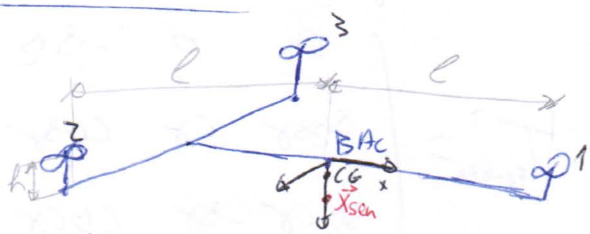
3 Copter 4 dof control Model.

○ Control Actions



- F_B = Body axes @ BAC
- F_H = Body Center and // F_I
- $F_I \equiv F_E$ = At local ground Origin. North-East-Down

No wind: $\vec{V}_w = 0$; $\vec{V} = \vec{V}_g = -\vec{V}^{air}$.

- CG ≠ BAC; $\vec{X}_{sen}$ = Sensors Positions; • Props ∈ BAC $X_B Y_B$ plane.

→ F_B ⇒ $\vec{\omega}_B = \begin{pmatrix} p \\ q \\ r \end{pmatrix}_{F_B}$; $\vec{V} = \vec{V}_{BAC} = \begin{pmatrix} u \\ v \\ w \end{pmatrix}_{F_B}$ = Vel of BAC about inertial ref.

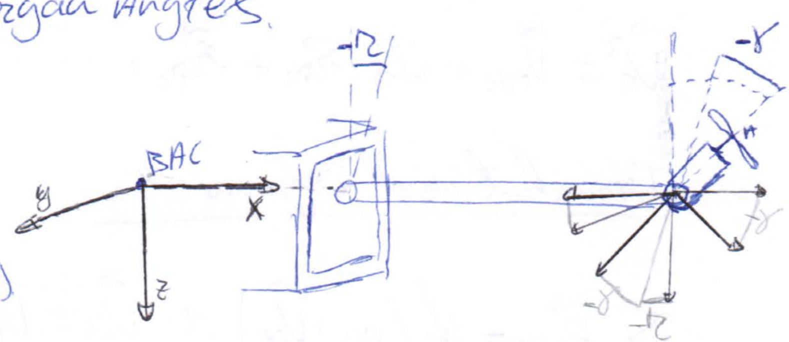
→ Firstly: Model of rigid body; Then Model of arms

Arms Kinematics: Tait-Bryan Angles.

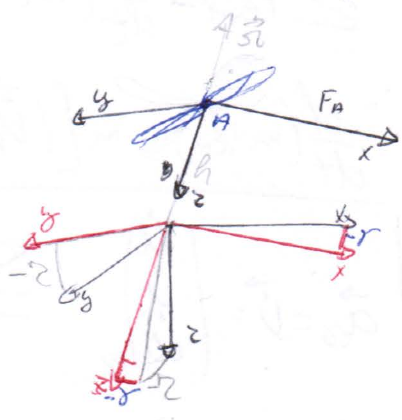
F_A = @ the center of the prop

z // hub of prop, $y_{F_A} \perp X_{F_B}$

Rotation order is: X, y, z; $(\eta, \gamma, 0)$ For Arm 1.



$$\vec{\omega}^1_{AB} = \begin{pmatrix} 0 \\ \dot{\gamma} \\ 0 \end{pmatrix}_{F_A} + \begin{pmatrix} \dot{\eta} \\ 0 \\ 0 \end{pmatrix}_{F_B}$$



$$\vec{\omega}^1_A = \vec{\omega}_B + \vec{\omega}^1_{AB};$$

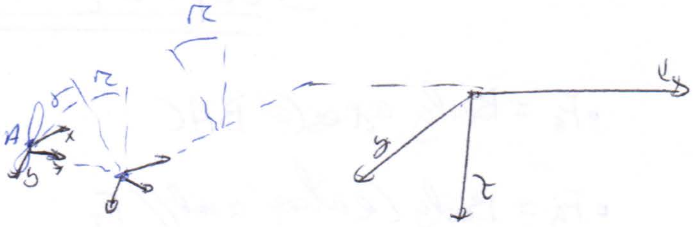
Arm 1) $\vec{r}_{P1} = \begin{pmatrix} l \\ 0 \\ 0 \end{pmatrix}_{F_B} + \begin{pmatrix} 0 \\ 0 \\ -h \end{pmatrix}_{F_A}$

