) \xi_i^T \tag{21}$$

$$\dot{\chi}_i = -k_2 \chi_i + \tau_i - \tau_{i0} \tag{22}$$

where W_i is the estimation of the optimal weight matrix W_i^*, $W_{\pi i}$ is defined by (18), $\chi_i \in R^3$ is generated by the auxiliary system (22), k_2 is a positive constant selected by the controller designer, δ_i is a positive constant, the robust term $\psi_i \in R^3$ is used to counteract the external disturbances and CNN approximation errors to be determined below, and $\tau_{ij} (j = 1, 2, 3)$ is defined as

$$\begin{aligned} \tau_{ij} &= \mathrm{sat}(\tau_{i0,j}, \tau_m, \tau_M) \\ &= \begin{cases} \tau_m, & \text{if } \tau_{i0,j} < \tau_m \\ \tau_{i0,j}, & \text{if } \tau_m \le \tau_{i0,j} \le \tau_M \\ \tau_M, & \text{if } \tau_{i0,j} > \tau_M. \end{cases} \end{aligned} \tag{23}$$

The introduction of an auxiliary system (22) is motivated by the work of [25]. Using the auxiliary signal χ_i, a modified adaptive law (21) is proposed, which can prevent the presence of input saturation from inhibiting the learning capabilities and memory of an online approximator in feedback control systems. Moreover, if $\tau_i = \tau_{i0}$, i.e., the designed controller τ_{i0} lies in the constrained region of the control input, then it is concluded from (22) that the signal χ_i remains zero or converges to zero asymptotically.

Define $\tilde{W}_i = W_i^* - W_i (i = 1, 2, \ldots, n)$, $\tilde{W}_{\pi i} = W_i^* - W_{\pi i}$, and

$$V_{Wi} = \frac{1}{\delta_i} \sum_{j=1}^{3} \sum_{k=1}^{N_1} \int_0^{\tilde{W}_{i,jk}} \Big(W_{i,jk}^* - \pi_{i,jk}(W_{i,jk}^* - w_{i,jk}) \Big)\, dw_{i,jk}. \tag{24}$$

Note that V_{Wi} is positive definite with respect to $\tilde{W}_{i,jk}$ for $W_{i,jk}^* \in [W_{i\min}, W_{i\max}]$. Furthermore, the time derivative of V_{Wi} is obtained as

$$\dot{V}_{Wi} = -\frac{1}{\delta_i} \sum_{j=1}^{3} \sum_{k=1}^{N_1} \tilde{W}_{\pi i,jk} \dot{W}_{i,jk}. \tag{25}$$

Applying the control law (20) to (12) results in

$$\begin{aligned} \dot{\alpha}_i &= \tau_i - \tau_{i0} + F_i(X_i) - W_{\pi i}\xi_i - k_2\alpha_i - \psi_i + \vartheta_i \\ &= \tau_i - \tau_{i0} + \tilde{W}_{\pi i}\xi_i - k_2\alpha_i - \psi_i + \vartheta_i + \varepsilon_i. \end{aligned} \tag{26}$$

Under Assumptions 2 and 4, we conclude that $\vartheta_i + \varepsilon_i$ is bounded. The robust controller $\psi_i (i = 1, 2, \ldots, n)$ in the control law (20) is defined as

$$\psi_i = \kappa_i \operatorname{sign}(\bar{\alpha}_i) \tag{27}$$

where $\bar{\alpha}_i = \alpha_i - \chi_i$, and κ_i is a positive constant satisfying $\kappa_i \geq \vartheta_{Mi} + \varepsilon_{Mi}$.

C. Stability Analysis

In order to study and analyze the stability of the overall closed-loop system, the following lemma is useful and presented.

Lemma 2: Let the matrix $W \in R^{3n \times 3n}$ be $W = (w_{ij})_{n \times n}$, where $w_{ii} = \sum_{j=0}^{n} a_{ij} I_3 \in R^{3\times 3}$ and $w_{ij} = -a_{ij} C_{ij} \in R^{3\times 3}$ for $j \neq i$. If the graph $\bar{G}$ is connected, then the matrix W is symmetric and positive definite.