$$T^1_{F_B \to F_A} = \begin{pmatrix} c\gamma & s\eta s\gamma & -c\eta s\gamma \\ 0 & c\eta & s\eta \\ s\gamma & -c\gamma s\eta & c\gamma c\eta \end{pmatrix}; \quad \Big(\Big)_{F_A} = T_{F_B \to F_A} \Big(\Big)_{F_B}$$

$$\dot{\vec{r}}_{P1} = \vec{\omega}^1_{AB} \times \begin{pmatrix} 0 \\ 0 \\ -h \end{pmatrix}_{F_A}; \quad \vec{V}^A_1 = \vec{V}_{BAC} + \dot{\vec{r}}_{P1} + \vec{\omega}_B \times \vec{r}_{P1}$$

Arm 2 and 3 / Rotations Y X Z

$$T^{2}_{Fe\to FA}=\begin{pmatrix} c\eta & 0 & -s\eta \\ s\eta s\gamma & c\gamma & c\eta s\gamma \\ s\eta c\gamma & -s\gamma & c\eta c\gamma \end{pmatrix}$$



$$\vec{r}_{P2}=\begin{pmatrix} -\frac{1}{2}l \\ \frac{\sqrt{3}}{2}l \\ 0 \end{pmatrix}_{Fe}+\begin{pmatrix} 0 \\ 0 \\ -h \end{pmatrix}_{FA};\ r_{P3}=\begin{pmatrix} -\frac{1}{2}l \\ -\frac{\sqrt{3}}{2}l \\ 0 \end{pmatrix}_{Fe}+\begin{pmatrix} 0 \\ 0 \\ -h \end{pmatrix}_{FA}$$

$$\vec{\omega}^{2}_{Ae}=\begin{pmatrix} 0 \\ \dot{\eta} \\ 0 \end{pmatrix}_{Fe}+\begin{pmatrix} \dot{\gamma} \\ 0 \\ 0 \end{pmatrix}_{FA}=\vec{\omega}^{3}_{Ae}\quad;\ \left(T^{2}_{Fe\to FA}\right)=\left(T^{2}_{FA\to Fe}\right)^{T}$$

$$\dot{\vec{r}}_{P2}=\vec{\omega}^{2}_{Ae}\times\begin{pmatrix} 0 \\ 0 \\ -h \end{pmatrix}_{FA}=\vec{\omega}^{2}_{Ae}\times\left(T_{FA\to Fe}\cdot\begin{pmatrix} 0 \\ 0 \\ -h \end{pmatrix}_{FA}\right)$$

$$\vec{\omega}^{2}_{Ae}=\begin{pmatrix} 0 \\ \dot{\eta} \\ 0 \end{pmatrix}_{Fe}+T^{2}_{FA\to Fe}\cdot\begin{pmatrix} \dot{\gamma} \\ 0 \\ 0 \end{pmatrix}_{FA};\ \vec{\omega}^{3}_{Ae}=\begin{pmatrix} 0 \\ \dot{\eta} \\ 0 \end{pmatrix}_{Fe}+T^{3}_{FA\to Fe}\cdot\begin{pmatrix} \dot{\gamma} \\ 0 \\ 0 \end{pmatrix}_{FA};$$

$$\vec{V}^{2}_{A}=\vec{V}_{BAC}+\vec{\omega}_{e}\cdot\vec{r}_{P2}+\dot{\vec{r}}_{P2};$$

Translational Dynamics

$$\sum\vec{F}_{ext}=\frac{d}{dt}(m\cdot\vec{V}_{CG})\Rightarrow\vec{V}_{CG}=\vec{V}_{BAC}+\vec{\omega}_{e}\times\vec{r}_{CG};\ \dot{\vec{V}}=\begin{pmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{pmatrix}_{Fe}$$

$$\frac{d}{dt}(m\vec{V}_{CG})=m\left[\left(\dot{\vec{V}}_{Fe}+\vec{\omega}_{e}\times\vec{V}\right)\times\left(\dot{\vec{\omega}}_{e}\times\vec{r}_{CG}+\vec{\omega}_{e}\times(\vec{\omega}_{e}\times\vec{r}_{CG})\right)\right]$$