Proof: It is obvious that the matrix W is symmetric. In order to prove that the matrix W is positive definite, we consider $V = x^T W x$, where $x = (x_1^T, x_2^T, \ldots, x_n^T)^T \in R^{3n}$ with $x_i \in R^3 (i = 1, 2, \ldots, n)$. It is easy to obtain that

$$V = \sum_{i=1}^{n} a_{i0} \|x_i\|^2 + \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} a_{ij} \|x_i - C_{ij} x_j\|^2 \geq 0. \tag{28}$$

Next, we show $V > 0$ for $x \neq 0$. We prove this by contradiction, i.e., if we can prove $V = 0$ if and only if $x = 0$, then we can conclude $V > 0$ for $x \neq 0$. It is to be noted that $x = 0$ is the solution of the equation $V = 0$, and we need to show that $x = 0$ is the only solution of $V = 0$. Since the graph $\bar{G}$ is connected, there is a path $(r_0, r_{j_1}), (r_{j_1}, r_{j_2}), \ldots, (r_{j_{m-1}}, r_{j_m}), (r_{j_m}, r_j)$ in $\bar{G}$ from the node r_0 to every other node $r_j (j = 1, 2, \ldots, n)$, which implies that $a_{j_1 0} > 0, a_{j_1 j_2} > 0, \ldots, a_{j_{m-1} j_m} > 0$, and $a_{j_m j} > 0$. Using (28), we can obtain that $x_j = 0$ for $\forall j = 1, 2, \ldots, n$, and hence we conclude $V = 0$ if and only if $x = 0$, which in turn implies that $V > 0$ for $x \neq 0$ and the matrix W is symmetric and positive definite. ■

The main result of this brief is stated in the following theorem.

Theorem 1: Consider that a group of n spacecraft is described by (1) and (2) and that Assumptions 1–4 are satisfied. If the graph $\bar{G}$ is connected, and the control law is defined by (20), where the adaptive law is provided by (21) and the robust term $\psi_i (i = 1, 2, \ldots, n)$ is given by (27), then $\bar{\alpha}_i$ converges to zero as $t \to \infty$. Furthermore, if $\tau_i = \tau_{i0}$, then $\alpha_i, \bar{q}_{i0}$ and ω_{i0} converge to zero as $t \to \infty$.

Proof: Consider the following Lyapunov function:

$$V = \sum_{i=1}^{n} \left(\frac{1}{2} \bar{\alpha}_i^T \bar{\alpha}_i + V_{Wi} \right). \tag{29}$$

The time derivative of the Lyapunov function (29) along with (12), (21), (22), (25), and (26) results in

$$\dot{V} = \sum_{i=1}^{n} \Big(-k_2 \bar{\alpha}_i^T \bar{\alpha}_i + \bar{\alpha}_i^T (\vartheta_i + \varepsilon_i - \psi_i) \Big) \leq -k_2 \bar{\alpha}^T \bar{\alpha} \tag{30}$$

where $\bar{\alpha} = (\bar{\alpha}_1^T, \ldots, \bar{\alpha}_n^T)^T \in R^{3n}$. According to Lyapunov theory, this demonstrates that $\bar{\alpha}$ is uniformly ultimately bounded. Integration of (30) from $t = 0$ to $t = \infty$ results in

$$\int_{t=0}^{\infty} \|\bar{\alpha}(t)\|^2 dt \leq \frac{V(0) - V(\infty)}{k_2}. \tag{31}$$

Noting that $V(t)$ is a nonincreasing function of time and low-bounded, this implies that $V(0) - V(\infty) < \infty$, and we have $\bar{\alpha} \in L_2$. The boundedness of $\bar{\alpha}$ implies $\bar{\alpha} \in L_\infty$ and $\dot{\bar{\alpha}} \in L_\infty$. Using Barbalat's lemma [36], we have $\lim_{t\to\infty} \bar{\alpha}(t) = 0$. Furthermore, if $\tau_i = \tau_{i0}$, then we conclude from (22) that χ_i converges to zero as $t \to \infty$, which implies that α_i converges to zero asymptotically. Next, we prove that ω_{i0} and $\bar{q}_{i0}$ converge to zero asymptotically if $\tau_i = \tau_{i0}$.