$$\boxed{\begin{aligned}\vec{a}_{CG}=\dot{\vec{V}}=\begin{pmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{pmatrix}_{Fe}&=\frac{\sum\vec{F}_{ext}}{m}-\vec{\omega}_{e}\times\vec{V}-\dot{\vec{\omega}}\times\vec{r}_{CG}-\vec{\omega}_{e}\times(\vec{\omega}_{e}\times\vec{r}_{CG})\\ \vec{a}_{sensor}=\vec{a}_{accel}&=\begin{pmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{pmatrix}_{Fe}+\vec{\omega}_{e}\times\vec{V}+\dot{\vec{\omega}}_{e}\times\vec{r}_{sens}+\vec{\omega}_{e}\times(\vec{\omega}_{e}\times\vec{r}_{sens})\end{aligned}}$$

$$\sum_i \vec{F}_{ext} = \vec{F}_g + \vec{F}_a + \sum_i^3 \vec{F}^{(i)}_{Arm} \; ;$$

$$\vec{F}_g = m \cdot \begin{pmatrix} 0 \\ 0 \\ g \end{pmatrix}_{Fh} \; ; \quad \vec{F}_a = \frac{1}{2} \rho \vec{v}(-\vec{v}) \begin{pmatrix} C_x A_x \\ C_y A_y \\ C_z A_z \end{pmatrix}_{Fe.}$$

$$\hookrightarrow \vec{F}_g = \left(m \cdot T_{Fh \to Fa} \cdot \begin{pmatrix} 0 \\ 0 \\ g \end{pmatrix}_{Fh.} \right|_{Fe}$$

$\sum \vec{F}^{(i)}_{Arm}$ = Forces of Arms. Later discussed.

- $\vec{X}_e$ = Position of 3copter about Fe

$$\frac{d}{\partial t} \vec{X}_e = \vec{v}_e = \begin{cases} T_{Fa \to Fh} \cdot \vec{v} \\ q \otimes \begin{pmatrix} 0 \\ \vec{v} \end{pmatrix} \otimes q^* \end{cases}$$

$$\rightarrow \vec{F}^{(i)}_{Arm} = \left(T^{(i)}_{FA \to Fa} \cdot \vec{F}^{(i)}_{prop} \right|_{Fe.} \; ; \quad \vec{F}^{(i)}_{prop} = \text{Given by Aerodynamic Model.}$$

Rotational dynamics

$$\boxed{\vec{\omega}_{sensors} = \vec{\omega}_a}$$

$$\vec{L} = I_{BAC}\cdot\vec{\omega}_a + \sum_i^3 \vec{L}^{(i)}_{Arm} \quad ; \quad \sum M_{ext} = \left(\frac{d}{dt}\vec{L}\right)_{F_I}$$

$$\boxed{I_{BAC}\,\dot{\vec{\omega}}_a + \vec{\omega}_a \times \left(I_{BAC}\cdot\vec{\omega}_a\right) = \sum M_{ext} - \sum^3 \underbrace{\left(\frac{d}{dt}\vec{L}^{(i)}_{Arm}\right)_{F_I}}_{M_{Gyro}}}$$

Considering $S_{Arm}, S_{FM}, S_{FA} = cte\ f(t)$

Each Arm has an inertia and Angular momentum.

Arm 1

$$\vec{L}^1_{Arm} = S^1_{Arm}\cdot\begin{pmatrix}\dot{\eta}\\0\\0\end{pmatrix}_{Fa} + \begin{pmatrix}0\\ S_{ForeArm}\cdot\dot{\gamma}\\ \pm S_{PM}\,\Omega_1\end{pmatrix}_{FA} ;$$

$$S^i_{Arm} = f(t,\eta,\gamma) ; \quad \boxed{\frac{\partial S_{Arm}}{\partial t,\partial\eta,\partial\gamma} \approx 0}$$