Since $\omega_i - C_{ij}\omega_j = \omega_{i0} - C_{ij}\omega_{j0}$, (9) can be reexpressed as

$$\begin{aligned} \sum_{j=0}^{n} a_{ij}\alpha_i &= \sum_{j=0}^{n} a_{ij}(\omega_{i0} + k_1\bar{q}_{i0}) - \sum_{j=1}^{n} a_{ij} C_{ij} (\omega_{j0} + k_1 \bar{q}_{j0}) \\ &= \sum_{j=0}^{n} a_{ij} s_{i0} - \sum_{j=1}^{n} a_{ij} C_{ij} s_{j0} \end{aligned} \tag{32}$$

where $s_{j0} = \omega_{j0} + k_1 \bar{q}_{j0}, j = 1, \ldots, n$. Define $s_0 = (s_{10}^T, \ldots, s_{n0}^T)^T \in R^{3n}$, $\bar{D} = \operatorname{diag}(\bar{d}_1, \ldots, \bar{d}_n) \in R^{n\times n}$, where $\bar{d}_i = \sum_{j=0}^{n} a_{ij}$, then (32) can be rewritten in a matrix form as

$$(\bar{D} \otimes I_3)\alpha = W s_0. \tag{33}$$

Since the matrix W is symmetric and positive definite, we obtain that $s_{i0} (i = 1, 2, \ldots, n)$ converges to zero as $t \to \infty$. Next, we consider the following Lyapunov function:

$$V_{i0} = 2(1 - q_{i0,4}) \qquad i = 1, 2, \ldots, n. \tag{34}$$

The first time-derivative of V_{i0} is

$$\dot{V}_{i0} = \bar{q}_{i0}^T \omega_{i0} = -k_1 \bar{q}_{i0}^T \bar{q}_{i0} \leq 0 \tag{35}$$

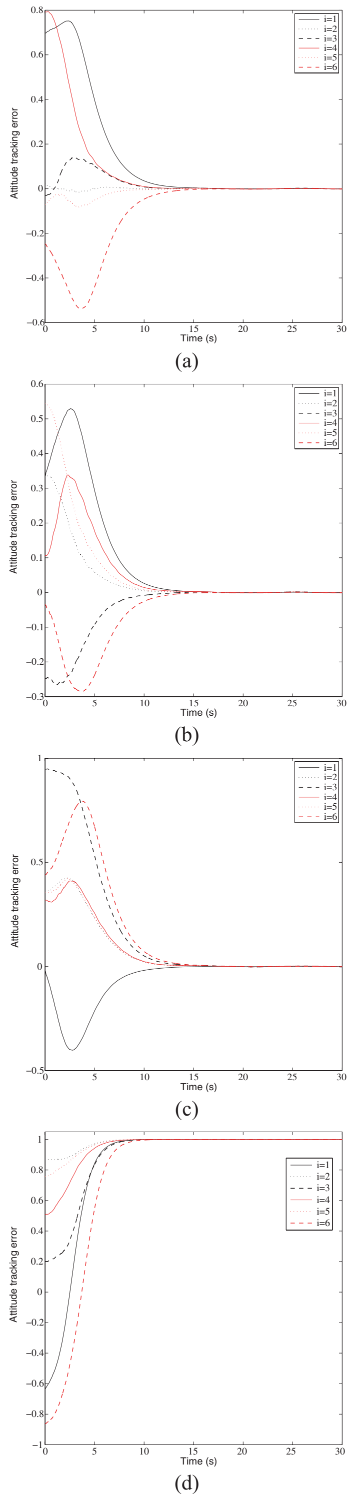


Fig. 1. Attitude tracking error $q_{i0}(i = 1, 2, \ldots, 6)$ (a) $q_{i0,1}$, (b) $q_{i0,2}$, (c) $q_{i0,3}$, and (d) $q_{i0,4}$.

which implies that $V_{i0}(t) \leq V_{i0}(0)(i = 1, 2, \ldots, n)$, and $\bar{q}_{i0}$ is bounded. In addition, we can verify that $\dot{\bar{q}}_{i0}$ is also bounded and so is $\ddot{\bar{q}}_{i0}$. Hence, using Barbalat's lemma [36], we conclude that $\bar{q}_{i0}(i = 1, 2, \ldots, n)$ converges to zero as $t \to \infty$, which in turn implies that ω_{i0} converges to zero as $t \to \infty$. ∎