$$S_{ForeArm} = f(m_{Motor}, \bar{\bar{I}}_{Prop}, h)$$

$$S_{PM} = S_{zProp} + S_{zMot}$$

$$\left(\frac{d}{dt}\vec{L}^1_{Arm}\right)_{F_I} = S^1_{Arm}\cdot\begin{pmatrix}\ddot{\eta}\\0\\0\end{pmatrix}_{Fa} + \vec{\omega}_a\times\left(S^1_{Arm}\cdot\begin{pmatrix}\dot{\eta}\\0\\0\end{pmatrix}\right)_{Fa} + \left(T^1_{FA\to Fa}\begin{pmatrix}0\\ S_{FA}\cdot\ddot{\gamma}\\ \pm S_{PM}\dot{\Omega}_1\end{pmatrix}_{FA}\right)_{Fa} +$$

$$\boxed{\vec{\omega}^1_A = \vec{\omega}_a + \begin{pmatrix}\dot{\eta}\\0\\0\end{pmatrix}_{Fa} + \left(T_{FA\to Fa}\cdot\begin{pmatrix}0\\ \dot{\gamma}\\0\end{pmatrix}_{FA}\right)_{Fa}} + \vec{\omega}^1_A \times \left(T^1_{FA\to Fa}\cdot\begin{pmatrix}0\\ S_{FA}\,\dot{\gamma}\\ \pm S_{PM}\,\Omega_1\end{pmatrix}_{FA}\right)_{Fa} \quad (3)$$

— Function of η,γ

Arm 2, 3

Taking in that $\dot{\vec{\eta}} = \begin{pmatrix}0\\ \dot{\eta}\\0\end{pmatrix}_{Fa}$

$$\vec{\omega}^2_A = \vec{\omega}_a + \begin{pmatrix}0\\ \dot{\eta}\\0\end{pmatrix}_{Fa} + \left(T^2_{FA\to Fa}\cdot\begin{pmatrix}\dot{\gamma}\\0\\0\end{pmatrix}_{FA}\right)_{Fa}$$

$$\vec{L}^2_{Arm} = S^2_{Arm}\cdot\begin{pmatrix}0\\ \dot{\eta}\\0\end{pmatrix}_{Fa} + \begin{pmatrix}S_{FA}\cdot\dot{\gamma}\\ 0\\ \pm S_{PM}\cdot\Omega_2\end{pmatrix}_{FA}$$

$$\left(\frac{d}{dt}\vec{L}^2_{Arm}\right)_{F_I} = S_{Arm}\cdot\begin{pmatrix}0\\ \ddot{\eta}\\0\end{pmatrix}_{Fa} + \vec{\omega}_a\times\left(S_{Arm}\cdot\begin{pmatrix}0\\ \dot{\eta}\\0\end{pmatrix}\right)_{Fa} + \left(T^2_{FA\to Fa}\cdot\begin{pmatrix}S_{FA}\,\ddot{\gamma}\\ 0\\ \pm S_{PM}\dot{\Omega}_2\end{pmatrix}_{FA}\right)_{Fa} + \vec{\omega}^2_A\times\left(T^2_{FA\to Fa}\cdot\begin{pmatrix}S_{FA}\,\dot{\gamma}\\ 0\\ \pm S_{PM}\,\Omega_2\end{pmatrix}_{FA}\right)_{Fa}$$

$$\sum \vec{M}_{ext} = \vec{M}_g + \cancel{\vec{M}_A}^{0} + \cancel{M_{AF}}^{0} + \sum_{i}^{3} \vec{M}^{i}_{PAF} + \sum_{i}^{3} \vec{M}^{i}_{PM}$$

- $\vec{M}_g = \vec{r}_{cg} \times \vec{F}_g$

- $\vec{M}^{i}_{PAF} = \vec{r}_{Pi} \times \vec{F}^{(i)}_{Arm.} = \left(r_{P1} \right)_{Fe} \times \left(T^{i}_{FA \to Fe} \cdot \vec{F}^{(i)}_{Prop} \right)_{Fe}$

$\to$ <u>Prop 1 / Arm 1</u>

$- f(r, t)$

$$\vec{M}^{1}_{PAF} = \vec{r}_{P1} \times \left(T^{1}_{FA \to Fe} \cdot \vec{F}^{1}_{Prop} \right)_{Fe} \; ; \; \vec{r}_{P1} = \begin{pmatrix} \ell \\ 0 \\ 0 \end{pmatrix}_{Fe} + T^{1}_{FA \to Fe} \cdot \begin{pmatrix} 0 \\ 0 \\ -h \end{pmatrix}_{FA}$$

- $\vec{M}^{(i)}_{PM} = \left(T^{i}_{FA \to Fe} \cdot M^{i}_{Prop} \right)_{Fe}$; M^{i}_{Prop} = Given by Aerodynamic Model.