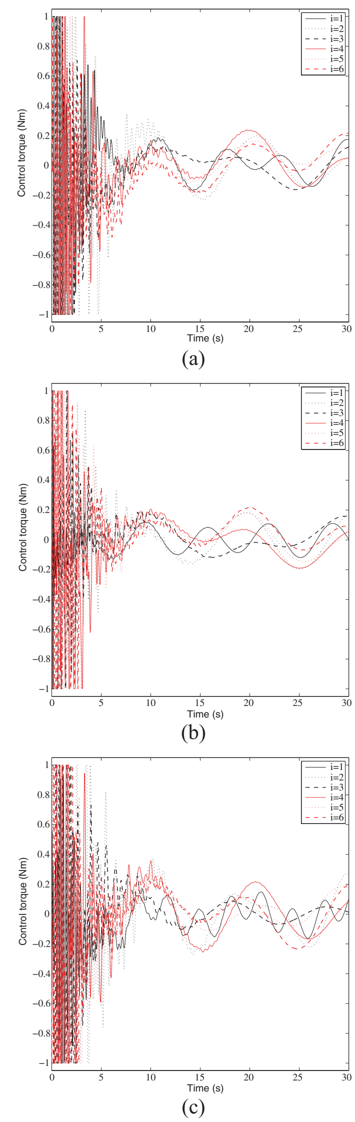
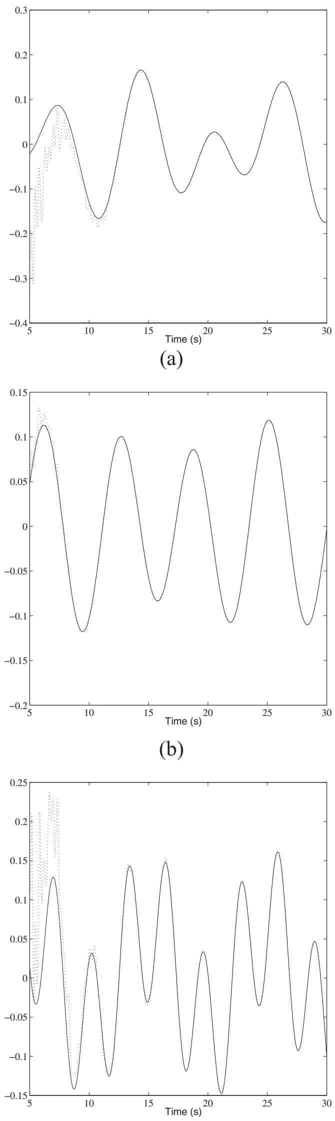


Fig. 2. Applied control torque $\tau_i(i = 1, \ldots, 6)$ (a) τ_{i1}, (b) τ_{i2}, and (c) τ_{i3}.

Remark 3: In order to avoid the chattering, the discontinuous signum function used in (5) and (27) can be replaced by the smooth hyperbolic tangent function [37], [38], for example, $\text{sign}(x)$ for $x \in R$ can be replaced by $\tanh(x/\epsilon)$, where ϵ is a small positive constant. In this case, the tracking errors α_i, $\bar{q}_{i0}$, and $\omega_{i0}(i = 1, 2, \ldots, n)$ converge to a bounded region rather than approaching zero asymptotically. Furthermore, the size of the bounded region is dependent on the controller parameters (e.g., k_1, k_2, and ϵ).

IV. Simulation Results

In this section, numerical simulations are presented to verify the effectiveness of the proposed controller. In the simulation, we consider a scenario where there are six spacecraft and one virtual leader. The spacecraft are modeled as rigid bodies, where the mass moment of inertia tensor for each spacecraft is chosen to be the same as in [12, Table I]. The common reference attitude q_0 is generated by $\dot{q}_0 = M(q_0)\omega_0/2$, where $\omega_0 = 0.1(\sin(0.2\pi t), \sin(0.2\pi t), \sin(0.2\pi t))^T$,



(a)

(b)

(c)

Fig. 3. Capture of the unknown function $F_{d1}+\vartheta_1$ by the CNN approximation in the proposed controller. (a) $F_{d1,1}+\vartheta_{11}$ (solid line) and its estimation (dotted line). (b) $F_{d1,2}+\vartheta_{12}$ (solid line) and its estimation (dotted line). (c) $F_{d1,3}+\vartheta_{13}$ (solid line) and its estimation (dotted line).

$\bar{q}_0(0)=(0.2,-0.15,0.3)^T$, and $q_{0,4}(0)=\sqrt{1-\bar{q}_0(0)^T\bar{q}_0(0)}$. The initial attitude of each spacecraft in the formation is considered to be

$$
\begin{aligned}
q_1(0) &= (\cos(-\pi/6),0,0,\sin(-\pi/6))^T\\
q_2(0) &= (0,\sin(\pi/6),0,\cos(\pi/6))^T\\
q_3(0) &= (0,0,\cos(\pi/6),\sin(\pi/6))^T\\
q_4(0) &= (\sin(\pi/4),0,0,\cos(\pi/4))^T\\
q_5(0) &= (0,\cos(\pi/4),0,\sin(\pi/4))^T\\
q_6(0) &= (0,0,\cos(-\pi/4),\sin(-\pi/4))^T
\end{aligned}
$$