<u>Attitude</u>

$$T_{FA \to Fe} = \begin{pmatrix} C\theta C\psi & C\theta S\psi & -S\theta \\ S\varphi S\theta C\psi - C\varphi S\psi & S\varphi S\theta S\psi + C\varphi C\psi & S\varphi C\theta \\ C\varphi S\theta C\psi + S\varphi S\psi & C\varphi S\theta S\psi - S\varphi C\psi & C\varphi C\theta \end{pmatrix}$$

With Quaternion:

$$q = \begin{pmatrix} C\varphi/_2 \, C\theta/_2 \, C\psi/_2 + S\varphi/_2 \, S\theta/_2 \, S\psi/_2 \\ C\psi/_2 \, C\theta/_2 \, S\varphi/_2 - S\psi/_2 \, S\theta/_2 \, C\varphi/_2 \\ C\varphi/_2 \, S\theta/_2 \, C\psi/_2 + S\varphi/_2 \, C\theta/_2 \, S\psi/_2 \\ C\varphi/_2 \, C\theta/_2 \, S\psi/_2 - S\varphi/_2 \, S\theta/_2 \, C\psi/_2 \end{pmatrix} ; \begin{cases} \varphi = \operatorname{atan2}\left(\dfrac{2q_3 q_4 + 2q_1 q_2}{2q_1^2 + 2q_4^2 - 1} \right) \\ \theta = -\sin^{-1}\left(2q_2 \cdot q_4 - q_1 q_3 \cdot 2 \right) \\ \psi = \operatorname{atan2}\left(\dfrac{2q_2 q_3 + 2q_1 q_4}{2q_1^2 + 2q_2^2 - 1} \right) \end{cases}$$

$$q_1^2 + q_2^2 + q_3^2 + q_4^2 = 1$$

$$\boxed{\frac{d}{dt}(q) = \frac{1}{2} q \otimes \begin{pmatrix} 0 \\ p \\ q \\ r \end{pmatrix}_{Fe}}$$

$$\left(\begin{pmatrix} 0 \\ \\ \\ \end{pmatrix}_{F_j} \right) = q \otimes \left(\begin{pmatrix} 0 \\ \\ \\ \end{pmatrix}_{Fe} \right) \otimes q^*$$

$$\begin{pmatrix} \dot{p} \\ \dot{\varphi} \\ \dot{r} \end{pmatrix}_{Fa} = \dot{\vec{\omega}}_{a} = I_{BAC}^{-1} \cdot \left(\sum M_{ext} - \sum \left(\frac{d}{dt} \vec{\mathcal{L}}^{i}_{Arm} \right)_{FI} - \vec{\omega}_a \times \left(I_{BAC} \cdot \vec{\omega}_a \right) \right)$$

$\hookrightarrow$ Equivalent Moments

Mass properties

- $m = cte$

- $I_{prop} = \begin{pmatrix} I_x & 0 & I_{xz} \\ 0 & I_y & 0 \\ I_{xz} & 0 & I_z \end{pmatrix}_{F_{prop}} \simeq \frac{1}{4} M_{eq} R^2 \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix}_{FA}$

$\Omega_i \gg \dot{\alpha}, \dot{\beta}, \dot{\gamma}$

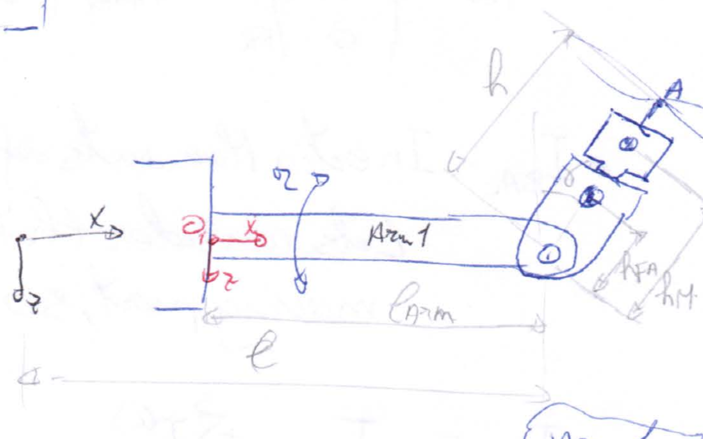
↳ Equivalent spinning Disc

$\boxed{M_{eq} \simeq \frac{1}{5} M_{prop}\,!}$

- $\boxed{J_{FA} = \frac{1}{4} M_{eq} R^2 + m_{prop} \cdot h^2 + m_{FA} \cdot h_{FA}^2 + m_H \cdot h_H^2}$ → In F_A!

- $J_{prop} = \frac{1}{2} M_{eq} R^2.$

- J^i_{Arm} = Depends on the arm geometry.



Arm 1

$$J^i_{Arm} = T^i_{FA \to FB} \cdot I_{prop} \cdot \left(T^i_{FA \to FB}\right)^T + I_{Tube} + I_{FA} + I_H + I_{steiner\,prop}$$

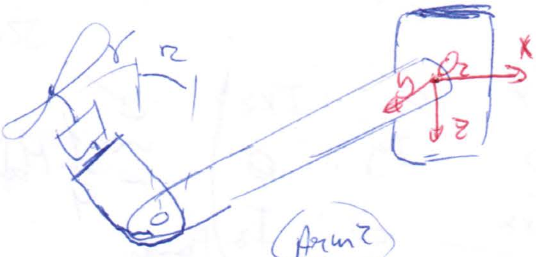
$$r^1_{FAO} = \begin{pmatrix} l_{Arm} \\ 0 \\ 0 \end{pmatrix}_{FB} + \begin{pmatrix} 0 \\ 0 \\ -h_{FA} \end{pmatrix}_{FA} ; \quad r^1_{HO} = \begin{pmatrix} l_{Arm} \\ 0 \\ 0 \end{pmatrix}_{FB} + \begin{pmatrix} 0 \\ 0 \\ -h_H \end{pmatrix}_{FA} ;$$

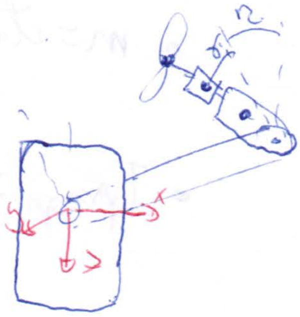
$$\vec{r}_{P1O} = \begin{pmatrix} l_{arm} \\ 0 \\ 0 \end{pmatrix}_{FB} + \begin{pmatrix} 0 \\ 0 \\ -h \end{pmatrix}_{FA} ;$$

$$\left(I_{FA}, I_H, I_{steiner\,prop}\right) = m_H \cdot \begin{pmatrix} r^1_{HO}(y)^2 + r^1_{HO}(z)^2 & r^1_{HO}(x) \cdot r^1_{HO}(y) & \\ & \ddots & \\ & & \end{pmatrix}$$

$$I_{Tube} = \begin{pmatrix} I_{xx} & 0 & I_{xz} \\ 0 & I_{yy} & 0 \\ I_{xz} & 0 & I_{zz} \end{pmatrix}_{FB} ; \quad \begin{aligned} I_{yy} &= I_{zz} = \frac{1}{3} M_{Tube} \cdot l_{Arm}^2 \\ I_{xx} &= \frac{1}{2} M_{Tube} \cdot R_{eq}^2 \\ I_{xz} &\simeq \frac{1}{10} I_{xx}\,! \end{aligned}$$

Arm 2, 3





$$J^2_{Arm} = T^2_{FA\to FR}\cdot J_{Arm}\cdot\left(T^2_{FA\to FR}\right)^2 + I_{Tube} + I_{Steiner\,Arm} + I_{FA} + I_M$$

$$\vec{r}_{P2O} = \begin{pmatrix} 0 \\ l_{Arm} \\ 0 \end{pmatrix}_{FR} + T^2_{FA\to FR}\cdot\begin{pmatrix} 0 \\ 0 \\ -h \end{pmatrix}_{FA} ; \quad \vec{r}_{P3O} = \begin{pmatrix} 0 \\ -l \\ 0 \end{pmatrix}_{FR} + \begin{pmatrix} 0 \\ 0 \\ h \end{pmatrix}_{FA}$$