and the initial angular velocity is chosen as $\omega_i(0) = (0,0,0)^T (i = 1,2,\ldots,6)$. The external disturbances are assumed to be $\vartheta_i = 0.1(\sin(t/i),\cos(t/i),\sin(2t/i))^T (i = 1,2,\ldots,6)$. The weighted adjacency matrix $A=(a_{ij})_{6\times 6}$ is taken as $a_{12}=a_{21}=0.2,\ a_{16}=a_{61}=0.3,\ a_{23}=a_{32}=0.3,\ a_{34}=a_{43}=0.3,\ a_{45}=a_{54}=0.4,\ a_{56}=a_{65}=0.3$, and $a_{ij}=0$ for other elements, and the adjacency matrix B is chosen as $B=\mathrm{diag}(0.5,0,0,0,0,0.5)$. The bounds of the control input are considered to be $\tau_m=-1$ Nm and $\tau_M=1$ Nm. The controller parameters are selected to be $\beta_1=10$, $\beta_2=0.5$, $k_1=k_2=1$, $\epsilon=0.01$, $\kappa_i=0$, and $\delta_i=50(i=1,2,\ldots,6)$. Note that $\kappa_i=0(i=1,2,\ldots,6)$ is considered in the simulation, i.e., the robust term ψ_i is not included in the controller (20). The projection operator $\pi_{i,jk}(i=1,\ldots,6,\ j=1,2,3,\ k=1,\ldots,22)$ is given by [35]

$$
\pi_{i,jk}(W_{i,jk}) = \begin{cases} W_{i\max}+\varepsilon_{Wi}\left(1-\exp\left(-\frac{W_{i,jk}-W_{i\max}}{\varepsilon_{Wi}}\right)\right), \\ \qquad\qquad \text{if } W_{i,jk} > W_{i\max} \\ W_{i,jk}, \qquad \text{if } W_{i,jk}\in[W_{i\min},W_{i\max}] \\ W_{i\min}-\varepsilon_{Wi}\left(1-\exp\left(\frac{W_{i,jk}-W_{i\min}}{\varepsilon_{Wi}}\right)\right), \\ \qquad\qquad \text{if } W_{i,jk} < W_{i\min} \end{cases} \tag{36}
$$

with $W_{i\min}=-1$, $W_{i\max}=1$, and $\varepsilon_{Wi}=0.01$. Fig. 1 shows the response of the station-keeping attitude errors $q_{0i}(i=1,2,\ldots,6)$, and Fig. 2 shows the bounded control torque τ_i. It is observed that the proposed control law (20), although the robust term $\psi_i(i=1,2,\ldots,6)$ is not included, guarantees a group of spacecraft to reach an agreement on a common time-varying reference attitude even when the common reference attitude is available only to a subset of the group members and there exist both structured and unstructured uncertainties acting on the spacecraft. An explanation for this observation lies in the fact that the CNN used in the controller has good capability to simultaneously capture the unknown functions F_{di} and the disturbance ϑ_i. Fig. 3 shows that the CNNs used in the controller have the capability to simultaneously capture the unknown desired nonlinear function F_{di} and external disturbances ϑ_i after the learning phase, (because of the page limitation, only $F_{d1}+\vartheta_1$ and its estimation are presented here). The simulation results show that the proposed distributed controller is robust against both structured and unstructured uncertainties even in the presence of control input saturation.

V. Conclusion

A quaternion-based distributed attitude coordination control scheme has been proposed for SFF under an undirected communication graph. A decentralized sliding-mode observer was introduced to generate the common reference attitude for each individual spacecraft when the reference attitude was available only to a subset of the group members. CNNs were used to approximate unknown nonlinear system functions, and the robust control technique was employed to counteract the CNN approximation errors and bounded external disturbances. Furthermore, a modified adaptive law was proposed such that the presence of input saturation caused neither instability in the closed-loop system nor "unlearning" in the online approximation process. The stability of the overall closed-loop system was guaranteed by the graph theory and Lyapunov approach. The results of the numerical simulation on a group of six spacecraft in the presence of parametric uncertainties and external disturbances reinforced the analytical results, and showed that the proposed control scheme guaranteed a group of spacecraft to track a common reference attitude without requiring all spacecraft to have access to the common reference attitude and in the presence of control input constraints. The issues of communication delay and noises will be investigated in future work.

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