- I_{BAC} = Inertia Moments of all the vehicle.
  Lets consider that the Motors are the most important moving part, so.

$$I_{BAC} = I_{BAC0} + \sum_i^3 I_M^{(i)}, \quad I_{BAC0} = \text{Inertia moments without Motors: To be measure!}$$

$$I_M^{(i)} = m_M\cdot\begin{pmatrix} r^1_M(y)^2 + r^1_M(z)^2 & & \\ & \ddots & \\ & & \ddots \end{pmatrix}$$

$$\vec{r}^{\,1}_M = \begin{pmatrix} 0 \\ l \\ 0 \end{pmatrix}_{FR} + T^1_{FA\to FR}\begin{pmatrix} 0 \\ 0 \\ -h_M \end{pmatrix}_{FA} ; \quad \vec{r}^{\,2}_M = \begin{pmatrix} -l/2 \\ \frac{\sqrt{3}\,l}{2} \\ 0 \end{pmatrix}_{FR} + T^2_{FA\to FR}\begin{pmatrix} 0 \\ 0 \\ h_M \end{pmatrix}_{FA}$$

- CG:

$$\vec{r}_{CG} = \frac{m_0\cdot\vec{r}_{CG0} + \sum^3 m_M\cdot\vec{r}^{\,i}_M}{m} = \frac{m_0}{m}\cdot\vec{r}_{CG0} + \frac{m_M}{m}\cdot\sum^3 \vec{r}^{\,i}_M$$

m_0 = Mass 3Copter without Motors.
$\vec{r}_{CG0}$ = CG location of 3Copter without Motors.
m_M = mass of each Motor

ESC + Motor Model

Find/Test the map: $\Omega = f(\text{Torque}, \text{Throttle})$

↳ This map is the static behaviour.

• The dynamics is modelled as a 1st order system

$$\boxed{\dot{\Omega} = \frac{f(\text{Torque}, \text{Throttle}) - \Omega}{\tau}}$$

; τ = in principle is constant but can be also $f(\text{Torque}, \text{Throttle})$

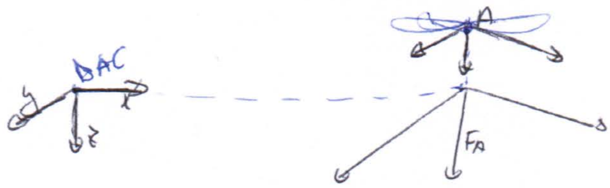
$$\begin{cases} \text{Torque} = J_{PM} \cdot \dot{\Omega} + \underbrace{\text{Torque}_{\text{propeller}}}_{M_{ZP}} \\ M_{ZP} = \rho \pi R^3 (\Omega \cdot R)^2 \cdot C_Q \end{cases}$$

Propeller Model

- Consider $\vec{V}_{w} = 0$; $\vec{V}_A^{Air} = -\vec{V}^A$

Arm 1

$$\vec{V}_1^A = \vec{V}_{BAC} + \dot{\vec{r}}_{P1} + \vec{\omega}_e \times \vec{r}_{P1}$$



$$\dot{r}_{P1} = \left(\begin{pmatrix} \dot{r} \\ 0 \\ 0 \end{pmatrix}_{F_B} + T^1_{F_A \to F_a} \cdot \begin{pmatrix} 0 \\ \dot{\gamma} \\ 0 \end{pmatrix}_{F_A} \right)_{F_a} \times \left| \begin{pmatrix} 0 \\ 0 \\ -h \end{pmatrix}_{F_A} \right|_{F_a}$$

$$\begin{cases} C_H = \frac{-\sigma a}{4} \mu \left(\theta_0 + \frac{\theta_1}{2}\right) \cdot \lambda + \frac{\sigma C_{do}}{4} \mu \\ C_T = \frac{\sigma a}{4} \left[\theta_0 \left(\frac{2}{3} + \mu^2\right) + \frac{\theta_1}{2}(1+\mu^2) + \lambda\right] \end{cases}$$

$$\vec{F}^i_{Prop} = \rho S (\Omega_i R)^2 \cdot \begin{vmatrix} C_H \frac{-\vec{V}^A_{i(x,y)}}{\| \quad \|} \\ -\frac{C_T}{1 - \frac{R^2}{16 \cdot z e^2}} \end{vmatrix}_{F_A}$$

$$\mu = \frac{\| \vec{V}^A_{i(x,y)} \|}{\Omega_i R} ; \quad \lambda = \lambda_i - \frac{\vec{V}^A_{iz}}{\Omega_i R} ; \quad \lambda_i = \frac{V_i}{\Omega_i R}.$$

$$\boxed{V_i = f\left(-V^A_{iz}, \|\vec{V}^A_{i(x,y)}\|, V_{io}\right)}$$

→ AIS included val Model

$$V_{io} = \sqrt{\frac{m \cdot g}{3 \cdot 2 \rho \pi R^2}}.$$

3 props

$$\vec{M}^i_{PAF} = \vec{r}_{Pi} \times \vec{F}^i_{Prop} \Rightarrow \left(\vec{r}_{Pi} \times \left(T^i_{FA \to Fa} \cdot \vec{F}^i_{Prop}\right)_{Fa} \right)_{Fa}$$

(1),(2)

$$\vec{M}^i_{PM} = \rho \pi R^3 (\Omega_i R)^2 \cdot \begin{vmatrix} \pm C_{MH} \frac{-V^A_{i(x,y)}}{| \quad |} \\ \pm C_{Qi} \end{vmatrix}$$

Los signos de esta TEP estan mal

$$\begin{cases} C_{MH} = \frac{\sigma a}{4} \left(\frac{2}{3}\theta_0 \mu + \frac{1}{2} \mu + \lambda + \frac{\theta_1}{2} \mu\right) \\ C_Q = \frac{\sigma a}{4} \cdot \left(\frac{2}{3}\theta_0 + \lambda + \frac{\theta_1}{2}\right) \lambda + \frac{\sigma C_{do}}{8}(1+\mu^2). \end{cases}$$

Angulos Euler

F_h (Inercial); F_a (Girado)

$$T_{F_h \to F_a} = \begin{pmatrix} c\theta c\psi & c\theta s\psi & -s\theta \\ s\varphi c\theta c\psi - c\varphi s\psi & s\varphi s\theta s\psi + c\varphi c\psi & s\varphi c\theta \\ c\varphi s\theta c\psi + s\varphi s\psi & c\varphi s\theta s\psi - s\varphi c\psi & c\varphi c\theta \end{pmatrix}.$$

$$(\ )_{F_a} = T_{F_h \to F_a} \cdot (\ )_{F_h}.$$

Con Cuaterniones:

$$T_{F_h \to F_a} = \begin{pmatrix} q_1^2 + q_2^2 - q_3^2 - q_4^2 & 2(q_2 q_3 + q_1 q_4) & 2(q_2 q_4 - q_1 q_3) \\ 2(q_2 q_3 - q_1 q_4) & q_1^2 - q_2^2 + q_3^2 - q_4^2 & 2(q_3 q_4 + q_1 q_2) \\ 2(q_2 q_4 + q_1 q_3) & 2(q_3 q_4 - q_1 q_2) & q_1^2 - q_2^2 - q_3^2 + q_4^2 \end{pmatrix}$$

$$q_1^2 + q_2^2 + q_3^2 + q_4^2 = 1$$

Con estas relaciones:

$$q = \begin{pmatrix} c\varphi/2\, c\theta/2\, c\psi/2 + s\psi/2\, s\theta/2\, s\varphi/2 \\ c\psi/2\, c\theta/2 \cdot s\varphi/2 - s\psi/2 \cdot s\theta/2\, c\varphi/2 \\ c\psi/2\, s\theta/2\, c\varphi/2 + s\psi/2\, c\theta/2\, s\varphi/2 \\ s\psi/2\, c\theta/2\, c\varphi/2 - c\psi/2\, s\theta/2\, s\varphi/2 \end{pmatrix}$$

$$\varphi = \operatorname{atan2}\left(\frac{2q_3 q_4 + 2q_1 q_2}{2q_1^2 + 2q_4^2 - 1} \right)$$

$$\theta = -\operatorname{asin}\left(2q_2 q_4 - 2q_1 q_3 \right)$$

$$\psi = \operatorname{atan2}\left(\frac{2q_2 q_3 + 2q_1 q_4}{2q_1^2 + 2q_2^2 - 1} \right).$$